Texas Council of Teachers of Mathematics
TCTM
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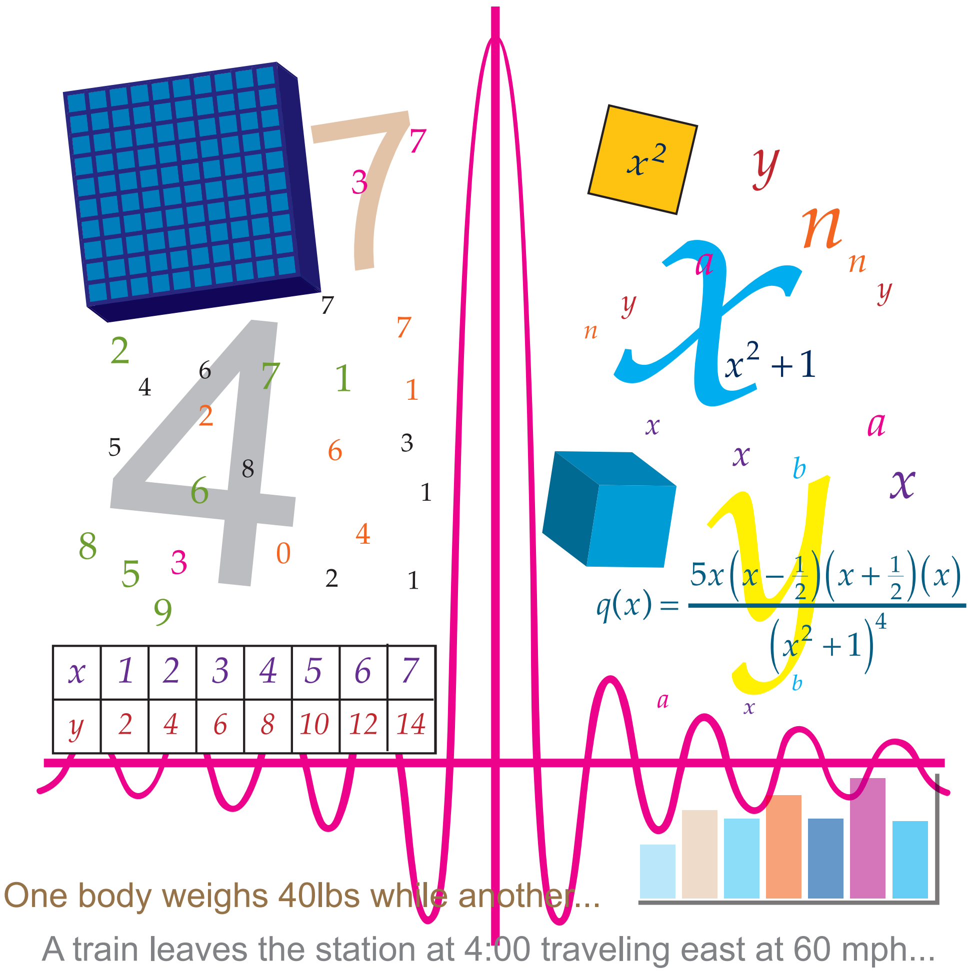
# Texas Mathematics Teacher

Volume LIII Issue 2 Fall 2006




# Selecting Representations

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**New NCTM Curriculum Focal Points!**

*see page 17*

**Check the Back Cover** for your membership card

**Choosing a Textbook for the Future**

*see page 6*

**Interested in Measurement in Elementary grades PK-4?**

*see page 10*

http://www.tctmonline.net/

# Texas Council of Teachers of Mathematics
# 2006-07 Mission and Goals Statements

## MISSION

To promote mathematics education in Texas

## GOALS

### Administration

- Streamline online membership registration through CAMT

### Publications

- Survey membership to identify what they want in the *Texas Mathematics Teacher* (TMT)
- Review and refine the TMT journal and the TCTM website
- Improve the review protocol, establish criteria for reviewers
- Provide tips for new teachers in the TMT and on the website

### Service

- Increase the donations toward Mathematics Specialist College Scholarships
- Staff CAMT with volunteers as necessary
- Advertise affiliated group conferences on the TCTM website, in the TMT and at CAMT

### Communication

- Maintain an e-mail list of members for timely announcements
- Communicate with affiliated groups in a timely manner

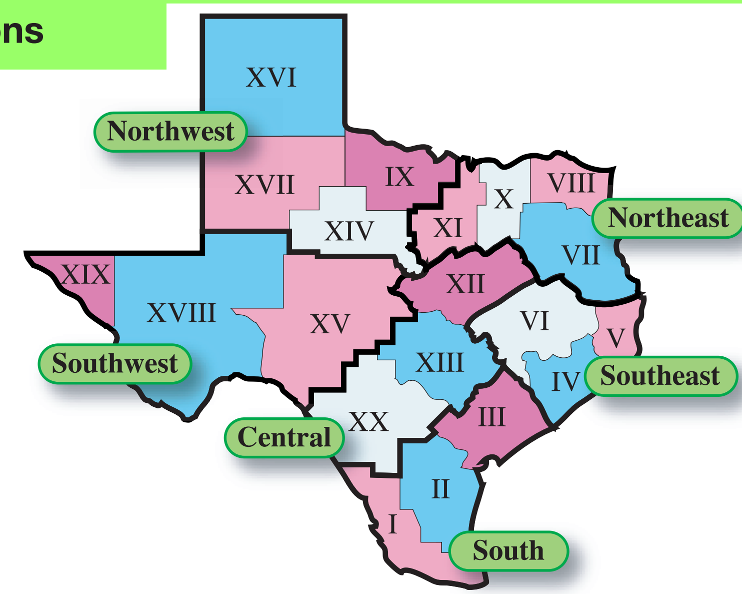
### Membership

- Encourage affiliated groups to include TCTM registration on their membership forms

### Public Relations

- Sponsor and staff the TCTM booth at CAMT
- Follow NCTM Advocacy Toolkit (2004) for increased voice of TCTM membership on issues relevant to our mission

## TCTM Regions




## TCTM Past-Presidents

| | | | | | |
|---|---|---|---|---|---|
| 1970-1972 | James E. Carson | 1982-1984 | Betty Travis | 1994-1996 | Diane McGowan |
| 1972-1974 | Shirley Ray | 1984-1986 | Ralph Cain | 1996-1998 | Basia Hall |
| 1974-1976 | W. A. Ashworth, Jr. | 1986-1988 | Maggie Dement | 1998-2000 | Pam Alexander |
| 1976-1978 | Shirley Cousins | 1988-1990 | Otto Bielss | 2000-2002 | Kathy Mittag |
| 1978-1980 | Anita Priest | 1990-1992 | Karen Hall | 2002-2006 | Cynthia Schneider |
| 1980-1982 | Patsy Johnson | 1992-1994 | Susan Thomas | | |

# Texas Mathematics Teacher

A PUBLICATION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

**Volume LIII Issue 2** **Fall 2006**

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## TCTM Applications

All applications are now available online at the TCTM website
*http://www.tctmonline.net/*

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***Texas Mathematics Teacher*** (ISSN# 0277-030X), the official journal of the Texas Council of Teachers of Mathematics, is published in the fall and spring. Editorial correspondence should be mailed or e-mailed to the editor.

**Call For Articles**

The ***Texas Mathematics Teacher*** seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Texas. All readers are encouraged to contribute articles and opinions for any section of the journal.

Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to the editor with a copy to the director. No author identification should appear on or in the manuscript. A cover letter containing author's name, address, affiliations, phone, e-mail address, and the article's intended audience should be included. After refereeing, authors will be notified of a publication decision.

Teachers are encouraged to submit articles for ***Voices From the Classroom,*** including inspirational stories, exemplary lessons, or management tools. If submitting a lesson, it should include identification of the appropriate grade level and any prerequisites.

Items for ***Lone Star News*** include, but are not limited to, NCTM affiliated group announcements, advertisements of upcoming professional meetings, and member updates.

Businesses interested in placing an **advertisement** for mathematics materials should contact Mary Alice Hatchett.

Deadline for submissions: Fall, July 1 Spring, January 1

# Letter from the President

Dear TCTM Members,

Thank you for the opportunity to serve Texas mathematics educators in the capacity of President of the Texas Council of Teachers of Mathematics (TCTM). The TCTM Board appreciates your support and continued dedication to mathematics education. TCTM has grown from 1,082 members in 2003 to over 6,500 today thanks to the leadership and dedication of Cindy Schneider, Past President, and the TCTM Board. The increase in membership has allowed the expansion of the number and amounts of scholarships awarded as well as make it possible to provide assistance when disaster strikes.

Currently eight $500 CAMTerships and four $1,500 Math Specialist Scholarships are awarded each year. Applications for these scholarships will be available online through the TCTM website in the spring. Don't miss out on this opportunity. Be sure to download and return your application.

With two devastating hurricanes striking the Gulf Coast region in 2005, people across the country found themselves wondering what they could do to help people whose lives were so dramatically turned upside down. The Indiana Council of Teachers of Mathematics (ICTM) is one such group.

At the National Council of Teachers of Mathematics (NCTM) Delegate Assembly in April 2006, ICTM members generously presented delegates from the Texas, Louisiana, and Mississippi councils with cash donations. The group's only requests were that we send the money to teachers and schools in regions that were severely affected by Hurricanes Rita and Katrina and that we let them know how the money helped support these teachers.

On behalf of TCTM, Past-President Cindy Schneider accepted a donation of $2,500 from ICTM. At the July 2006 Board of Directors meeting, the Board voted to match these funds for a total of $5,000 to send to schools in Southeast and East Texas that have been affected by Hurricane Rita.

After much discussion, the Board decided to enlist local experts' help in identifying some of the schools in Region 5 that sustained the most damage and continue to have lingering immediate needs. TCTM is an organization of mathematics teachers, for mathematics teachers. We may not be able to physically reconstruct the schools. But we can help teachers, supported by the generosity of ICTM and increase in TCTM memberships, begin to restore their supplies of instructional materials that were lost in the storm and its aftermath.

Record numbers attended CAMT 2006 in Houston. Over 6,640 mathematics educators enjoyed attending about 640 sessions featuring speakers from Texas and literally from coast to coast. The CAMT Preconference this year, which pulled in record attendance, featured strategies to make mathematics accessible to English language learners and students with special needs. Participants also enjoyed browsing the exhibit hall and previewing instructional materials for the 2006-2007 school year. We look forward to seeing everyone in San Antonio in June 2007.

Exciting changes are happening with the adoption of House Bill 1 in spring 2006. Among these changes is the requirement for four years of both mathematics and science at the high school level. Current requirements only call for three years of mathematics and two years of science. The expansion of these requirements will serve to increase the relevance and rigor of mathematics and science education for all students in Texas. Also, a fourth year of high school mathematics will better equip students with the tools necessary to compete in today's complex, changing, and expanding job market. Students will also gain a stronger background empowering them for success in higher education coursework.

Again, thank you all for the opportunity to serve this tremendous organization. I look forward to the next two years as we work together to improve mathematics teaching in Texas.

