Texas Mathematics Teacher

Volume LXIII Issue 2 Fall/Winter 2017/2018

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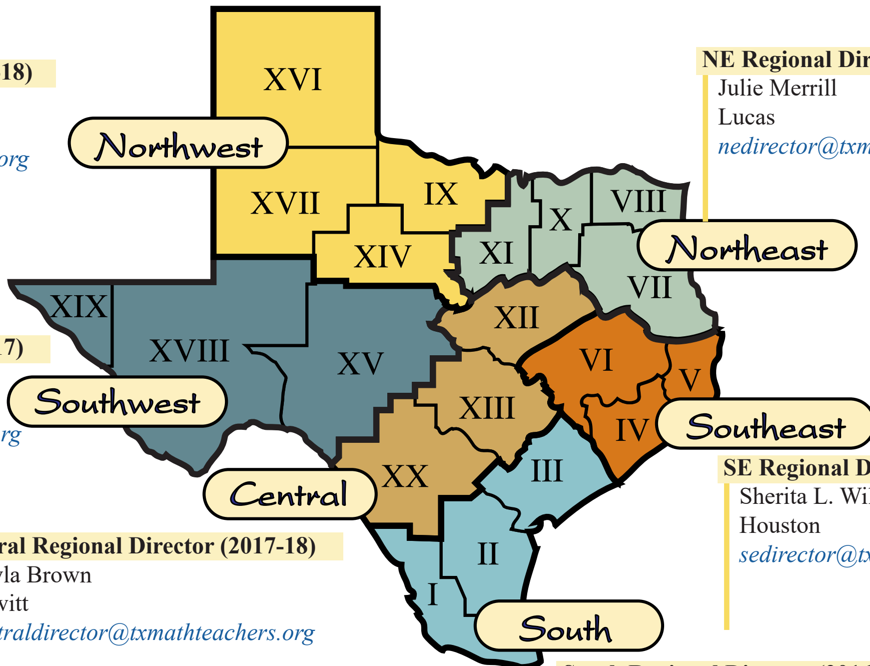

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Texas Mathematics Teacher

A PUBLICATION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Volume LXIII Issue 2 **Fall/Winter 2017/2018**

Cover photos by Brandy Crowley

Articles

Voices from the Classroom & Activities

TCTM Applications

All applications (including TCTM membership) are available online at: *www.txmathteachers.org.*

Departments

General Information

From the Editors

We are delighted to share our first issue as editors of the journal! In this issue you will find the latest research-based practices, ready to use classroom activities, ideas for implementing technology, and support for developing and sustaining a professional learning community of mathematics educators. Additionally, you will find legislative and local affiliate updates. We hope you will find the issue beneficial as you lead in mathematics education across Texas to support student learning. We would like to thank Cynthia Schneider, as past editor, for her support and guidance in publishing this first issue. We could not have done it without her! Further we thank the authors, reviewers, assistant editor, graphics designer, and editorial panel for their efforts to ensure a quality resource for your use. We look forward to continued collaborations and encourage you to send in your ideas to editor@txmathteachers.org for future issues!

Trena L. Wilkerson, Editor
Rachelle M. Rogers, Associate Editor

Letter from the President

Dear TCTM Colleagues,

This issue of TMT signifies the official transition of journal editorship. Dr. Trena Wilkerson and Dr. Rachelle Rogers now serve as Editor and Associate Editor, respectively. The journal is now "housed" at Baylor University and will be distributed to members through the TCTM web site and available in hard copy at various events. Current members will receive log in information to access the current issue and all archived issues, available on the TCTM web site. It will be important to keep your membership current so that you do not miss a single issue! If you have colleagues who are not members, encourage them to join so that they can enjoy the wonderful resources included in each and every issue of TMT.

TCTM leadership has worked hard this past year to elevate the level of communication with members. Each month, an eBlast newsletter is sent by the president via email to current members. If you have not received this email, be sure to check your spam/junk folder or you might verify that your membership is current. The email is sent from president@txmathteachers.org, so be sure to add this to your "safe senders" list. In addition, TCTM has a FaceBook page and Twitter account (@txmathteachers), so be sure to "Like" our page and "Follow" our tweets! This will provide the most up to date announcements, alert to special opportunities, and helpful resources.

We hope you have had a successful start to this new school year and are "off and running" with helping your students achieve great strides in learning mathematics. The primary mission of TCTM is to support K-12 teachers of mathematics, so please send us ideas of how we can best support you and your colleagues. Be sure to make your plans now for participation in CAMT 2018 which will be held in Houston and encourage a team from your school to attend. This is where you can interact directly with your TCTM leadership, so we hope to see you there!

All my best for a great school year!

Sandi Cooper
President, 2016 - 2018

Website: www.txmathteachers.org
Facebook: www.facebook.com/TexasCTM/
Twitter: www.twitter.com/txmathteachers

TCTM Leader Spotlight

Each year since 1995, TCTM has accepted nominations for two awards for leaders in our professional community. The TCTM Leadership Award is presented to a TCTM member who is nominated by a TCTM affiliate. The second award, the E. Glenadine Gibb Achievement Award, is presented to someone nominated by a TCTM member. The following individuals have been honored and we wish to acknowledge their former and ongoing contributions this year in the leader spotlight. If you wish to nominate someone for 2018, please download the forms from our website. **Please submit your nomination by Dec. 31, 2017**.

Our prior awardees are:

Year	Leadership(local/state)	Gibb (state/national)
1995	Mary Alice Hatchett	Iris Carl
1996	Betty Forte	Cathy Seeley
1997	Diane McGowan	Pam Chandler
1998	----	----
1999	Linda Shaub	Eva Gates
2000	Lloy Lizcano	Bill Hopkins
2001	Susan Hull	Pam Alexander
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2014	Caren Sorrells	Noemi Rodriguez-Lopez
2015	Jennifer Hylemon	Bea Luchin
2016	Robb Wilson	Trena Wilkerson
2017	Anita Hopkins	James A. M. Álvarez

Apply now for a MET Grant, Scholarship, or Award!

NCTM's Mathematics Education Trust (MET) channels the generosity of contributors through the creation and funding of grants, awards, honors, and other projects that support the improvement of mathematics teaching and learning.

MET provides funds to support classroom teachers in the areas of improving classroom practices and increasing mathematical knowledge. MET also sponsors activities for prospective teachers and NCTM Affiliates, as well as recognizing the lifetime achievement of leaders of mathematics education. Grant, scholarship, and award funding ranges from $1,500 to $4,000 and can be used for conferences, workshops, seminars; research and in-service training in mathematics coursework; or professional development activities. The deadline is November 3, 2017.

If you are a teacher, prospective teacher, or school administrator and would like more information about MET grants, scholarships, and awards, please visit their website,

www.nctm.org/Grants/

Affiliate Groups

These are local affiliated groups in Texas. If you are actively involved with them, please send future meeting and conference information to Trena Wilkerson at editor@txmathteachers.org so we may publicize your events. Contact information for each group is also available on the NCTM website, www.nctm.org. Contact information for regional directors is located on the inside front cover of this publication.

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Texas South Plains CTM
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Julie Merrill, Regional Director

East Texas CTM
Contact: Martha Godwin, *mgodwin@qcisd.net*.

Greater Dallas CTM
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Sherita Wilson-Rodgers, Regional Director

Fort Bend CTM
Contact: Alena McClanahan, *alena.mcclanahan@fortbend.k12.tx.us*.

SOUTH TEXAS REGION
Service Centers 1, 2, 3
Faye Bruun, Regional Director

The South Texas Region is on Project Share! The group is "Texas Council of Teachers of Mathematics: South Region."

Coastal CTM
Contact: Faye Bruun, *faye.bruun@tamucc.edu*,
or visit sci.tamucc.edu/MATH/CTM/.

ME by the SEa, Texas A&M Corpus Christi, June 15, 2018

CTM @ Texas A&M University at Corpus Christi (Student Affiliate)
Contact faculty advisor Faye Bruun, *faye.bruun@tamucc.edu.*

CTM @ Texas A&M University at Kingsville (Student Affiliate)
Contact NCTM Representative: Susan Sabrio

Rio Grande Valley CTM
Contact: Velma Sanchez at *vesatea10@hotmail.com*,
or visit *www.rgvctm.org*.

The Rio Grande Valley Council of Teachers of Mathematics invites you to participate in their 52nd Annual Conference, Saturday, November 4, 2017. The conference will be held at the University of Texas – Rio Grande Valley located at 1201 W. University Drive, Edinburg, Texas, 78539

CENTRAL TEXAS REGION
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Contact: ADCTM president Linda Gann, *linda.gann@boerne-isd.net*

Central Texas CTM
Contact: President of CTCTM Kayla Brown,
kayla.brown@midwayisd.org, or visit *www.ctctm.org*.
(CTCTM) Spring Conference: January 27, 2018

"There is geometry in the humming of the strings, there is music in the spacing of the spheres."
~ Pythagoras (c. 570–495 BC)

STATEWIDE

Texas Association of Supervisors of Mathematics (TASM)
TxASMOnline@gmail.com, or visit *www.tasmonline.net*.

The Association of Mathematics Teacher Educators of Texas (AMTE-TX)
Contact Sarah Quebec-Fuentes at: *s.quebec.fuentes@tcu.edu*,
or visit *www.amte-tx.org*.

NATIONAL

National Council of Teachers of Mathematics (NCTM)
visit: *nctm.org*.

National Council of Supervisors of Mathematics (NCSM)
visit *www.mathedleadership.org*.

Teaching Students to Understand and Solve Word Problems

> "Students with learning difficulties often need motivation to remain attentive and to increase positive behavior."

To demonstrate competency in mathematics, students need to understand how to set up and solve word problems. Word problems include a combination of words and numbers that require interpretation by the student (e.g., *Sadie has 14 dolls. Hayden has 21 dolls. How many dolls do the girls have together?*). In Texas, as early as third grade, students learn to set up and solve word problems to establish mathematical proficiency (e.g., TEKS 3.4A). Many students, however, experience difficulty with word problems (Jitendra et al., 2013; Kong & Orosco, 2015). To support students who struggle to solve word problems, we developed and refined a word-problem program (i.e., Pirate Math) specifically for students in Texas.

We designed the Pirate Math program to help third-graders who experience difficulty solving word problems set up and solve three types of addition and subtraction word problems. Several of the students in our study received special education services in mathematics, but most received mathematics instruction in the general education classroom. Therefore, the program is designed for students with word-problem difficulty and can be delivered in a variety of settings (e.g., interventionists in special education or teachers in general education). Originally developed at Vanderbilt University (Fuchs et al., 2008; Powell et al., 2015), Pirate Math has successfully improved the word-problem solving performance of students in second and third grade. Like pirates who search for a treasure marked with an "X," students in the Pirate Math program find "X" by solving equations (e.g., 6 + X = 13) created to represent the structure of the word problem (e.g., *Reece had 6 notebooks. Then, her aunt gave her some notebooks for her birthday. Now, Reece has 13 notebooks. How many notebooks did she get for her birthday?*). This practice aligns with TEKS 3.5A in which students are expected to represent addition and subtraction problems with equations.

Addition and Subtraction Schemas

For addition and subtraction word problems, there are three word-problem schemas (i.e., problem types). In lieu of describing a word problem as "addition" or "subtraction," a schema allows a student to understand the underlying structure of the word problem. The operation does not define the word problem – the schema does.

The three word-problem schemas include Total, Difference, and Change (Fuchs et al., 2008; Powell, 2011; Riley & Greeno, 1988). In Total problems, also called combine or part-part-whole problems, students combine amounts for a total (e.g., *Donna ordered 7 pizzas. Michelle also ordered some pizzas. If they ordered 15 pizzas altogether, how many pizzas does Michelle have*?). Students may solve Total problems where the total is missing or one of the parts is missing. In Difference problems, also called compare problems, students compare two amounts for a difference (e.g., *Andrew bought 25 records. John bought 18 records. How many fewer records did John buy*?). Students may solve Difference problems where the difference is missing, the greater amount is missing, or the amount that is less is missing. In Change problems, students start with an amount and the amount increases or decreases to a new end amount (e.g., *Kristi picked 22 flowers. Then, her friend Laura gave her 17 more flowers. How many flowers does Kristi have now*?). Students may solve Change problems where the start amount is missing, the change amount is missing, or the end amount is missing.

Overview of Lessons

The Pirate Math intervention includes over 50 one-on-one lessons, implemented three times a week, with each session lasting about 30 min. Each lesson consists of (1) Math Fact Flashcards, (2) tutor-led activity about the equal sign (i.e., Equation Quest), (3) tutor-led lesson featuring schema instruction (i.e., Buccaneer Problems), (4) Shipshape Sorting, and (5) Jolly Roger Review. Research supports that the use of a set of brief activities can be an effective strategy for students with mathematics difficulties to sustain attention and learn content material (Zheng, Flynn, & Swanson, 2013).

In Math Fact Flashcards, the tutor shows flashcards to the student during two, one-minute timings. The student graphs the highest score from the two trials on a bar graph. In Equation Quest, the tutor and student complete activities related to solving equations and the meaning of the equal sign. In the Buccaneer Problems, the tutor provides scaffolded instruction to the student to set up and solve three additive word-problem schemas: Total, Difference, and Change. In Shipshape Sorting, the tutor reads aloud word-problem cards for 1 min as the student identifies the problem type as Total, Difference, or Change. In Jolly Roger Review, the student participates in a paper-and-pencil review, which consists of completing 9 addition and subtraction math problems in 1 min and a word problem in 2 min. Table 1 features the general outline of the Pirate Math lessons.

Table 1

Pirate Math Unit and Lesson Overview

Unit	Days	Topics Covered
Introduction	1-4	• Solve basic addition problems, with and without regrouping • Solve basic subtraction problems, with and without regrouping • Label graphs (i.e., bar graphs, pictographs, pictographs with multiplier, tables) • Introduce and discuss meaning of the equal sign
Total	5-16	• Introduce Total problems (P1 + P2 = T) • Solve Total problems with total missing (e.g., 5 + 4 = X) • Solve Total problems with one part missing (e.g., 5 + X = 9) • Introduce three-part Total problems (P1 + P2 + P3 = T) • Solve three-part Total problems with total or one part missing (e.g., 5 + 4 + 2 = X; 5 + X + 2 = 11) • Solve Total problems with graphs (i.e., bar, pictographs, pictographs with multipliers, tables) • Use cubes to balance both sides of the equal sign • Use cubes to solve missing addend problems • Draw pictures to solve missing addend problems • Draw pictures to balance both sides of the equal sign • Solve equations by balancing sides and isolating the X • Solve Total equations by balancing sides • Practice computation (addition and subtraction with and without regrouping)
Difference	17-33	• Introduce Difference problems (G – L = D) • Introduce and practice identifying compare words (e.g., more, fewer, less, taller, smaller, faster) • Solve Difference problems with difference missing (e.g., 8 – 4 = X) • Solve Difference problems with the greater amount missing (e.g., X – 4 = 4) • Solve Difference problems with the lesser amount missing (e.g., 8 – X = 4) • Solve Total problems (i.e., total missing, part missing, three-part Total problems with total or part missing) • Solve Total and Difference problems with graphs (i.e., bar graphs, pictographs, pictographs with multipliers, tables) • Use cubes to balance both sides of the equal sign with subtraction problems • Solve Total and Difference equations by balancing sides • Draw pictures to solve missing addend, minuend, and subtrahend problems
Change	34-42	• Introduce Change increase and Change decrease problems (ST +/– C = E) • Review the word *more* and how *more* can be used in a Difference or Change problems • Solve Change increase and Change decrease problems with end amount missing (e.g., 8 + 5 = X; 8 – 5 = X) • Solve Change increase and Change decrease problems with change missing (e.g., 8 + X = 13; 8 – X = 3) • Solve Change increase and Change decrease problems with start amount missing (e.g., X + 5 = 13; X – 5 = 3) • Introduce double Change problems (e.g., 8 + 5 + 5 = 13; 10 – 5 – 5 = X; 10 + 5 – 5 = X) • Solve double Change problems with two increases, two decreases, and one increase and one decrease • Solve Total, Difference, and Change problems with and without graphs • Use cubes to balance both sides of the equal sign with addition and subtraction problems • Solve Total, Difference, and Change equations by balancing sides • Solve equations with more than two addends (e.g., 2 + 2 + 2 = X + 4) • Solve equations with minuend (e.g., X – 5 = 10 – 7) or subtrahend missing (e.g., 8 – X = 10 – 7) • Choose numbers for equations and solve (e.g., X – __ = __)
Review	43-51	• Solve Total, Difference, and Change problems with and without graphs • Solve Total, Difference, and Change equations by balancing sides • Solve equations with more than two addends (e.g., 2 + 2 + 2 = X + 4) • Solve equations with minuend (e.g., X – 5 = 10 – 7) or subtrahend missing (e.g., 8 – X = 10 – 7) • Choose numbers for equations and solve (e.g., X – __ = __)

Description of Daily Activities

Activity #1: Math Fact Flashcards

Research suggests that fact fluency is an important skill for future success in mathematics (National Mathematics Advisory Panel, 2008). We introduce Pirate Math lessons with a fluency-building activity where students are presented with addition and subtraction flashcards with minuend and subtrahends ranging from 0 to 9. On Days 1 through 4 of the program, the tutor teaches the student a counting-up strategy for learning addition and subtraction facts. The student is prompted to implement the counting-up strategy throughout the lessons (Tournaki, 2003). In the flashcard activity, the student has 1 min to complete each of two trials. After the initial 1 min trial, the tutor and student count the number of flashcards answered correctly. The tutor also provides immediate, corrective feedback to the student by reviewing the counting-up strategy for any noted errors. Prior to starting the second one-minute trial, the tutor challenges the student to beat his or her previous score. The tutor encourages the student as a way to alleviate the tedious nature of the fluency building activity and motivate him to improve his or her score (Fuchs et al., 2008). At the end of the second one-minute timing, the tutor and student graph the highest score (see Figure 1). The graph serves as a self-regulation tool for setting future goals and monitoring student progress (Fuchs et al., 1997).

