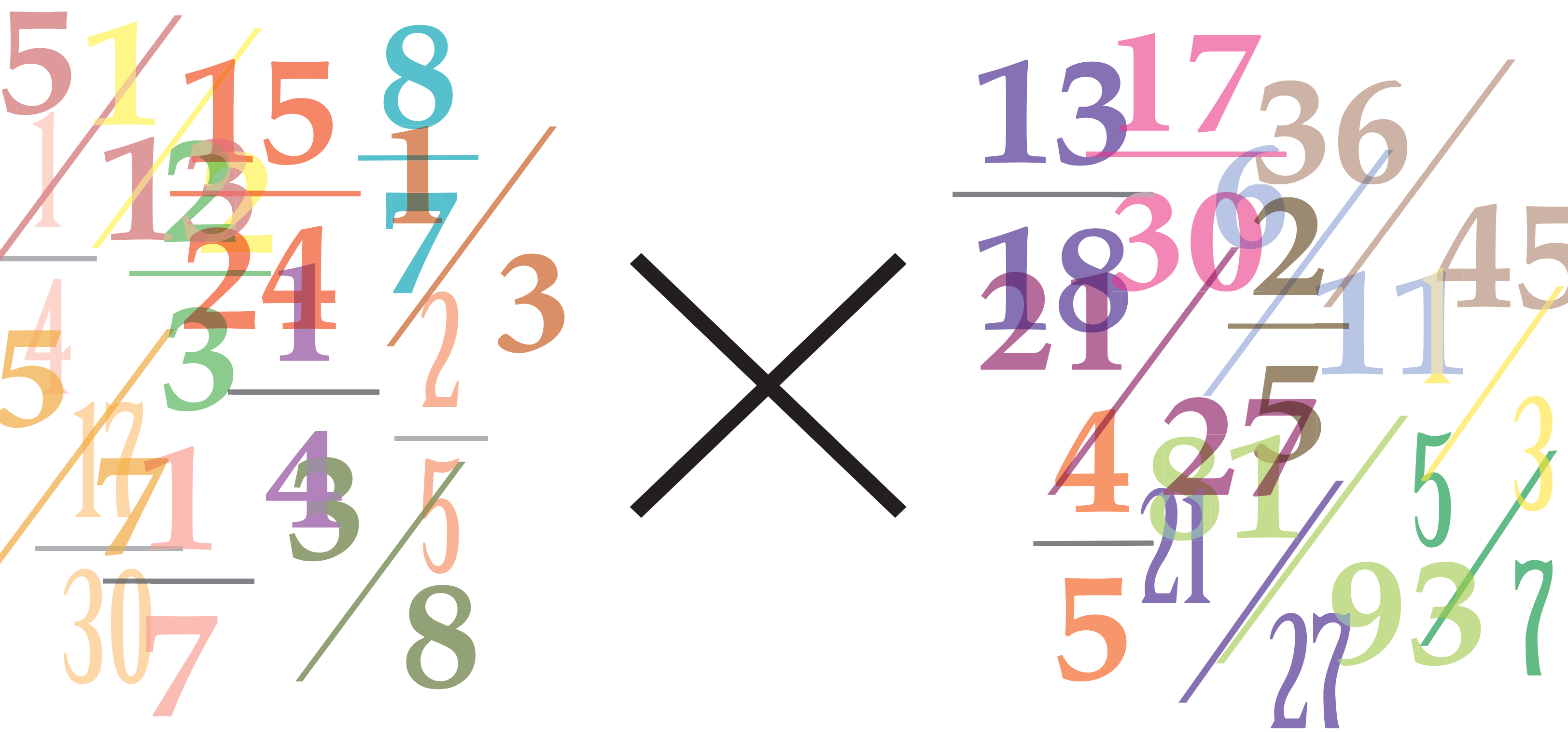
Texas Mathematics Teacher

Volume LII Issue 1 Spring 2005



Fraction Multiplication

A visual guide

page 6

Check the Back Cover
for your membership card
and renewal date

TCTM Elections
Candidates and Ballot inside
vote by June 1, 2005

Volunteer for CAMT
Math Alive! at Dallas
see *page 21*

http://www.tctmonline.net/

Texas Council of Teachers of Mathematics
2005 Mission and Goals Statements

MISSION

To promote mathematics education in Texas

GOALS

Administration

- Investigate online membership registration through CAMT and/or the TCTM website

Publications

- Survey membership to identify what they want in the Texas Mathematics Teacher (TMT)
- Review and redesign the TMT journal and the TCTM website
- Improve the review protocol, establish criteria for reviewers

Service

- Increase the donations toward Mathematics Specialist College Scholarships
- Staff CAMT with volunteers as necessary
- Advertise affiliated group conferences on the TCTM website, in the TMT and at CAMT

Communication

- Maintain an e-mail list of members for timely announcements
- Communicate with affiliated groups in a timely manner

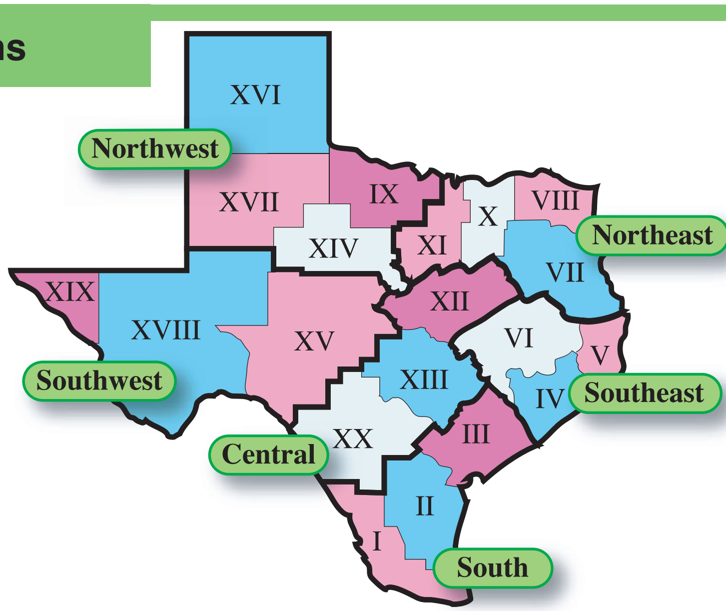
Membership

- Encourage affiliated groups to include TCTM registration on their membership forms

Public Relations

- Sponsor and staff the TCTM booth at CAMT
- Follow NCTM *Advocacy Toolkit* (2004) for increased voice of TCTM membership on issues relevant to our mission

TCTM Regions



TCTM Past-Presidents

1970-1972	James E. Carson	1982-1984	Betty Travis	1994-1996	Diane McGowan
1972-1974	Shirley Ray	1984-1986	Ralph Cain	1996-1998	Basia Hall
1974-1976	W. A. Ashworth, Jr.	1986-1988	Maggie Dement	1998-2000	Pam Alexander
1976-1978	Shirley Cousins	1988-1990	Otto Bielss	2000-2002	Kathy Mittag
1978-1980	Anita Priest	1990-1992	Karen Hall		
1980-1982	Patsy Johnson	1992-1994	Susan Thomas		

Texas Mathematics Teacher

A PUBLICATION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Volume LII Issue 1 Spring 2005

Articles

Features

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Texas Mathematics Teacher, the official journal of the Texas Council of Teachers of Mathematics, is published in the fall and spring. Editorial correspondence should be mailed or e-mailed to the editor.

Call For Articles

The ***Texas Mathematics Teacher*** seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Texas. All readers are encouraged to contribute articles and opinions for any section of the journal.

Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to the editor with a copy to the director. No author identification should appear on or in the manuscript. A cover letter containing author's name, address, affiliations, phone, e-mail address, and the article's intended audience should be included. After refereeing, authors will be notified of a publication decision.

Teachers are encouraged to submit articles for ***Voices From the Classroom,*** including inspirational stories, exemplary lessons, or management tools. If submitting a lesson, it should include identification of the appropriate grade level and any prerequisites.

Items for ***Lone Star News*** include, but are not limited to, NCTM affiliated group announcements, advertisements of upcoming professional meetings, and member updates.

Businesses interested in placing an **advertisement** for mathematics materials should contact Mary Alice Hatchett.

Deadline for submissions: Fall, July 1 Spring, January 1

Letter from the President

Dear TCTM Members,

Included in this issue are the candidates running for office on the TCTM Board. Please return your ballot by May 31, 2005. In preparing for this election, I paused to reflect on my role and responsibilities. As president of the Texas affiliate, my responsibilities include management of the Board and support of Texas teachers of mathematics. Both jobs are important. Just as in the classroom, however, sometimes one job takes a back seat to another. In my case, management seems to take up more of my time than support. In my work for TCTM, management includes soliciting and evaluating our scholarship, CAMTership, and leadership applications (please see the forms and deadlines in this journal). My supporting responsibilities, as I see them, are to provide classroom teachers of mathematics with inspiration, reflection, information and political leadership. Like most of us, I will aspire to accomplish them all, but perhaps some will be done more effectively on one day than another.

Under the heading of management, please note the new registration policy for CAMT. In order to encourage CAMT participants to join TCTM, the other two sponsoring organizations of CAMT (TASM and the TX-Section MAA) agreed to allow a reduced registration fee if the participant paid a $10.00 membership to TCTM for the upcoming school year. Our normal membership fee will remain at $13.00 if the application is sent directly to us. This should increase our numbers and thus improve our ability to represent Texas mathematics educators to the Texas Education Agency and the Texas legislature.

As for political leadership, TCTM's Government Relations Committee (myself, Jo Ann Wheeler of ESC 4, and Nita Keesee of Abilene ISD) met this spring to discuss questions on several issues related to education policy and mathematics education. We will be sending a letter requesting information about the development and implementation of the Texas Math Initiative. Please look for an update on this in the fall journal.

At this year's annual NCTM conference, Cathy Seeley announced plans by the Council to clarify the Standards by grade level rather than grade band. This, of course, will be a challenge to represent what and when topics should be taught. Rather than trying to be comprehensive, these recommendations will focus on a few big ideas per grade level, rather than the entire list of possibilities.

Upcoming local meetings are described in the Lone Star News. If you have a meeting you would like included, please send the information to me as soon as you have the dates.

I close with the recommendation that you read this current journal and reflect on your practice. What can you change today about instruction that will incorporate the lessons learned herein? Do you have examples or stories that you feel would help another teacher? If so, please write them up and send them to me. Your story may be the most inspirational message our colleagues hear this year. You make a difference in your students' lives everyday. Let's start making a difference in our colleagues' lives as well.

Have a great May and see you in Dallas at CAMT!

Sincerely,

Cynthia L. Schneider
TCTM President 2004-2006

Lone Star News

Affiliate Groups

These are local affiliated groups in Texas. If you are actively involved with them, please send future meeting and conference information to Cynthia Schneider at *<cschneider@satx.rr.com>* so we may publicize your events. Contact information for each group is available on the NCTM website,

http://www.nctm.org

Contact information for regional directors is located on the inside back cover.