Sincerely,

Jo Ann Wheeler
TCTM President 2006-2008

# Lone Star News

## Affiliate Groups

These are local affiliated groups in Texas. If you are actively involved with them, please send future meeting and conference information to Cynthia Schneider at *<cschneider@satx.rr.com>* so we may publicize your events. Contact information for each group is available on the NCTM website,

*http://www.nctm.org*

Contact information for regional directors is located on the inside back cover.

### SOUTHWEST REGION: *Service Centers 15, 18, 19*

Rebecca Ontiveros, Regional Director

**Greater El Paso CTM**

Annual fall conference on October 21, 2006 at EPCC Transmountain Campus. Contact: Lori Correll <llcorrel@episd.org>

### SOUTHEAST REGION: *Service Centers 4, 5, 6*

Paul Gray, Regional Director

**Fort Bend CTM**

Holds a short meeting in August, a fall mini-conference, a spring mini-conference and an end-of-year banquet to serve the districts of Alief, Fort Bend, Katy, and Stafford. Contact: Jan Moore, *<Jan.Moore@fortbend.k12.tx.us>* or Susan Cinque, *<olsoncinque@alltel.net>.*

**Houston CTM**

**1960 Area CTM**

Holds two meetings and one competition a year to serve the districts of Aldine, Klein, Katy, Humble, Tomball, Spring, and Cypress-Fairbanks. Provides scholarships for students in mathematics education and awards for local mathematics education leaders. Contact: Sheila Cunningham, *<scunningham@kleinisd.net>*.

### NORTHWEST REGION: *Service Centers 9, 14, 16, 17*

Nita Keesee, Regional Director

**Big Country CTM & Science**

Will hold their annual conference in January 2007. Contact: Leslie Koske, *<lkoske@esc14.net>* or 325-675-8661.

**Texas South Plains CTM**

Thirteenth Annual Panhandle Area Mathematics and Science Conference, September 30, 2006, Canyon, TX. Contact: Gilberto Antunez, *<gantunez@mail.wtamu.edu>*, or see *http://www.wtamu.edu/academic/ess/edu/*

### NORTHEAST REGION: *Service Centers 7, 8, 10, 11*

Shirl Chapman, Regional Director

**East Texas CTM**

**Red River CTM**

STEAM (Successfully Training Educators As Mathematicians) is held every four years at the campuses of Texas A&M University-Texarkana and Texarkana College, Contact: Debra Walsh, *<dwalsh@redwater.esc8.net>* or Susie Howdeshell, *<showdeshell@pgisd.net>* or see *http://www.tamut.edu/~rrcmath/*

**Greater Dallas CTM**

Holds two mathematics contests (W. K. McNabb Mathematics Contests) for students in grades 7 - 12 - one in the fall (early Nov.) and one in the spring (early April). A banquet in May is held for the winners. Contact: Tom Butts, *<tbutts@utdallas.edu>* .

### SOUTH TEXAS REGION: *Service Centers 1, 2, 3*

Barba Patton, Regional Director

You are invited to join a listserv which will be for the purpose of promoting math education. The listserv will be a means to communicating your concerns about math education, share teaching ideas, etc. This listserv is managed by Dr. Barba Patton, Assistant Professor of Education at the University of Houston-Victoria. To join, send an email to imailsrv@listserv.uhv.edu Leave the subject line blank and put in the message subscribe Math at MathED-L your full name (example subscribe Math at MathED-L Barba Patton). Barba's email is *<pattonb@uhv.edu.>*

**CTM @ Texas A&M University at Corpus Christi (Student Affiliate)**

**CTM @ Texas A&M University at Kingsville (Student Affiliate)**

**Rio Grande Valley CTM**

The 41st annual conference, Saturday November 18, 2006, at the University of Texas - Pan American, Edinburg, Texas, from 8:00 to 4:00 p.m. Contact: Frank Rivera, *<f.rivera@ljisd.esc1.net>* or see *http://www.rgvctm.org*

### CENTRAL TEXAS REGION: *Service Centers 12, 13, 20*

David Hughes, Regional Director

**Austin Area CTM**

Will hold a fall conference on October 21, 2006. Contact: Carol Lindell, *<clindell@taylor.isd.tenet.edu>* .

**Alamo District CTM**

Normally holds a fall and spring conference. Contact: Kathy Mittag, *<kmittag@utsa.edu>*, or see *http://www.adctm.net*

**Central Texas CTM**

CTCTM will hold a fall meeting on Tuesday evening, October 3 and a spring mini-conference on Saturday morning, Febuary 24, 2007, in Waco at the Region 12 Service Center. Contact: Tommy Bryan *<Tommy_Bryan@baylor.edu>* or see *http://www.baylor.edu/soe/ctctm*

### NON-AFFILIATED CONFERENCES

### STATEWIDE

Texas Association of Supervisor's of Mathematics (TASM) Spring Meeting Feburary 5-6, 2007 in Austin. For membership and registration information, please see *http://www.tasmonline.net/*
(Membership is required to register for this meeting.)

NCTM Regional Conference in Houston, November 29-December 1, 2007.

FEATURE

# Choosing a Textbook for the Future

This school year, secondary mathematics teachers have the opportunity to participate in the adoption of instructional materials that will be used for several years. There are several processes available to help selection committees review the many choices available to them. The following information is meant to provide some examples of the over-arching questions underlying the review process and to help committees consider criteria they may wish to include in their deliberations.

### What's most important? The Texas Essential Knowledge and Skills (TEKS)

The state textbook review committees do a great job in the time they have, but they don't do your job for you. The TEKS must be interpreted in their global sense as well as their smallest details. It's not enough that student expectations be "covered;" what is important is that the materials support the depth and rigor of the TEKS. For example, what representations are used? Do you see graphs, tables, symbols and words in contextual situations? Are connections made between them? How are problem solving and communication handled? How hard are the problems? What progression can you observe with respect to rigor?

### Criteria

One set of criteria your committee may wish to consider is the following:

For materials to earn a "quality" rating,

- Include the major goals of developing students' problem-solving, reasoning, and communication abilities;
- Emphasize the development of conceptual understanding and the connections among topics;
- Allow ample opportunities for students to apply the mathematics in realistic and meaningful situations;
- Reflect high expectations for ALL students;
- Include appropriate student assignments;
- Promote students' active involvement in learning mathematics;
- Reflect an appropriate developmental sequence;
- Provide alternative assessment instruments and methods;
- Integrate the use of technology
- Reflect current research in mathematics education; and above all
- Teach all the TEKS (including Basic Understandings, Strands, Knowledge and Skills, and Student Expectations).



Another example of criteria generated by Region 4 ESC for starting points for discussion include:

| Score | | | Criteria | |
|---|---|---|---|---|
| Low | 0 | 1 | Does not meet depth and complexity of the SE. | Addresses knowledge level components |
| Low-Middle | 1 | 2 | Addresses the "surface" of the depth and complexity of the SE | Addresses knowledge and comprehension level components |
| Middle- High | 2 | 3 | Addresses the general intent of the depth and complexity of the SE | Addresses knowledge, comprehension, and application level components |
| High | 3 | 4 | Addresses the intent of the depth and complexity of the SE and the application of the SE to related SEs | Addresses knowledge, comprehension, application, and analysis level components. |

(Note: references to the SE include the knowledge statements.)

After the first cut, dig even deeper.

- How well do the texts align to the introductory paragraphs for the TEKS?
- How well do the texts align vertically? Is the scoring consistent across a series or are the gaps within the series?
- How "user-friendly" are the ancillary materials, especially those that are technology-related?
- How well do these texts address the "top 10" student needs as defined by students?
- How well do these texts address the "top 10" teacher needs as defined by teachers?



### Mathematics Textbook Evaluation Form : An Example

Here is an example of one district's scoring guide (note, this is not an recommendation or endorsement, but is provided to the reader as an example of how a scoring guide could be created by your district.) Reprinted with permission.

Rate each criterion on a scale of 1-4, where 4 is superior, 3 is above average, 2 is below average, and 1 is unsatisfactory.

A. Curricular Emphasis

1. Primary emphasis throughout is on integration of problem solving requiring higher level thinking skills.

2. Focus is on concepts and applications, rather than memorization of procedures.

3. Conceptual development precedes instruction on computational algorithms.

4. Many opportunities are provided for students to communicate about mathematics.

5. Estimation is emphasized throughout the problem-solving process, including estimation strategies that develop number sense.

6. Mathematics concepts are related to each other, to other curriculum areas, and to the students' daily lives.

7. Testing materials match instructional materials, with a strong emphasis on problem solving.

8. There is greater depth of instruction on the TEKS than on supplemental topics.

9. Review of previously learned topics is integrated throughout the text in relevant, meaningful contexts.

B. Instructional Strategies

10. The text addresses a variety of problem solving strategies.

11. The text uses a variety of approaches to develop concepts.

12. The text consistently uses manipulative materials to develop concepts.

13. The text develops concepts from concrete to connecting to abstract.

14. The text integrates the use of a calculator or computer as a problem-solving tool when appropriate.

15. The text contains lessons designed for cooperative group work.

16. The text uses an appropriate, interesting format.

C. Teacher Materials

17. Materials include clearly defined objectives.

18. Materials include specific instructions on how to teach problem solving strategies.

19. Materials include support for ways to use calculators and computers to teach concepts and for problem solving.

20. Materials include suggestions on appropriate student groupings for instruction, including strategies for the use of small groups.

21. Materials include resources for parent/family involvement.

22. Materials include suggestions for how to modify methods, materials, and/or pace for various student populations.

23. Materials include a variety of evaluation procedures for assessing student mastery.

### Tips for managing the textbook committee

It's as much about the process of review as it is the materials being reviewed. Don't begin your process with pre-conceived ideas; don't bring to the table anecdotes about what you may have heard from others. Let everyone draw his or her own conclusions. It is tempting to let others do the work, but participating in the process ensures that many perspectives have been included. Have all committee members look at the materials independently and build a pro/con list or use a rubric for group discussion. Many of the analysis tools available include rubrics for scoring the materials, including those mentioned above from the Dana Center or Region 4 ESC. This keeps the review process consistent among all reviewers.

### Key Components

- Who – Will you use volunteers or mandate participation? Will there be teachers with differing years of experience on the committee?
- When – How often will your textbook committee meet? What amount of time can you really devote to this? Set expectations that are reasonable, which will leave everyone feeling this was a constructive and doable task with a positive outcome.
- How – Will everyone look at every resource? Will you use a rubric? Or a well-designed, thorough checklist? Are there special criteria for a particular course? For example, for the Algebra I resource, if your scope and sequence is a functions-based approach, do your instructional materials mirror that focus?
- Buy In – If there is disagreement, do you have a plan to negotiate the outcome so everyone feels they were heard, but accepting of the final outcome?
- Time management – Do you have plan for the meetings? Can you limit the amount of wasted time? Latecomers and participants that don't complete their assignment, both lead to a slow-down of the process. How can you reduce the likelihood of these occurrences?

### The Students

What will the kids think? Do you want to ask them? We seldom hear about including the number-one stakeholder in this discussion. High school students, especially, might give your committee some insightful feedback on the accessibility of the text and appeal of the contexts. Include middle-school students; they have opinions too.

### Do's and Don'ts

Based on stories shared about the previous textbook adoption cycle, here is a brief list of suggested do's and don'ts.