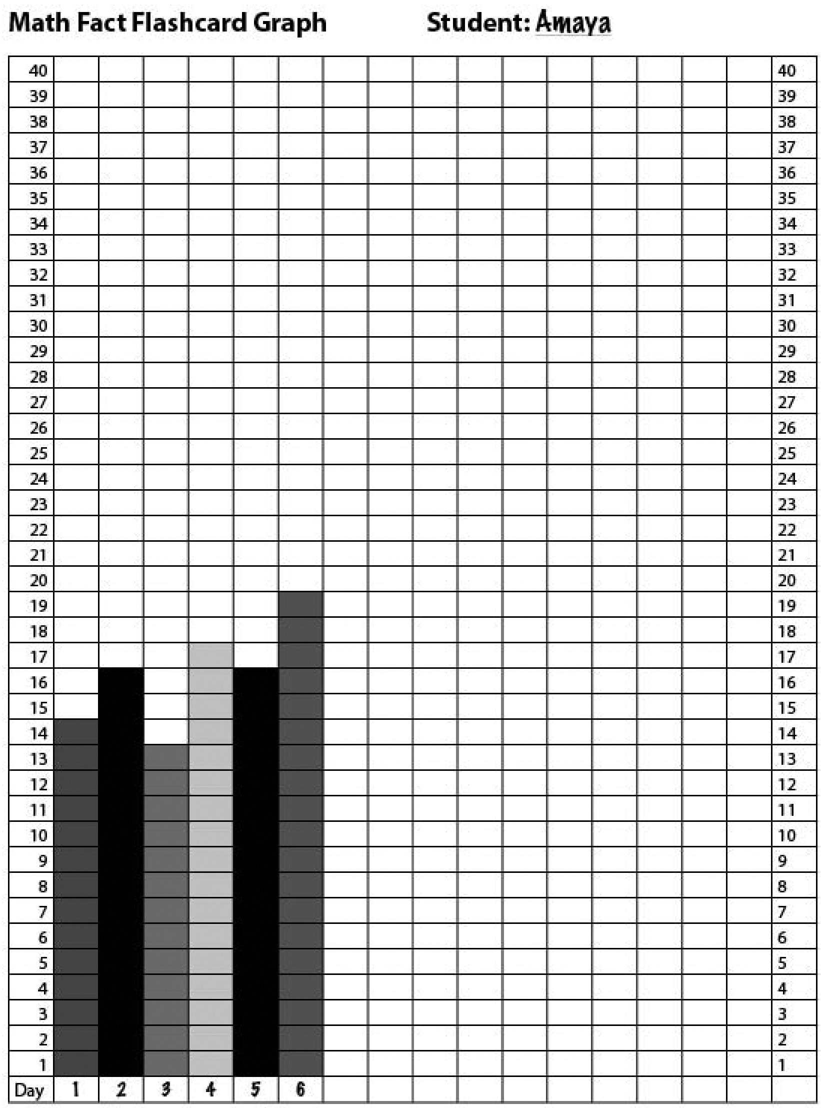

Figure 1. Flashcard graph.

Activity #2: Equation Quest

Students often misinterpret the meaning of common mathematical symbols (Driver & Powell, 2015). For example, students may associate the equal sign as an operational indicator (e.g., "do something" or "write the answer") instead of understanding the equal sign as a relational symbol indicating balance between two sides of the equal sign (Powell, 2012). To equip students with a better understanding of the equal sign, the Pirate Math curriculum incorporates explicit instruction on the equal sign through an activity called Equation Quest. During this 2-min activity, the tutor reintroduces the common symbol and teaches the student to understand the meaning of the equal sign as *the same as*. Through a sequence of activities, the student learns that the equal sign acts as a balance between two sides of an equation and does not solely signal a calculation.

Equation Quest follows the concrete-representational-abstract (CRA) framework (Miller & Hudson, 2006) to teach the student about the equal sign, first using standard equations (e.g., 4 + 7 = X) followed by non-standard equations (e.g., X = 8 + 2 or 5 + 4 = X + 7). The combination of standard and nonstandard equations helps students develop better pre-algebraic reasoning (McNeil & Alibali, 2005; Powell, Driver, & Julian, 2015). In the initial concrete phase, the tutor and student work with manipulatives (i.e., cubes) to determine if the number of cubes on the left side of the equation is *the same as* the number of cubes on the right side of the equation (see Figure 2). The tutor prompts the student to add or subtract cubes from one side of the equation to make both sides *the same*. As the lessons progress, the tutor and student enter the representational phase of the framework where the student draws shapes to represent the equations. After the student masters the concept of the equal sign using cubes and drawings, the abstract phase ensues. In the abstract phase, the student only uses numbers and symbols to determine if the left side of the equation is the *same as* the right side of the equation. The student learns a sequence of steps to balance an equation, using variables (e.g., "X") to represent missing numbers. To provide consistency across lessons and student participants, the tutor maintains the language of the equal sign (e.g., *the same as*) throughout the Pirate Math lessons.

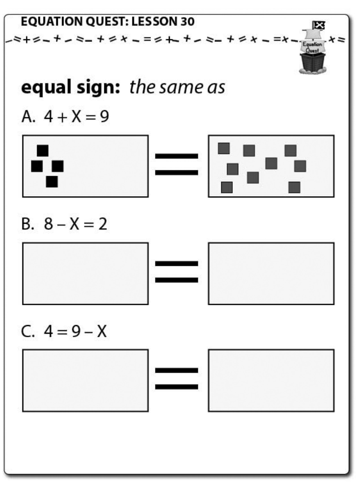

Figure 2. Sample Equation Quest activity.

Activity #3: Buccaneer Problems

The third activity consists of tutor-led schema instruction through Buccaneer Problems. Days 1 through 4 of the Buccaneer Problem lessons include a review of addition and subtraction skills using the *counting up* strategy to assist with fact-retrieval difficulties and regrouping for double-digit calculations. On Day 5, the student begins to apply the learned fluency and equation-solving skills to solve word problems. Each lesson includes a review of previously learned material, is highly interactive, and requires the student to verbalize his or her thinking.

When approaching any word problem, the student learns to think through the problem before solving it by first checking for a table or a graph and numbering it (if applicable), and then *RUNning* through the problem. To *RUN*, the student must Read the problem, Underline the label and cross out irrelevant information, and Name the problem type (i.e., choose the correct schema to use) by asking questions about the problem. To determine if a problem is a Total problem, the student asks: *Are two or more parts being put together for a total*? For Difference problems, the student asks: *Are two amounts being compared for a difference*? For Change problems, the student asks: *Is there a starting amount that increases or decreases to a new amount*? The student then employs a written series of steps to solve the problem. The steps are specific to each of the three schemas, but for all problems, the student is taught to use an equation to represent the problem and to mark "X" to represent the missing information. For the young pirate, "X" represents the treasure for which they are searching.

Total problems. The Total schema first is introduced on Day 5. The missing information (i.e., "X") may be the total or one of the parts. After checking for a table or a graph and *RUNning* through the problem to organize and prepare to solve, the student uses five steps to find the solution of Total problems: (1) Write P1 + P2 = T (i.e., part 1 + part 2 = the total), (2) Find T, (3) Find P1 and P2, (4) Write the signs, and (5) Find X. When these steps are combined with the equation-solving skills learned in Equation Quest, the student's paper looks similar to Figure 3. For Total problems with more than two parts, the student is taught to simply alter the equation (i.e., P1 + P2 + P3 = T).

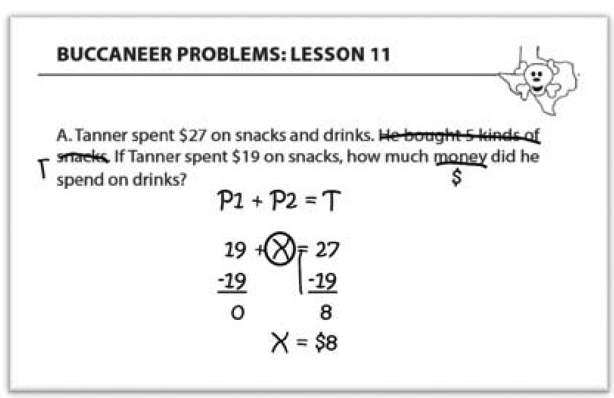

Figure 3. Total problem example.

Difference problems. In Difference problems, the student learns to compare an amount that is greater and an amount that is less to find the difference. The missing information (i.e., "X") for Difference problems may be the amount that is greater, the amount that is less, or the difference. The most important problem attribute used for identifying this schema and correctly solving the problem is the compare sentence. The student learns to find the compare sentence by looking for a compare word (e.g., words like *more*, *less*, or *fewer*, or other words like *older*, *shorter*, or *faster*), and then using the compare sentence to determine which quantities are greater and less, and whether the difference is given or missing. There are six steps to solving a difference problem: (1) Write G – L = D (i.e., amount that is greater – amount that is less = difference), (2) Put brackets around the compare sentence and label G and L, (3) Find D, (4) Find G and L, (5) Write the signs, and (6) Find X (see Figure 4).

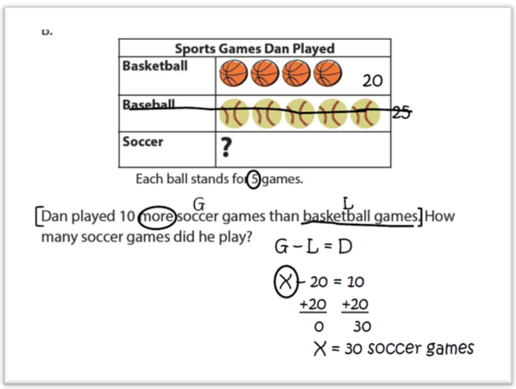

Figure 4. Difference problem example.

Change problems. In Change problems, there is a starting amount, then at a later time something happens to increase or decrease the starting amount, so the ending amount is changed. The missing information (i.e., "X") may be the starting amount, the change amount, or the end amount. Change problems are multi-step because students must determine whether the change increases or decreases the starting amount and then add or subtract accordingly. The six steps used to solve a Change problem are: (1) Write ST +/– C = E (i.e., start amount +/– change amount = end amount), (2) Find ST, (3) Find C, (4) Find E, (5) Write the signs, and (6) Find X. If there is more than one change within the problem, the student is simply taught to alter the equation to reflect the information in the problem (ST + C – C = E; see Figure 5).

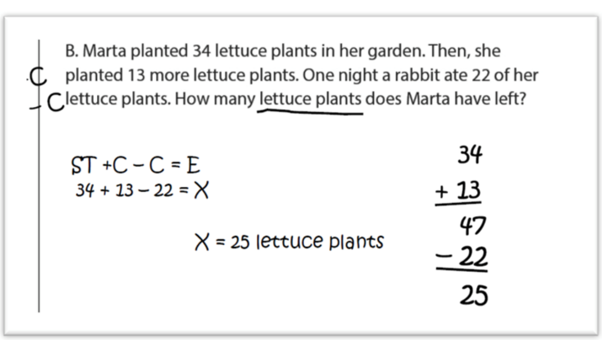

Figure 5. Change problem example.

Activity #4: Shipshape Sorting
Shipshape Sorting is a timed activity that allows the student to practice identifying word- problem schemas learned during the Buccaneer problems. Before the sorting activity begins, a mat with four squares is placed in front the student. Each square is labeled with a word-problem type letter (i.e., T for Total, D for Difference, or C for Change) or the question mark symbol (see Figure 6). The tutor reviews the three word-problem schemas, and explains the directions to place each word-problem card on the square with the corresponding problem type letter (i.e., T, D, or C). If a student is uncertain about the problem type, he should place the card on the square with the question mark symbol. The tutor reminds the student to sort the word-problem cards and to not solve any of the word problems. The tutor sets the timer for one minute and reads the first word-problem card aloud before handing it to the student. The tutor waits for the student to place the card on the mat before reading the next word-problem card. After one minute, the tutor reviews the sorting cards on the mat by providing immediate, corrective feedback to the student on at least three of the word-problem cards. Shipshape Sorting provides a valuable opportunity for the student to practice identifying word-problem schemas. Efficiently and accurately sorting word-problem schemas proves helpful for selecting the appropriate equation (i.e., Total, Difference, or Change equation) to solve each word problem.

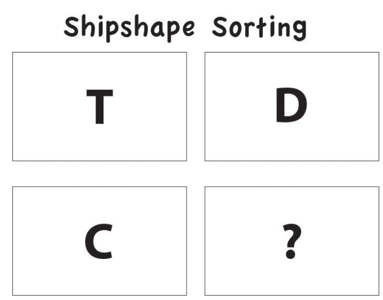

Figure 6. Sorting mat.

Activity #5: Jolly Roger Review
The Jolly Roger Review includes a brief, timed review of the lesson content. Low-stakes practice testing outside the classroom has been shown to improve test scores for students across grade and achievement levels (Dunlosky, Rawson, Marsh, Nathan, & Willingham, 2013). Specifically, practice testing directly improves test-taking ability by providing student exposure to the test-taking environment and indirectly by promoting knowledge of the material (Dunlosky et al., 2013). Low-stakes practice testing proves especially beneficial for students when provided in conjunction with immediate feedback (Dunlosky et al., 2013). The Jolly Roger Review activity incorporates such components of low-stakes practice testing. First, the student has one minute to answer up to nine computational math problems (i.e., single and double-digit addition and/or subtraction problems) or write appropriate equations for the three word-problem schemas (e.g., Total equation: P1 + P2 = T).

Next, the student has 2 min to complete a word-problem using the appropriate schema steps taught during the Buccaneer Problem activity (see Figure 7). The student performs the timed review autonomously and then receives content-rich feedback from the tutor, which reinforces mastered content. The Jolly Roger Review activity promotes independent practice of previously learned material and preparation for the test-taking environment.

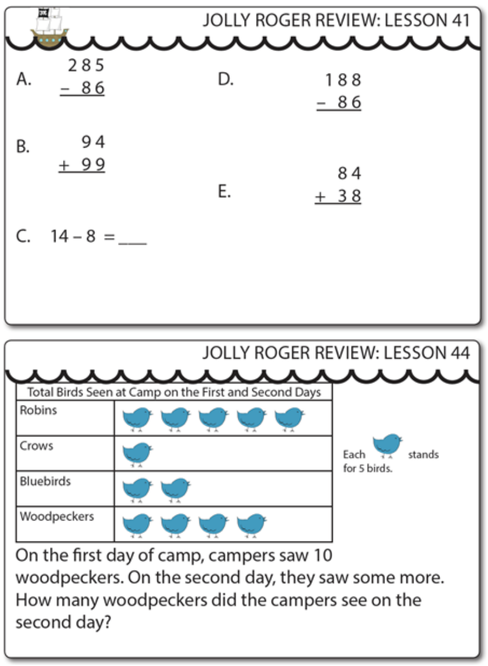

Figure 7. Sample Jolly Roger Review.