SOUTHWEST REGION: *Service Centers 15, 18, 19*

Alicia Torres, Regional Director

Greater El Paso CTM
Annual fall conference tentatively set for October 22, 2005; regular meetings throughout the year. Contact: Bob Kimball *<kimball2rc@gmail.com>*.

SOUTHEAST REGION: *Service Centers 4, 5, 6*

Judy Rice, Regional Director

Fort Bend CTM
Holds a short meeting in August, a fall mini-conference, a spring mini-conference and an end-of-year banquet to serve the districts of Alief, Fort Bend, Katy, and Stafford. No dates for 2005-06 have yet to be established. Contact: Jan Moore, *<Jan.Moore@fortbend.k12.tx.us>* or Susan Cinque, *<olsoncinque@alltel.net>*.

Houston CTM
Members interested in reorganizing this affiliate should contact Cynthia Schneider at *<cschneider@satx.rr.com>*.

1960 Area CTM
Holds two meetings and one competition a year to serve the districts of Aldine, Klein, Katy, Humble, Tomball, Spring, and Cypress-Fairbanks. No dates for 2005-06 have yet to be established. Provides scholarships for students in mathematics education and awards for local mathematics education leaders. Contact: Sheila Cunningham, *<scunningham@kleinisd.net>*.

NORTHWEST REGION: *Service Centers 9, 14, 16, 17*

Nita Keesee, Regional Director

Big Country CTM & Science
Holds an annual conference in late January or early February. Contact: Leslie Koske, *<lkoske@esc14.net>* or 325-675-8661.

Texas South Plains CTM
Twelth Annual Panhandle Area Mathematics and Science Conference, September 24, 2005, Canyon, TX. Contact: Gilberto Antunez, *<gantunez@mail.wtamu.edu>*, or see *http://www.wtamu.edu/academic/ess/edu/*

NORTHEAST REGION: *Service Centers 7, 8, 10, 11*

Jacqueline Weilmuenster, Regional Director

East Texas CTM

Red River CTM
STEAM IV Conference (Successfully Training Educators As Mathematicians) for the Promotion of Mathematics Education, Friday, October 21, 2005, at the campuses of Texas A&M University-Texarkana and Texarkana College, $25 per person, Contact: Debra Walsh, *<dwalsh@redwater.esc8.net>* or Susie Howdeshell, *<showdeshell@pgisd.net>* or see *http://www.tamut.edu/~rrcmath/*

Greater Dallas CTM
Holds two mathematics contests (W. K. McNabb Mathematics Contests) for students in grades 7 - 12 - one in the fall (early Nov.) and one in the spring (early April). A banquet in May is held for the winners. Contact: Tom Butts, *<tbutts@utdallas.edu>* .

SOUTH TEXAS REGION: *Service Centers 1, 2, 3*

Sheryl Roehl, Regional Director

CTM @ Texas A&M University at Corpus Christi (Student Affiliate)

CTM @ Texas A&M University at Kingsville (Student Affiliate)

Rio Grande Valley CTM
The 40th annual conference, Mathematics Today - Gearing Up for the Future, Saturday November 19, 2005, at the University of Texas - Pan American, Edinburg, Texas, from 8:00 to 4:00 p.m. Contact: Nancy Trapp, *<nancy.trapp@lyfordcisd.net>* or see *http://www.rgvctm.org*

CENTRAL TEXAS REGION: *Service Centers 12, 13, 20*

Patricia Rossman and Scott Fay , Co-Regional Directors

Austin Area CTM
Holds a fall conference in October and spring meeting in April. Dates for 2005-06 have not yet been set. Contact: Carol Lindell, *<clindell@taylor.isd.tenet.edu>* .

Alamo District CTM
Normally holds a fall and spring conference. Dates for 2005-06 have not yet been set. Contact: Kathy Mittag, *<kmittag@utsa.edu>*, or see *http://www.adctm.net*

Central Texas CTM
Holds a fall meeting in October and a spring mini-conference in March in Waco. Contact: Trena Wilkerson, *<Trena_Wilkerson@baylor.edu>* or see *http://www.baylor.edu/soe/ctctm*

NON-AFFILIATED CONFERENCES

The Fourth Annual San Antonio Science and Mathematics Saturday, "Teaching REAL Science and Mathematics," Saturday October 15, 2005, at San Antonio College, San Antonio, TX from 7:30 a.m. to 4:30 p.m. Catered breakfast and lunch are included. Six hours of CE credit will be given. Cost is $20.00 for certified teachers and $15.00 for college students or student teachers. Contact: Roger Kramer, *<roger.kramer@harlandale.net>* .

Teaching the Hard Stuff to Diverse Groups of Students, a national conference, July 13-15, 2005, Adam's Mark Hotel, Dallas, TX. Contact: Laura Maldonado *<Laura.Maldonado@mail.utexas.edu>* or see *http://www.utdanacenter.org/conference*.

Teachers Teaching with Technology Conference, October 21-22, 2005, Lubbock Memorial Civic Center, Lubbock, TX. Contact: Beverly Anderson, *<banderson@esc17.net>*.

STATEWIDE

Texas Association of Supervisor's of Mathematics (TASM) Fall Meeting October 6-7, 2005 in Austin. For membership and registration information, please see

http://www.tasmonline.net/

Membership is required to register for this meeting.

Fraction Multiplication:
Using Visual Representations to Justify the Traditional Algorithm

Introduction

"Numerator times numerator over denominator times denominator…" As children, this was the traditional algorithm for multiplying two fractions that most of us were taught for the first time somewhere around the fifth grade. It was easy enough to remember the algorithm, but who knew why that particular strategy would get us the right answer? Thus was born my quest to develop a concrete strategy to illuminate the "numerator times numerator over denominator times denominator" algorithm in a way that would support understanding why the fraction multiplication algorithm makes sense.

The strategy I developed, and report here, includes the "paper folding" model that is a favorite illustration among mathematics educators, but it goes further than most paper folding activities. This article also:

- Constructs fraction multiplication as an extension of whole number multiplication;
- Facilitates the development of strong estimation strategies and skills;
- Illuminates not just that the algorithm works (by illustrating the product), but also why the algorithm makes sense; and
- Sets a foundation upon which a concrete model to illuminate the logic of the fraction division algorithm can be constructed.

This article does not present a formal lesson plan for teaching fraction multiplication to naïve[1] learners. Instead, it presents to the experienced teacher a development of the logic and reasoning involved in fraction multiplication and the step-by-step construction of a model that can be used as a guide when developing lessons for naïve learners.

The first part of this article sets a foundation for the subsequent parts with an introduction to the language and meaning of multiplication and fractions. The next part uses that foundation to construct insight into what a fraction multiplication statement like $\frac{3}{4} \times \frac{7}{9}$ is communicating. Finally, the last part of the article illuminates the logic that explains why the "numerator times numerator over denominator times denominator" algorithm makes sense.

There are two strategies that I have discovered which are very helpful when developing this model with the students[2] in my classes. The first is to begin with very simple examples and then methodically increase the complexity of those examples while emphasizing the logic that links one to the next. This helps reduce cognitive load and makes it seem like each new idea is just a minor variation on an idea that is already familiar and comfortable. The second strategy is to use formal language very deliberately as a tool to illuminate the connections from one example to the next. Throughout this article I will use both of these strategies to develop the logic of the fraction multiplication algorithm. This is not meant to insult the reader, but rather to provide a comprehensive guide for constructing the model from beginning to end.

PART 1: Setting the Foundations

Revisiting the Language And Meaning Of Fractional Representation

Students will need to be comfortable with the information communicated by the numerator and the denominator of a fraction. This means that numerator and denominator must be more than vocabulary words with an informal definition like "the numerator is on top and the denominator is on the bottom." Students must be aware that each number in the fraction communicates important information and what that information is. I help my students think about it this way:

$\frac{3}{5}$	The denominator tells us how many equal sized parts in our unit (or how many equal sized groups are in our whole).

[1] A naïve learner is defined as a student who has no previous knowledge of a concept. This is in contrast to an experienced learner who is revisiting a concept that is already familiar.

[2] Pre-service teachers in both mathematics content and mathematics methods coursework and experienced in-service teachers participating in professional development courses and workshops.

It is important for students to understand what is meant by the words *unit* and *whole* within the context of fractions. Generally, it is more comfortable to talk about parts in our unit when the context is a single item (like a pie or a candy bar) and groups in our whole when the context is a set of discrete items (like marbles in a cup). In either case, the unit and the whole represents what we begin with. In my experience, students are able to use the terms unit and whole interchangeably whether the context is a single unit or a set of discrete units, so I usually choose the term that is most appropriate for the given context.

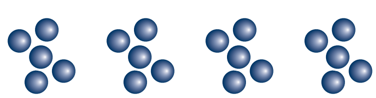
It is also important to emphasize the concept of equal sized. When I teach fractions I continually emphasize equal sized. This is a fractional concept that is frequently mentioned when fractions are first introduced, but then neglected. The concept of equal sized parts in our unit will be important throughout later development of the invert and multiply algorithm for fraction division.

$\frac{3}{5}$ The numerator tells us how many of the equal sized parts are of interest.

Once students are comfortable with the information communicated by the denominator, the information communicated by the numerator is easy.

Revisiting The Language of Multiplication

In all multiplication statements, one factor communicates the size of each group, and the other factor communicates the number of groups. When developing the concept of fraction multiplication, it is very helpful be very consistent in the use of the mathematical language. For example, suppose I have the following pictorial representation of a multiplicative context:



Because of the commutative property, it would be correct to represent this picture in two ways:

$$5 \times 4 = 20 \quad \text{and} \quad 4 \times 5 = 20$$

For the purposes of this lesson, however, it will be very helpful to match each factor with the way we would naturally use words to describe the pictorial or concrete representation: "We have four groups with five marbles in each group" or, in a more abbreviated statement, "We have four groups of five." The literal symbolic representation that parallels the verbal statement would be:

$$4 \times 5 = 20$$

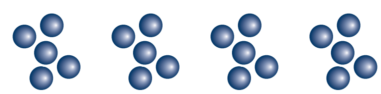
Thus, in order to match natural language, I will very consistently write the factor representing the number of groups on the left side of the multiplication symbol and the factor representing the size of each group on the right side of the multiplication symbol:

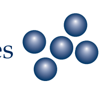
number of groups | size of each group

$$4 \times 5 = 20$$

I am also going to introduce a way to think about the factors that may seem trivial now but will prove to be helpful later when one or both factors are fractions.