- Consider the TEKS
- Don't base selection on appearance
- Consider the whole book/instructional material
- Don't select based strictly on one element (e.g. I like their tests).
- Do consider supplemental materials, but only with respect to their content and how they support the main resource
- Don't select based only on supplemental materials – these have not been reviewed by the TEA Textbook Committees
- Consider technology connections related to the mathematics
- Don't overvalue the technology support for teachers (i.e. it's not about bells and whistles)
- Consider how this will serve ALL your students
- Don't think about just one group of students

This is one of the most important tasks in which a teacher may participate. As with most endeavors, planning and time are important factors that will contribute to a successful outcome. ■

# 2006 Award Recipients

Each year since 1995, TCTM has accepted nominations for two awards for leaders in our professional community. The TCTM Leadership Award is presented to a TCTM member who is nominated by a TCTM affiliate. The second award, the E. Glenadine Gibb Achievement Award, is presented to someone nominated by a TCTM member. The following individuals have been honored and we wish to acknowledge their former and ongoing contributions this year in the leader spotlight. If you wish to nominate someone this year, please see the forms on our website.

Our prior awardees are

| **Year** | **Leadership(local/state)** | **Gibb (state/national)** |
|---|---|---|
| 1995 | Mary Alice Hatchett | Iris Carl |
| 1996 | Betty Forte | Cathy Seeley |
| 1997 | Diane McGowan | Pam Chandler |
| 1998 | ---- | ---- |
| 1999 | Linda Shaub | Eva Gates |
| 2000 | Lloy Lizcano | Bill Hopkins |
| 2001 | Susan Hull | Pam Alexander |
| 2002 | Janie Schielack | Judy Kelley |
| 2003 | Bonnie McNemar | Dinah Chancellor |
| 2004 | Dixie Ross | Jacqueline Weilmuenster |
| 2005 | Barbara "Basia" Hall | Barrie Madison |

## TCTM Leadership Award

Nancy Trapp

Honored for her service in mathematics education in Texas to improve professional development and empower teachers to provide the best teaching environment for all students, **Nancy Trapp** of Lyford ISD received the 2006 TCTM Leadership Award. She was recognized for her contributions to the improvement of mathematics education in Texas at the 2006 CAMT luncheon in Houston.

## TCTM E. Glenadine Gibb Achievement Award

Lois Gordon Moseley

Honored for her service in mathematics education at the state and national level to empower teachers to provide the best teaching environment for all students, **Lois Gordon Moseley** received the 2006 E. Glenadine Gibb Award from the Texas Council of Teachers of Mathematics. She was recognized for her contributions to the improvement of mathematics education in Texas at the 2006 CAMT luncheon in Houston.

## Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST)

The Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST) identifies outstanding mathematics and science teachers, kindergarten through 12th grade, in each state and the four U.S. jurisdictions. These teachers serve as models for their colleagues and leaders in the improvement of science and mathematics education. The 2006 Texas nominees are:

- **Kristine Botterman** is a 6th grade math teacher at Cross Timbers Middle School in Grapevine/ Colleyville ISD.
- **Kathy Letchworth** is a 6th grade math teacher at Frankford Middle School in Plano ISD.
- **Kit B'Smith** is a 6th grade math teacher at Pflugerville Middle School in Pflugerville ISD.

The 2006 PAEMST Awardees will be announced at the beginning of April 2007. Each Presidential Awardee will receive a $10,000 award from the National Science Foundation. Each award recipient will also be invited to attend, along with a guest, recognition events in Washington, D.C. These events will include an award ceremony, a Presidential Citation, meetings with leaders in government and education, sessions to share ideas and teaching experiences, and receptions and banquets to honor recipients.

The competition alternates each year between teachers of grades K-6 and teachers of grades 7-12. The nomination form for 2007 (7-12 teachers) can be downloaded at :

*http://www.paemst.org/*

Nomination forms must be submitted to Norma Torres-Martinez at the Texas Education Agency prior to the application being sent to qualified candidates. E-mail her at

*<Norma.Torres-Martinez@tea.state.tx.us>*

if you would like to nominate a colleague.

# 2006 Award Recipients

## TCTM CAMTership

Seven $500.00 CAMTerships were awarded this past summer by TCTM. We would like to extend our congratulations to each of the following recipients. All recipients volunteered two hours of their time at CAMT and attended the annual TCTM Business Meeting and Breakfast as guests of TCTM. If you have been teaching for five or fewer years, look for the CAMTership application in this *Texas Mathematics Teacher*. The CAMTership is intended to encourage beginning teachers to attend CAMT by helping cover part of the expenses associated with attending the conference.

**Brenda Best Kubiak**

Cypress-Fairbanks ISD

**Gloria Joyce Quesada**

Port Arthur ISD

**picture not available**

**Julie Anne Chavez**

San Elizario ISD

**Catherine Collins**

Pflugerville ISD

**Kym Celestine-Franklin**

West Orange-Cove CISD

**David G Fratto**

Abilene ISD

**Roger Reliford**

Fort Worth ISD

## TCTM Mathematics Specialist Scholarship

Three Texas students were awarded the $1500 TCTM Mathematics Specialist Scholarship for 2006-07.

**Jordan Byrd**

**Jordan Byrd**

*East Texas Baptist University*

Student Teaching in Marshall ISD

**Cheryl Lyn Mansell**

**Cheryl Lyn Mansell**

*Angelo State University*

Student Teaching in San Angelo ISD

**Rayna Glasser**

**Rayna Glasser**

*Texas Woman's University*

# TEKS: A Mathematical Continuum for the Early Grades

## Introduction

The State Board of Education adopted the Texas Essential Knowledge and Skills (TEKS) in 1997 as a mandated core curriculum with a change of focus from what should be taught to what should be learned at each grade level (Goodson, 1997). The TEKS, as well as the Prekindergarten Curriculum Guidelines (TEA, 1999), form the foundation for the district scope and sequence that enables teachers to design lesson plans to meet state standards. It is useful for teachers to examine the continuum of standards across grade levels to identify the knowledge and skills that come before and after their particular grade level. This vertical alignment (Huie, et al., 2004) through consecutive grade levels can provide focus for building on previous knowledge and preparing a foundation for future learning. Highly effective schools have been shown to enable vertical teaming in planning and implementing the curriculum (Kitchen, DePree, Celedón-Pattichis & Brinkerhoff, 2004). This article will offer an example of teacher vertical teaming in examining the continuum of standards in mathematics at the preschool and elementary levels.

## Background

A small urban elementary school in south Texas is a dual language partnership school offering the following classes: Prekindergarten for 3-year olds, Prekindergarten for 4-year olds, Kindergarten, and first through third grades. The school has six classroom teachers, a Special Education teacher, and two paraprofessionals. The faculty has an average of ten years of teaching experience. The school enrolls 128 students of which 90% are minority, 49% with limited English proficiency, and 49% economically disadvantaged.

A mathematics faculty member partnered with the school principal, an education faculty member, to offer professional development training in mathematics for seven teachers in grade levels pre-Kindergarten through third grade. During the first training session, teachers completed a questionnaire regarding a previous year's training and what they were expecting and needing from the present training. The third grade teacher responded that she wanted to "get a really solid idea of where we are all coming from through the grade levels". This request helped to focus a training objective to understand that the TEKS build on each other from grade level to grade level, and thus there is a need for vertical alignment. Over the subsequent training meetings, teachers developed a continuum of state standards for all five mathematics strands across six grade levels, pre-Kindergarten through fourth grade. This allowed each teacher to see what came before and after her particular grade level. Preschool teachers used their experience and insight to adapt the Kindergarten guidelines into an extension of standards for the preschool curriculum.

## Measurement Continuum Grid

The mathematics faculty member designed the first continuum grid for the teachers in response to her own personal need to grasp state mathematics standards after relocating to a new state. Measurement was chosen because this mathematical strand had resulted in the lowest TAKS scores in third grade the previous year. This first grid focused on listing student expectations in subcategories for each mathematical topic in the TEKS and Guidelines. However, measurement is not a focal point in the prekindergarten guidelines. The prekindergarten teachers agreed that exposure to measurement concepts such as weight and temperature would help prepare the students for encountering those skills in kindergarten. This lead to a discussion of whether the mathematical concept on the grid was intended to expose students to a new topic, introduce the concept, provide practice, or build on a previously learned concept. The continuum grid was then color-coded to reflect these decisions, with parenthetical suggestions for exposure or maintenance in empty grid cells.

It was immediately obvious that there was a "hole" in the third grade TEKS for capacity (the recently revised 2005 TEKS have since filled this hole). The district scope and sequence included capacity standards for third grade, and the third grade teacher knew her students needed to continue to measure liquid and dry ingredients and select appropriate tools for measurement. She was already maintaining those knowledge and skills between the second and fourth grades. This is an example of the prioritization of concepts, where capacity is not a focal point for third grade, but students must continue to build a foundation of basic understandings in capacity until it returns as a focal point in fourth grade (TEA, 2005).

The teachers then designed the continuum grids for the other four mathematical strands, beginning with Patterns, Relationships and Algebraic Thinking. The related TEKS for this strand were printed in large font for each grade level and cut into separate strips. Four large sheets of chart paper were glued together to make a wall-sized poster, and a large grid array was drawn with grade levels pre-Kindergarten through fourth grade listed across the top. During the next training session, teachers discussed the Patterns, Relationships and Algebraic Thinking standards for their grade level and identified common topics that were then listed down the far left column. Teachers glued each standard into the appropriate row and column for grade level and topic. Standards that crossed topics or skills were connected with arrows to appropriate boxes. Later, the faculty member transferred the outcomes to single-sheet document for distribution to teachers.

On the Patterns, Relationships and Algebraic Thinking continuum, it appears that patterns stop abruptly after second grade. However, the upper grade teachers were quick to point out that the sound, movement and concrete patterns used through second grade were subsumed into number patterns such as fact families, multiples, and place value, in the higher grades. This increased the teachers' understanding that grade level TEKS were related and built on the earlier standards.

## Vertical Teaming

One of the Prekindergarten teachers brought up a concern during the first training session. She said that as hard as she tried to teach and model the concepts of tall, short, long, and high, her students only used the words "big" or "bigger". She shared the multiple contexts that she had tried to use to teach the children. The students understand what she is saying and doing, but their replies all use the word bigger instead of the modeled vocabulary of taller, shorter, longer, or higher. She asked the other teachers what they were observing in their classes and questioned if it was too early to require this diversification of language. The kindergarten teacher replied that her students used the appropriate language in lesson responses but sometimes among their casual conversations they reverted to the word bigger. Other teachers indicated that students were using such measurement vocabulary correctly. This vertical team discussion supported the preschool teacher in exposing her students to new vocabulary in different contexts, and reassured her that students were beginning to develop the ability to discern the differences between the specific terms. Later in the year, she reported that her students were beginning to use the new vocabulary words correctly.

## Results

One of the Prekindergarten teachers described an activity based on the children's book, *If You Give a Mouse a Cookie*, which included counting raisins onto paper plate "cookies". The children placed raisins on their cookies according to the number rolled on a number cube. Then they chose how many raisins to eat before adding raisins again. The teacher made sure that having zero raisins occurred at some point in the activity. This allowed her to formatively assess the developmental readiness of her students to deal with the concept and notation of zero. The kindergarten teacher then announced that she would begin the next fall with the same activity for both review and to build on the previous knowledge of the students.