Motivation

Students with learning difficulties often need motivation to remain attentive and to increase positive behavior associated with learning (Fuchs et al., 2008). Token economy systems have demonstrated reduced problematic behavior and regulated attention in third-grade students with learning disabilities (Fuchs et al., 2008; Higgins, Williams, & McLaughlin, 2001). The Pirate Math program incorporates a token-based reward system and goal-driven tasks to promote student attention and encourage student motivation. At the beginning of each lesson, the tutor reviews the Pirate Math rules (e.g., listening to the instructor, staying in the seat, working hard) with the student. When a student follows the Pirate Math rules, he receives gold coins throughout the lesson. At the end of lesson, the student counts the number of gold coins earned and colors the appropriate number of coins on a treasure map (see Figure 8). When the student colors all of the coins on the treasure map, he reaches the treasure chest and receives a prize of his or her choosing from the treasure box. The behavioral component of the Pirate Math program decreases challenging behaviors and fosters an environment conducive to learning.

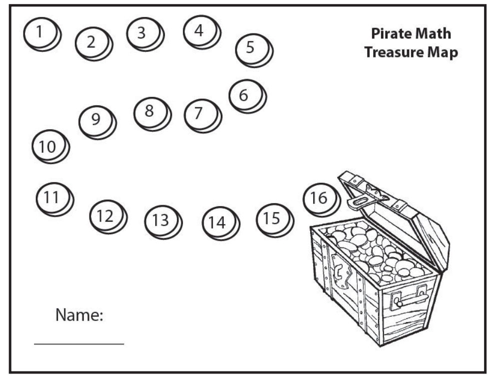

Figure 8. Treasure map.

Initial Results

During the 2015-2016 school year, we tutored third-grade students with mathematics difficulty in public schools in Austin, Texas. After screening over 1,100 general education students, we defined mathematics difficulty as performance below the 13th percentile on a test of word problems. Of the 133 students that we identified with mathematics difficulty, we provided individual tutoring to 88 of these students. A few of these students received special education services but the majority received instruction in general education without a diagnosis of a mathematics disability. At posttest, students receiving the version of Pirate Math described in this article demonstrated a 21-point gain (from pretest) on an assessment of single- and double-digit additive word problems, whereas students not receiving Pirate Math only demonstrated an 8-point gain. These results indicated that students who receive Pirate Math improve significantly on word-problem measures. We are continuing this research during the next few school years and look forward to sharing results with teachers across Texas and the United States.

Pirate Math in Your Classroom

In this article, we described the Pirate Math program and the strategies embedded within the program. Even though our focus is third grade, we believe teachers across the elementary grades could implement several of the Pirate Math strategies in their classrooms. For example, teachers can introduce the additive schemas (i.e., Total, Difference, Change) to students and help students understand word problems by schema rather than operation. Beyond third grade, teachers could introduce the multiplicative schemas (Xin, Jitendra, & Deatline-Buchman, 2005).

Teachers can also provide students with an attack strategy (e.g., RUN) in which students are encouraged to read a word problem before doing any other work. The attack strategy also helps students work methodically through any word problem. Additionally, the findings suggest that teachers use a fluency practice activity to help students who have difficulty with automaticity of mathematics facts. When students become more fluent with math facts, the computation within word problems becomes easier. Teachers can also provide instruction about the equal sign meaning the same as, which may help students solve different types of addition and subtraction equations and establish pre-algebraic reasoning. All of these strategies could be used to contribute to a deeper understanding of word problems in mathematics.

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Publishing Opportunities for the Texas Mathematics Teacher

Call For Articles

Texas Mathematics Teacher seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Texas. All readers are encouraged to contribute articles. Manuscripts should adhere to the general publishing guidelines listed below. **Deadline for submissions: Fall, July 1; Spring, January 1**. Please contact our journal staff with your article and information at editor@txmathteachers.org.

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Teachers are encouraged to submit articles for Voices From the Classroom, including inspirational stories, exemplary lessons, or management tools. Educators are encouraged to submit classroom activities or lessons for the journal. If submitting a lesson, it should include identification of the appropriate grade level and any prerequisites. Deadline for submissions: **Fall, July 1; Spring, January 1**. Please contact our journal staff with your story and information.

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Apply or nominate a peer to become a reviewer by sending an email to our journal staff. Teachers are especially encouraged to apply.

Call for Lone Star News

Announcements for the Lone Star News include, but are not limited to, NCTM affiliated group announcements, announcements of upcoming professional meetings, and member updates. Please keep us and your members informed, contact our journal and web staff with your affiliated group's updates.

Additional Calls

The Texas Mathematics Teacher seeks submissions for the following departments. All readers are encouraged to contribute.

- Recommended Reading/Resources/Software/App
- On the Cover
- Puzzle Corner
- Quotes for Thought

Any images submitted for On the Cover must be original to the author and include 3-5 mathematical problems connected to the cover picture(s). Puzzle Corner submissions should include the solution.

Call for Advertisements:

For-Profit Organizations and businesses interested in placing an advertisement for mathematics materials should contact the journal staff. Advertisements do not imply endorsement by TCTM's board, editorial staff or members. Deadline for submissions:
Fall, September 1; Spring, February 1.

General Guidelines for Submission

As part of the submission process, authors are required to check their submission compliance with each of the following items. Submissions that do not adhere to these guidelines may be returned to authors.

General Preparation Checklist

- The manuscript has not been previously published and is not simultaneously being considered for publication elsewhere.
- The manuscript is in Microsoft Word format.
- All graphics, tables, and figures should be embedded in the manuscript. In addition, graphics should be saved as a jpg and attached as a separate document.
- The manuscript follows the most current APA guidelines with in-text citations as well as references at the end of the article. When possible, DOIs should be utilized in the references.
- The manuscript is double spaced, Times New Roman, size 12 font, with 1 inch margins on all sides.
- The manuscript's title page should include the title of the article/activity, author name, affiliation, mailing address, email address, phone number, and the intended target audience. No author names should appear on the manuscript after the title page.
- The author should indicate whether his or her email address can be published with the article.

Receipt of manuscripts will be acknowledged. If the manuscript is accepted for publication, the editor/reviewers may make suggestions or revisions in consultation with the principal author. Receipts for all copyrighted materials

Leadership Awards presented at the TCTM business meeting at CAMT 2017

The E. Glenadine Gibb Achievement Award was presented to Dr. James A. M. Álvarez, Professor & Distinguished Teaching Professor at the University of Texas-Arlington, for his contribution to the improvement of mathematics education at the state and/or national level.

The TCTM Leadership Award was presented to Anita Hopkins, Executive Director of CAMT, for her contributions to the improvement of mathematics education at the local and state level.

Diameter

Lawrence Mark Lesser

Geometer's
Parameter:
Iambic (yes...)
Diameter!

A circle's span
Across the round,
There is a line
So eas'ly found.

The center's pierced
By this fine line;
And that is what
Makes it confined.

Each vertex of
Triangle sits:
We know it's right
If long side is

Diameter,
So Thales claimed:
A theorem now
Does bear his name.

Diameter
That strikes a chord
At angle right
Bisects its world.

We'll find its length
If we divide
Circumference
By our friend π!

QUESTIONS:

1. What is the meaning of the prefix "di" in the word "diameter"? And, what other examples from science or mathematics can you think of that use the prefix "di" in the same way?
2. The most common form of poetic meter in English is iambic pentameter, in which each line of verse has five iambic metrical feet that each consist of one unstressed syllable followed by one stressed syllable. Therefore, in light of the answer to the previous question, what form would poetic lines in "iambic diameter" have?
3. Give a definition for a diameter of a circle. (Note that we say "a" diameter since a circle has an infinite number of them!)
4. State Thales' theorem and draw an associated picture for it. Who was Thales ("THAY-leez")?
5. How can you prove Thales' theorem?
6. If a diameter is perpendicular to a chord, what does it do to that chord? Draw a picture representing the associated theorem and see if you can find a proof.
7. For a given circle, what is the relationship between a diameter and circumference?

Find answers page 31

Making Math Workshop Work

Small Group as the Central Focus of Math Workshop

Small group instruction for mathematics is imperative to ensure success for all students. Dr. Nicki Newton, author of *Guided Math in Action*, verifies "In small groups, you can meet learners where they are and take them where they need to go" (Newton, 2013 pg. 11). I studied small group instructional practices and worked feverishly to create small group lessons, design and develop meaningful workstations, and set up an environment which I was positive would be buzzing with mathematical conversations. Implementation occurred and I thought I was well prepared, but it was not what I had envisioned and my students were not being as successful as I had hoped. What was going wrong? I found myself rushing to "fit it all in," often sacrificing the concrete step of instruction and skipping key skills needed to learn the overall concept. I needed a systematic planning process to utilize every minute of instructional time around the primary focus of math workshop, small group instruction.

Step 1: Breakdown the Student Expectations

Student expectations (SE) are *full* and can be overwhelming for a teacher. As educational professionals, we know the transition from concrete to pictorial to abstract (CPA) is best for students when learning any new mathematical concept. With that in mind, I started breaking down each SE based on each specific skill in addition to the necessary mathematical vocabulary related to that skill. Once my students learned each skill, they could put the skills together and master the concept successfully. For example, in SE 3.3E (The student is expected to solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8.) students must be able to: define and explain "partition" and "recipient", represent partitioning of an object and a set of objects from a word problem, complete partitioning word problems using pictorial models, and represent the partitioning of an object/set as a fraction with a denominator of 2, 3, 4, 6, and 8. To monitor my students' understanding of these skills, I created short, simple checklists (Figure 1) based on the SE breakdown and utilized the checklists during small group instruction.

"Student expectations are *full* and can be overwhelming for a teacher."

The checklists allowed me to keep track of individual student progress as well as what I had accomplished instructionally with each small group of students.

Small Group Checklist

SE: 3.3E Solve problems involving partitioning an object or a set of objects among two or more recipients using pictorial representations of fractions with denominators of 2, 3, 4, 6, and 8. (RC1: Supporting)[3.3E]

Groups	Monday	Tuesday	Wednesday	Thursday	Friday
*Mateo Kaitlyn Daniel Emma Makayla	o Partition o Object/Set o Fraction o Pictorial	o Partition o Object/Set o Fraction o Pictorial	o Partition o Object/Set o Fraction o Pictorial	o Partition o Object/Set o Fraction o Pictorial	o Partition o Object/Set o Fraction o Pictorial
Comments					

SE Breakdown:

- Be able to define and explain "partition".
- Be able to represent partitioning of an object and a set of objects from word problem.
- Be able to complete partitioning word problem using pictorial model.
- Be able to represent the partitioning of object/set as a fraction with denominator of 2, 3, 4, 6, and 8.

Figure 1

Step 2: Use Anchor Charts as Instructional Tools

Providing concrete experiences for students is crucial in order to develop a memorable and relatable mathematical skill. Every phase of instruction serves as a tool toward successful implementation of small group instruction. Anchor charts were used during instruction, but I was unintentionally skipping the most important part of the learning process, concrete experiences. My students were constantly writing down sentence stems and mathematical vocabulary, yet they could not use the language orally or in written format when asked to do so independently. It occurred to me that my students had no experiential connection between the concept and the mathematical language.

I decided to connect the use of anchor charts and vocabulary along with a hands-on, manipulative-based experience. Anchor charts in my classroom were then created to represent the hands-on activity used to introduce each lesson and facilitate independence during workstations. The anchor charts now reflected the students' definitions of new concepts and the students' procedures of how a problem could be solved. The mathematical conversations and justifications within my classroom began to flourish. When I worked with small groups, I noticed my students referring to anchor charts to help their peers and to solve problems in workstations (Figure 2).

When looking at sample assessment items, it is clear that students need to understand specific mathematical language in order to develop proficiency in mathematics. I began consistently using anchor charts along with manipulative-based exploration to lead into small group instruction.

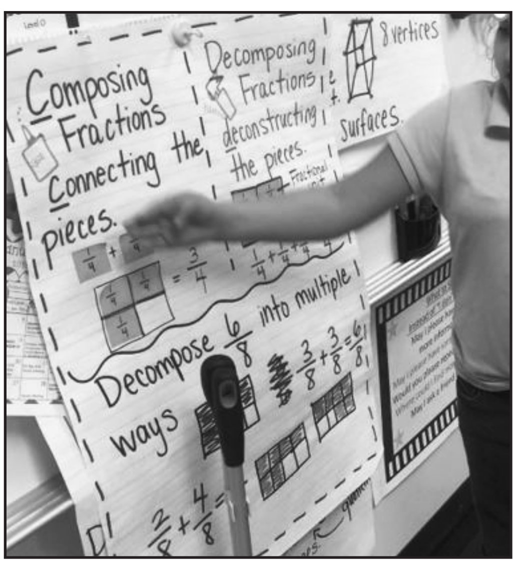

Figure 2

Step 3: Use an Instructional Flow that Maximizes Teaching Time

One of my lesson goals was to create an instructional flow from concrete to pictorial to abstract which mirrored a flow from whole group to small group to workstations. To avoid getting "stuck" in whole group, I began using small group instruction as an avenue to teach workstations. A short segment of small group time was used to directly connect the whole group concept to the following day's workstation. During whole group on Day 1, the concept was introduced concretely with the anchor chart as academic vocabulary support. During small group on Day 1, students had immediate practice of the concept taught along with multiple opportunities to practice and use academic language.

> "...allow time for each step of concrete, pictorial, and abstract problem solving."

In order to avoid lost instructional time during whole group, I used the workstation for Day 2 as the immediate practice component during small group on Day 1. Activities were selected that encouraged use of manipulatives, allowed for varying problem solving strategies, provided opportunities for enrichment, could be replicated with different numbers and scenarios, and offered a quick form of assessment. Day 2 built upon the skill taught on Day 1, but with an emphasis on pictorial representations when solving problems. I often used the same activity, which provided a connection between the manipulative-based practices on Day 1 to a pictorial representation on Day 2. Mathematical language was continually incorporated by having students talk about the procedure used to solve each problem. Students were expected to justify their work using complete sentences and sentence stems.

Leading up to and including Day 3, the lessons continually increase in the level of rigor incorporating all skills needed to master the mathematical concept. On Day 3, it was my goal for students to be solving abstract word problems related to real-world situations and providing explanations of work using mathematical language. At the end of Day 3, I provided an independent task for students to complete in order to assess student understanding of the mathematical concept. This three-day pattern of planning was repeated for each SE. To better organize my instruction and ensure that I followed through with this process, I created a lesson planning template (Figure 3) to more easily visualize this linear model or "flow" of teaching.

	Monday Day 1	Tuesday Day 2	Wednesday Day 3
Whole Group Lesson	**Concrete**: Anchor Chart with fraction circles and cookies pg. 3 standard clarification. (See video) Cindy, Jessi, and Karen all want to equally share one cookie.	**Pictorial:** Thinking map with word problem (set of objects). 5 friends, 3 cookies. • The ________has been partitioned into______equal parts. • Each fractional part represents______ of the whole. • Each_______is___ of the ________.	Abstract: (Independent) Provide word problem.
Small Group Lesson	Focus on meaning of partitioning and recipients. Use fraction strips to represent the whole and recipients.	Fair Sharing and Fractions (Engaging Math pg. 45)	Brownie Bites activity incorporate question stems.
Workstation 2 Current SE	3.3E Foldable standard clarification pg 17	3.3E Foldable standard clarification pg 17 Start: Sharing Candy Bars (Engaging Math pg. 43)	Sharing Candy Bars (Engaging Math pg. 43)

Figure 3

Reflections and Next Steps

Overall, this systematic planning process has improved the math workshop environment not only for my students, but for me as well. When planning instruction, I now continually allow time for each step of concrete, pictorial, and abstract problem solving. There is always room for growth and reflection in education; therefore, I will continue to improve my math workshop planning efforts in order to best meet the needs of all of my students.

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Mathematics Instructional Coach • Laura Bush Elementary, Pasadena ISD

"A new, a vast, and a powerful language is developed for the future use of analysis, in which to wield its truths so that these may become of more speedy and accurate practical application for the purposes of mankind than the means hitherto in our possession have rendered possible."
~Ada Lovelace, (1815-1852)

TCTM Government Relations Update

As always, there is a lot going on at the state level with regards to policymaking and mathematics education issues. Currently, there are a few updates from both the Texas Legislature and the State Board of Education that may impact mathematics education, and thus our membership, teachers, and most importantly, our students.