In the picture below a blue dot represents one marble and each group has five distinct marbles:



We can think of each marble as a unit and thus each group of marbles  has five units. I could represent this idea by saying "I have five groups of one unit" or "I have five groups of one." A literal symbolic translation would be:

$$5 \times 1 = 5$$

This statement preserves the orientation and meaning of each factor as presented above:

number of groups | size of each group

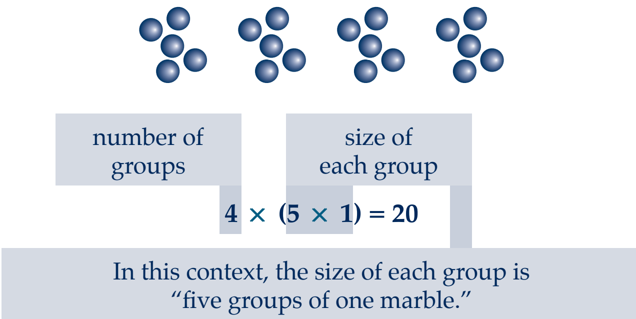
$$5 \times 1 = 5$$

In this context, the size of each group is one marble.

An "extended" verbal description of the picture above could thus be "I have four groups with five groups of one marble." The literal symbolic translation would then be:

$$4 \times (5 \times 1) = 20$$

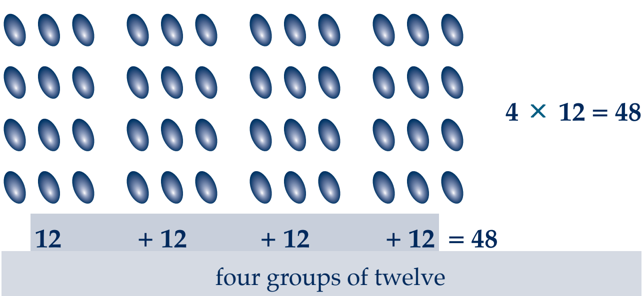
Again, this statement preserves the consistent orientation and meaning for each factor:

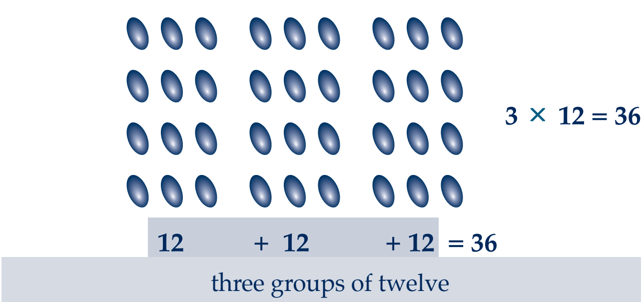


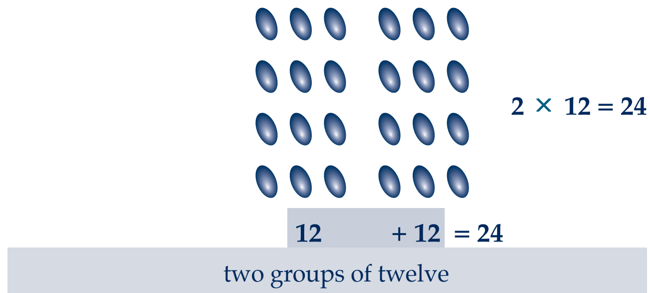
At this point you may be scratching your head wondering why I have introduced this artificially complicated way to think about a simple multiplication statement. What I am doing here is setting up a strategy to connect *language* with a visual image that will prove helpful later on when one or both factors are fractions.

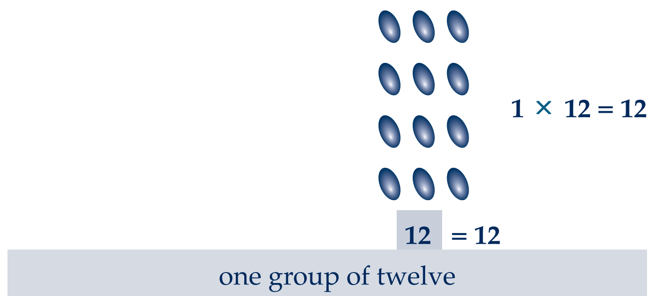
PART 2: Linking Fraction Multiplication to Whole Number Multiplication

I have discovered that a nice way to develop a conceptual understanding of fraction multiplication is to set up a pattern using pictures to illustrate whole number multiplication, and then work my way from there to the fractional example. For example, I might begin with 4 groups of 12 jelly beans and work my way down to 1 group of 12 jelly beans, and then to $\frac{1}{2}$ of a group of 12 jelly beans:

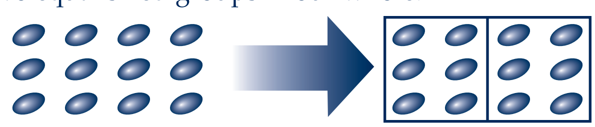




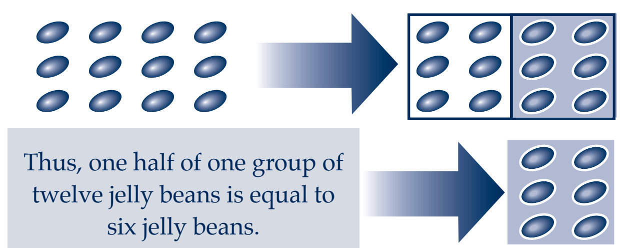




Now that this pattern is set up I can ask, "What if I don't want a complete group of twelve jelly beans, but instead I only want a *part* of the group of 12 jelly beans? What if I wanted only one half of the group? What would that look like?" Because of the pattern set up above, and the consistent use of language, the answer to the last questions becomes fairly intuitive. The denominator in $\frac{1}{2}$ tells us that we want to make two equal sized groups in our whole:



The numerator tell us we are interested in only one of those two groups:



I would represent this using symbolic notation this way:

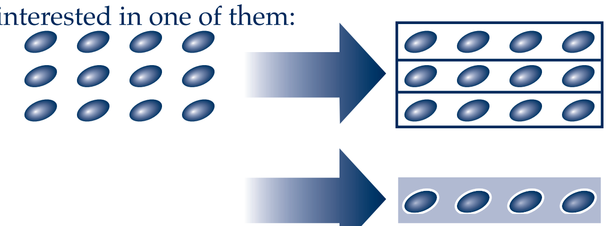
The number of groups is $\frac{1}{2}$ of a complete group. | The size of the group is 12.

$\frac{1}{2} \times 12 = 6$

So $\frac{1}{2}$ of a complete group of 12 jelly beans equals 6 jelly beans.

Here is a second example: Suppose this time I only want $\frac{1}{3}$ of the group of 12 jelly beans. How many jelly beans would $\frac{1}{3}$ of a group of 12 jelly beans be?

The $\frac{1}{3}$ tells us that we want to organize the 12 jelly beans into three equal sized groups and we are interested in one of them:



Thus, one third of one group of twelve jelly beans is equal to four jelly beans and we would represent it this way:

The number of groups is $\frac{1}{3}$ of a complete group.

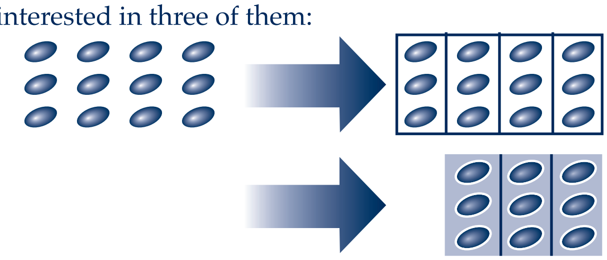
The size of the group is 12.

$\frac{1}{3} \times \mathbf{12} = \mathbf{4}$

So $\frac{1}{3}$ of a complete group of 12 jelly beans equals 4 jelly beans.

One last example just to drive this idea home: Suppose that now I want $\frac{3}{4}$ of the group of 12 jelly beans. How many jelly beans would $\frac{3}{4}$ of a group of 12 jelly beans be?

The $\frac{3}{4}$ tells us that we want to organize the 12 jelly beans into four equal sized groups and we are interested in three of them:



Thus, three fourths of one group of twelve jelly beans is equal to nine jelly beans and we would represent it this way:

The number of groups is $\frac{3}{4}$ of a complete group.

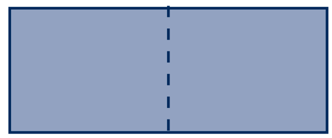
The size of the group is 12.

$\frac{3}{4} \times \mathbf{12} = \mathbf{9}$

So $\frac{3}{4}$ of a complete group of 12 jelly beans equals 9 jelly beans.

The next step is to be sure the students extend this understanding to a context in which the second factor, the size of each group, is less than one. To illustrate what a fraction multiplication statement is communicating when both factors are fractions, I might start with an example like this:

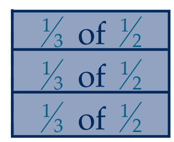
Suppose I am getting ready to make a quilt and in step one, the directions tell me to cut a piece of fabric that is one half of one yard long:



One yard of fabric

One half of one yard of fabric

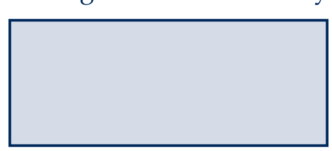
In step two, the directions tell me to cut $\frac{1}{2}$ of that yard of fabric into three equal pieces so that each piece is $\frac{1}{3}$ of the $\frac{1}{2}$ yard of fabric:



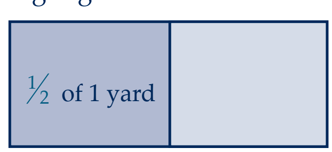
Now I am wondering, "What portion of the initial yard of fabric is just one of the small pieces I created?"

A helpful way to "see" the answer to this question is to look at the $\frac{1}{3}$ of the $\frac{1}{2}$ of the fabric within the context of the original piece of fabric – the quantity that represents the whole (or unit).

We begin with one whole yard of fabric:

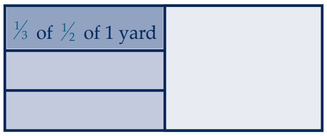


Next $\frac{1}{2}$ of one yard of the original piece of fabric highlighted:



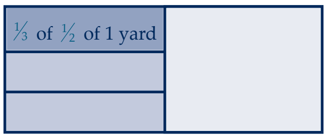
$\frac{1}{2}$

Then $\frac{1}{3}$ of that $\frac{1}{2}$ is highlighted:



$$\frac{1}{3} \times \frac{1}{2}$$

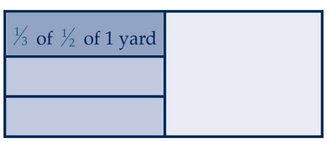
The question was "What portion of the initial yard of fabric is $\frac{1}{2}$ of $\frac{1}{3}$ of that yard?" If I extend the horizontal lines across the rest of the fabric, I can show six pieces that are all the same size as the $\frac{1}{3}$ of $\frac{1}{2}$ piece:



$$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

Thus, I am able to show that $\frac{1}{3}$ of $\frac{1}{2}$ of 1 yard of fabric is $\frac{1}{6}$ of a yard.