One of the teachers owned a sand table which had been in storage for some time due to lack of space. Discussions turned to capacity, and how to support student learning through concrete activities. Her offer to share the sand table sparked a wave of resource sharing that will build a continuity of experiences across grade levels. The table was filled with rice (less messy than sand and fairly inexpensive for 50 pounds) and various different-shaped containers and measuring tools were acquired. Teachers designed activities appropriate to their measurement topic and their grade level. Students were excited to see a new perspective on measurement and responded eagerly to the new activities.

The third grade teacher wanted the whole class to be able to participate at the same time, so she purchased plastic dish tubs and beans so that pairs of students could interact and discuss their explorations. The kindergarten teacher extended the activities onto the playground, where her class filled various buckets with sand. With one particularly large bucket, the students soon tired of taking turns shoveling sand. The ensuing discussion centered on why the larger bucket took a longer time and more work than the smaller buckets. The process of dumping the buckets reinforced the concept as the children dumped the smaller buckets but the teacher had to help to push the large bucket over and scoop out the sand because it was too heavy to dump.

One of the prekindergarten teachers designed a capacity activity using various ordinary household objects. A large-mouth jar was shown, and the students were asked to guess how many of the proposed item would fit into the jar to fill it up. Beginning with bear counters, students counted and recorded how many bears would fit into the jar. Next, they predicted how many cotton balls and then Styrofoam packing peanuts would fit. The students noticed that more cotton balls and packing peanuts could be put into the jar because they could be squished. The counting bears did not squish to fill in the empty spaces between.

## Conclusion

Teachers stated that listening to what others do in their classrooms, sharing lesson ideas, and completing the continuum grids were very helpful and the most beneficial portion of the training. They were able to develop and understand a common mathematical language among themselves, and pass that understanding on to their students. They realized that the TEKS build across grade levels, that gaps sometimes occur, but that they could be aware of such gaps and maintain those skills for the upcoming grade level. (Editors's note: Recognizing that the U.S. curriculum in international studies is seen to be a 'mile wide and an inch deep,' it is not necessary that every concept be fully developed in every grade level; the newly released Focal Points from NCTM will help K-8 teachers across the country understand where to spend more time on topics while maintaining others.) The teachers realized the need for constant communication about what is happening in their mathematics classroom, and that building on previous knowledge, skills, and mathematical activities can strengthen their students' understanding and retention of mathematical concepts and vocabulary. They were able to identify areas on the grid where the sudden stop or start of concepts were related, such as the disappearance of the pattern standard after the second grade – where it reemerges as function in the same grade. This illustrates the importance of the foundational preparation for later mathematical topics in middle and high school.

## For the Future

Teachers can be proactive in organizing their time and efforts for vertical alignment of standards. Asking for administrative support, space, and time will enable teachers to help themselves and each other. Continuum grids can be developed for one or all mathematical strands, and should go beyond student expectations to also include the overarching knowledge and skills that contain the expected student learning outcomes, as well as reading and understanding the introductory paragraphs that shape the standards for each grade.

For example, one of the third grade measurement TEKS states: "The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions" (TEA, 2005). Rather than one single concept to be learned, this statement involves complex and interwoven skills and understandings to produce a product that shows that the student has been able to construct a deeper understanding of measurement and apply it correctly in a real world context.

With the publication of the revised TEKS, the continuum grids need to be revised to reflect the changes in the standards for elementary mathematics. Figure 1 is an example of a revised continuum grid for Measurement across Prekindergarten through third grade, including the knowledge and skills statements. Teachers may consider whether students are being exposed to the topic, or whether the topic is being introduced, practiced, or building on previous learning as they review the grid. It is hoped that elementary teachers involved in vertical teaming for mathematics instruction will be able to create their own continuum grids for the other mathematical strands.

■

**References**


Goodson, J. J. (1997, October). Texas Redefines the Three R's: The New Public School Curriculum, House Research Organization Focus Report No. 75-19, October 7, 1997.

Heinz, K., Marroquin, C., & Gough, J. (2003). Factors impacting teachers' abilities to select and set up cognitively demanding mathematics tasks. Final Report from a Regent's Initiative Collaborative Research Grant. Retrieved December 12, 2005, from *http://falcon.tamucc.edu/~eduweb/academyDoc/Heinz-MarroquinFinalReport.doc*

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 28(5), 524-549.

Herbst, P. G. (2003). Using novel tasks in teaching mathematics: Three tensions affecting the work of the teacher. *American Educational Research Journal*, 40(1), 197-238.

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., Olivier, A, & Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational Researcher,* 25(4), 12-21.

Huie, S. B., Buttram, J. L., Deviney, F. P., Murphy, K. M. & Ramos, M. A. (2004). Alignment in SEDL's working systemically model: A research report. Austin, TX: Southwest Educational Development Laboratory. Retrieved December 27, 2006, from *http://www.sedl.org/ rel/resources/ws-report-summary04.pdf*

Kitchen, R., DePree, J., Celedón-Pattichis, S. & Brinkerhoff, J. (2004, Spring). High achieving schools initiative: Final report. Educating New Mexico, University of New Mexico. Retrieved December 27, 2005, from *http://educating-nm.coe.unm.edu/2004spring/ downloads/hp_final_report2.pdf*

National Council of Teachers of Mathematics (NCTM). (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

Shaw, J., & Blake, S. (1998). *Mathematics for Young Children.* Upper Saddle River, NJ: Merrill.

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal*, 33(2), 455-488.

Texas Education Agency. (2005). K-12 Mathematics TEKS: Texas Essential Knowledge and Skills. Austin, TX: The Charles A. Dana Center.

Texas Education Agency. (1999). Prekindergarten Curriculum Guidelines. Retrieved August 15, 2006, from *http://www.tea.state.tx.us/curriculum/early/prekguide.pdf*

Van de Walle, J. A. (2004). *Elementary and Middle School Mathematics: Teaching Developmentally*. Boston, MA: Pearson Education, Inc.



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# TEKS Continuum Grid for Measurement across PK–4

| | PRE-K | KINDERGARTEN | 1st GRADE |
|---|---|---|---|
| Focal pt | FP: Measurement | | |
| Introduction | • Make decisions about size by directly looking, touching & comparing objects while building language to express size relationships | • Build a foundation of basic understandings in measurement<br>• Begin to develop measurement concepts by identifying & comparing attributes of objects & situations | |
| Size, length, width, area | • Cover an area with shapes such as tiles<br>• Begin to make size comparisons between objects (taller than, smaller than)<br>• Begin using tools to imitate measurement<br>• Begin to order 2-3 objects by size such as largest to smallest | KS: Use comparative language to solve problems and answer questions of length & area<br>SE: • Compare/order 2-3 concrete objects by length (shorter/longer/same)<br>• Compare areas of two flat surfaces (covers more/ covers less/covers same) | KS: Use comparative language to solve problems & answer questions of length & area<br>KS: Select & use non-standard units to describe length<br>SE: • Estimate/measure length with nonstandard units<br>• Compare & order 2+ concrete objects by length from longest to shortest<br>• Describe relationship between unit size & number of units needed to measure length of object<br>• Compare & order areas of 2+ more flat surfaces |
| Capacity & volume | • Fill a shape with solids or liquids such as ice cubes or water | KS: Use comparative language to solve problems and answer questions of capacity<br>SE: • Compare & order 2 containers by capacity (holds more/less/same) | KS: Use comparative language to solve problems & answer questions of capacity<br>SE: • Compare & order 2+ containers by capacity (holds most to least) |
| Weight & mass | (Exposure to weight concepts) | KS: Use comparative language to solve problems and answer questions of weight/mass<br>SE: • Compare & order 2 objects according to weight & mass (lighter/ heavier/ equal) | KS: Use comparative language to solve problems & answer questions of weight/ mass<br>SE: • Compare & order 2+ objects by weight & mass (heaviest to lightest) |
| Temperature | (Exposure to temperature concepts) | KS: Understand time can be measured<br>KS: Use comparative language to solve problems & answer questions of temperature<br>SE: • Compare situations & objects by relative temperature | KS: Use comparative language to solve problems & answer questions of temperature<br>SE: • Compare & order 2+ objects according to relative temperatures (hottest to coldest) |
| Time | • Begin to categorize time intervals<br>• Use time language in everyday situations (in morning, after snack) | KS: Describe, compare & order events using time<br>SE: • Compare event duration (more/less time than)<br>• Sequence up to 3 events<br>• Read calendar using days, weeks, months<br>(Exposure to clocks) | KS: Describe & compare situations using time<br>SE: • Read time to hour & half-hour on analog & digital clocks<br>• Order 3+ events according to duration |

# TEKS Continuum Grid for Measurement across PK–4

| 2nd GRADE | 3rd GRADE | 4th GRADE | |
|---|---|---|---|
| FP: Use measurement processes | FP: Standardize language & procedures in measurement | | Focal pt |
| | | | Introduction |
| KS: Use comparative language to solve problems & answer questions of length & area<br>KS: Select & use non-standard units to describe length & area<br>KS: Recognize & use models that approximate standard units of length<br>SE: • Identify & measure concrete models that approximate standard units of length<br>• Select non-standard unit to find area of 2-D surface | KS: Use comparative language to solve problems & answer questions of length & area<br>KS: Select & use non-standard units to describe length & area<br>SE: • Use linear measurement tools to estimate & measure lengths using standard units<br>• Use standard units to find perimeter of shape<br>• Use concrete & pictorial models of square units to find area of 2-D surfaces | KS: Estimate & measure to solve problems involving length, perimeter & area<br>SE: • Estimate & use measurement tools to find length, perimeter & area<br>• Perform simple conversions between units of length | Size, length, width, area |
| KS: Use comparative language to solve problems & answer questions of capacity<br>KS: Select & use non-standard units to describe capacity<br>KS: Recognize & use models that approximate standard units of capacity<br>SE: • Select non-standard unit to measure capacity of given container<br>(Exposure to volume) | KS: Use comparative language to solve problems & answer questions of capacity<br>KS: Select & use non-standard units to describe capacity/volume<br>SE: • Identify concrete models that approximate standard units to measure capacity<br>• Use concrete models that approximate cubic units to find volume of given container/3-D figure | KS: Use measurement tools to measure capacity/volume<br>SE: • Estimate & measure capacity with standard & metric units<br>• Perform simple conversions between units of capacity<br>• Use concrete models of standard cubic units to measure volume<br>• Estimate volume in cubic units | Capacity & volume |
| KS: Use comparative language to solve problems & answer questions of weight/ mass<br>KS: Select & use non-standard units to describe weight & mass<br>KS: Recognize & use models that approximate standard units of weight/ mass<br>SE: • Select non-standard units to determine weight & mass of given object | KS: Use comparative language to solve problems & answer questions of weight/ mass<br>KS: Select & use non-standard units to describe weight/mass<br>SE: • Identify concrete models that approximate standard units & use to measure weight & mass | KS: Use measurement tools to measure weight/ mass<br>SE: • Estimate & measure weight using standard units of oz/lb/g/kg<br>• Perform simple conversions between units of weight<br>• Explain difference between weight & mass | Weight & mass |
| KS: Use standard tools to estimate & measure temperature in degrees Fahrenheit<br>SE: • Read a thermometer to gather data | KS: Measure temperature in degrees Fahrenheit to solve problems<br>SE: • Use thermometer to measure temperature | KS: Measure temperature in degrees Fahrenheit & Celsius<br>SE: • Use thermometer to measure temperature & changes in temperature | Temperature |
| KS: Recognize & use models that approximate standard units of time<br>KS: Use standard tools to estimate & measure time<br>SE: • Read/write times on analog & digital clocks in 5-minute increments<br>• Describe activities that take approximately 1 second, 1 minute, 1 hour | KS: Read & write time to solve problems<br>SE: • Tell & write time on analog & digital clocks | KS: Measure time<br>SE: • Use tools (clock w/gears, stopwatch) to solve problems involving elapsed time | Time |

FEATURE

# Voices from the Classroom

## Tips for Teachers: Communicating with Parents

These words of wisdom come from NCTM members and publications.