Texas Legislature
The Texas Legislature met for its regular session in Spring 2017 and again during the summer for a special session. Many bills were filed in the regular session that ended in May 2017. As is customary in the Texas Legislature, most of these bills did not become law. Dr. Cynthia Schneider, TCTM's previous Government Relations Representative, and I watched several bills that had the potential to impact Texas students.

One bill that did pass and will have a positive impact on our students is **SB826**. This bill fixed a prerequisite problem by removing the language in HB5 that defined "advanced mathematics" as a course taken *after* successful completion of Algebra 1 and Geometry. As a result, students can now take their "advanced mathematics" Foundation or Endorsement course at any time in their high school course pathway, as long as the prerequisites for the course (as identified in the TEKS) are honored. For example, students may take Algebra 2 prior to or concurrently with Geometry. Likewise with Math Models, Algebraic Reasoning, or any of the other courses that qualify as "advanced mathematics" in TEA's graduation requirements section. *Governor Abbott signed this bill into law and it takes effect for the 2017-18 school year*.

Other bills did not pass but contain ideas that are likely to reemerge in the 2019 Legislature. Mathematics TEKS revision and STAAR requirements were among the ideas being debated. I expect to see these ideas again.

State Board of Education
In June 2017, the State Board of Education approved a timeline to address TEKS revision and adoption of instructional materials (see their detailed document on the TEA website: http://tea.texas.gov/index2.aspx?id=25769817636).

For mathematics, the current plan calls for the following process to revise the mathematics TEKS:

- 2021-22 school year – review existing mathematics TEKS and issue Proclamation 2024 for K-12 mathematics instructional materials.
- 2022-23 school year – Adopt revised mathematics TEKS and SBOE reviews/adopts instructional materials for K-12 mathematics.
- 2023-24 school year – Districts review and adopt instructional materials for K-12 mathematics.
- 2024-25 school year – Implement both revised K-12 mathematics TEKS and instructional materials.

Based on this timeline, in four years, we will start the review/revision process for the mathematics TEKS again so that we can implement them in seven years.

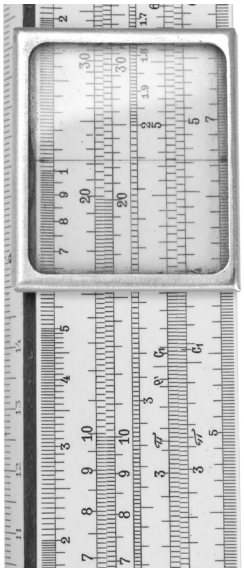
Participate in the Process!
As always, we encourage you to participate in both the legislative process and interact with the State Board of Education! I have learned over the years that our elected officials genuinely do want to hear what people, in particular classroom teachers, think about issues directly affecting them. Phone calls, emails, and letters are always welcomed by legislators and SBOE members.

Find out who represents you using the Texas Legislature Online directory.
Visit http://www.fyi.legis.state.tx.us/Home.aspx and enter your home address. You will then see a list of all of your state and national elected officials and links to their contact information.

You can also testify at the SBOE meetings either in person or by providing written testimony on particular agenda items that the SBOE may be discussing. Agendas for the SBOE are found online at http://tea.texas.gov/sboe/.

Dr. Paul Gray, TCTM Government Relations Representative

Teaching "Old Math" Using New Technology:
Selecting, Implementing, and Maximizing for Learning



Technology of today allows teachers of mathematics to dynamically present mathematical content in such a way as to foster students' imagination and understanding of mathematics. New technologies have consistently helped advance mathematical discovery and improve the teaching and learning of this subject. The National Council of Teachers of Mathematics (NCTM) has advocated the use of new technologies since its origin in 1920. While the slide rule was a popular piece of new technology in the early 20th century (Breckenridge, 1921), the calculator and computer ushered in the 21st century (NCTM, 1980). Current technologies are seen as tools to help students gain a better conceptual understanding of mathematics (NCTM, 2014), and tools like the iPad bring this magnificent subject to the touch of a finger.

Defining what technology is can be challenging. Some teachers in today's mathematics classrooms would define technology as any digital tool used in teaching and learning. While the International Society for Technology in Education (ISTE) does not have a formal definition of technology, in 2016 they emphasized how their standards for using technology with students centered on improving pedagogy and not just "throwing digital tools into the classroom" (p. 2). Teachers of mathematics must understand that effective technology is more than just a simple act of using a digital tool.

When technological resources, grounded in scientific knowledge, are selected and properly integrated in the mathematics classroom, the cognitive demand of mathematical exploration can be maximized both in the teaching and learning of this beautiful subject. Successful technology integration begins with teachers of mathematics properly selecting, implementing, and maximizing the impact of technology in their mathematics classrooms. What then should we do as teachers of mathematics?

Selecting Technology

First of all, teachers of mathematics must select their technology. Selecting technology to enhance the teaching and learning of mathematics should not be done haphazardly. Today's technological market is saturated with resources ranging from handheld devices to interactive projection screens, thus teachers of mathematics need to find a technology that they will use and implement. The selection of technology should not be made without ample research and an adequate plan of implementation. Ideally, a teacher should:

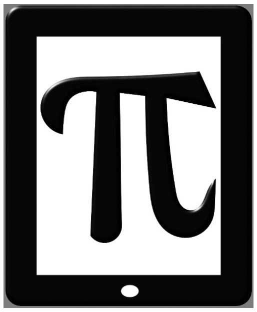
1. **Research**: Teachers should learn about available software for computers, interactive whiteboards and large touchscreen displays, tablet or mobile devices, calculators, and other interactive devices available for educational purposes by reading articles, exploring websites, and listening to colleagues. Some teachers of mathematics may be unaware of the technological tools that are available because of lack of time to investigate or perhaps failure to seek new ways of teaching.

2. **Observe**: Teachers should attend conferences, participate in trainings, and seek reviews and opinions from other educators who have used the technology before purchasing technology. Teachers can learn what technology can do from those that produce it, but teachers should not simply just take the word of the manufacturer selling the device but look for opportunities to see the technology being used before investing in their classrooms.

3. **Explore**: Teachers need to explore how certain technology works before any large investment is made. Once a teacher has researched how a piece of technology can be used in the teaching and learning of mathematics, the teacher should find a way to practice with the device. Teachers must spend time exploring, testing, and becoming knowledgeable about the ways their chosen technology can enhance the teaching and learning experiences in mathematics classrooms prior to implementing with students.

4. **Purchase, Continually Explore, and Plan**: After teachers have taken the time to know how their technology can engage students in the learning of mathematics, they can make their selection. That is only the start as teachers continuously explore the capabilities of this technology, and plan when and how the technology will be implemented into their classes.

Implementing Technology



Secondly, teachers of mathematics must implement their technology. Implementing technology into the classroom is an art, a science, and a requirement of the teaching profession. The creativity of a teacher guides the art of technology implementation. The usage of research supported instructional strategies makes up the science of this implementation. The profession of technology implementation comes from local, state, and national policies that govern this implementation. The state of Texas has moved to a teacher-appraisal system entitled the Texas Teacher Evaluation and Support System (T-TESS) (Texas Education Agency, 2017) that requires teachers to plan and implement instructional lessons incorporating technology in order to earn a proficient rating, which is the middle rating out of five. Teachers of mathematics can use the following categories of implementation as a guide when deciding how their technology can be incorporated in their classroom:

1. **Calculator-Based Implementation**: Today's classrooms have multiple types of electronic calculators ranging from handheld graphing calculators from companies like Texas Instruments[1] or Casio[2] to web-based graphing calculator applications like Desmos[3] and Meta-Calculator[4]. Teachers have multiple ways of implementing technology for mathematical explorations using a plethora of hand-held and computer-based calculators.

2. **Inquiry-Based Implementation**: Teachers of mathematics must provide learning experiences for students that center on the understanding of mathematics through conceptual inquiry. This conceptual understanding is grounded in a structure that asks teachers to seek ways to concretely model mathematical concepts and help students build a bridge between those concepts to the more abstract and procedural understanding. Teachers could use interactive whiteboards such as the SMART Board[5], Promethean Board[6], and MimioBoard[7] to engage students as they physically model mathematical concepts using digital manipulatives projected onto the touchscreen from web-based sites like the National Library of Virtual Manipulatives[8] and Shodor Interactivate[9]. Teachers could also wirelessly mirror and project their iPad[10] screen for a whole class exploration that allows students to physically explore the relationship between the circumference and diameter of a circle using an application such as Geogebra[11] or the Geometer's Sketchpad Explorer[12]. Inquiry lessons in mathematics classrooms are imperative, and technology can help those inquiries become dynamic and interactive.

3. **Resource-Based Implementation**: Teachers of mathematics can use technology as a tool to help students research, remediate, and remember mathematical definitions, concepts, and procedures. Students can explore key vocabulary in statistics using an application like Quizlet[13] or Brainscape[14] that allows teachers to generate their own sets of flashcards or other study resources for students. Students could use these resource-based programs using a class set of Surface Pro Tablets[15] or Macbook Air laptops[16]. Students could use their handheld mobile device like Google's Nexus[17] or the Samsung Galaxy[18] to individually view a series of videos from a popular site like Khan Academy[19] or MathVids.com[20] in order to review or remediate certain concepts not yet mastered. Students can also work at individual Apple[21] computers or personal desktop computers running Windows Operating System[22] to practice procedural skills from software packages downloadable or navigable from websites like IXL[23] and Dreambox[24]. Teachers of mathematics can use their technology to engage students in resource-based curricula in order to build their knowledge base in the subject.

4. **Presentation-Based Implementation**: The evolution of presentation media in the teaching of mathematics has moved from clay tablets having cuneiform, to papyrus scrolls, to chalk on slate boards, to whiteboards and overhead projectors, to modern electronic tablets being wirelessly projected inside of classrooms. Teachers of mathematics can have students use a document camera to showcase their work or they can use an iPad Pro and the Explain Everything[25] application to take a photograph of student work and have students use an Apple Pencil[26] to demonstrate their thinking associated with their work. Teachers can use digital manipulative applications from Brainingcamp[27] and Ventura Educational Systems[28] as students explore hands-on manipulatives at their desks. Teachers can project results from data gathering tools like scientific probes from Vernier[29] or Pasco[30] as students explore real-world applications of mathematics. New technologies of today have presentation capabilities more engaging and interactive than the sand and stick lessons during the days of Pythagoras.

5. **Game-Based Implementation**: Teachers of mathematics can use popular games as a catalyst for mathematical exploration or have a game be the primary theme of a unit or even a full curriculum. Teachers can use a game like Angry Birds[31] as a means for students to become engaged in parabolas and vectors (Lamb, 2014). The teacher could have every student equipped with a Chromebook[32] and the game Minecraft[33] to explore structures created by students based on volume and surface area of each student's world. Games can be excellent launching events or primary themes for engaging students in learning mathematics.

Maximizing the Impact of Technology

Finally, teachers of mathematics are tasked to maximize the impact of their implemented technology. Maximizing the impact technology can have on student learning is the responsibility of the teacher. Technology is a tool that can help students engage in mathematics at their highest level of cognitive demand. If teachers use technology as a way for students to merely reach a solution faster, then the technology is being severely under utilized. If systems of equations are simplified down to a series of keystrokes on a graphing calculator, then the concept of linear combinations can be lost on students. Teachers of mathematics have the obligation of pushing students to reach their highest level of understanding in mathematics classrooms and technology should be used to help students reach these high levels and not simply 'do the math' for them.

Teachers need to become the experts of their chosen technology and not waver from their chosen technology. Other similar technologies may be available, but teachers should "own" their technology and be the expert of that technology. A teacher using an Interactive Whiteboard should know everything they can about this piece of technology
A teacher using an iPad cannot get caught up in the latest app of the day, but rather, be an expert in the apps that significantly enhance student learning. They must commit to their technology and become an expert of that technology.

Mathematicians are continually adding to the body of knowledge that is mathematics. Modern mathematicians are discovering content that requires a foundation learned in the K-12 mathematics curriculum. One can point to works of Euclid 2,000 years ago, Newton in the late 17th century, and Euler nearly 250 years ago as influential mathematicians of this foundational mathematics we teach in K-12. As far back as 5,000 years ago, ancient Egyptians, Babylonians, and Chinese were carving, recording, and documenting mathematics that is still being taught in today's K-12 classrooms. This doesn't mean that the mathematics is not relevant. It is quite the opposite. This merely illustrates that teachers of mathematics must understand the curriculum in K-12 classrooms is filled with old mathematics, and we are called to teach this old mathematics using new technologies in order to inspire our current and future generations.

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(listed in order that they appeared in the article)

[1]**Texas Instruments -**
https://education.ti.com/en/products
[2]**Casio -**
https://www.casio.com/products/calculators/graphing
[3]**Desmos -**
https://www.desmos.com/
[4]**Meta-Calculator –**
www.meta-calculator.com
[5]**SMART Board -**
https://education.smarttech.com
[6]**Promethean Board –**
https://www.prometheanworld.com
[7]**MimioBoard -**
http://www.mimio.com
[8]**National Library of Virtual Manipulatives -**
http://nlvm.usu.edu/en/nav/vlibrary.html
[9]**Shodor Interactivate -**
http://www.shodor.org/interactivate
[10]**iPad -**
https://www.apple.com/ipad
[11]**Geogebra -**
https://www.geogebra.org
[12]**The Geometer's Sketchpad Explorer -**
http://www.dynamicgeometry.com
[13]**Quizlet -**
https://quizlet.com
[14]**Brainscape -**
https://www.brainscape.com
[15]**Surface Pro -**
https://www.microsoft.com/en-us/surface
[16]**Macbook Air -**
https://www.apple.com/macbook-air
[17]**Google Nexus -**
https://www.google.com/nexus/
[18]**Samsung Galaxy -**
http://www.samsung.com/us/mobile/tablets/
[19]**Khan Academy -**
https://www.khanacademy.org
[20]**MathVids.com -**
http://mathvids.com
[21]**Apple -**
http://www.apple.com
[22]**Windows -**
https://www.microsoft.com/en-us/windows/
[23]**IXL -**
https://www.ixl.com
[24]**Dreambox -**
http://www.dreambox.com
[25]**Explain Everything -**
https://explaineverything.com/download/
[26]**Apple Pencil -**
https://www.apple.com/apple-pencil
[27]**Brainingcamp -**
https://www.brainingcamp.com/index.html
[28]**Ventura Educational Systems -**
http://www.venturaes.com/iosapps/index.html
[29]**Vernier -**
https://www.vernier.com/products/sensors/
[30]**Pasco -**
https://www.pasco.com
[31]**Angry Birds -**
https://www.angrybirds.com/games/
[32]**Chromebook -**
https://www.google.com/chromebook/
[33]**Minecraft -**
https://minecraft.net/en-us/

On the Cover: Find Mathematics in Bridges

Bridges are a vital part of most communities. They allow us the freedom to quickly travel over obstacles such as rivers. Bridges have evolved over the years.

In 1870, the 475-foot Waco Suspension Bridge was the first bridge built over the Brazos River at a cost of $141,000. The suspension bridge made it possible for people and cattle to cross the river even in times of floods. Originally, the Waco Suspension Bridge was a toll bridge. It cost 10 cents for every person riding an animal, 5 cents for cattle and people walking across, and 3 cents for animals such as sheep, hogs, or goats. Many people disliked the toll, so McLennan County purchased the bridge for $75,000. They sold the bridge to the City of Waco for $1 providing that the city was responsible for its maintenance.

It wasn't until 1902 that another bridge was built over the Brazos River. The Washington Avenue Bridge is a 450-foot truss bridge. The bridge's deck is 21.7 feet wide. Unlike the Waco Suspension Bridge, it is still used for vehicle traffic to this day.

In more recent years, one Waco bridge has evolved into a light show. The I-35 access road bridge in front of Baylor's McLane Stadium has a light system that changes color. Baylor University spent $500,000 for the lighting system as a way to welcome people as they cross the Brazos River.

What math do you see when you look at these beautiful bridges? What math was necessary to create these bridges? Here are a few questions to get your students thinking about mathematics found in bridges:

1. Which bridge cost more: the Waco Suspension Bridge or the I-35 access road bridge? Explain your thinking. How much more did it cost?

2. Did McLennan County lose or gain money when they sold the Waco Suspension Bridge to the City of Waco? Why do you think they chose to do this? What would have happened, financially, if they had not sold the bridge? Why do you think that Waco did not purchase the bridge in the first place?

3. What would be the toll if 8 cowboys riding horses took 250 head of cattle across the Waco Suspension Bridge as they followed the Chisolm Trail?