What is important to emphasize here is that, while the portion we began with was $\frac{1}{2}$ of a yard, the unit (or whole) is the full (or one) yard of fabric. This is where that strange model I introduced earlier starts to become useful. In order to keep track of the unit against which the product is compared, it is helpful to think of the factor on the right side of the times sign as not just $\frac{1}{2}$, but rather $\frac{1}{2}$ of one yard of fabric or ($\frac{1}{2}$ x 1). Thus:



Number of groups of the beginning quantity that we want.

The size of the group is $\frac{1}{2}$ of one whole.

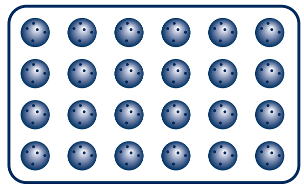
$$\frac{1}{3} \times \left(\frac{1}{2} \times 1\right) = \frac{1}{6}$$

The product represents the total amount $\frac{1}{3}$ of $\frac{1}{2}$, in comparison to the whole, which in this case is the original 1 yard of fabric.

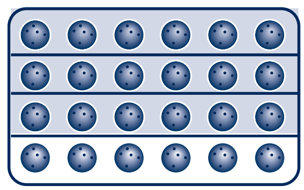
To further illustrate how keeping track of the whole this way is helpful, consider this example:

> Last week I baked a batch of twenty-four cookies. I decided to take $\frac{3}{4}$ of them to work to share with my colleagues and keep $\frac{1}{4}$ for a midnight snack at home. To my surprise, within five minutes of arriving at work my colleagues had eaten $\frac{2}{3}$ of the cookies I brought. What portion of the original batch of cookies did my colleagues eat in those five minutes?

In this example, my unit is one batch of cookies and it does not matter how many cookies are in that batch.

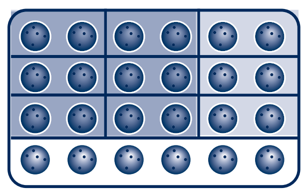


One batch of cookies



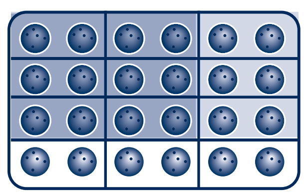
Three fourths of the one batch that went to work.

$$\left(\frac{3}{4} \times 1\right)$$

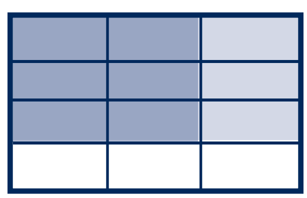


Two thirds of the three fourths of the one batch that got eaten.

$$\frac{2}{3} \times \left(\frac{3}{4} \times 1\right)$$



(Note: The cookies are distracting so I'll eliminate the cookies to show the fraction of the batch that got eaten.)



The darker portion, or six of the twelve equal sized spaces in the picture illustrates the portion of the batch that got eaten. Thus, 6/12 of the batch was eaten in the first five minutes.

$$\frac{2}{3} \times \left(\frac{3}{4} \times 1\right) = \frac{6}{12}$$

Since reducing fractions adds no value to the conceptual development, and it may in fact distract from the desired emphasis, I leave all fractions as they are. In the example above, I would not even mention that $\frac{6}{12}$ could or should be reduced.

In summary, the language attached to each number in the symbolic representation is exactly parallel to all of the preceeding examples:

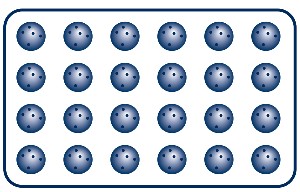
Number of groups of the beginning quantity that we want.

The size of the group is $\frac{3}{4}$ of one whole.

$$\frac{2}{3} \times \left(\frac{3}{4} \times \mathbf{1} \right) = \frac{6}{12}$$

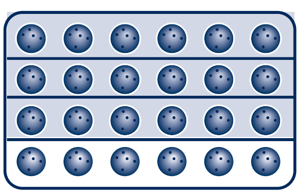
The product represents the total amount, $\frac{2}{3}$ of $\frac{3}{4}$, in comparison to the whole, which in this case is one batch of cookies.

Suppose I change the ultimate question just slightly to ask, "How many cookies did my colleagues eat in those first five minutes?" This minor modification causes my unit to change. In this new question my unit is one cookie and I begin with 24 units. I can think of the 24 as "24 groups of 1 cookie." The expressions below show what the symbolic statement looks like if we keep the unit represented.



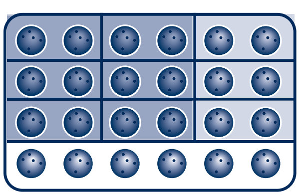
One batch of 24 cookies

$$24$$
$$(24 \times 1)$$



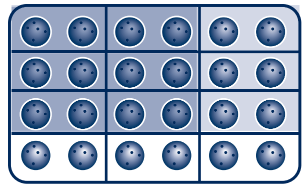
Three fourths of the one batch that went to work.

$$\left(\frac{3}{4} \times 24\right)$$
$$\left(\frac{3}{4} \times (24 \times 1)\right)$$



Two thirds of the three fourths of the one batch that got eaten.

$$\frac{2}{3} \times \left(\frac{3}{4} \times 24\right)$$
$$\frac{2}{3} \times \left(\frac{3}{4} \times (24 \times 1)\right)$$



This time one cookie is our unit so seeing the cookies to show the fraction of the batch that got eaten is helpful. The darker portion under 12 of the cookies illustrates the number of cookies that got eaten. Thus, 12 of the 24 cookies were eaten in the first five minutes.

$$\frac{2}{3} \times \left(\frac{3}{4} \times 24\right) = 12$$
$$\frac{2}{3} \times \left(\frac{3}{4} \times (24 \times 1)\right) = (12 \times 1)$$

The language attached to each number in the symbolic representation is again exactly parallel to all of the preceding examples:

Number of groups of the beginning quantity that we want.

The size of the group is $\frac{3}{4}$ of the 24 cookies, which is a total of 18 cookies.

$$\frac{2}{3} \times \left(\frac{3}{4} \times \mathbf{24} \right) = \mathbf{12}$$

The product represents the total amount, $\frac{2}{3}$ of $\frac{3}{4}$, in comparison to the whole. In this case the whole (or unit) is one cookie, and we have 12 groups of one cookie.

Before moving on to the next section which finally gets into justifying the algorithm itself, I want to point out the connection between the two questions above. For the question that asks "What portion of the original batch of cookies were eaten?" the answer was $\frac{6}{12}$. For the alternate question that asks "How many of the cookies were eaten?" the answer was 12. Note that both answers represent $\frac{1}{2}$ of the whole batch of cookies whether it is $\frac{1}{2}$ of a batch ($\frac{6}{12}$), or $\frac{1}{2}$ of the initial quantity of cookies ($\frac{12}{24}$).

PART 3: Developing Estimation Skills

When students learn how to do fraction multiplication by simply memorizing the algorithm, especially when the problems given are devoid of context, very few of them have any way to estimate the product or analyze the reasonableness of the product they have calculated. One great benefit of understanding the meaning of fraction multiplication – what information each factor is communicating and what the multiplication statement as a whole is communicating, is that it makes it so much easier for students to use that information to come up with a reasonable estimate for the product.

Before I begin to construct the justification for the fraction multiplication algorithm with my students, I take some time to work through the reasoning that one would use to estimate the product. I will use the problem below to illustrate this conceptual development for an estimation strategy.

I have $\frac{3}{4}$ of a sheet cake left over from my birthday party. I want to eat $\frac{1}{6}$ now and put $\frac{5}{6}$ in the freezer for later. I'm wondering:

What portion of the whole sheet cake will go in the freezer?

It takes a bit of initial analysis to generate a reasonable estimate. That analysis includes determining the unit, determining the size of each group, and then determining how many groups we have. We can link this analysis to the general form of the multiplication statement:

number of groups — size of each group

$$\mathbf{a} \times \mathbf{b} = \mathbf{p}$$

product

The first thing to think about is "What is the unit (or the whole)?" In this case, the whole would be one complete, uncut birthday sheet cake. The second thing to think about is the size of the group we begin with. In the general form of the statement shown above, this would be factor **b**. Relative to the birthday cake problem, is the size of the group one whole cake? Is it more than one whole cake? Less than one whole cake? In this problem, it tells us that we do not have the whole cake to work with. We have less than one whole cake. In fact, what we have to work with is only $\frac{3}{4}$ of the cake.

number of groups — size of each group

$$\mathbf{a} \times \frac{3}{4} = \mathbf{p}$$

product

The third thing to think about is, "How many groups of the beginning amount do we have?" Do we have one complete group? More than one complete group (i.e., multiple groups)? Less than one complete group (i.e., only a portion of a group)? In this problem, we are thinking about the part of the group that we are going to put in the freezer. The problem tells us that we are going to put $\frac{5}{6}$ of what we have (which is $\frac{3}{4}$ of the whole cake) into the freezer, so we are going to have less than one complete group of the cake that we have left.

number of groups — size of each group

$$\frac{5}{6} \times \frac{3}{4} = \mathbf{p}$$

product

Now we are ready for some reasoning. These are the kinds of questions I ask:

Q: *We are beginning with $\frac{3}{4}$ of a cake, so less than one whole cake. If we want to put $\frac{5}{6}$ of what we have in the freezer, will we be putting in the freezer more than $\frac{3}{4}$ of the cake or less than $\frac{3}{4}$ of the cake?*

A: *Less than $\frac{3}{4}$ of the cake.*

Q: *Okay. So we will be putting less than $\frac{3}{4}$ of the cake into the freezer. Will we be putting a lot less than $\frac{3}{4}$ of the cake in the freezer or only a little less?*

A: *Only a little less.*

Q: *How do you know that it is only a little less?*

A: *Well, we want $\frac{5}{6}$ of what we have to go in the freezer and $\frac{6}{6}$ would be all of what we have, so $\frac{5}{6}$ is only a little less than $\frac{3}{4}$ of the leftover cake we have.*

Q: *Good. So if only a little less than $\frac{3}{4}$ of the cake will go into the freezer, about how much of the whole cake will go into the freezer?*

At this point students may come up with a variety of estimates depending on their sophistication: Some may say $\frac{2}{4}$ of the cake since that is the next "lowest" fraction if we keep 4 as the denominator. Others may think of $\frac{3}{4}$ as $\frac{6}{8}$ and suggest that about $\frac{5}{8}$ of the cake would go in the freezer believing that perhaps the product should be closer to $\frac{3}{4}$ than $\frac{2}{4}$.

Q: *The estimates you have suggested are $\frac{2}{4}$ and $\frac{5}{8}$. These both seem to be close to $\frac{1}{2}$. One of the estimates is just a little more than $\frac{1}{2}$. Would it seem reasonable* *then to estimate that our product (the portion of the whole cake that will go in the freezer), will be just a bit more than $\frac{1}{2}$ of the cake, but definitely less than $\frac{3}{4}$ of the cake?*

Below is the construction of a pictorial representation of the problem along with the symbolic representation:

1

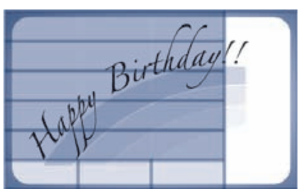
This is our unit (our whole).