Feel free to supplement these tips with ones that have worked for you by emailing feedback@nctm.org.

- Make the first step positive. Take note of something positive about each of your students within the first week of school. Then, call their parents and say, "Hi! I'm Mr./Mrs. _____, your child's teacher. Your son/daughter did _ (insert good thing here)_ in class today, and I wanted you to know." Follow up by telling them how to reach you if they ever have questions, and that you look forward to meeting them and working together to help their child succeed.
- Give parents your school phone number rather than your home phone number.
- Create a separate e-mail account (Yahoo, Hotmail, and others allow you to do this for free) for use with school-related business. An address like MrsSmithAlgebra@yahoo.com will make the address easy for parents to remember and for you to separate school and other e-mail.
- E-mail students' grades weekly to their parents, and maintain a web page filled with information for both parents and students.
- Have parents e-mail you. Assign students homework to have their parent/guardian fill out an information form asking for contact information including an e-mail address. Award a bonus point if they also send you an e-mail with their child's name and class period in the subject line-then you can just move the reply to the appropriate folder in your e-mail program.
- Send a postcard. Have the students create their own mathematical collage postcard on a note card (if you put two cards together with the collage facing out, it runs through the laminator and comes out nicely). Then send it home with a positive comment on it sometime during the year.
- Send a parent newsletter or e-newsletter to communicate the mathematical goals that you have set for students and the ways in which you are helping students reach those goals.
- Host a family math night. Create an opportunity for students to shine and parents to share in the mathematical experiences that their children are receiving in your classroom.
- Recruit parent volunteers to serve as tutors, guest speakers, and general classroom helpers.

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*<www.nctm.org/teachmath/tips.htm>*

FEATURE

# TCTM E-mail Communications

## Timely announcements are sent to our membership using e-mail.

If you have an e-mail address, please be sure it is on file and up-to-date with TCTM. If you do not have an e-mail address, please let us know by indicating this on your membership application. TCTM members that have e-mail and have not received e-mail messages from the president, Jo Ann Wheeler, in the last month, should contact the membership chair at <*cschneider@satx.rr.com*> or by phone at 512-475-9713. Also note, if your server is not accepting our messages due to security, we would like to work with you on this issue.

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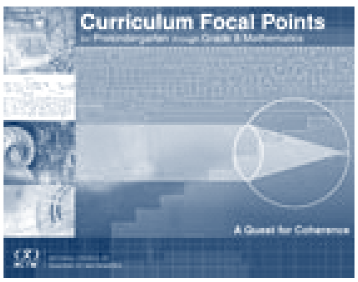
# Skip Fennell announces Curriculum Focal Points to NCTM

Dear NCTM Members:

I am pleased to announce that *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics: A Quest for Coherence* was released on September 12. The Curriculum Focal Points are the next step in the implementation of the Standards. The focal points fully support the Council's Principles and Standards for School Mathematics. The appendix in Curriculum Focal Points directly links the focal points to virtually all the expectations in Principles and Standards.

Curriculum Focal Points presents the most important mathematical topics for each grade level. A focal point specifies the mathematical content that a student needs to understand thoroughly for future mathematics learning. The focal points are compatible with the original Standards and represent the next step in realizing the vision set forth in *Principles and Standards for School Mathematics* in 2000. The focal points are intended for use by mathematics leaders as they examine their state and local mathematics expectations and seriously consider what is important at each grade level. This discussion, dialogue, or perhaps debate is designed to influence the next generation of curriculum frameworks, textbooks, and assessments.




Unfortunately, some of the media coverage has raised questions and caused concern among our members. Despite several conversations with a reporter from the *Wall Street Journal* explaining what the Curriculum Focal Points are and are not, a September 12 *Wall Street Journal* article did not represent the substance or intent of the focal points. The focal points are not about the basics; they are about important foundational topics. The Council has always supported learning the basics. Students should learn and be able to recall basic facts and become computationally fluent, but such knowledge and skills should be acquired with understanding. Unfortunately, some of the other news media have followed the *Wall Street Journal's* lead and have similarly misrepresented the Curriculum Focal Points.

The Council's goal is to support teachers in guiding students to learn mathematics with understanding. Organizing a curriculum around a set of focal points can provide students with a connected, coherent, ever expanding body of mathematical knowledge. The focal points describe what should be the focus of what students should know and understand thoroughly.

I encourage you to explore the complete Curriculum Focal Points and related resources. You can view a video overview and introduction to Curriculum Focal Points, and you can see answers to some questions or submit your own questions about the focal points. The news release on the focal points and a video of the news conference at the National Press Club announcing the release, as well as an article on the focal points from *Education Week*, are also on the Curriculum Focal Points section of the NCTM Web site.

Sincerely,

Francis (Skip) Fennell
*President*

Editors Note: For more information see *<www.nctm.org/focalpoints/>*

# TEA Talks

## Hot News

For additional information, refer to the websites listed

- **TEKS Refinements for Grades 6-Exit Level**
  The TEKS refinements for grades 6 through high school that were adopted by the State Board of Education in February 2005 had some minor effects on the TAKS tests for these grades. The spring 2006 tests did not assess skills/concepts that had been removed from the TEKS, i.e., narrower focus, limitations, etc. The spring 2006 tests did assess skills/concepts that contained clarified language in addition to skills/concepts that were not changed. The spring 2007 tests may include field-test items that assess additional skills/concepts, and the spring 2008 tests may include live items (for accountability) that assess additional skills/concepts.

- **TEKS Refinements for Grades 3-5**
  The TEKS refinements for grades 3-5 that were adopted by the State Board of Education in October 2005 also affected the TAKS tests for these grades. The spring 2007 tests will not assess skills/concepts that have been removed from the TEKS, i.e., narrower focus, limitations, etc. The spring 2007 tests will assess skills/concepts that contain clarified language in addition to skills/concepts that were not changed. The spring 2008 tests may include field-test items that assess additional skills/concepts, and the spring 2009 tests may include live items (for accountability) that assess additional skills/concepts.

- **Online Testing in 2006-2007**
  Online testing continues to grow as districts, test administrators, and students become more familiar with the online testing capabilities. The following online initiatives are planned for the 2006-2007 school year.
  - October 2006 TAKS exit level math, ELA, social studies, and science
  - Spring 2007 TAKS grade 7 math and reading
  - Spring 2007 TAKS grade 8 math, reading, social studies, and science
  - Spring 2007 TAKS grade 9 math and reading
  - Spring 2007 TAKS grade 10 math, ELA, social studies, and science
  - July 2007 TAKS exit level math, ELA, social studies, and science

- **End-of-Course Testing Opportunities**
  The Algebra I End-of-Course Exam will continue to be offered online only for students enrolled in Algebra I. The testing window to administer the Algebra I EOC exam is May 7-25, 2007. A highlight of this online only opportunity is that districts receive results within 24 hours of submitting the exam. The Geometry End-of-Course Exam is being developed to be administered online for students enrolled in Geometry. The field-test window for the Geometry EOC exam is April 23-May 18, 2007.

■

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*Director of TAKS Math & Science*
*• Texas Education Agency*

**Note the Early Date!**

# CAMT 2007

## The Magnitude of Mathematics June 28 – 30, 2007

CAMT 2007 will be held June 28-30, 2007, at the Henry B. Gonzalez Convention Center in San Antonio, Texas. The Program Chair is Sandra Browning of Seguin ISD. Program information will be available online next spring (probably May 1) at:

*<www.tenet.edu/camt/>*

Many favorite speakers such as Kim Sutton, Randy Charles, Marcy Cook, Chris Trioli, Juanita Copley and many others are returning in 2007. There will also be a strong strand on sessions for teachers of English Language Learners (ELL). Math-A-Rama will be all three mornings, and STEPS will be two full days. We will be using an easy online registration process. Look for information via your email.

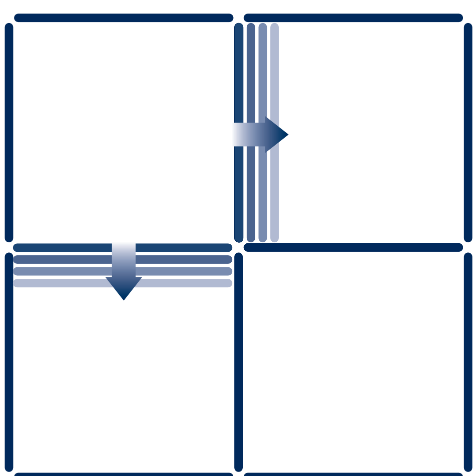


# Puzzle Corner

## Sticks #7 Answer

Arrange 12 craft sticks to form the original figure. Remove two sticks to form two similar squares.

Shown is a diagram of a solution.

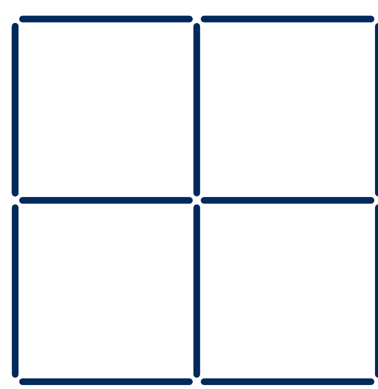


## Sticks #8 Puzzle

We are interested in how your students responded to this problem and how they explained or justified their reasoning. Please e-mail copies of your students' work, include your name, grade level, campus name and district name to Mary Alice Hatchett, Director of Publications, *Texas Mathematics Teacher*. Selected submissions will be acknowledged and published in subsequent issues.

*Please prepare a sketch of your solution*

Arrange 12 craft sticks to form the following figure.



Move four sticks to make two squares.

# Selecting Representations

To adapt a hip-hop cultural expression, we math teachers "gotta represent!" In our context, NCTM (2000, p. 67) explains: "The term representation refers both to process and product—in other words, to the act of capturing a mathematical concept or relationship in some form and to the form itself." Representation is also the most recently added standard of NCTM (2000) for school mathematics, the focus of the 2001 NCTM Yearbook, and the 2006-07 NCTM Professional Development Focus of the Year.