4. Mr. Roberts crossed the Waco Suspension bridge and had to pay 55 cents. How many people riding animals, people walking, cattle, and/or sheep could Mr. Roberts have with him? Is there more than one way to solve this problem?

5. What is the area of the Washington Avenue Bridge's deck? If the width of an average car is 7 feet and the length of an average car is 16 feet, about how many cars could fit on the Washington Avenue Bridge?

6. Portions of the Washington Avenue Bridge are metal squares with two diagonal metal pieces intersecting in the middle of the square. If the length of the square is 12 feet, what is the length of the diagonal? How much metal did it take to create one square and its diagonals?

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The Pathway to Protractors, and Beyond

> *"...the right angle tool had done its job and helped students develop a mental image of a right angle and compare that mental image..."*

In Texas, students are introduced to the process of measuring angles when they are in the fourth grade. Protractors are introduced at this stage, and a search of the Texas Essential Knowledge and Skills (TEKS, Texas Education Agency, 2012), reveals that the word "protractor" only appears within standards for fourth grade mathematics. In this article, I will highlight the pathway we took in my fourth-grade classroom to develop an understanding of how and why a protractor is used. I will also provide some recommendations for continuing the use of the protractor in later grades, when its use would be beneficial in light of the TEKS.

In addition to learning how to measure angles, fourth grade is also the year when students first receive formal instruction on the concept of angle. By the end of fourth grade, students need to be able to answer these two questions related to angles:

1. What is an angle?
2. What is the measurement of a given angle?

We teachers should take precaution to treat these questions separately, and devote sufficient time to both of them. Ideally, students should have an understanding of what an angle is before they begin using a protractor to measure the angle. Unfortunately, students develop misconceptions that unnecessarily entangle these two topics. In a 2003 study, Keiser, Klee, and Fitch asked 78 sixth-grade students to write a definition of angle. They found that 23 of 78 students (29.5%) emphasized the measure or number of degrees as the definition of an angle. In the same way that we want students to understand that the area of a shape is not the same as the formula for the area of that shape, we want them to know that an angle is not the same as the measure of the angle.

First steps: Developing the angle concept

For this reason, I spent about a week helping my fourth-grade students develop an initial understanding of the angle concept. To do this, students needed to coordinate multiple ideas, as an angle involves two linear objects, the point at which they meet, and the opening or relative inclination between them. We began with concrete examples, following the suggestion of Mitchelmore (1998), with items such as, "scissors, fans, road junctions, furniture corners, and wall intersections in which the two arms of the angle are obvious" (p. 281). Afterward, we moved to looking at pictures and making drawings of angles. By the end of the week, most of my students were correctly identifying angles as corners or openings between straight objects.

Next, we started comparing the relative sizes of angles. I used a number of different activities and tasks to introduce students to right, acute, and obtuse angles. Several tasks were found in Putting Essential Understanding of Geometry and Measurement into Practice in Grades 3–5 (Chval, Lannin, & Jones, 2016), with objectives such as finding the smallest angle, identifying right angles, and describing obtuse angles given examples of angles that were obtuse and examples of angles that were not obtuse. We used paper folding to create a "right angle tool." (See "Finding Right, Acute, and Obtuse Angles: Grade 4 Module 4 Lesson 2" at https://www.youtube.com/watch?v=g7K4zztMXT0). Students used their right angle tools to compare angles and classify them as right, acute, obtuse, or straight. Throughout the next few lessons, many students became proficient in visually identifying angles as acute, right, obtuse, or straight; the right angle tool had done its job and helped students develop a mental image of a right angle and compare that mental image to other angles.

Measuring angles

Once students were accurately comparing angle sizes and classifying angles as acute, right, obtuse, or straight, I was ready to begin instruction on measuring angles. The measurement process for angles is similar to that of other attributes, such as length or weight. To effectively estimate angle measures and use measurement tools (e.g., protractors), students need to be familiar with the units of measure (Van de Walle, Karp, & Bay-Williams, 2010). For that reason, I incorporated students' prior knowledge from skateboarding, skiing, and other sports that a complete rotation measured 360 degrees, and a half rotation, or straight angle, measured 180 degrees. We fit pattern blocks into straight angles to find measures of those angles in degrees. Because two squares fit into a straight angle, the angles in squares had a measure of 90 degrees; three equilateral triangles fit into a straight angle, so those angles had a measure of 60 degrees. At this point, they understood that an angle that measured only one degree (denoted 1°) would be very small.

After students had several experiences of measuring angles with nonstandard units like pattern blocks and paper wedges (Wilson & Adams, 1992), I introduced the protractor as a measurement tool. We discussed the appropriate way to use a protractor, which included the following steps:

1. Visually classify the angle as acute, right, obtuse, or straight.
2. Place the hole of the protractor on top of the vertex of the angle to be measured.
3. Align the baseline that passes through the hole of the protractor with one leg of the angle to be measured.
4. The other leg of the angle should pass through the curved part of the protractor. If it does not, extend that leg with a straightedge.
5. There are two scales on the protractor, so the leg that crosses the curved part of the protractor may yield two different numbers. Use the measurement that fits your classification of the angle. For example, if the line passes through 70° and 110° and the angle was acute, use 70° for the measure. If the angle was obtuse, use 110°. To verify this measurement, we may count up (perhaps in 10° increments) beginning at the baseline with 0° and ending at the other leg of the angle.

My students had multiple opportunities to measure angles with a protractor and draw angles with a given measurement. I did notice that the space provided in our textbooks was insufficient, or the figures too small, to easily perform this process. For that reason, I created larger drawings so that students would not need to extend the legs of the angle. This allowed me to see how well they could use protractors without having to alter the diagram. (Of course, it is important that students eventually learn to extend the sides so that all angles can be measured. I simply didn't want to focus on that during this early stage in their development.)

Assessment of protractor use
I conducted several formative assessments of students' protractor use during this unit. Some closely resembled their learning activities, such as when I asked them to measure given angles with a protractor, or use a protractor to draw an angle with a specified measure (such as 55°). I also utilized writing prompts based on images of a protractor and an angle, as shown in Figure 1 and Figure 2. Students were shown the image, and asked to find the measure of the angle in the picture.

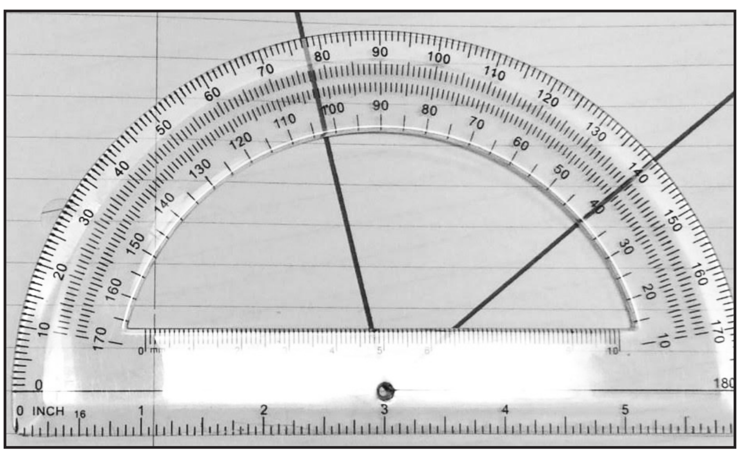

Figure 1. Find the measure of the angle in the picture.

I began with the picture in Figure 1. Right away, students noticed that the baseline of the protractor was not aligned with either leg of the angle. I agreed with them, but we were not able to alter the picture. I encouraged them, saying that they knew enough about math and measurement to figure out this problem. I asked them to write in their journals about how they found the measure. After some time, they shared their responses with the class. I was glad to see that several students gave an answer close to 62°. Several students indicated that they had counted the marks on the protractor – usually in groups of 10°, as those were marked on the protractor. I asked if anyone had used subtraction, but no one admitted to doing so. For this reason, I challenged them to work in pairs and find a way that used subtraction; several pairs realized that we could subtract in different ways and obtain 62°. Some used the outer scale, with 140° – 78°; many preferred using the inner scale with 102° – 40°.

That evening, I pondered my students' growing understanding of measuring angles. I wanted to encourage them, but also help them move beyond counting. Therefore, I created the picture in Figure 2, where a portion of the protractor is covered, and a glare obscured some of the numbers on the scale.

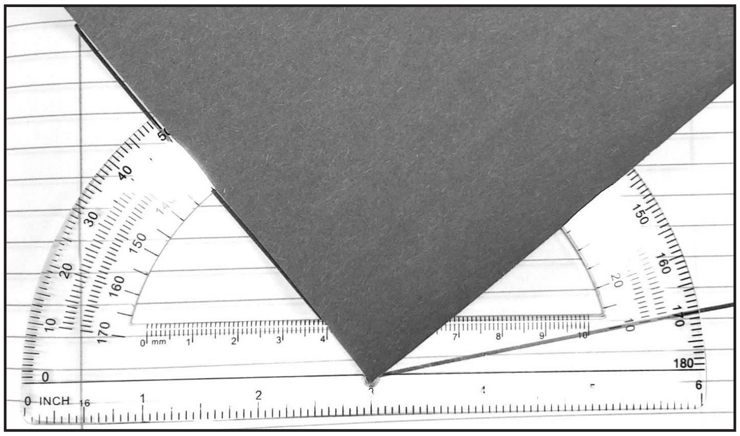

Figure 2. Find the measure of the angle in the picture, where part of the protractor is obscured.

The two most common measures for this angle were 120° and 130°. The work of five different students (identified with pseudonyms) is shown in Figures 3–7. Both Alf (Figure 3) and Betty (Figure 4) recognize that the legs of the angle cross the protractor at the 50° mark and the 170° marks, but they arrive at different results. Alf does indicate that the angle of interest is between these marks, but he doesn't provide any details of how he obtained the value of 120°. We might assume that Alf subtracted, but he may also have counted.

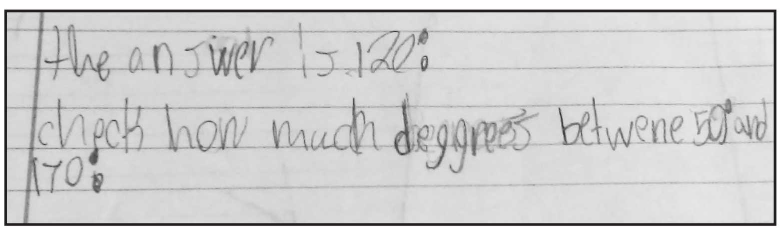

Figure 3. Alf's reasoning that the angle measures 120°

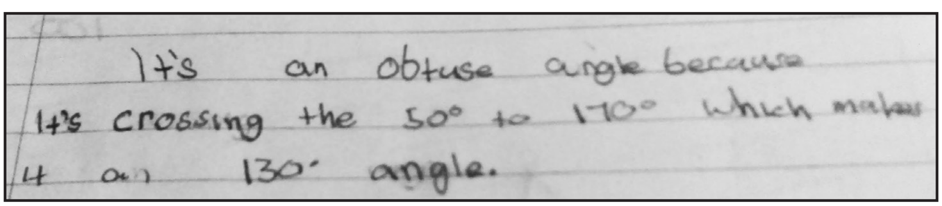

Figure 4. Betty's reasoning that the angle measures 130°

Both Cammie (Figure 5) and Delta (Figure 6) used counting from 50° to 170°. Cammie stated that she arrived at 130°, and that the angle is obtuse. While she is correct about the classification, her measurement is incorrect. As a teacher, I wonder how she counted. Delta provides more insight to her counting process, and says that she skip counted. Her writing further indicates that she used groups of tens. It appears that her initial measure was 130°, which leads me to believe that in her counting she paired the 10° with 50°, 20° with 60°, 30° with 70°, and so on, ending with 130° paired with 170°. If this is the case, it is incorrect, as 0° should be paired with the starting point of 50°, 10° with 60°, and so on. It is both amusing and confusing that Delta wrote the angle measured 130° or 120°. This makes it appear that she is unsure about which approach to counting she could use.

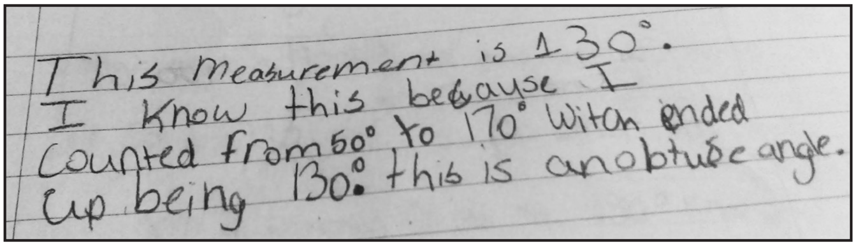

Figure 5. Cammie's reasoning that the angle measures 130°

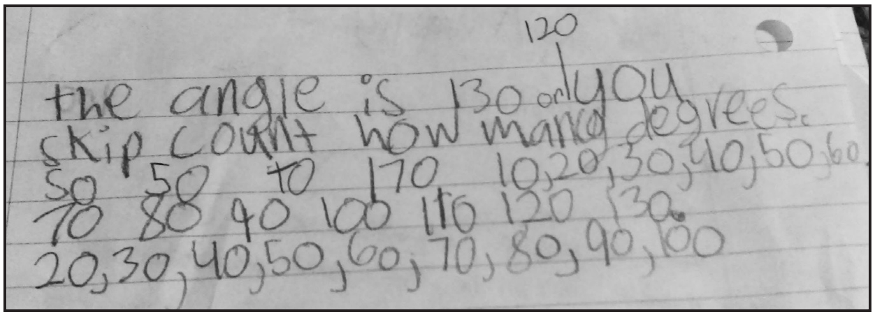

Figure 6. Delta's reasoning that the angle measures 130° or 120°

Epsi (Figure 7) provided the clearest explanation for how she obtained 120°. Her process describes using the parts of the protractor outside of the angle to be measured. She sees that 50° is outside of the angle on the left-hand side, and combines that amount with the 10° outside of the angle on the right-hand side. She subtracts this combined amount from the 180° shown by the entire protractor to arrive at her result of 120°.

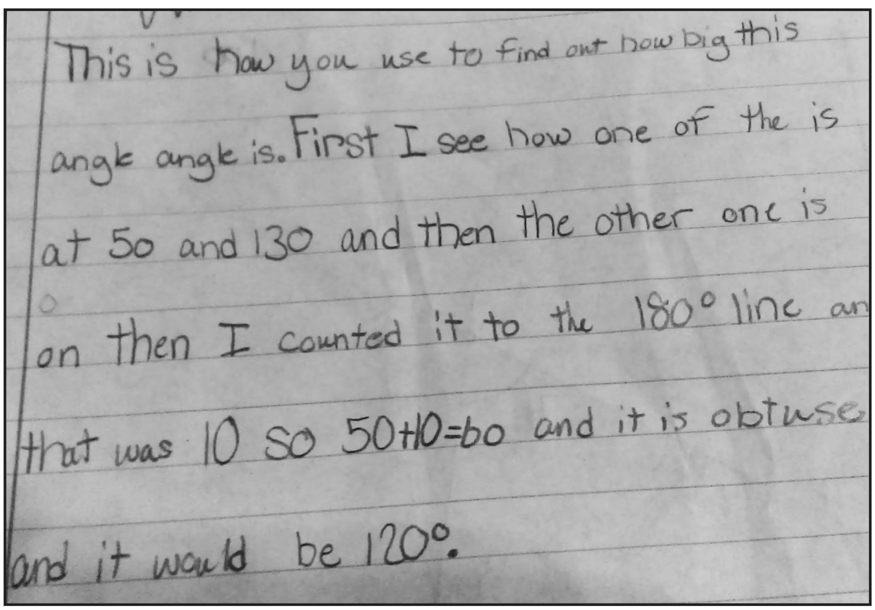

Figure 7. Epsi's reasoning that the angle measures 120°

Reading my students' journal entries gave me insight into how they understood the measurement process and how they applied their understanding to unfamiliar or novel situations. Students who used counting (such as Cammie and Delta) used a form of iterating units, where they repeatedly place units together with no gaps or overlaps, to measure. No student explicitly used subtraction, although 170° – 50° would yield the correct result. All the same, I was satisfied with the writing that I received, because it allowed me to assess my students' measurement understanding. If a student had simply written 170° – 50° = 120°, I would not know why he chose to subtract or what he understood about measurement.

Going beyond fourth grade

Even though the word "protractor" does not appear in the TEKS outside of the fourth grade, it is one of the tools that students may select to solve problems, as mentioned at each grade level within the Mathematical Process Standards. Here are a few examples.