When we begin, we only have $\frac{3}{4}$ of the whole cake:

$\frac{3}{4}$

The size of the group we begin with (just $\frac{3}{4}$ of the whole cake).

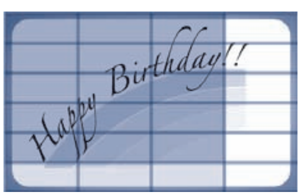
Next, I want to look at the portion of the leftover sheet cake that will go in the freezer:



$\frac{5}{6}$ of $\frac{3}{4}$

The number of groups we want (not all of the $\frac{3}{4}$, but slightly less than that).

Finally, I can look at the portion of the cake I want to freeze in comparison to the whole cake:



$\frac{5}{6}$ of $\frac{3}{4}$ is $\frac{15}{24}$

The total we have in comparison to the whole.

So $\frac{5}{6}$ of $\frac{3}{4}$ of the whole sheet cake, which is the part that will go in the freezer, is $\frac{15}{24}$ of the whole sheet cake. The symbolic representation for the problem would be:

$$\frac{5}{6} \times \frac{3}{4} = \frac{15}{24}$$

When we compare the estimate for our product (something just a little more than one half) and the actual product, we discover that our estimate was definitely in the right ballpark.

PART 4: A Visual Justification for the Fraction Multiplication Algorithm

Justifying the "Numerator Times Numerator" Part of the Algorithm

Now that the foundations of language, understanding and estimation are set, it is time to begin to construct the model that illustrates why the "numerator times numerator over denominator times denominator" makes sense.

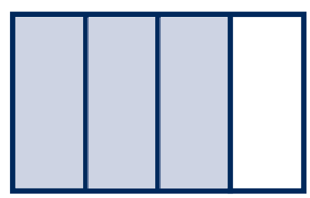
Recall the birthday cake problem from above, $\frac{3}{4}$ of the cake was left over from the party and I wanted to put $\frac{5}{6}$ of the leftover cake in the freezer. We arrived at this symbolic statement:

$$\frac{5}{6} \times \frac{3}{4} = \frac{15}{24}$$

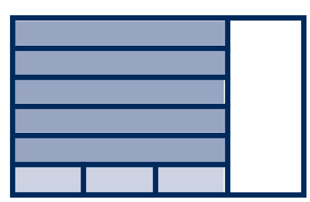
The fraction multiplication algorithm says to multiply the two numerators together to get the numerator of the product and then multiply the two denominators to get the denominator of the product. Thus, we have the "numerator times numerator over denominator times denominator" chant. But why would it make sense to multiply numerators and then multiply denominators? To answer this question we will look at the numerators first:

$$\frac{5}{\ } \times \frac{3}{\ } = \frac{15}{\ }$$

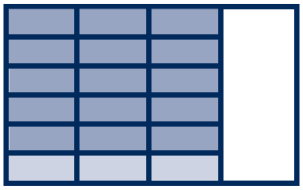
We began with one whole cake and then separated it into four equal pieces and shaded three of them to represent $\frac{3}{4}$:



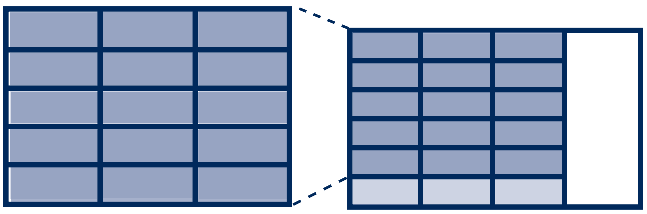
Next, we took just the shaded part representing three fourths and separated that into six equal pieces. We then shaded five of them to represent $\frac{5}{6}$ of the $\frac{3}{4}$.



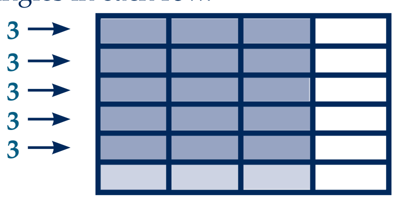
In the next picture I brought forward the lines that were hidden behind the $\frac{5}{6}$ shading:



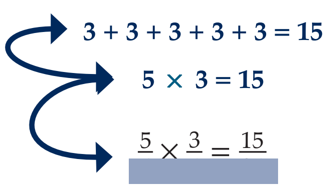
To justify the "numerator times numerator" part of the algorithm we need to focus on the section of the picture that illustrates the $\frac{5}{6}$ of $\frac{3}{4}$ section:



This part of the picture shows three columns (from the 3 in $\frac{3}{4}$) and five rows (from the 5 in $\frac{5}{6}$. That construction creates an array of equal sized rectangles. We can describe the array as having five rows with three rectangles in each row.

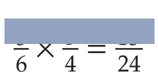


To determine the number of rectangles, we can add three together five times (five groups of three) to get a total of 15 rectangles. Of course I can represent the repeated addition statement using multiplication notation instead and there we have the product of our numerators:

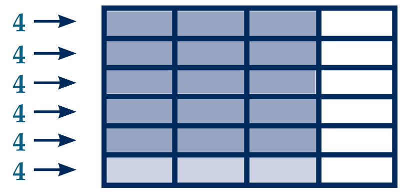


Justifying the "Denominator Times Denominator" Part of the Algorithm

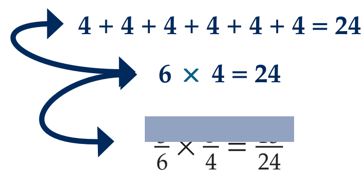
The final task in this article is to justify the "denominator times denominator part of the algorithm:



The strategy for the justification is actually quite similar to the justification just shown for the numerator, but we instead focus on the entire array rather than on just a portion of the array. In the entire array there are four columns (from the 4 in $\frac{3}{4}$) and six rows (from the 6 in $\frac{5}{6}$). We can describe the array as having six rows with four equal sized rectangles in each row.



To determine the number of rectangles we can add four together six times (six groups of four) to get a total of 24 rectangles. I can represent the repeated addition statement using multiplication notation instead:



And there we have the product of our denominators!

SUMMARY:

Both national and state standards documents, and the modern needs of our economy, suggest that students need to learn mathematics with understanding. Unfortunately, much of the logic that underpins the traditional fraction multiplication was forgotten during the 19th and 20th centuries when mathematics instruction focused solely on the memorization of algorithms and the development of quick and accurate computational skills. This article has provided a line of reasoning, supported by visual images, that teachers can use to help their students learn fraction multiplication with understanding. ■

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TCTM Leader Spotlight

Jacqueline Weilmuenster, 2004 E. Glenadine Gibb Awardee

Jacqueline Weilmuenster is the Coordinator of Mathematics for grades kindergarten through twelve in Grapevine-Colleyville Independent School District, Grapevine, Texas. It is her responsibility to direct and monitor the mathematics program for a diverse population of almost 14,000 students and provide staff development and vertical team curriculum and assessment writing for over 300 teachers.

With all of this responsibility, Jacqueline still remains very active professionally. She serves as the Northeast Regional Director for the Texas Council of Teachers of Mathematics. She is also the Conference for the Advancement of Mathematics Teaching (CAMT) representative for TCTM on the CAMT governing board. In this capacity she served as the chair of the CAMT Board from 2002-2004. With 25 years of teaching experience in Texas, Tennessee, and New Jersey, her expertise is both sought and recognized. Jacqueline is a recipient of the E. Glenadine Gibb Achievement Award for TCTM, as well as being twice named Teacher of the Year. She has also served on the steering committee as a member for the Algebra I End-of-Course state exam; contributing writer for the staff development for the Algebra I Assessments Institute and the problem bank for Problem Solving in the Middle School for the Texas Mathematics Center for Educator Development at the Charles A. Dana Center. She has been a reviewer and contributor for Middle School Math for Scott-Foresman; curriculum fellow for Project ABCD sponsored by Texas ASCD; and writing team fellow for Project Pass sponsored by a coalition between GTE and the NFL.

Jacqueline, in her role as Coordinator and mathematics leader, is aware of several issues and challenges facing today's educational leaders. In addressing these issues, her goal is to effectively link student achievement, teacher preparation and practice, curriculum, and assessment in a way that is attainable for school districts. In this way, she is helping to guarantee students a continuum of standards and expectations from kindergarten through college. She sees the need to support teachers with "systemic structures that allow for content study, collaborative lesson planning, and shared instructional practices that maximize their effectiveness with students, given the constraints of time and funds." This can be achieved by "learning how to use action research and assessment data to guide decisions regarding change; assisting teachers to utilize the training they receive in order to increase their students' understanding."

Jacqueline believes that the most valuable professional development she has received is from the "TEXTEAMS resources from the Dana Center at the University of Texas at Austin, especially the Rethinking Middle School Mathematics: Algebraic Thinking Institute for its groundbreaking focus on multi-representations of functions and Rethinking High School Mathematics: Algebra 2000 & Beyond Institute for the depth of its concept-building activities." She has combined this training with "the philosophies of student engagement and inquiry by Marilyn Burns, cooperative learning by Johnson, Johnson, and Holubec, and student empowerment in Cognitively Guided Instruction by Carpenter, Franke, and Levi. " She believes the National Council of Supervisors of Mathematics (NCSM) conference sessions have provided her with the most practical and yet visionary training for her past position as teacher specialist and now as the mathematics coordinator.

It is wonderful to acknowledge Jacqueline for her ongoing volunteer involvement with the CAMT. Her contributions to the organization of this conference have provided over 6,000 teachers a year with extensive opportunities to learn from nationally renowned leaders and to collaborate with other professionals from across Texas. CAMT has long served as a first class experience for educators, due to the dedication and efforts of the Texas Association for Supervisors of Mathematics, in partnership with the Texas Council of Teachers of Mathematics, and the Texas section of the Mathematical Association of America. The partnership includes the participation of the Texas Education Agency, providing updates on assessments and state standards, and of the Dana Center, offering leadership and data analysis training for administrators and teacher leaders. Jacqueline states that "CAMT was one of my earliest and most exciting learning opportunities as a math teacher and it is an honor to help others enjoy the same experience."

Thank you Jacqueline for your hard work and dedication.

■

TEA Talks

Hot News

For additional information, refer to the websites listed

- The revised TEKS for grades 6-12 mathematics have been approved by the State Board of Education. The implementation date for all schools to begin using these documents is the 2006-2007 school year. Campuses may elect to begin implementation during the 2005-2006 school year. A copy of the revised TEKS for these levels may be found at

  http://www.tea.state.tx.us/curriculum/math/ch111_amendments_proposed.pdf

- The TEKS for elementary mathematics are currently in the revision process. Mathematicians and mathematics education professors have recently completed the expert review process and the Agency is revising the TEKS based on this review. The Agency will present the proposed changes to the State Board of Education at the end of April. First reading will occur at the July SBOE meeting, followed by final reading and adoption at the September SBOE meeting, pending board approval. The implementation date for the revised elementary TEKS has not been established.