Choike (2000, p. 557) elaborates, "Students should be taught the value of representing mathematics verbally in words, numerically in tables, visually in graphs, and algebraically in symbols….A teaching strategy that connects the various forms of multiple representations to describe mathematics is an effective strategy for reaching out to students with different learning styles….Multiple representation is also a problem-solving strategy…an effective means for gaining insight into, and understanding of, a problem situation." This also connects to the Texas Essential Knowledge and Skills (TEKS), of course. For example, section 6.12 of the 6th grade TEKS states: The student is expected to "communicate mathematical ideas using language, efficient tools, appropriate units, and graphical, numerical, physical, or algebraic mathematical models; and evaluate the effectiveness of different representations to communicate ideas."

For the purposes of this article we shall refer to algebraic representation as analytic representation, which is used more frequently in the literature cited here. Hughes-Hallett (1991, p. 121) relates that, "One of the guiding principles is the 'Rule of Three,' which says that wherever possible topics should be taught graphically and numerically, as well as analytically. The aim is to produce a course where the three points of view are balanced…" Some mathematics educators add a verbal representation to make a 'Rule of Four', and sometimes the labels for representations are interpreted differently. The example in Table 1 shows the terms we will use in this article.

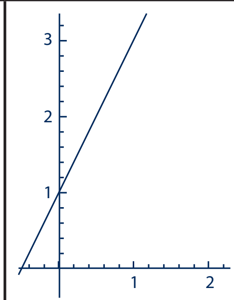
| Abbreviation in this article | Representation Category Name | Actual Representation |
|---|---|---|
| N | Numerical (or Tabular) | x: 0, 1, 2, 3, 4<br>y: 1, 3, 5, 7, 9 |
| G | Graphical |  |
| A | Analytic (or Algebraic) | f(x) = 2x + 1 |
| V | Verbal | "output is one more than double the input" |

**Table 1: Multiple Representations of a Linear Function**

## Teaching with Representation: One versus Many

In the 1990's, the second author implemented a pilot quasi-experimental design with seventy Russian high school precalculus students on solving inverse trigonometric identities such as arctan( $\frac{1}{2}$ ) + arctan( $\frac{1}{3}$ ) = $\frac{\pi}{4}$ using multiple representations (Tchoshanov, 1997), as in Figure 1. The analytic/algebraic proof (where L and R denote the "left-hand side" and "right-hand side", respectively) is the more familiar representation to most teachers and students. It is interesting to consider the geometric representation, which can be constructed on a 5x5 geoboard. The tangent of the angle with one marking (in the smaller shaded right triangle with 1-2-$\sqrt{5}$ sides) is $\frac{1}{2}$. The tangent of the angle with two markings (in the larger shaded right triangle with 1-3-$\sqrt{10}$ sides) is $\frac{1}{3}$. Those adjacent marked angles combine to form a larger angle in an isosceles right triangle so this new angle must be 45º or $\frac{\pi}{4}$ radians.

The experiment consisted of two studies. The first study was focused on the effect of single and combined representational modes on students' understanding and consisted of 3 groups. The first group of students was taught with a traditional analytic (algebraic) approach to trigonometric problem solving and proof. The second group was taught with a concrete/graphical approach using enactive (i.e., geoboard as manipulative aid) and graphical representations. The third group was taught with a combination of analytic and graphical means using translations among different representational modes.

As reported in Pape & Tchoshanov (2001), the combination group scored 26% higher than the graphical group and 43% higher than the analytic group. This experiment also showed that students in the "pure" (analytic or graphical) groups "stuck" to one particular mode of representation; they were reluctant to use different representations. For instance, students in the pure-graphical group tried to avoid any analytic solutions: they were "comfortable" if and only if they could use graphical techniques. Therefore, we realized that any intensive use of only one particular mode of representation does not improve students' conceptual understanding. Students in the combination group were much more flexible "switching" from one mode of representation to another in search of better understanding of mathematical concept. This observation supports findings from similar studies, such as Lesh, Post, and Behr (1987, p. 38) who state that "good problem solvers tend to be sufficiently flexible in their use of a variety of relevant representational systems that they instinctively switch to the most convenient representation... at any given point in the solution process." Wilson (1994) found that being able to translate between multiple representations was deeply related to conceptual understanding.

NCTM (2000, p. 69) notes that "Different representations often illuminate different aspects of a complex concept or relationship," giving the example of various representations of fractions, including: sectors of a circle, fraction bars, points on a number line, and ratios of discrete elements of a set. Readers may wish to take a moment and think about what aspects of the fraction concept each of these representations emphasizes and what aspects are ignored. For example, fraction bars "convey the part-whole interpretation of fractions" but not "other interpretations of fraction, such as ratio, indicated division, or fraction as number."(ibid).

Multiple representations can be insightful even for the counting numbers. For example, Zazkis and Gadowsky (2001) ask which are perfect squares or cubes in the following set of numbers:

$36^{200}$, $36^{300}$, $36^{400}$, $36^{500}$, $36^{600}$, $36^{700}$

They also ask for which of the following numbers can

**Prove the Identity**

$\arctan(\tfrac{1}{2}) + \arctan(\tfrac{1}{3}) = \tfrac{\pi}{4}$

**Analytic/ Algebraic Proof**

**1.** $\tan(L) = \tan(R)$

**2.** $\tan(L) = \tan(\arctan \tfrac{1}{2} + \arctan \tfrac{1}{3})$

$$= \frac{\tan(\arctan \tfrac{1}{2}) + \tan(\arctan \tfrac{1}{3})}{1 - \tan(\arctan \tfrac{1}{2}) \cdot \tan(\arctan \tfrac{1}{3})}$$

$$= \frac{\tfrac{1}{2} + \tfrac{1}{3}}{1 - \tfrac{1}{2} \cdot \tfrac{1}{3}}$$

**3.** $\tan(R) = \tan(\tfrac{\pi}{4}) = 1$

**Q.E.D.**

**Visual/ Geometric Proof**

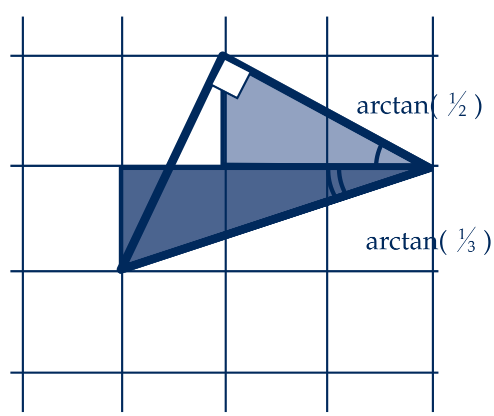



**Figure 1: Trigonometric Identity Using Multiple Representations**

one assess divisibility by 13 without calculation:

$13^{50}$, $50^{13}$, 1000000000, $123456\times13 + 3$,
$39 + 654321\times13$, $36\times7654 + 3\times4567$, $24\times35\times56$,
$24\times135\times56$

(See representative solutions to these questions at the end of the article.) Teachers of all grade levels can create a surprisingly large variety of representations for even a single number such as 1.

## Sequence of Representations within a Collection

With evidence suggesting that a combination of representations is better, the next natural question for us was whether the sequence of representations within that combination may be significant. So the second study (later in the same semester) with the same classes of high school pre-calculus students was aimed at the effect that representational sequence has on students' understanding with respect to techniques of solving inverse trigonometric identities. We considered three different representational sequences, each placing the analytic representation in a different position (with numeric always preceding graphical).

The mean classroom test scores (out of 100 possible points) of the groups given the N-G-A, N-A-G and A-N-G sequences were 76, 85, and 91, respectively. These results appear to go against a dominant view among educators (especially those influenced by Piaget or Bruner) that mathematical activities should always be structured from concrete to abstract (i.e., the N-G-A sequence) in order to develop students' understanding of mathematical concepts and ultimately to improve students' performance. The learning model of Bruner (1966) is based upon three levels of engagement with representations: enactive (e.g., manipulating concrete materials), iconic (e.g., pictures and graphs), and then symbolic (e.g., analytic and algebraic).

However, Krutetskii (1976) shows that the differences in mathematical performance depend on mostly abstractness-oriented characteristics of the mathematical cast of mind. Vasilii Davydov (1990) first examined the effectiveness of the method of going from analytic to concrete by teaching algebra concepts to typical Russian elementary school students in the early 1970's. Studies on Davydov's method have found that "the Russian students (from Davydov's program) have a profound grasp of mathematical structure, confidence, and the ability to extend their knowledge well beyond the levels at which they had been instructed" (Zeigenhagen, 2000). More discussion and interpretation on sequencing issues appears in Lesser and Tchoshanov (2005) and Van Patten, Chao & Reigeluth (1986).

Another reason the N-G-A sequence espoused by so many mathematics educators may not always be the best could be due to the particular mathematical content at hand. For example, consider Simpson's Paradox, which says that a comparison between two groups can be reversed when data is aggregated. Understanding this possibility is important for quantitative literacy, is listed as essential for citizenship (NCED 2001), and can be explored with only fraction arithmetic (see Table 2). As Lesser (2001, p. 131) notes: "It is routine to verify that within each department, women are hired at a higher rate than men, since $30 \div 80 = 0.375 > 0.25 = 5 \div 20$ and $15 \div 20 = 0.75 > 0.625 = 50 \div 80$, yet are hired at a lower rate than men for the overall situation: $(30 + 15) \div 100 = 0.45 < 0.55 = (5 + 50) \div 100$."

| | Social sciences | | Physical sciences | |
|---|---|---|---|---|
| | males | females | males | females |
| Hired | 5 | 30 | 50 | 15 |
| Not hired | 15 | 50 | 30 | 5 |
| Total | 20 | 80 | 80 | 20 |

**Table 2: Hiring Data by Gender and Department**

Curriculum virtually always begins (and often also ends) with the numerical representation, which "is undeniably effective in demonstrating that Simpson's paradox can happen but limited in offering insight into how it can happen." (ibid). Of the ten or so representations in Lesser (2001), the most commonly used representation by statisticians is a graphical representation that can be called the trapezoidal representation (Tan 1986; Lesser 2004, 2005b), which seems to go against the end-goal of an abstract representation. As the y-intercepts of Figure 2 show, the female hiring rates for each department are higher than the respective male hiring rates, but because of different gender mixtures for each department, the overall female hiring rate is nevertheless lower than the overall male hiring rate. Further discussion of this (and other representations) are in Lesser (2001, 2005b). Finally, it should be noted that several attendees at Lesser (2005a, 2005c) noted "blurring" between the three categories, such as finding both graphical and analytic features in the trapezoidal, circle graph, vector geometry and probability representations of Simpson's Paradox from Lesser (2001), suggesting a modified "continuum" model of representation categorization.

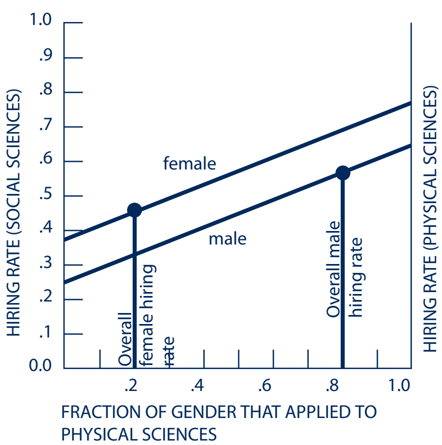



**Figure 2: Trapezoidal Representation Adapted from Tan (1986)**

## Teacher Experiences and Reflections

For some especially rich mathematical phenomena, the number of distinct representations may be too large to expect a teacher to have time to use all of them. Therefore, it is necessary to learn which representations might be more effective than others, as Lesser (2001) explored in pilot studies with pre-service secondary teachers (at a public research university and at a public comprehensive university) on exploring an abundance of representations of Simpson's Paradox. It can be helpful to reflect upon these criteria (National Research Council (NRC) 2001, pp. 99-101) when selecting representation:

- transparency ("how easily can the idea be seen through the representation?"),
- efficiency ("does the representation support efficient communication and use?"),
- generality ("does the representation apply to broad classes of objects?"),
- clarity ("is the representation unambiguous and easy to use?"), and
- precision ("how close is the representation to the exact value?").