- In sixth grade, students may use protractors to measure each of the angles in a triangle and determine their sum. It is possible that these students may need a quick refresher on using a protractor in order to efficiently measure the angles of several triangles.

- Students in seventh grade could use protractors to create circle graphs by hand. They can also measure the angles in a circle graph and use proportional reasoning to determine the proportion of the data indicated by each sector.

- In eighth grade, students can use protractors to verify that the corresponding angles in a shape and its image under dilation are congruent. They could also draw the image of a shape under rotations by a given angle.

- Students in high school may use a protractor to verify results found by using trigonometric relationships (such as finding the measure of an acute angle of a right triangle when two side lengths are given).

In each of the above examples, the protractor is a good concrete tool that could be used in the initial stages of instruction. For example, most people (including me) use technology to create circle graphs. However, even though technology provides an efficient and effective method for creating a circle graph, students will benefit from initially using their own hands and concrete materials to form their object. Once students understand the process that underlies the technology, then technology should be used to make the graph.

Angle measurement and protractors are useful in activities related to medicine. Kurz, Yanik, and Lee (2015) describe an activity for elementary students which describes how doctors measure angles to diagnose scoliosis in children. Protractors can also be used to create other tools useful for navigation and surveying. Howe and Badillo (2010) describe how to create a sighting box; they also describe an activity which requires students to determine the locations of several flags. Maxwell (2006) describes how to use a protractor to create a clinometer that is used to indirectly measure the heights of tall objects.

Conclusion

Angles and angle measurement are closely related, but distinct topics. An angle is a geometric object formed by the intersection of two straight objects, while angle measurement relates to the process of quantifying the "spread of the angle's rays" (Van de Walle, Karp, & Bay-Williams, 2010, p. 386). Fourth grade teachers should take a careful approach in helping students understand what angles are, and how to measure them. This requires special support at each step of the measurement process. Formative assessments, such as journal prompts, may allow insight into a student's thinking process. Finally, protractors should continue to be used at later grades for appropriate tasks that involve measuring angles.

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Application Information

2018-19 Mathematics Preservice Teacher Scholarship

There are five $2000 scholarships available for 2018-19. Any student attending a Texas college or university - public or private - and who plans on student teaching during the 2018-19 school year in order to pursue teacher certification at the elementary, middle or secondary level with a specialization or teaching field in mathematics is eligible to apply. A GPA of 3.0 overall and 3.25 in all courses that apply to the degree (or certification) is required. Look for the scholarship application online at *www.txmathteachers.org*. The application deadline is May 1, 2018. Winners will be announced in July 2018.

2018 TCTM Grant

This grant is for K-12 educators, university faculty and NCTM affiliate groups in Texas. Please note, pre-service teachers are not included as they can apply for the Mathematics Preservice Teacher Scholarship. The grant can be awarded to an individual, a group of teachers or to another NCTM or NCSM affiliate organization, if they are in Texas. Grant requests up to $800 will be accepted.

Uses include (1) improving mathematics classroom(s), or (2) helping your school achieve its goals related to mathematics, or (3) promoting mathematics teaching and learning, or (4) improving your ability to teach mathematics.

The online application may be found at www.txmathteachers.org. The application deadline is November 30, 2017. Awardees will be notified by January 31, 2018.

NCTM Membership

What's an easy way to support TCTM?

Join NCTM or renew your NCTM membership!

Sign up for your NCTM membership and use the link on the web form to indicate TCTM as the affiliate you wish to receive a rebate! Go to www.nctm.org.

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Please remember, you cannot join your local affiliates from the NCTM website. You must join the local affiliates directly by the process they have established. You may join TCTM by either attending the CAMT conference as a paid participant, or by using our membership form found online at www.txmathteachers.org.

TCTM Membership

Join TCTM or renew your membership!

Please join TCTM each year! Your membership includes this journal as well as updates on state and national opportunities such as grants, competitions, or professional development. You may join TCTM by either attending the CAMT conference as a paid participant, or by using our membership form found online at www.txmathteachers.org. If you are a paid participant at CAMT your TCTM membership is automatic for the school year following CAMT.

Remember to renew your membership if you do not attend CAMT or are not a paid participant. Our current membership dues are only $13.00 per year. If you are a new or returning member, please find our membership form online at www.txmathteachers.org. Just fill out the form and mail your check to our current treasurer. Sorry, we are not able to process electronic payments, but you can join or renew for multiple years. You may also donate to our scholarship fund at any time.

Maximizing Team Planning: Purposeful Math TEKS-Based Instruction

As more campuses implement professional learning communities (PLC), educators are able to collaborate in order to ensure learners deepen their understanding through purposeful, rigorous lessons. The following article describes this process from the Instructional Coach's point of view.

STEP 1. DECONSTRUCTING EACH STUDENT EXPECTATION:

Two specific questions helped in guiding our beginning discussions:

I. **What** do we expect our students to learn?
II. **How** will we know when they have learned it?

In order to answer the first question, grade level teams were brought together with the beginning task of simply reading each mathematics student expectation individually. Together, each grade level read the student expectation (SE) out loud in order to discuss and agree on what the ultimate goal of student learning would be. As the instructional coach, I came alongside each team to provide some pacing and guiding questions. It was of the essence that each team take their time in analyzing each student expectation in order for every member to grasp a good understanding of **WHAT** students are expected to learn. Teachers were prompted to circle each of the verbs as well as underline the most critical parts of the SE to use in their lesson planning (see Figure 1).

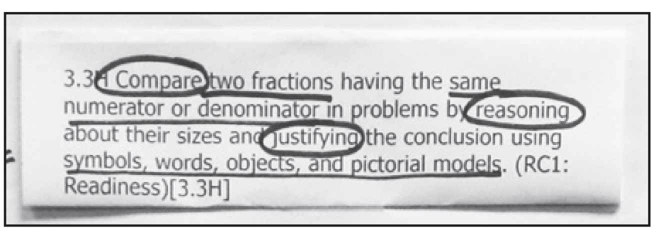


Figure 1

The second guiding question of **HOW** students will prove their learning prompted teachers to take a closer look at how each SE would be assessed in our district-provided nine-week assessments. With the end in mind, teachers better understood that they needed to pay close attention to the previously circled verbs and take into close consideration which parts of the SE would need to be taught first, perhaps with special emphasis, in order to "layer" their instruction for students' learning to be optimal.

STEP 2. PLANNING WITH LESSON PROGRESSION IN MIND:

With each SE now having been fully dissected and corresponding assessment items also found and reviewed, teachers were ready to begin planning for instruction. The first component for their lesson would be to take note of any new academic vocabulary needed to introduce and model during their lesson(s) structure. Secondly, but most importantly, days of instruction were critical to note in order to plan with lesson progression in mind. In essence teachers were now being driven to plan out day-to-day activities according to how many days were allotted for each student expectation in the district scope and sequence (see Figure 2). Teachers were guided to think about their students' prior knowledge in order to decide how they would present their lesson and what type of activities would be best for whole group instruction. The idea of "layering" instruction for the purpose of optimal student learning became key for discussion.

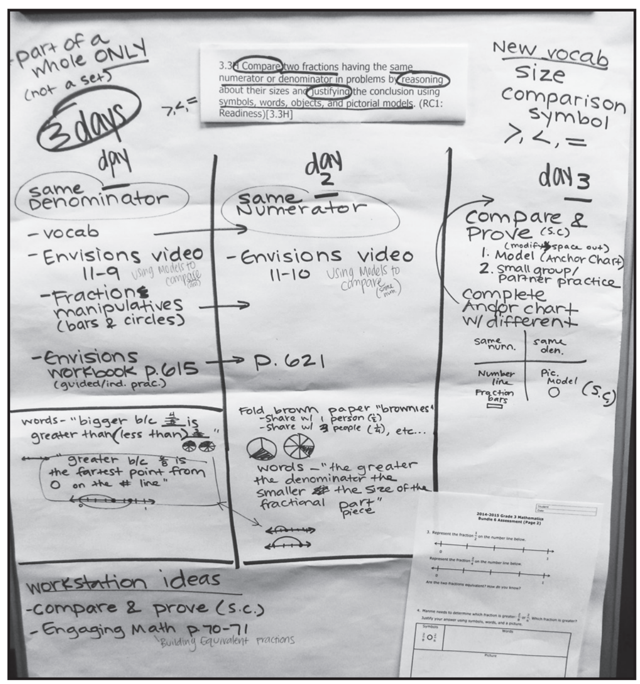


Figure 2

STEP 3: SELECTING RESOURCES FOR EACH DAY OF INSTRUCTION

With mathematics resources by their side, teachers were able to begin looking at a variety of activities and mapping out what their first day of instruction would include versus their second or third and perhaps last day of instruction for that particular SE. This piece of planning spurred teachers to look critically at each activity in order to ensure that the specific verbs stated in the SE were embedded in the activities for students' learning. Teams were able to prioritize activities and look at tweaking some activities in order to target all parts of the SE. Teachers' conversations during this piece of planning spurred further discussion of what exactly those verbs meant and what the action would look like once in the students' hands. In addition to selecting whole group and small group activities, teachers were able to identify activities that would serve well as work stations during their small group/guided math time.

In addition to gaining a better understanding of what students are expected to learn, teachers were able to walk away with a framework for what their mathematics instruction would look like for the following nine weeks. With the main lesson expectation and activities/work stations mapped out, teachers felt they were able to spend more time on making the necessary adjustments for their specific students. Teachers came together to "divide and conquer" the workload of copying activities, making academic vocabulary definition card games, and gathering necessary manipulatives for instruction and workstations.

Posters from planning sessions became part of weekly planning meetings where teachers were able to discuss, tweak, and plan with a common understanding of their grade level SE's (see Figure 3).

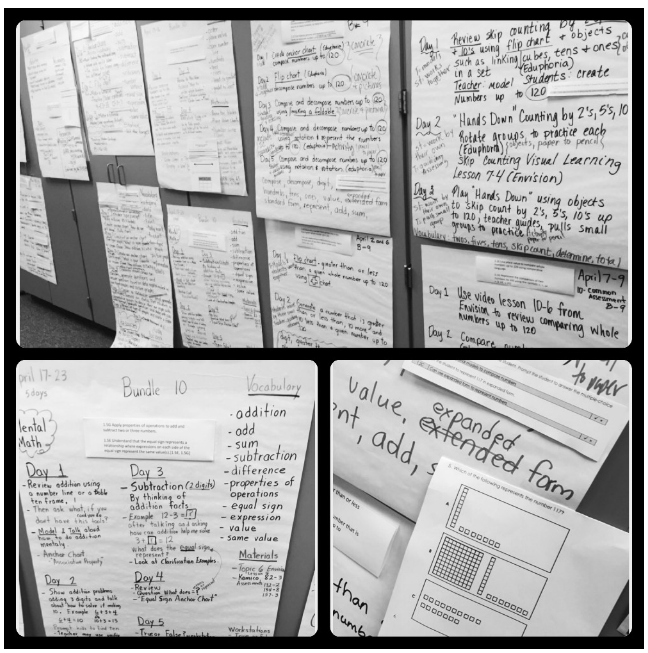


Figure 3

The initial goal of better understanding students' learning expectations and planning accordingly had surprisingly been surpassed – each team's time together accounted for great dialogue about mathematics instruction. Both new and veteran teachers were able to come together and in essence "start from scratch" in order to learn and grow as a grade level team. Collaborative planning for mathematics instruction is now a "must have."

Veronica Rodriguez-August • vrodriguez-august@pasadenaisd.org • www.twitter.com/VMAugust13
Mathematics Instructional Coach • Genoa Elementary, Pasadena ISD

Diameter continued from page 14

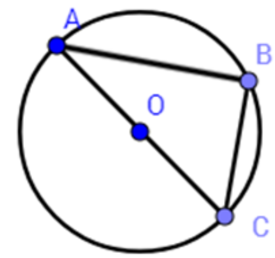
ANSWERS:

1. The prefix "di" often means two. This appears in many words in the sciences such as biology (e.g., diploid) or chemistry (e.g., a carbon dioxide molecule has two oxygen atoms). In logic or mathematics, a dichotomy divides a whole into two parts that are mutually exclusive and collectively exhaustive. For a circle, a diameter has the length of two radii and could be formed by two collinear radii. Also, the prefix "dia" can mean "through," and a diameter goes through the center of a circle and a diagonal is a segment that passes through two nonadjacent angles of a polygon.

2. Iambic diameter, which is the poetic meter this poem uses, involves each line of verse having two iambic metrical feet. For example, if we capitalized the stressed syllables, the poem's last line would be read "by OUR friend PI."

3. The most common way to define a diameter of a circle is any straight line segment that passes through the center of the circle and whose endpoints lie on the circle. (One could also say a diameter is a chord with the longest length.)

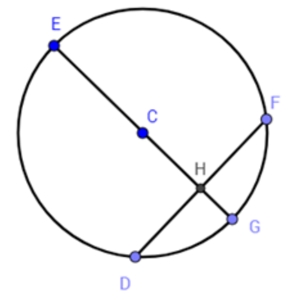
4. Thales of Miletus was a Greek geometer and philosopher from the 6th century BCE and online sources of biographical information about him include http://www-groups.dcs.st-and.ac.uk/history/Biographies/Thales.html As evoked by the fourth and fifth stanzas of the poem, the theorem states that if A, B, and C are points on a circle where the line segment AC is a diameter of the circle, then angle ABC is a right angle. Using dynamic geometry software, students could drag point B around the circle and see that angle ABC remains a right angle.



5. Thales' theorem (which is a special case of the inscribed angle theorem) can be proven in several ways, but we will outline a way that is arguably the most accessible to students, using the above diagram with radius OB drawn. Since all radii of a circle have the same length, OA = OB = OC, which means ΔBOA and ΔBOC are isosceles triangles. Since the base angles of an isosceles triangle are congruent, let's say that the measure of angles OAB and OBA each equal x degrees and the measure of angles OBC and OCB each equal y degrees. The sum of the interior angles of any triangle is 180^o, and when we set that equal to $x + (x + y) + y$, we see that $x + y$ (which is the measure of angle ABC) must equal 90^o, so ΔABC is a right triangle.

6. In circle C, add the appropriate symbol to show that diameter EG is perpendicular to FD. We ask the reader to complete a proof that FD is therefore bisected from these hints: Draw radii CF and CD. Explain why ΔCHF and ΔCHD are congruent right triangles. From Corresponding Parts of Congruent Triangles are Congruent (CPCTC), it now follows that FH = DH and so FD is bisected. (Can you also explain how we know that arc FD is bisected?)

7. $C = \pi * d$.



Author's Note: I am a Professor at The University of Texas at El Paso and a former full-time high school teacher at Houston's Emery HS, where I taught algebra, geometry, precalculus, and calculus. In the January 2017 issue of *Journal of Humanistic Mathematics*, I published a mathematical poem called "The Algebra Teacher Writes In Verse". From subsequent musing about what kind of verse a geometry teacher might write, I wrote "Diameter" – a poem about a circle's diameter written in the visual form of a circle's diameter, using the poetic meter of iambic diameter! While I hope "Diameter" is interesting as a stand-alone poem, I invite teachers to consider also preceding, accompanying, or following the poem with these questions for exploration I wrote to help students make connections within geometry as well as among language, mathematics, and science.