- The Agency is receiving inquiries regarding the changes to TAKS as a result of the changes to the TEKS for elementary and secondary mathematics. TEA Student Assessment and Curriculum Divisions are in the process of developing a timeline that reflects the changes within the Texas Assessment Program. Careful consideration will be given to districts as implementation begins and all students are exposed to the revised mathematics TEKS. When timelines are finalized, they will be posted on the TEA Student Assessment website. At this time, calculators will not be used on the TAKS for grades 6-8, even though "graphing technology" is now required within the state standards.

- Given the curriculum changes of the revised TEKS, the Agency has contracted with a university to create four TEKS Implementation modules. The modules will inform teachers about the changes in the TEKS, the intent of the content changes, and build teacher content knowledge within specific TEKS strands. These two-day trainings will also provide assessment information to assist teachers in developing valid measures to determine student success within a given student expectation statement. These modules will be provided through the Education Service Center in your region.

- Improving Student Achievement in Mathematics through Professional Development Grants will soon be available for districts creating partnerships to support mathematics professional development during the 2005-2006 school year. The deadline for grant submission is June 9, 2005. The Agency anticipates awarding one hundred grants up to 150,000 dollars each. For more information, visit

  http://www.tea.state.tx.us/opge/disc/

- Beginning this spring 5th grade students who take TAKS or SDAA II will be subject to all Student Success Initiative (SSI) requirements for both reading and mathematics. The SSI requirements include multiple opportunities to test, accelerated instruction when students are not successful, and automatic retention if students are not successful after three testing opportunities. There are amendments to the SSI Commissioner's Rules. These amendments address eligibility for SSI requirements, out-of-district accelerated instruction and testing, and additional considerations for the grade placement committee (GPC). Please refer to the 2004-2005 GPC Manual, which is in districts and on the TEA website at

  http://www.tea.state.tx.us/student.assessment/resources/ssi/index.html

  when making decisions for SSI students.

- An alternative mathematics assessment process will be implemented this spring for LEP-exempt students who are enrolled in grades included in AYP calculations (Grades 3-8 and 10). This alternative assessment process will enable students who are LEP-exempt under Texas state policy to take the mathematics portion of the TAKS or SDAA II tests with linguistic accommodations. The secure mathematics tests, not released tests, will be used. The same performance standards will apply to these administrations as to the regular administrations. This new alternative assessment process will be referred to as linguistically accommodated testing, or LAT. For more information please refer to the LAT District and Campus Coordinator Manual Supplement, which is available on the TEA website at

  http://www.tea.state.tx.us/student.assessment/admin/rpte/index.html.

- Texas is moving toward expanded use of computer-administered testing and reporting in its comprehensive assessment program. Several online testing initiatives will be conducted this spring. These initiatives include a new version of the voluntary online Algebra I end-of-course assessment; a comparability study of online and paper-and-pencil versions of the Grade 8 TAKS tests, including field-testing of Grade 8 science; and an extra opportunity for students to take the Exit Level TAKS tests online. The Algebra I end-of-course will be TAKS-aligned and will be offered during a testing window in April-June. The online Grade 8 TAKS test will be offered to districts that volunteered to test online. This comparability study will occur the week of April 11-15, 2005. The online Exit Level TAKS tests will be offered to selected students in districts that volunteered to test online. This extra opportunity to test will occur the week of June 6-10, 2005.

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Paula Moeller • <Paula.Moeller@tea.state.tx.us>
Director of Mathematics • Texas Education Agency

Recommended Readings and Resources

Powerful Practices in Mathematics and Science by Carpenter and Romberg

Powerful Practices in Mathematics and Science (2004) by Carpenter and Romberg provides two research-based CD-ROMs and a monograph that features findings of several years of classroom research conducted through the National Center for Improving Student Learning and Achievement in Mathematics and Science. The goal of these materials is to provide examples of what is possible in mathematics and science education. The key pedagogical concepts addressed are modeling, generalization and justification. The first CD contains a 35-minute introductory video. The second CD extends from the introduction with nine classroom excerpts. The classroom episodes show the ways that the powerful teaching and learning practices of modeling, generalization, and justification strengthen students' learning and understanding of complex mathematics and science ideas to the benefit of elementary and secondary students.

Professional development leaders and teachers will find *Powerful Practices* a valuable resource. This product is currently available free of charge at:

http://www.learningpt.org/msc/products/practices.htm

■

TCTM Breakfast and Business Meeting at CAMT 2005

Wednesday July 13, 2005, 7:00 a.m. – 8:30 a.m.
Adam's Mark Hotel

Come join us for the annual TCTM breakfast and business meeting. This is your opportunity to meet the current and incoming board members and be a part of TCTM decisions. Plus, there will be great door prizes from the vendors you see at CAMT. This breakfast is for members only (no children or other guests). Breakfast tickets must be reserved in advance; there will be no tickets available at CAMT. Please mail your contact information, along with your check for the breakfast, to the address below no later than June 1, 2005. Your e-ticket for the breakfast will be e-mailed to you no later than June 30.

Member Information

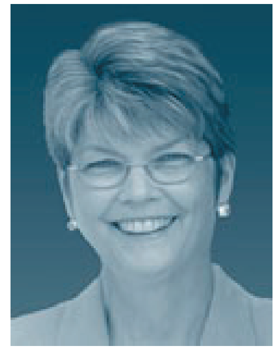
Name:	Last	First	Middle
Address:	Number and street		Apt. number
	City		Zip Code
Phone :	Home Phone	Work Phone	Email address
Affiliation:	District or Professional Affiliation		ESC

☐ Enclosed please find my $15.00 check, payable to TCTM, for the breakfast.

Please mail your check and form to **Cynthia Schneider,**
234 Preston Hollow,
New Braunfels, TX 78132

Who's Doing the Talking?

Message from NCTM's President for Affiliates



In the November 2004 President's Message for the *NCTM News Bulletin* (available at *www.nctm.org/news/president*), I suggested that the most important factor in a student's mathematics learning (after teacher expectations) is the student's active engagement in the learning process. One clue to whether students are engaged in learning can be found by looking into classrooms and noticing who is doing the most talking—the teacher or the students.

Most of us learned to teach the same way we ourselves were taught. Often, the classrooms we experienced as learners were teacher-centered, with students expected to listen, take notes, do homework, and answer test questions based on what was presented to us by the teacher. Along the way, there may have been a few absolutely wonderful teachers who drew us into their teaching through entertaining and nonroutine variations on the traditional lecture model. Other times we suffered through boring mathematics presentations where we were not engaged in our own learning. Fortunately, many of us were successful as students in this type of lecture-based classroom, and we may even have become somewhat proficient using a similar model of teaching ourselves. While we were learning, however, many other students were never engaged in mathematics through lectures, even with the most energetic teacher.

Today we are called to teach challenging mathematics to a much wider range of students than ever before. Teachers tell me that fewer and fewer of even their more successful students respond positively to teacher-centered, lecture-based teaching. In recent curriculum projects based on NCTM's *Principles and Standards for School Mathematics*, a different teaching model is emerging. Often, students are expected to work in small groups around engaging tasks, either in real settings or in interesting mathematical contexts. Although it is possible to use these excellent materials in a teacher-centered classroom, far greater gains are found when the teacher gives students a greater role in the learning process.

The teachers who are most effective with these materials offer guidance and probing questions instead of telling students all the things they are supposed to learn. In this kind of student-centered classroom, the teacher's role is to set the stage, organize the task, ask good questions, and help students connect their experience to the mathematics being addressed. Much of this work happens with the class as a whole, but there is usually a period of intense student activity where students interact around the mathematics in pairs or small groups. This new teacher role calls for sophisticated knowledge of both mathematics and learning, and it takes at least as much preparation as a good lecture or content presentation. But the payoff is immense. When students have the opportunity to figure out an approach to a problem; discuss, argue, and justify their ideas; and wrestle with challenging mathematics, they are truly engaged in their learning. They are hooked into the mathematics. They are much more likely to be able to remember what they learn and apply it to other situations than they would if they were simply told how to solve a particular type of problem.

To determine how engaged your own students are, take an objective look at your classroom and ask yourself who's doing the talking. If the teacher's voice is the voice usually heard, how engaged are students? If the classroom is largely quiet, how engaged are students? If only short fill-in-the-blank kinds of responses are expected from students, how engaged are students? Even if students are heard, if only a few students have the opportunity to make comments or offer possible answers to the teacher's questions, how engaged are the rest of the students?

Shifting the focus of the classroom to include more student engagement does create a noisier classroom. In fact, it may appear to be less structured or orderly than a teacher-directed classroom. After all, this type of learning environment involves lots of students talking, often at the same time, as they work in small groups. Learning to see the benefits of this apparent disorder is an important step for a teacher shifting toward more student engagement. Noise and student involvement do not have to turn into chaos or lack of structure. On the contrary, effective teachers learn to manage such classrooms with clearly spelled-out expectations for student behavior and student participation. Students have well-defined roles in their groups, and the teacher serves as an organized facilitator. The result is that students learn with real understanding.

If you are accustomed to teaching in a teacher-directed classroom, it may be challenging to shift to a more student-centered style. You will likely need to go through appropriate professional development that will ideally include some kind of long-term support. But the payoff for you and your students will be tremendous as you hear a higher level of mathematics conversation and as you see for yourself a higher level of student learning.

■

Cathy Seeley • <cseeley@nctm.org>
President, National Council of Teachers of Mathematics

This article is provided as a service to Affiliates of NCTM. If you are not currently a member of NCTM, find out how to be part of this national professional community committed to a high-quality mathematics education for every student at http://www.nctm.org/

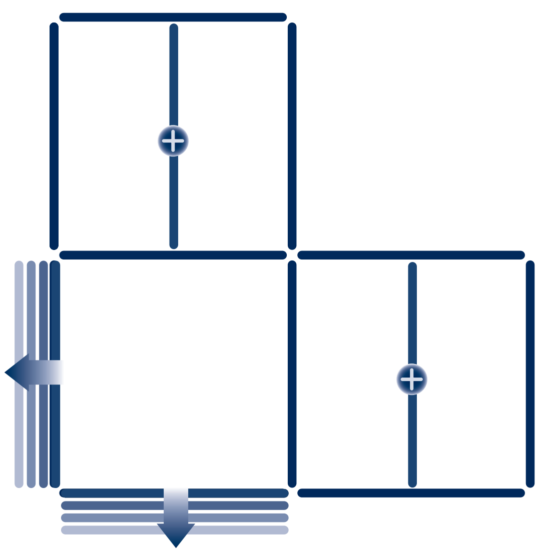
FEATURE

Puzzle Corner

Sticks #4 Answer

Arrange ten craft sticks to form the origional figure. Move two sticks to form four congruent rectangles.