To find out what real-life classroom factors might also influence teacher choice of representations or representational sequence, we surveyed mathematics teachers at a public middle school in El Paso County in spring 2005. Here are the factors that teachers said might influence their choice (in parentheses is the number of teachers that chose that factor from our list; we also gave them an option of "other", but no one used it):

- learning style of students (7)
- teaching/ presentation style of teacher (7)
- particular math content involved (6)
- time constraints (6)
- learning goals (4)
- alignment with standardized tests or other assessment (3)

The differences between the NRC criteria and the classroom teachers' criteria lead the authors to consider how all the criteria may be used most effectively. In the future, it would be worthwhile to examine further how the selection of particular representations and representational sequences may depend on each of these various factors.

## Professional Development

Under a 2005-06 Texas Education Agency grant, the authors delivered professional development workshops to help middle school teachers' increase their students' achievement in mathematics. Using item analysis of TAKS test data, the workshops engaged teachers in analyzing student error patterns and adapting pedagogy. As presented at the 2006 Conference for the Advancement of Mathematics Teaching and described in Lesser and Tchoshanov (in press),

> Each session was launched by teacher reflections upon low-performing items from one of the 6 middle school TAKS objectives.

Teacher exploration of items went beyond teaching-to-the-test to unpacking big conceptual ideas and strategies (e.g., multiple representations) to help improve achievement on a much larger collection of items, and situate this understanding in a larger set of curriculum objectives and in the K-12 continuum.

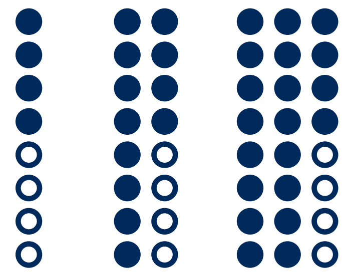
While the average increase (from 2005 to 2006) in percentage points of our workshop's teachers' students passing the mathematics TAKS was significant (about 10 points), it is impossible to say how much of this was due to the particular strategy of multiple representations from our training.

## One Example

An example of a problem discussed in the workshops was this released item (#12) from the Spring 2004 7th grade mathematics TAKS test: "Generate a sequence that follows the rule 8*n*-4, where *n* represents the position of a term in the sequence". The choices were:

(F) 16, 12, 8, 4, 0, …..
(G) 8, 16, 24, 32, 40,….
(H) 4, 16, 64, 216, 1024,….
(J) 4, 12, 20, 28, 36,….

The inservice teachers discussed possible reasons why 7th graders' success on this item was only 29%, barely better than what would be expected by random guessing. A plurality of students (42%) chose the incorrect answer (F). Trying this problem out with their students, teachers observed that students had difficulty "looking for the pattern from position to value of term" (as one teacher put it in her written reflections). The teachers agreed that students able to think of $8n–4$ using multiple representations would be better equipped to solve the problem, and they generated many representations of this problem in our workshop discussion. For example, by representing $8n–4$ as a line, the slope of 8 would suggest an incremental gain of 8 units when moving from one term to the next of a sequence (this observation alone already eliminates choices F and H). Teachers found they could also represent $8n–4$ by forming an 8-by-*n* rectangular array of stones and then remove 4 stones. Using ● for a stone and ○ for a removed stone, here's what the pattern looks like for $n = 1, 2, 3$:



One teacher discovered that the strategy of actively manipulating a given representation could also aid solving this problem. He noted that adding "4" to 8*n*-4 should result in a sequence of multiples of 8 and sure enough, only choice (J) yields multiples of 8 when 4 is added to all sequence values.

Representations will likely continue to be a rich focus for teachers and researchers seeking to understand and tap the power of representations to enhance student learning. Teachers whose students are struggling with algebraic representations may want to explore some of the resources that are available to teach mathematical ideas without using the algebraic representation. This can start as simple as illustrating the distributive property using a rectangle divided into two "subrectangles," and can progress through a wide variety of further examples, such as those catalogued in Nelsen (1993, 2000). Statistical literacy textbooks such as Utts (2005) introduce basic concepts and even computations (e.g., for standard deviation) with their algebraic formulas deliberately delayed until the end of each chapter, after students gain intuition from other representations first. But even as educators explore what representations or what sequence of representations works best in each situation, it is helpful to keep in mind that the richest conceptual understanding and problem solving strength comes when students are able to translate readily from any representation to any other representation.

## Solutions:

Though too big for our calculators, $36^{300}$ can be seen to be a square and a cube because the number can be represented as $(36^{150})^2$ and $(36^{100})^3$, respectively.

We see that $50^{13}$ is not a multiple of 13 because its unique prime factorization is a product of powers of only the primes 2 and 5: $(2\times5^2)^{13}$. From the division algorithm, we see that the representation $123456\times13 + 3$ has a nonzero remainder (3) when divided by 13. From the distributive property, we see that $39 + 654321\times13 = 13(3 + 654321)$, a representation that indicates a multiple of 13.

**References**


Bruner, J. (1966). *Toward a theory of instruction.* Cambridge, MA: Belknap Press.

Choike, J.R. (2000). Teaching strategies for algebra for all. *Mathematics Teacher*, 93(7), 556-60.

Davydov, V. (1990). *Types of generalization in instruction: Logical and psychological problems in the structuring of school curricular*. Reston, VA: NCTM.

Hughes-Hallett, D. (1991). Visualization and calculus reform. In W. Zimmermann & S. Cunningham (Eds.), *Visualization in Teaching and Learning Mathematics,* MAA Notes No. 19, 121-126.

Krutetskii, V. (1976). T*he psychology of mathematical abilities in schoolchildren*. Chicago: University of Chicago Press.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of Representation in the Teaching and Learning of Mathematics* (pp. 33-40). Hillsdale, NY: Lawrence Erlbaum Associates.

Lesser, L. (2001). Representations of reversal: An exploration of Simpson's Paradox. In A. Cuoco & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 129-145). Reston, VA: NCTM.

Lesser, L. (2004). Letter to the Editor. *The American Statistician,* 58(4), 362.

Lesser, L. (2005a). New teachers and the newest NCTM Standard: Real-world, research-based reflections on representation, session, 9th annual national conference of the Association of Mathematics Teacher Educators, Dallas, TX.

Lesser, L. (2005b). Illumination through representation: An exploration across the grades. *Statistics Teacher Network*, 66, pp. 3-5.

Lesser, L. (2005c). Preparing secondary teachers to use multiple representations: Representing a three-way table of data. Mathematical Association of America PMET workshop, Texas Southern University, Houston, TX, July 2005.

Lesser, L. and Tchoshanov, M. (2005). The effect of representation and representational sequence on students' understanding. In Lloyd, G. M., Wilson, M., Wilkins, J.L.M., & Behm, S.L. (Eds.). *Proceedings of the 27th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*.

Lesser, L. and Tchoshanov, M. (in press). Evidence-based professional development partnership with middle schools to improve student test achievement. *Proceedings of the 28th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*.

National Council on Education in the Disciplines (2001). *Mathematics and democracy: The case for quantitative literacy*. Princeton: Woodrow Wilson National Fellowship Foundation.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Research Council (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Nelsen, R.B. (1993) *Proofs without words: Exercises in visual thinking.* Washington, DC: Mathematical Association of America.

Nelsen, R.B. (2000) *Proofs without words II: More exercises in visual thinking*. Washington, DC: Mathematical Association of America.

Pape, S., & Tchoshanov, M. (2001). The role of representation(s) in developing mathematical understanding. *Theory into Practice*, 40(2), 118-127.

Tall, D., & Vinner, S. (1981). Concept image and concept definition in mathematics with particular references to limits and continuity. *Educational Studies in Mathematics*, 12(2), 151-169.

Tan, A. (1986). A geometric interpretation of Simpson's paradox. *College Mathematics Journal*, 17, 340-341.

Tchoshanov, M. (1997). *Visual mathematics*. Kazan: ABAK.

Utts, J. M. (2005). *Seeing through statistics (3rd ed.)*, Pacific Groves, CA: Brooks/Cole.

Van Patten, J., Chao, C. and Reigeluth, C. (1986). A review of strategies for sequencing and synthesizing instruction. *Review of Educational Research*, 56(4), 437-471.

Wilson, M. (1994). One preservice secondary teacher's understanding of function: The impact of a course integrating mathematical content and pedagogy. *Journal for Research in Mathematics Education*, 25, 346-370.

Zazkis, R. & Gadowsky, K. (2001). Attending to transparent features of opaque representations of natural numbers. In A. Cuoco & F. Curcio (Eds.), *The roles of representation in school mathematics* (pp. 44-52). Reston, VA: NCTM.

Ziegenhagen, N. (2000). *Vygotskian math at Susquehanna School, Binghamton, New York*, http://www.bestpractices.org/DiscoveryGrants1997/VygotskianMath.html.

■

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FEATURE

# NCTM Membership

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FEATURE

# 2006 Scholarship Donors

**The TCTM Board wishes to thank the following contributors for their very generous donations to the TCTM scholarship fund.**

## Major Contributor

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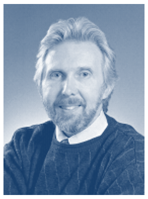
# Recommended Readings and Resources

*Saving Our Students, Saving Our Schools: 50 Proven Strategies for Revitalizing At-Risk Students and Low-Performing Schools*

If you attended the CAMT luncheon this past July, you heard the keynote speakers, Robert Barr and William Parrett, presentation on Strategies That Work for Underachieving Students of Poverty.

Much of their presentation was based on their understanding and current research, published in 2003, *Saving Our Students, Saving Our Schools: 50 Proven Strategies for Revitalizing At-Risk Students and Low-Performing Schools.* This book offers a dynamic approach to meeting the challenges faced by today's students and their schools.

This significant resource explains how to effectively apply and use these strategies. While these strategies are specifically designed for the student at risk, they can also be used effectively to reach out to every student.



**William Parrett**

**Robert Barr**



# Legislative Update and Advocacy

*News from the September 2006 State Board of Education Meeting.*

As you may know, during the third special session, the Texas Legislature passed House Bill 1. In this bill, the legislators called for four years of mathematics and science credits for all students graduating under the Recommended High School Plan. Implementing this law is now in the hands of the State Board of Education (SBOE). One issue that is of great importance to our field is whether or not the fourth course for mathematics have an Algebra II prerequisite or not. Also of importance is whether or not the SBOE will allow the course Mathematical Models with Applications (MMA) to count as as one of these credits. The TCTM President and Board have expressed an opinion that the fourth course not require Algebra II and that the MMWA course remain as a choice for students toward the four credits. Your opinion on these issues matters. Please contact your SBOE member and inform them of the impact these decisions will have on you as a classroom teacher and on your students.