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The University of Texas at El Paso

TCTM 2017 Mission, Focus and Goal Statements

Mission of the Texas Council of Teachers of Mathematics:

To promote mathematics education in Texas

To support this mission, TCTM has five **focus areas**:

Recruit and Retain Mathematics Teachers	Curriculum and Instruction Support	Advocacy	Promote Communication among Teachers	Serve as Partner Affiliate for NCTM

TCTM activities will align to the five strategic goals. Goals of the organization include six strands:

Administration
- Streamline online membership registration through CAMT

Publications
- Survey membership to identify what they want in the Texas Mathematics Teacher (TMT)
- Review and refine the TMT journal and the TCTM website
- Improve the review protocol, establish criteria for reviewers
- Provide tips for new teachers in the TMT and on the website

Service
- Increase the donations toward Mathematics Specialist College Scholarships
- Staff CAMT with volunteers as necessary
- Advertise affiliated group conferences on the TCTM website, in the TMT and at CAMT

Communication
- Maintain an e-mail list of members for timely announcements
- Communicate with affiliated groups in a timely manner

Membership
- Encourage affiliated groups to include TCTM registration on their membership forms

Public Relations
- Sponsor and staff the TCTM booth at CAMT
- Follow NCTM Advocacy Toolkit (2004) for increased voice of TCTM membership on issues relevant to our mission

TCTM Past-Presidents

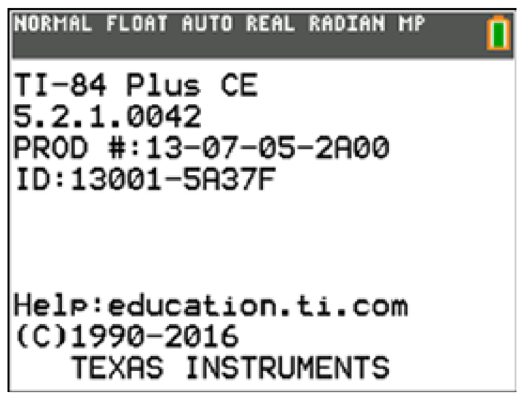
1970-1972	James E. Carson
1972-1974	Shirley Ray
1974-1976	W. A. Ashworth, Jr.
1976-1978	Shirley Cousins
1978-1980	Anita Priest
1980-1982	Patsy Johnson
1982-1984	Betty Travis
1984-1986	Ralph Cain
1986-1988	Maggie Dement
1988-1990	Otto Bielss
1990-1992	Karen Hall
1992-1994	Susan Thomas
1994-1996	Diane McGowan
1996-1998	Basia Hall
1998-2000	Pam Alexander
2000-2002	Kathy Mittag
2002-2006	Cynthia L. Schneider
2006-2008	Jo Ann Wheeler
2008-2010	Paul Gray
2010-2012	Nancy Trapp
2012-2014	Mary Alice Hatchett
2015-2017	Martha Godwin

TI-84 Technology Tips for the Classroom

With school back in session, it is time to dust off those TI-84 Plus™ graphing calculators and start pressing buttons again. Whether you teach middle school mathematics, Algebra I, or any other math content course; there are many functionality features on the TI-84 Plus family of graphing calculators that can be very useful in a math classroom. The TI-84 Plus family of graphing calculators includes the TI-84 Plus, the TI-84 Plus Silver Edition™, the TI-84 Plus C Silver Edition™ and the TI-84 Plus CE™. This article will focus on three features that are useful, but often forgotten by math educators or students: Operating Systems (OS), Graphing Databases (GDB), and Grouping.

Operating System (OS)
Since the graphing calculator could also be considered similar to a handheld programmable computer, it uses an OS in much the same way as a computer. An operating system is the program that manages the other programs in a calculator. To find out what OS is currently on your TI-84 Plus graphing calculator, simply press the key 2nd and then the + key. You will then see Screen 1.
The 1:About menu item is highlighted, so now press the ENTER key. Once this is done, Screen 2 shows the model of calculator on the first line and then the current OS on the second line. Additional identification information is shown on the other lines noted on the screen. To avoid confusion, all graphing calculators used in the same room should have the same OS. This is important if students bring their own graphing calculators to class.

Screen 1

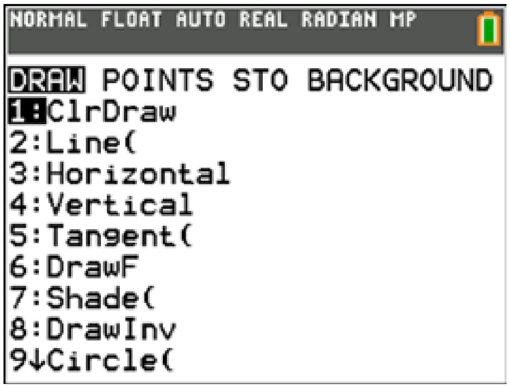


Screen 2

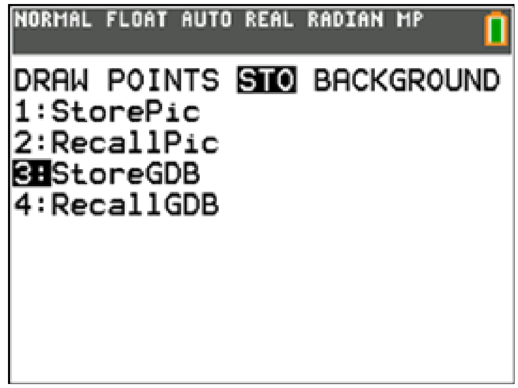
To see if you have the most current OS for your model of TI calculator, you can go to www.education.ti.com/latest. If you need to upgrade to the latest operating system (if none of your students or campus math teachers know how), the easiest way is to contact the TI-CARES Service Center at 1-800-TI-CARES or 1-800-842-2737. You should always try to have the latest OS in order to take advantage of the latest functionality available for your graphing calculator.

Graphing Databases (GDB)
Another useful feature of the TI-84 Plus family is Graphing Databases (GDB). A GDB is a calculator state that contains the set of elements that defines a particular graph. These elements include the graphing mode, window variables, format settings, all functions in the Y= editor, and the graph style for each Y= function. You can recreate the graph from these elements. Up to 10 GDBs can be stored in variables GDB1 through GDB0, and can be recalled at a later time to recreate those graphs. A unique GDB can be saved for each class period. This is particularly convenient when saving calculator settings from one day to the next. No data settings in the GDB are lost as students use the graphing calculator from one class period to another.

To store a graphing database, go to the home screen and press 2nd PRGM to access the "**DRAW**" menu shown in Screen 3 below. Then press the right arrow twice to select the "**STO**" menu. Now select **3:StoreGDB** as shown in Screen 4.
On the new screen select the number key



Screen 3



Screen 4

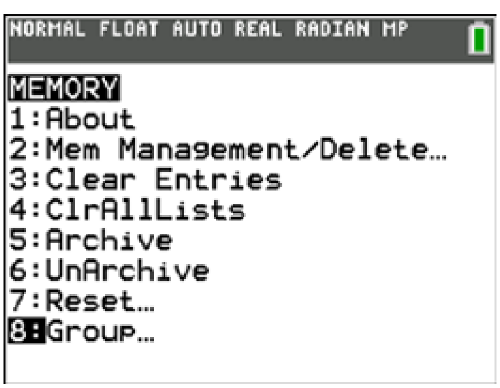
(from 1 to 9, or 0) of the GDB variable to which you want to store the graph database. As an example, if you enter 3 (perhaps to indicate Period 3), the TI-84 Plus will store the GDB to GDB3. To recall a graph database, select 2nd PRGM and then the "**STO**" menu. Now select **4:RecallGDB**, enter the number of the GDB you want to recall and then press ENTER. For example, if you enter 3, the TI-84 Plus will recall the GDB stored in GDB3.

At this point, it does not appear that anything has happened, but when you recall a GDB, it replaces all existing Y= functions. You may need to consider storing the current Y= functions to another database before recalling a stored GDB. Although this process looks complicated at first, it becomes very fast and easy on the second or third use. It also makes a great way to save calculator settings just before the bell rings to end the class period. Graphing databases are stored in RAM memory, so they can easily be overwritten by selecting the same number again.

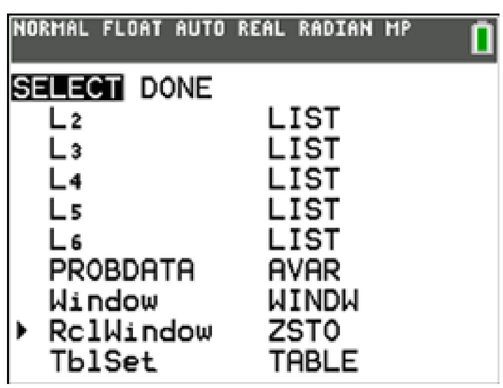
Grouping

And finally, another time saver and a great way to save even more data than a graphing database is grouping and ungrouping variables. Grouping allows you to make a copy of two or more variables residing in RAM on the calculator and then store them as a group to be recalled later. This feature is very useful for times when the lesson is not quite over, but the bell is about to ring to end the class period. Students save the current variable settings on their calculators in a group and then can reopen the group later without having to do any manual data entry. This saves instructional time without much button pushing.

Basically, as you work through a lesson or homework problem in class, pressing a variety of keys, the graphing calculator saves the various entries made in RAM memory. There are times when you may want to save these data settings to avoid having to enter them again. When you are ready to save, have the students select 2nd + to open the "**MEMORY**" menu and you will see Screen 5 below. Now go to **8:Group**. On a grayscale (non-color screen) TI-84 Plus, the **8:Group** will not appear on the first page due to the larger pixilation on the grayscale unit. On the new screen select **1:Create New**, and then name the group. As it requires a minimum of three letters, it is suggested that you select "AAA" for Period 1, "BBB" for Period 2, etc, then press ENTER. Now you will need to select what to save in the group. **Select 2:All-**, which shows all variables of all types that are available but not selected as partially seen in Screen 6.

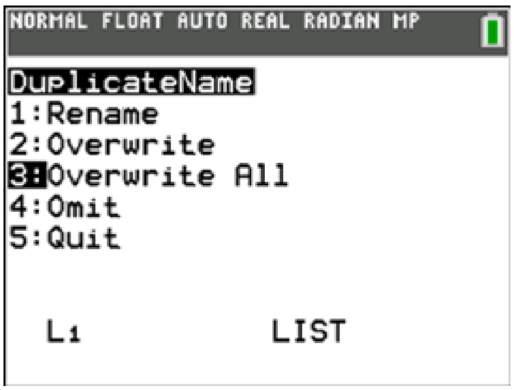


Screen 5



Screen 6

Press [▲] and [▼] to move the selection cursor next to the first item you want to copy into a group, and then press [ENTER]. A small square will remain to the left of all variables selected for grouping. Repeat the selection process until all variables for the new group are selected and then press [▶] to display the **DONE** menu. Press [ENTER] to complete the grouping process. Ungrouping allows you to reenter the variables copied in a group. Once again select **8:Group** from the "**MEMORY**" menu ([2nd] [+]). During the ungrouping action, if a duplicate variable name is detected in RAM, the "**DUPLICATE NAME**" menu is displayed. The easiest action is to press the **3:Overwrite All** option as shown in Screen 7. All of the selected variables have now been entered and are ready for use. Once you have used it several times with students, the process is very simple and fast.



Screen 7

Added Tip! Screen Capture

All of the screen images used in this article feature the TI-84 Plus CE graphing calculator. Images for the grayscale TI-84 Plus models will look very similar. This functionality is also available on the TI-83 Plus graphing calculator, as well. The images were created using the screen capture feature available on any of TI's graphing calculator software programs including TI Connect™ or the TI-84 Plus SmartView™ emulator software. If you have any questions about Operating Systems, Graphing Databases, Grouping variables, or screen captures; a call to TI-CARES should provide any clarification or guidance that might be needed. Additional information is also available at the TI website – www.education.ti.com or www.education.ti.com/84ce

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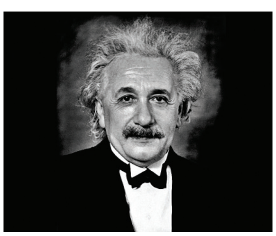


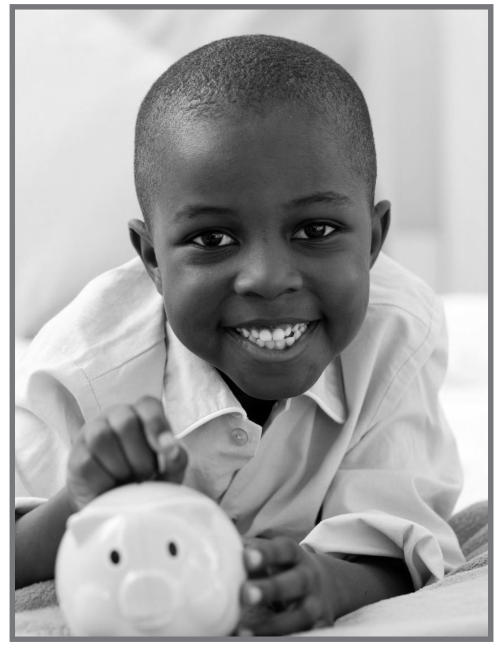
"Pure mathematics is, in its way, the poetry of logical ideas."

~ Albert Einstein (1879-1955)

Putting the Literacy in Financial Literacy:

Activities for 5th - 8th Grades

Personal financial literacy, a component of the mathematics Texas Essential Knowledge and Skills (Texas Education Agency, 2012) requires a student to understand financial concepts so they can make informed decisions about the "management of money" (Noctor, Stoney, and Stradling, 1992, p.4). Being financially literate will allow students to understand the complex economic relations that govern our society, and will encourage them to make savvy decisions in their personal lives, as well as in their role as a citizen of the United States (Whitin & Whitin, 2004). By fusing children's literacy strategies with math methods, students can connect financial literacy concepts to real-life situations through critical thinking and problem solving (Gastón, 2008). In this article, we begin by discussing children's literature and an approach to writing activities that connect with 5th grade personal finance TEKS (see sidebar) and conclude with activities designed to help students master the essential concepts for understanding taxes, income, and budgeting. Although we designed these activities around 5th grade personal financial literacy TEKS, the activities can be easily modified to cover the same strand through 8th grade.



Concepts Required to Understand Personal Finance

At least nine conceptual understandings are required to build a strong understanding of financial literacy at the 5th-8th grade levels.

- Income Tax
- Payroll Tax
- Sales Tax
- Property Tax
- Gross Income
- Net Income
- Methods of Payment
- Systems for Budgeting
- Balancing a Budget

Income Tax refers to the tax paid to the federal government by workers. The 16th Amendment in the U.S. Constitution allows the federal government to directly tax income (TurboTax, 2016), and the percentage of income paid to the federal government is determined by marital status and level of income, which is written into law. The 2017 tax brackets (Pomerleau, 2016) can be found on the Tax Foundation's website listed in the references. Currently, the state of Texas does not have a state income tax. *Payroll Tax* refers to the tax an employer withholds from a worker's paycheck. In the state of Texas, employers typically withhold Social Security (a retirement program) and Medicare (a health insurance program for the elderly) taxes. *Sales Tax* refers to a tax local and state governments collect on the consumption of goods and services. The state of Texas has a 6.25% sales tax rate, but local jurisdictions can impose an additional sales tax up to 2%. *Property Tax* is a tax a citizen pays if they own real estate. Property taxes are paid to the county tax collector and are often included in a homeowner's mortgage payment. Payments are collected monthly by the lender that holds the loan note and then deposited into an escrow account. At the end of the year, the lender sends a check to the county tax collector to pay the homeowner's property tax. Property tax rates vary depending on the county. *Gross Income* is a worker's total individual income before any taxes or other deductions are taken out. *Net Income* is the amount that is left after all taxes and deductions have been subtracted. *Methods of Payment* are ways a consumer chooses to pay for goods and services. These can include check, credit card, debit card, and electronic payments, such as PayPal. *Systems for budgeting* are plans for how a worker's money will be spent on bills, as well as on additional goods and services. Systems for budgeting come in a variety of forms that include using an envelope system, a paper system, or even an electronic system such as an app on a smart device. *Balancing a budget* refers to a management of money where a person's income is equal to the amount they spend.

Texas Essential Knowledge and Skills (TEKS)-Personal Finance Standards 5th Grade

(Texas Education Agency, 2012)

(10) Personal financial literacy. The student applies mathematical process standards to manage one's financial resources effectively for lifetime financial security. The student is expected to:

(A) define income tax, payroll tax, sales tax, and property tax;

(B) explain the difference between gross income and net income;

(C) identify the advantages and disadvantages of different methods of payment, including check, credit card, debit card, and electronic payments;

(D) develop a system for keeping and using financial records;

(E) describe actions that might be taken to balance a budget when expenses exceed income; and

(F) balance a simple budget.

Literacy Activities to Develop Understanding of Personal Finance

Using literacy strategies in conjunction with math instruction can increase students' critical thinking skills about mathematical concepts (Glover, 2016). By reading and writing about personal financial literacy, students can gain a stronger understanding of math terms and formulas (Adu-Gyamfi, Bosse, & Faulconer, 2010), and the instructor can use literacy activities as a form of informal assessment throughout a unit. In this section, we discuss math journaling, doodle sketching, multiple-approach learning stations, narrative storytelling, and literature that support the development of a student's understanding of the term and concepts previously listed, and while the activities provided have been geared towards the 5^{th} grade classroom, each can readily be modified to fit the personal financial literacy TEKS for 6^{th} through 8^{th} grades.