Shown is a diagram of a solution

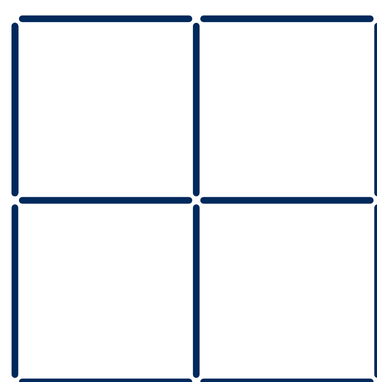


Sticks #5 Puzzle

We are interested in how your students responded to this problem and how they explained or justified their reasoning. Please e-mail copies of your students' work, include your name, grade level, campus name and district name to Mary Alice Hatchett, Director of Publications, *Texas Mathematics Teacher*. Selected submissions will be acknowledged and published in subsequent issues.

Please prepare a sketch of your solution

Arrange 12 craft sticks to form the following figure



Rearrange three sticks to form a figure that has three congruent squares.

TCTM CAMTership Application

Deadline : May 31, 2005

Eligibility: Six $300 CAMTerships will be awarded to teachers with five or fewer years teaching experience who are members of TCTM and have not attended CAMT before. CAMTerships will be awarded to teachers in each of the following grade levels: K - 4, 5 - 8, and 9 - 12. Winners will be determined by random drawing of names and will be notified after June 1, 2005. Winners will be asked to work for two hours at registration or the NCTM material sales booth and will be TCTM's guest at our breakfast, where the checks will be presented. Good luck!

Name: Last | First | Middle

Address: Number and street | Apt. number

City | State | Zip Code

Contact: (   ) Home Phone | (   ) Work Phone | Email Address

Affiliation: District or Professional Affiliation | ESC

Are you a member of TCTM? **Y** **N**
note: If you are not a member of TCTM, you must enclose a $13 check with this application to apply for membership.

Have you attended CAMT before? **Y** **N**

How long have you been teaching?

Describe your teaching responsibilities.

Send your completed application to:

by mail: **Cynthia Schneider,**
234 Preston Hollow,
New Braunfels, TX 78132

by fax: **(512) 232-1855**
ATTN: Cynthia Schneider

by email: ***<cschneider@satx.rr.com>***

CAMT 2005

Volunteer by May 31, 2005 to work at CAMT 2005 in Dallas!

All members of TCTM should take an active role to help make CAMT successful. Come work 'behind the scenes' to facilitate the experience for all participants. Please examine the times below and volunteer to serve. There are two locations for volunteers to work. We need you! Identify the time slot(s) when you can help and e-mail the date and time(s) on or before May 31, 2005 to Cynthia Schneider by using the address listed below. We will confirm your assigned time via e-mail or phone on or before June 30.

Volunteer Information

Name:	Last	First	Middle
Address:	Number and street		Apt. number
	City	State	Zip Code
Contact:	( ) Home Phone	( ) Work Phone	Email Address
Affiliation:	District or Professional Affiliation		ESC

Registration Area

Day					
Sunday July 10	1:30 p.m. – 3:30 p.m.	3:30 p.m. – 5:00 p.m.			
Monday July 11	6:30 a.m. – 9:00 a.m.	9:00 a.m. – 11:00 a.m	11:00 a.m. – 1:00 p.m.	1:00 p.m. – 3:00 p.m.	3:00 p.m. – 5:00 p.m.
Tuesday July 12	6:30 a.m. – 9:00 a.m.	9:00 a.m. – 11:00 a.m.	11:00 a.m. – 1:00 p.m.	1:00 p.m. – 3:00 p.m.	3:00 p.m. – 5:00 p.m.
Wednesday July 13	7:00 a.m. – 9:00 a.m.	9:00 a.m. – 11:00 a.m.	11:00 a.m. – 1:00 p.m.	1:00 p.m. – 3:00 p.m.	3:00 p.m. – 5:00 p.m.

Exhibits Area –TCTM Booth

Day				
Monday July 11	9:45 a.m. – 12:00 p.m.	12:00 p.m. – 2:00 p.m.	2:00 p.m. – 4:00 p.m.	4:00 p.m. – 6:00 p.m.
Tuesday July 12	8:45 a.m. – 11:00 a.m.	11:00 a.m. – 1:00 p.m.	1:00 p.m. – 3:00 p.m.	3:00 p.m. – 5:00 p.m.
Wednesday July 13	8:45 a.m. – 11:00 a.m.	11:00 a.m. – 1:00 p.m.	1 – 2:00 p.m.	

Please submit your form to Cynthia Schneider

by mail: **Cynthia Schneider, 234 Preston Hollow, New Braunfels, TX 78132**

by fax: **(512) 232-1855 ATTN: Cynthia Schneider**

by email: ***<cschneider@satx.rr.com>***

APPLICATION

TCTM Mathematics Specialist Scholarship

Amount: $1500 **Deadline: May 31, 2005**

Eligibility: Any student attending a Texas college or university – public or private – and who plans on student teaching during the 2005-06 school year in order to pursue teacher certification at the elementary, middle or secondary level with a specialization or teaching field in mathematics is eligible to apply. A GPA of 3.0 overall and 3.25 in all courses that apply to the degree (or certification) is required.

Applicant Information

Name:	Last	First	Middle
Address:	Number and Street		Apt. number
	City	State	Zip Code
Contact:	( ) Home Phone	( ) Work Phone	Email Address
Personal:	Social Security Number		Birth Date

College Information

What college or university do you plan to attend? If you are awarded this scholarship, TCTM's treasurer will send a check directly to the business office of the college. We need the college's complete address.

Name:	College or University		
Address:	Number and Street		
	City	State	Zip Code

You must submit three (3) copies of each of the following documents:

1. Completed application form.
2. One official college transcript and two copies.
3. Two letters of recommendation:
   - One from either a mathematics or mathematics education professor you have taken coursework from and is not related to you.
   - One from a K-12 classroom teacher of mathematics you have worked with recently or that was a former teacher of yours and is not related to you.
   - It is required that at least one of these recommendations come from a current member of TCTM, it is preferred that both recommendations come from current members of TCTM.
4. An essay of 1,500 words or more that describes your philosophy of teaching mathematics and how you will implement this philosophy with your future students. Specific examples of how you will teach a mathematics concept are required to illustrate your teaching philosophy. Or you may write an essay that explains a specific mathematics topic or concept, for example, a paper on proportionality.

Please submit all materials in one envelope to:

by mail: **Cynthia Schneider**
234 Preston Hollow
New Braunfels, TX 78132

by fax: **(512) 232-1855**
ATTN: Cynthia Schneider

TCTM Leadership Award Application

Deadline: May 31, 2005

Eligibility: The TCTM Leadership Award is presented to a TCTM member who is nominated by a TCTM Affiliated Group. This person is to be honored for his/her contributions to the improvement of mathematics education at the local and state level. He/she has designed innovative staff development and has promoted the local TCTM Affiliated mathematics council.

Information about the **TCTM member** nominating a candidate

Name: Last | First | Middle

Address: Number and street | Apt. number

City | State | Zip Code

Contact: ( ) Home Phone | ( ) Work Phone | Email Address

Affiliation: District or Professional Affiliation | ESC

Are you a member of TCTM? Y N

Are you a member of NCTM? Y N

Information about the **nominee**

Name: Last | First | Middle

Address: Number and street | Apt. number

City | State | Zip Code

Contact: ( ) Home Phone | ( ) Work Phone | Email Address

Affiliation: District or Professional Affiliation | ESC

Is the nominee a member of TCTM? Y N

Is the nominee a member of NCTM? Y N

Is the nominee retired? Y N

Applications should include 3 pages:

- ☐ Completed application form
- ☐ One-page, one-sided, typed biographical sheet including:
  - Name of nominee
  - Professional activities
  - National offices or committees
  - State TCTM offices held
  - Local TCTM-Affiliated Group offices held
  - Staff Development
  - Honors/awards
- ☐ One-page, one-sided essay indicating why the nominee should be honored for his/her contribution to the improvement of mathematics education at the state/national level

Send the completed application, biographical sketch, and essay to

by mail: **Cynthia Schneider,**
234 Preston Hollow,
New Braunfels, TX 78132

by fax: **(512) 232-1855**
ATTN: Cynthia Schneider

by email:
<cschneider@satx.rr.com>

Texas Council of Teachers of Mathematics Membership Form

Applicant Information

Name: Last | First | Middle

Address: Number and street | Apt. number

City | State | Zip Code

Contact: ( ) Home Phone | ( ) Work Phone | Email Address

Affiliation: District or Professional Affiliation | ESC

Individual TCTM Membership

Cost : $13.00 per year

Membership includes 1 copy of the biannual TMT journal.

Circle area(s) of interest: **K-2** | **3-5** | **6-8** | **9-12** | **College**

Circle one : New Member | Renewal | Change of Address

[ ] year(s) x $13.00 = $

Professional TCTM Membership

Cost : $40.00 per year

For schools, institutions, or affiliated groups. Membership includes 3 copies of the TMT journal.

Circle one : New Member | Renewal | Change of Address

[ ] year(s) x $40.00 = $

National Council of Teachers of Mathematics Membership

Circle one : New Member | Renewal | Change of Address

Full Individual membership includes a print subscription to the NCTM News Bulletin and one NCTM Journal. Select one journal below.

Additional print journals may be selected to enhance your membership, and includes online access.

Journal	Full Individual membership	Additional print journal
Teaching Children Mathematics	$72	$30
Mathematics Teaching in the Middle School	$72	$30
Mathematics Teacher	$72	$30
Journal for Research in Mathematics Education	$94	$52

Membership Dues $

Additional Journals $

Amount Due NCTM $

Scholarship Donations

TCTM awards scholarships to college students planning to pursue a career in mathematics teaching either as a mathematics specialist in elementary school or as a secondary school teacher with certification in mathematics. Your contributions in any amount are greatly appreciated. Please write a separate check for scholarship donations.

Scholarship Donations $

Make check(s) payable to TCTM and mail to:

TCTM Treasurer
2833 Broken Bough Trail
Abilene, TX 79606

TOTAL AMOUNT DUE $

TCTM E. Glenadine Gibb Achievement Award Application

Deadline: May 31, 2005

Eligibility: The E. Glenadine Gibb Achievement Award is presented to someone nominated by a TCTM member to be honored for his/her contribution to the improvement of mathematics education at the state and/or national level.