As part of our support for members, TCTM has included a link to an advocacy website that will help you reach out to your elected officials and state agencies. With the legislature returning to session in January and the actions of the State Board of Education on changes to the Recommended High School Plan, we encourage all TCTM members to voice their opinion. If you want to contact a SBOE member, go to the TCTM website,

*<www.tctmonline.net>*

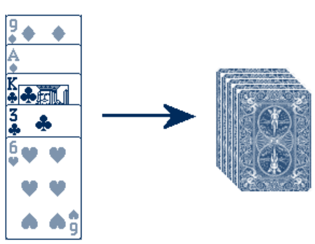
click on members only, then click on the link under Legislative Action. Click on View next to TX Officials and Agencies, then scroll down to Department of Education. This will open up the list of board members and an envelope next to their name. Click on the envelope to send a message.

# Solution for Cards and Codes in the Classroom

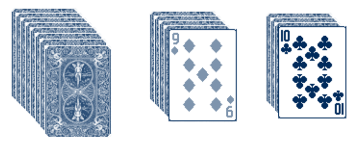
*Dear Readers: The card trick problem in our last issue of the TMT gave readers a chance to submit their own answer. Here is a reprint of the problem, and the answer submitted by one of our readers to question 4. - Editor*

Card tricks are another source of mathematical investigation. Most card tricks that are not the sleight-of-hand variety can be easily explained with the help of mathematics. Surprisingly, the mathematics is usually extremely elementary. The following example is one such trick.

*Activity 3.*
Here is an interesting card trick. You will need an ordinary deck of playing cards (52 cards). Each card will have the face value shown on the card: the Aces will have a value of one, the Jacks will have the value of eleven, Queens have the value of 12, and the Kings have a value of 13. Shuffle the deck of cards. Turn the top card face up on the table. Note the face value of the card. Start counting from this number and add cards face up to that stack until the count reaches 13. Once you reach 13, turn the entire pile face down. Note that the only face card value that you are concerned with is the first card you turned face up. The rest of the cards should be thought of as placeholders.



Repeat this process until you cannot complete the count. For example, if you had 4 cards left in your hand and the card you turned over was the 4 of spades, you would not have enough cards to get to 13. Have a member of the class point to three of the stacks. Collect all the stacks of cards that were not pointed out by the class member and place them back in your hand. Have another member of the class pick two of the three stacks and turn over the top card of each stack.



Note the face values of the two cards that were turned up and add these two numbers. Remove that number of cards from your hand. Now remove ten additional cards from your hand. Count the number of cards left in your hand; this will be the face value of the top card in the stack that was not chosen.

1. Suppose the top card in one of your stacks was a 7 of hearts. How many cards would be in your stack?
2. What if your top card was the Jack of clubs? How many cards would be in that stack?
3. Can you find a general rule for the number of cards in a stack if you know the value of the top card?
4. Can you explain how this trick works? The removal of ten cards at the end of the trick may be a hint as to how the trick works.

*Selected Solutions from this Activity*

1. 7
2. 3
3. Fourteen minus the face-value of the card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**Solution to question 4**
**by Maurice Stadler • Memorial Middle School**

I enjoyed the card trick described in the Spring 2006 issue of *Texas Mathematics Teacher*. Following is my explanation of why the trick works.

Of the three remaining stacks, let $n_1$ and $n_2$ represent the values of the cards that are turned up on the two chosen stacks, and let m represent the value of the top card in the stack that was not chosen. There are $14 - n_1$ and $14 - n_2$ cards in the two chosen stacks and $14 - m$ cards in the stack that was not chosen. The magician then removes $n_1 + n_2 + 10$ cards from her hand. Subtract those expressions from fifty-two to find the number of cards remaining in the magician's hand; the result is m:

$$52 - (14 - n_1 + 14 - n_2 + 14 - m + n_1 + n_2 + 10) = m$$

Therefore, the number of cards in the magician's hand is m, the value of the top card in the stack that was not chosen.

■

FEATURE

# A Measurement Book List for the Early Grades

This list of books for early childhood/elementary grades is provided to our readers to complement the article "TEKS: A Mathematical Continuum for the Early Grades" on page 10. If you have not already read these books, check them out. This list was compiled by Mary Alice Hatchett.

| | | | |
|---|---|---|---|
| 1 | A Pig is Big | Florian, Douglas | 2000 |
| 2 | Bats Around the Clock | Appelt, Kathi | 2000 |
| 3 | Game Time | Murphy, Stuart J. | 2000 |
| 4 | How Tall, How Short, How Far Away | Adler, David A. | 2000 |
| 5 | Lulu's Lemonade | Derubertis, Barbara | 2000 |
| 6 | Marvin Measures Up | Browning, Dave | 2000 |
| 7 | Marvin Weighs In | Browning, Dave | 2000 |
| 8 | Measuring Penny | Leedy, Loreen | 2000 |
| 9 | Tell Me How Far It Is | Willis, Shirley | 2000 |
| 10 | Wilma Unlimited | Krull, Kathleen | 2000 |
| 11 | All In a Day | Anno, Misumasa | 2001 |
| 12 | Carrie Measures Up | Aber, Linda Williams | 2001 |
| 13 | Racing Around | Murphy, Stuart J. | 2001 |
| 14 | What's Up with That Cup | Keenan, Sheila | 2001 |
| 15 | A House for Hermit Crab | Carle, Eric | 2002 |
| 16 | Chickens on the Move | Pollack, Pamela | 2002 |
| 17 | How Do You Know What Time It Is? | Wells, Robert E. | 2002 |
| 18 | Pigs on the Move | Axelrod, Amy | 2002 |
| 19 | Sam's Sneaker Squares | Gabriel, Nat | 2002 |
| 20 | Hershey's Milk Chocolate Weights and Measures | Pallotta, Jerry | 2003 |
| 21 | Inchworm and a Half | Pinczes, Elinor J. | 2003 |
| 22 | Mapping Penny's World | Leedy, Loreen | 2003 |
| 23 | The Borrowers (Chap 1) | Norton, Mary | 2003 |
| 24 | The Man Who Walked Between the Towers | Gerstein, Mordicai | 2003 |
| 25 | Fannie in the Kitchen: The Whole Story from Soup to Nuts of How Fannie Farmer Invented Recipes with Precise Measurements | Hopkinson, Deborah | 2003 |
| 26 | Mighty Maddie | Murphy, Stuart J. | 2004 |
| 27 | Block City | Stevenson, Robert Louis | 2004 |
| 28 | Hamster Champs | Murphy, Stuart J. | 2005 |
| 29 | Keep Your Distance | Herman, Gail | 2005 |
| 30 | My Friend Rabbit | Rohmann, Eric | 2005 |
| 31 | Polly's Pen Pal | Murphy, Stuart J. | 2005 |
| 32 | The 100-Pound Problem | Dussling, Jennifer | 2005 |
| 33 | Too Tall Tina | Pitino, Donna Marie | 2005 |
| 34 | Millions to Measure | Schwartz, David M. | 2006 |
| 35 | Sir Cumference and the Isle of Immeter | Neuschwander, Cindy | 2006 |

# Texas Science, Technology, Engineering, and Math Initiative (T-STEM)

The following information has been downloaded verbatim from the T-STEM website at

*<www.tea.state.tx.us/ed_init/thsp/tstem/index.html>*

For more information, such as the goals for academies and centers, please review the information online.

**Texas Science, Technology, Engineering, and Math Initiative Goals and Outcomes**

- To develop the nation's leading innovation economy workforce by aligning high school, postsecondary education, and economic development activities
- To establish 35 Texas Science, Technology, Engineering, and Math Academies in areas of high need across the state, each year producing 3,500 Texas high school graduates from diverse backgrounds, with the preparation to pursue study and careers in science, technology, engineering, and math related fields
- To create 5-6 Texas Science, Technology, Engineering, and Math Centers across the state that will support the transformation of teaching methods, teacher preparation, and instruction in the science, technology, engineering, and math fields
- To establish a statewide best practices network for science, technology, engineering, and math education to promote broad dissemination and adoption of promising practices from the initiative and to improve math and science performance for students across Texas

**Texas Science, Technology, Engineering, and Math Academy Implementation Grants**

Three campuses were awarded implementation grants to become Texas Science, Technology, Engineering, and Math Academies in fall 2006. Burnham Wood Charter School in El Paso , which is opening a new campus that will serve grades 6-12, received a grant award of $700,000. YES College Preparatory Schools in Houston also received $700,000 to convert its Southeast campus, which serves grades 6-12, into a T-STEM academy. New Deal ISD in Lubbock, which will be creating a small learning community serving grades 9-12 at New Deal High School, received $80,000.

**Texas Science, Technology, Engineering, and Math Academy Start-up Grants**

Start-up grants for T-STEM academies were awarded to four districts: Northeast ISD in San Antonio ; Richardson ISD; Dallas ISD and Corpus Christi ISD. The four start-up grantees will use the 2006-2007 school year as a planning year and will open the academies in the fall of 2007. Northeast ISD, which is creating academies at Nimitz Middle School and Lee High School , and Corpus Christi ISD, which is creating academies at Cunningham Middle School and Moody High School , are each receiving grants of $750,000. Richardson ISD, which is creating a T-STEM academy at Berkner High School , and Dallas ISD, which will create an academy at Conrad High School , are each receiving a grant of $480,000.

**Additional T-STEM Academy Grant Recipients**

Previous T-STEM academy grant recipients include Manor ISD near Austin ; Harmony Science Charter School with campuses in El Paso , Fort Worth , Houston and San Antonio ; Waxahachie ISD, and KIPP, Inc. in Houston . Manor ISD and Harmony Science Charter School will open T-STEM academies in the fall 2006. Waxahachie ISD and KIPP, Inc. will open academies in the fall of 2007.

Three T-STEM academy grants were awarded in the fall of 2005 to early innovators: A.J. Moore Academy in Waco ISD; Carver High School in Aldine ISD; and the Academy of Irving in Irving ISD.

**T-STEM Center Grants**

Grants were awarded for the creation of five regional T-STEM centers that will develop new science, technology, engineering and math instructional materials. The centers will also provide professional development training to teachers and school leaders, and will evaluate the practices used at T-STEM academies to identify successful practices that can be duplicated in other schools throughout Texas . Each fiscal agent is being awarded a grant of $1 million and will have a number of partners in this effort. They will use the 2006-2007 school year as a planning year and will open in 2007. Education Service Center Region 1 will partner with the University of Texas -Pan American, 13 school districts in Region 1 and the Charles A. Dana Center . Education Service Center Region 13 will partner with Education Service Center 20 in San Antonio, the University of Texas at Austin College of Engineering, San Antonio ISD, and Taylor ISD. Texas A&M will partner with Dallas ISD and Education Service Center Region 10 in Richardson . Texas Tech will partner with Lubbock ISD and Education Service Center Regions 15, 16, 17 and 18, which are housed in San Angelo , Amarillo, Lubbock and Midland , respectively. University of Texas at El Paso will partner with 12 school districts in the El Paso area and Education Service Center Region 19 in El Paso.

■

# Texas Council of Teachers of Mathematics
# Executive Board 2006 - 2007

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