Keeping a math journal—— In their *Executive Summary*, The National Council of Teachers of Mathematics (2016) recommends writing as a method for assessing students' ability to "communicate their ideas and results effectively" (p. 5). Math journaling is a method of writing that includes informally jotting down a student's thoughts on math problems using words, pictures, diagrams, and formulas. These journals can serve as a record for documenting a student's problem-solving capabilities, a place for the student and teacher to communicate about mathematical concept learning in the classroom, and a place for students to organize their thoughts about mathematical concepts (Glover, 2016). To promote an understanding of personal finance terminology, a teacher could ask students questions such as:

- As a 5^{th} grader, on what do you prefer to spend your money?
    - Rank those items in order of importance to you.
- Create an algebraic formula for calculating sales tax in the city of (insert name of local city), Texas.
    - What is the total cost of a flat screen TV, a movie ticket, and a hoverboard?
    - What method of payment would you prefer to use to purchase each one? Explain the positive and negative factors of each method.
    - Based on an allowance of $10 a week, how long would you have to save to purchase each of those items?
- What goods and/or services do you think your federal and state taxes pay for?
- Why do you think the state government takes a percentage of a worker's income each pay period?
- What is the importance of creating a balanced budget? Explain your response.

Word Listing + Doodle Sketching—— In her book, *The Doodle Revolution: Unlock the Power to Think Differently*, Brown (2015) explains that doodling in a content area combined with listening or writing math word lists helps student with creative problem-solving and long-term retention of the vocabulary and information while improving comprehension. Word listing while math doodling is an activity that can be used in conjunction with keeping the math journal, note taking, or creating math word list portfolios or word walls (See example in Appendix A). Both portfolios and word walls, often associated in literacy classes with content literacy improvement, can be created and designed by students for students to develop and track their growth in math vocabulary and concept learning.

Math Multiple Approach Learning Station—— Peer group sharing, creating, and practicing in a multiple approach learning station, allows for low-pressure exploration of personal finance concepts and situations and provides reluctant students with the ability to try out ideas on paper, an organizer or a whiteboard without the pressure of making a mistake in front of the entire class. Noting the lowered risk with small group actions and activity posits that students' math verbal communication skills increase and improve as students practice and work to convince one another why their personal finance solutions are correct. This approach is encouraged as personal finance teachers often focus on teaching facts and procedures, rather than teaching them the larger context needed in order to make wise, information-based decisions (Clow, 2009).

Narrative Story-Telling with Personal Finance Concepts—— Willis (2010) suggests that using narratives to explain mathematical concepts resonates as students relate to common story structures they have heard their whole lives. Research indicates that narrative story-telling coupled with math content results in more engaging material for students that impacts higher content recall (Britton, 2008). In the following section, several book selections along with the research for read-alouds and picture books are recommended for instruction.

Literature to Support the Concepts—— Reading both print and digital literature provides a strong basis for building a conceptual understanding of financial mathematics that can lead to a deep and thorough understanding of key principles. Listed below are some excellent literature resources for teaching personal finance concepts, K-12.

- *Esperanza Rising* (2000) by Pam Munoz Ryan. In this engaging Pura Belpre award-winning book, a young girl must flee to California with her mother during the Great Depression and through many financial hardships must learn how to manage a tight budget.
- *Jimmy's Stars* (2008) by Mary Ann Rodman. During the World War II effort, an eleven-year-old girl must make emotional and difficult adjustments to her life. This short narrative examines the types of sacrifices and adjustments that must be made by all members of a family, adults as well as children.
- *Last Stop on Market Street* (2015) by Matt de la Pena. In this Caldecott honor book and Newbery Award winner, a young boy questions his grandma about many points concerning poverty-stricken areas, riding a bus and other things that are associated with not having a car.
- *Inside Out and Back Again* (2011) by Thanhha Lai. This national book award winner details the trials and tribulations of Ha, a young girl, and her family who must flee poverty in war torn Saigon to come to America and endure harrowing hardships to survive in Alabama.
- *A Chair for My Mother* (1982) by Vera B. Williams. This colorful narrative tells the endearing story of how a child and her mother save their spare change to purchase a new armchair one year after a fire turned all their belongings into "charcoal and ashes." The story illustrates the concept of how long it takes save for big-budget items.
- *Ruby Holler* (2002) by Sharon Creech. In this leaving the orphanage story, twin boys go from living a life of hardships to learning about living with adventure and new-found luxury.
- *Okay for Now* (2011) by Gary D. Schmidt. A young son learns how to survive by rising above adversity and living through the financial troubles of his own family.
- *Paying Taxes (True Books: Civics)* (2012) by Sarah Decapua. This expository, small chapter book explains what our taxes support, what types of taxes we pay, the process for filing federal taxes, and includes a discussion about whether or not taxes are fair. This text makes the complex ideas about different types of taxes understandable to 5th graders.

Expanding Understanding of Personal Finance

The main ideas in the 5th grade TEKS in the personal finance strand address different types of taxes, deductions from gross income, and creating a balanced budget. Thus, the activities we describe are designed to help students develop conceptual understandings of some of those terms. The goal was to create engaging activities that give students a space to practice the mathematical computations required to become proficient in the personal finance TEKS.

Activity 1 - Budgeting School Supplies for Hogwarts. Place students in groups of six. Let each student roll the dice to select their character. The student who rolls a one is Harry Potter, two is Ron Weasley, three is Hermione Granger, four is Draco Malfoy, five is Neville Longbottom, and six is Oliver Wood. Each student is given a budget and a list of required and optional school supplies (generated by the teacher) to be purchased in Diagon Alley along with a price sheet for the goods available. The students must work as a team to determine which items they can afford on their specified budget (see Appendix B for Budget Amounts and Tables).

Activity 2-Calculating Deductions. Using the following Income Tax Chart calculate each deduction from your selected gross income to compute your net income. For example, choose an income amount between $0 and $20,000 and compute each deduction. Next, add all the deductions and subtract from the gross income to find the net income. Do the calculations again for an income between $20,001 and $40,000. Do a third income between $40,001 and $60,000. Compare your deduction amounts and your net income amounts relative to the gross incomes (see Appendix C for graph and worksheet).

Activity 3- Property Tax The new movie theatre in town is offering luxury loungers for sale. Movie-goers now have the option of purchasing a luxury lounger plot, but must pay property tax on the lounger. Using the seat map as a guide, choose a seat you would like to own. Then, using the sale and rental prices in the table and how often you go to the movies every year, determine if it is a better deal to own a seat and pay property taxes on it or to rent a seat (See Appendix D for seating chart and prices).

"By seamlessly integrating children's literacy strategies with math methods, students can acquire and apply critical thinking and problem-solving skills..."

Answer the following questions in your math journal:

1) How often do you go to the movies each month? Each year?
2) What is purchase price, including property tax for the seat of your choice?
3) How much you would pay for the seat of your choice with the property tax? Write an equation to support your solution.
4) If the theater lets you make quarterly payments how much would each payment be?
5) Based upon how frequently you go to the movies and what seat you want, would you be saving money, breaking even, or paying too much? Explain your reasoning.
6) If you would be paying too much, but you still want to own your seat, what could you do?
7) What would you suggest for someone who goes to the movies once a month? Explain your reasoning.
8) Compare the purchase price (including tax) with the rental price for each of the four categories. Considering the category, how much could someone save if they went to the movies 3 times a month?
9) For seat owners the theater offers a snack deal for $6 per movie attendance consisting of a medium drink, a medium popcorn, and a small box of candy. The regular price of a medium drink is $2.75, a medium popcorn is $3.00 and a small box of candy is $1.00. However, if you want this snack deal you are limited to two snack deals a month and need to pay for it with your seat purchase. You cannot carry over unused snack deals to another month.
   How much would this add each month? How much would it cost for the year? Explain why you would or would not be interested.
10) If you did purchase the snack deal in #9 and included it in your quarterly payments, how much would your quarterly payments be?
11) If you wanted the snack deal in #9, but did not have enough funds, what would you consider doing differently so you could have the snack deal?
12) For each of the following annual entertainment budgets what decisions would you make about renting or buying a seat and about the snack deal? Discuss your reasoning with a partner.
    a) $250
    b) $300
    c) $400
    d) $500
    e) $600

Activity 4: Apps for Budgeting

1. **Rooster Money –** This is a free app (Android and Apple) for children to manage money (allowance) with an effective method for parents to teach children about money and budgeting. This app allows the parent to enter funds into the child's mock account. This system allows children to monitor their allowance and know how much money they have saved or spent by looking at their statement. Children can also enter savings goals and how much they need to contribute to reach their goal by a certain date.

2. **Our Home** - Chores and Rewards **–** This is a free app (Android and Apple) that motivates children to complete chores to earn points. The parents enter rewards and the cost of each. Chores and points are entered into the app by the parents and the children select which ones they want to complete. Upon completion the child receives the designated points. Points can be saved to be exchanged for a points-based selected reward or cash. Examples of rewards include purchasing a new toy or game, selecting a favorite restaurant, choosing the movie for family night, or selecting a game or activity for the family to play.

3. **Account Book** - This app (Android) is designed to teach money management starting with young children. Although it has many features, only those that are developmentally appropriate need to be used. It records income, expenses, loans, debits and credits. It also has the option of tracking upcoming bills and transactions. One's account can be shared with others to view. It can be used to share the family budget with all members or create a separate budget for each family member.

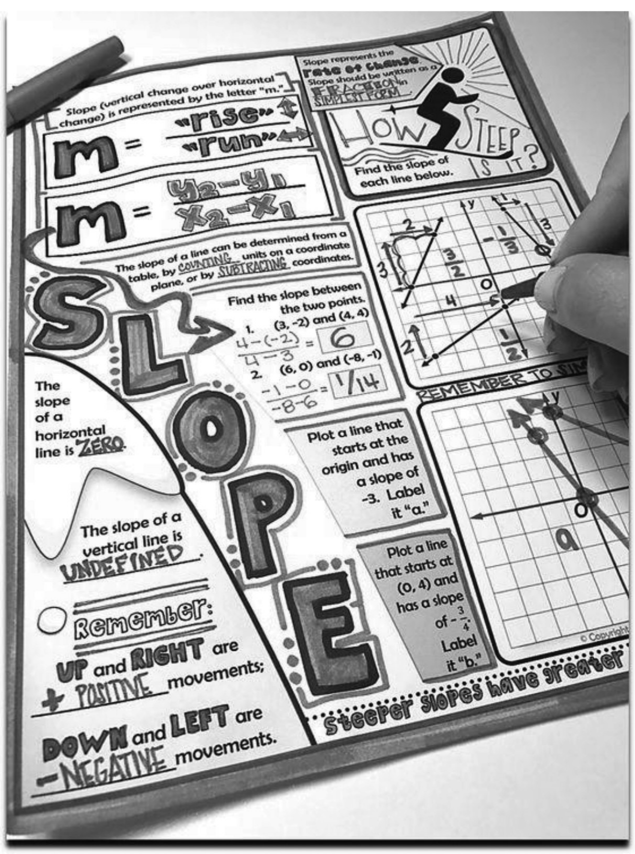
4. **Bankaroo-virtual banking for kids** - This free app (Android and Apple) supports the interaction of parents and children. The parent initially sets up the features of the app, then each child enters his name and selects an avatar. The parent assigns the child a chore along with the funds to be paid upon completion. The parent can record earned money or allowances and deduct funds as children spend money they have saved. Children can set financial goals, then they can enter how soon they want to reach the goal. The app allows the child to check their savings towards a goal. Until the financial goal is reached the child is unable to purchase the desired reward.

5. **Amazing Coin (USD)** - This app (Amazon and Apple) provides 3 free games to try before purchase. It has nine interactive games about coins, counting, payments, and making change. Each correct answer rewards the player with a quarter. Rewards allow the player to buy faux food to store in their game refrigerator.

Conclusion

With the requirement of personal financial literacy added recently to the mathematics TEKS, teachers who are now required to teach financial concepts for understanding must explicitly teach students to make informed decisions concerning money management. By providing students experiences with financial literacy activities such as suggested in this article, students not only gain greater understanding of the complex economic relationships that govern our society, but are encouraged to make intelligent decisions in their personal and professional lives and to participate as responsible citizens.By seamlessly integrating children's literacy strategies with math methods, students can acquire and apply critical thinking and problem-solving skills to taxes, income and budgeting, and connect financial literacy concepts to real-life situations which will serve them well in our global society.

Appendix A
Example of Doodle Notes from Math Giraffe (2017)

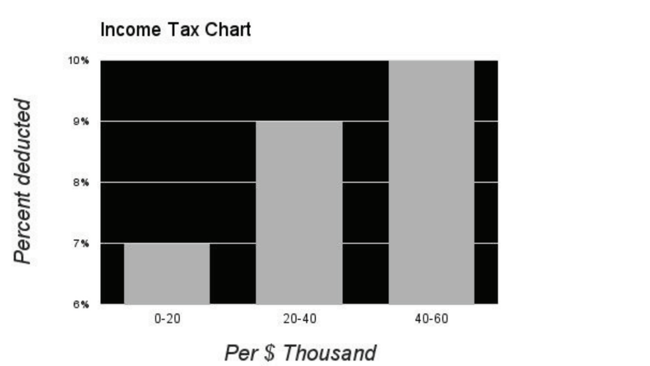


Appendix B
Budgeting School Supplies for Hogwarts

Name	Budget
Harry Potter	$305
Neville Longbottom	$150
Ron Weasley	$85
Oliver Wood	$250
Hermione Granger	$225
Draco Malfoy	$350

Type	Item Name	New	Used
Wands	Dragon Heartstring Core	$45	$25
	Phoenix Feather Core	$50	$30
	Unicorn Tail-hair Core	$60	$40
Robes	Standard Black	$15	$5
Brooms	Clean Sweep 7	$75	$35
	Nimbus 2000	$80	$45
	Silver Arrow	$60	$25
Broom Add-Ons	Cruise Control	$25	
	Reverse Camera	$20	
	Sirius Satellite Radio	$30	
Textbooks	Potions Book	$20	$10
	Spell Book	$25	$15
	Book on Mystical Creatures	$15	$5

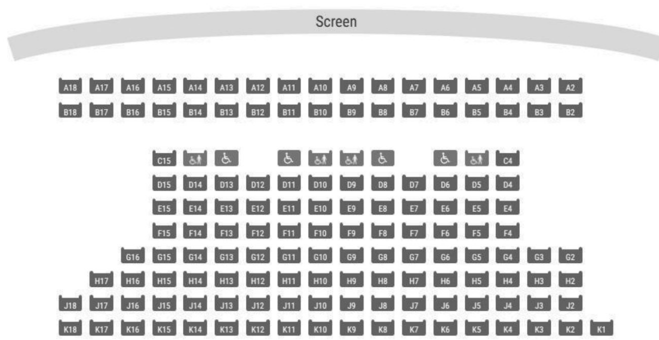
Appendix C
Calculating Deductions



Income Range	0-$20,000	$20,000-$40,000	$40,000-$60,000
Gross Income (choose an amount within that range)			
Deductions			
Federal Income Tax (use the graph to determine the percentage based on the income bracket)			
Medicare (1.5%)			
Social Security (6.2%)			
Total Deductions			
Net Income			

Appendix D
Calculating Property Tax

Row	Seat Numbers	Purchase Price per year (10% discount)	Property Tax Rate per year 1.26 %	Rental Price per movie attendance
A, B	2-18	$ 260	$ 3.28	$5
C, D, E, J, K	2-18	$ 364	$ 4.59	$7
F G H	4-6, 13-15 2-6, 13-16 2-6, 13-17	$ 416	$ 5.24	$8
F, G, H	7-12	$ 468	$ 5.90	$9



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Since 1971, the Texas Council of Teachers of Mathematics (TCTM) has produced the journal Texas Mathematics Teacher for our members. Our mission is to promote mathematics education in Texas. In the journal we accomplish this by publishing peer-reviewed articles by leading authors and local news from around the state. TCTM is committed to improving mathematics instruction at all levels. We place an emphasis on classroom activities that are aligned to the Texas Essential Knowledge and Skills and the NCTM Principles and Standards for School Mathematics. The Texas Mathematics Teacher seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Texas. All readers are encouraged to contribute articles and opinions for any section of the journal. Teachers are encouraged to submit articles for Voices from the Classroom, including inspirational stories, exemplary lessons, or management tools. More specific guidelines for submissions may be found below. Original artwork on the cover is another way teachers may contribute. We publish the journal twice each school year, in the fall and spring semesters. Our website archives the journals in PDF format. If you wish to view past issues, please see www.txmathteachers.org

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