Information about the **TCTM member** nominating a candidate

Name: Last | First | Middle

Address: Number and street | Apt. number

City | State | Zip Code

Contact: ( ) Home Phone | ( ) Work Phone | Email Address

Affiliation: District or Professional Affiliation | ESC

Are you a member of TCTM? Y N

Are you a member of NCTM? Y N

Information about the **nominee**

Name: Last | First | Middle

Address: Number and street | Apt. number

City | State | Zip Code

Contact: ( ) Home Phone | ( ) Work Phone | Email Address

Affiliation: District or Professional Affiliation | ESC

Is the nominee a member of TCTM? Y N

Is the nominee a member of NCTM? Y N

Is the nominee retired? Y N

- ☐ Completed application form
- ☐ One-page, one-sided, typed biographical sheet including:
  - Name of nominee
  - Professional activities
  - National offices or committees
  - State TCTM offices held
  - Local TCTM-Affiliated Group offices held
  - Staff Development
  - Honors/awards
- ☐ One-page, one-sided essay indicating why the nominee should be honored for his/her contribution to the improvement of mathematics education at the state/national level

Please submit the completed application, biographical sketch, and essay

by mail: **Cynthia Schneider, 234 Preston Hollow, New Braunfels, TX 78132**

by fax: **(512) 232-1855 ATTN: Cynthia Schneider**

by email: ***<cschneider@satx.rr.com>***

Rational Functions and Crossing Asymptotes

In this article, we present an investigation of the common student misconception that "*the graph of a function cannot cross its horizontal asymptotes*." The investigation leads toward a process for building an example of a rational function whose graph crosses its horizontal asymptote as many times as desired. We begin the investigation with a discussion of vertical asymptotes and their properties that may be causes for the misconception about horizontal asymptotes.
Before we proceed we recall the following definitions:

Definition (1). A **rational function** is a function of the form $R(x)=\frac{p(x)}{q(x)}$ where p and q are polynomial functions and q is not the zero polynomial. The domain is all real numbers except those for which the denominator is zero.

Definition (2). For a rational function R, the line $x=c$ is a vertical asymptote of the graph of R if as x approaches some number c, the values $|R(x)|\to\infty$.

Definition (3). For a function f, the line $y=L$ is a horizontal asymptote of the graph of f if as $x\to\infty$ or as $x\to-\infty$, the values $f(x)$ approach some fixed number L.

In general, teachers and students have a solid notion that the graphs of rational functions cannot cross their vertical asymptotes. A counterexample that emphasizes this notion can be created by investigating the following question: *If the graph of a function were able to cross its vertical asymptotes what would the graph of that function look like?* A teacher can produce the following graph:

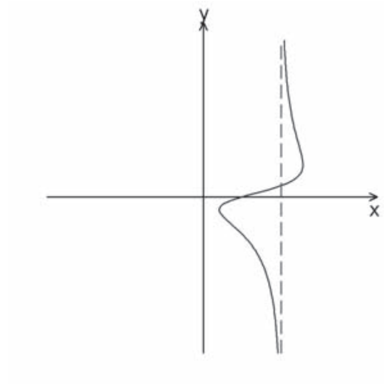


Figure 1

With this graph, we illustrate that if crossing vertical asymptotes were possible, then the expression under consideration in Figure 1 would not be a function. That is, a crossing would impose at least one x-coordinate for which there is more than one y-coordinate; i.e. the graph would not satisfy the vertical line test. We note that in order to satisfy the definition of vertical asymptote, the values of the function near the vertical asymptote must approach $\pm\infty$ which forces any crossing to return toward the line $x=c$.

More questions seem to arise among students and teachers when asked if it is possible for the graph of a function to cross its horizontal asymptotes. We have seen that many students begin calculus with the misconception that functions cannot cross asymptotes of *any* kind. This may be due to an overgeneralization by students of the fact that vertical asymptotes cannot be crossed, to a lack of appropriate learning experiences that expose students to functions that actually do cross their horizontal asymptotes, or to the

Connections

When modeling free motions of a vibrating mass connected to a dashpot on an elastic spring, the solutions to the equations represent the displacement of the mass from the static equilibrium position (y=0) with respect to time. These solutions can cross their horizontal asymptotes. In particular, in the case of an underdamped-spring-mass system, the solution modeling the motion intersects its horizontal asymptote (y=0) infinitely many times.

Although the solutions to the damped-spring-mass system equations involve trigonometric and exponential functions, the notion that horizontal asymptotes can be intersected may be investigated prior to a course in pre-calculus by examining horizontal asymptotes on rational functions.

fact that many teachers also hold this misconception. When pressed on their understanding, we find that teachers and students will recall seeing or graphing functions that cross horizontal asymptotes once. Most are skeptical when asked to consider the following task:

Task (1) Give an example of a rational function, q_n, that has a horizontal asymptote and crosses this asymptote n times.

This task has been used by the author in a problem-solving setting with students and in professional development workshops delivered to teachers and college instructors. It is assumed that problem-solvers (students or teachers) will use a graphing calculator to examine patterns and for experimentation.

Investigating this Task (1) accomplishes several goals:

- requires students and/or teachers to investigate cases where a horizontal asymptote is crossed more than once;
- emphasizes the importance of the end-behavior when considering horizontal asymptotes; and
- integrates ideas about transformations in simplifying possible solutions.

These goals can be linked to the Algebra Standard for Grades 9-12 in *Principles and Standards for School Mathematics* (NCTM 2000, pp 296-306) where it is stated that instructional programs from prekindergarten through grade 12 should enable students to "Understand patterns, relations, and functions" and that in grades 9-12 all students should—

- analyze functions of one variable by investigating rates of change, intercepts, zeroes, asymptotes, and local and global behavior;
- understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions; and
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions. (NCTM 2000, p 296)

The Texas Essential Knowledge and Skills (TEKS) in Precalculus that play a role in this task are:

The student is expected to:

(1D) recognize and use connections among significant points of a function (roots, maximum points, and minimum points), the graph of a function, and the symbolic representation of a function; and

(1E) investigate continuity, end behavior, vertical and horizontal asymptotes, and limits and connect these characteristics to the graph of a function.

(2A) apply basic transformations, including $a \cdot f(x)$, $f(x)+d$, $f(x-c)$, $f(b \cdot x)$, $|f(x)|$, $f(|x|)$, to the parent functions. (TEA 1997, pp C15-C16)

The TEKS in Algebra II that relate to this task are:

(E1) The student uses quotients to describe the graphs of rational functions, describes limitations on the domains and ranges, and examines asymptotic behavior.

(E2) The student analyzes various representations of rational functions with respect to problem situations. (TEA 1997, p C-9)

Here, we focus our investigation on rational functions so that the roles of the zeros of the numerator and denominator come to the forefront in the investigation. If we were to consider quotients involving trigonometric functions we can see, for example, that $v(x) = \begin{cases} \frac{\sin x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$ crosses its horizontal asymptote 18 times on the interval $-30 \le x \le 30$ (see Figure 2). In fact, the graph of v crosses its horizontal asymptote, y = 0, infinitely many times.

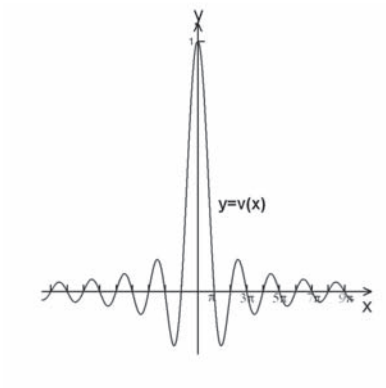


Figure 2

Note that in Definition (2) we define vertical asymptote for a *rational function* because we want to focus our investigation at the precalculus and Algebra II level. In general, the line $x = c$ is a vertical asymptote for a function f if $|f(x)| \to \infty$ when x approaches c from either the right *or* the left. Figure 3 illustrates a function f with vertical asymptote at $x = c$; however, f is not a rational function. Although f *intersects* its vertical asymptote it does not "cross" the asymptote. This intersection can occur at most once. *We note that a "crossing" is an intersection but an intersection is not always a crossing.*

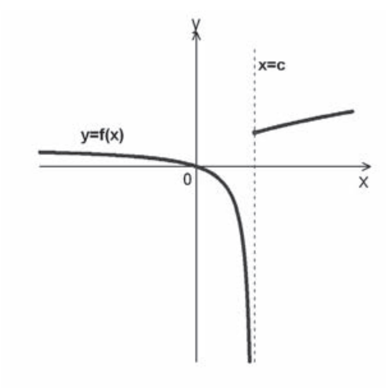


Figure 3

As problem-solvers grapple with Task (1), scaffolding exercises are then introduced to build toward constructing a general example of a rational function that crosses its horizontal asymptote, say $y = L$, where L is any real number, n times. Research suggests that students who employ an example generation learning strategy are more effective in attaining initial understanding of a new concept (Dahlberg & Housman, 1997).

We may preface scaffolding exercises for Task (1) with an informal discussion about horizontal asymptotes. That is, if we claim that a function f has a horizontal asymptote at $y = L$, then we see that the graph of the function should be getting arbitrarily close to this horizontal line ($y = L$) for large $|x|$. Moreover, if f has a horizontal asymptote at $y = L$ and we wanted to know if there were an interval on the x-axis over which the $y = f(x)$ values were within, say 0.02 of the y-value ($y = L$) of the horizontal asymptote—we are guaranteed that we can find this interval and it would be of the form (a, ∞) or $(-\infty, b)$. For example, in Figure 4, we see that the function f has a horizontal asymptote at $y = L$ (we can also see that $L = 0.01$ in this case) and that on the interval $(7.005, \infty)$ the values of $y = f(x)$ are within 0.02 of L. That is, we are guaranteed that we can "trap" the function values in a horizontal strip (of width 0.04) about $y = L$ when $x > 7.005$. We determined the value 7.005 by finding the intersection points of the curves $y = L + 0.02$ and $y = f(x)$ and the curves $y = L - 0.02$ and $y = f(x)$ and then choosing the intersection point with the largest x-value to determine the interval $(7.005, \infty)$. From the graph and the calculations of the intersection points, we see that f converges relatively quickly to $y = L$, but f will cross $y = L$ infinitely many times. We could say that if $x > 7.005$ then "L is a good approximation for $f(x)$ "or $f(x) \approx L$ or $|f(x) - L| < 0.02$.

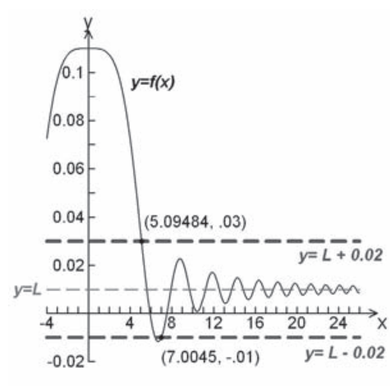


Figure 4

Now, the scaffolding exercises begin with asking problem-solvers to generate examples of rational functions given specific conditions on their asymptotes and intercepts such as the following:

(A) Give an example of a rational function that has vertical asymptotes at $x = 0$ and $x = 4$ and a horizontal asymptote at $y = 1$.

(B) Give an example of a rational function that has a vertical asymptote at $x = 1$, a horizontal asymptote at $y = 1$, and an x-intercept at 2.

It is assumed that problem-solvers have had experiences graphing and analyzing rational functions when given a defining expression for the rational function and that they would have had experiences that lead to their understanding of a theorem that states that a rational function, $R(x) = \frac{p(x)}{q(x)}$, in lowest terms will have a vertical asymptote at $x = r$ if $x - r$ is a factor of the denominator q.

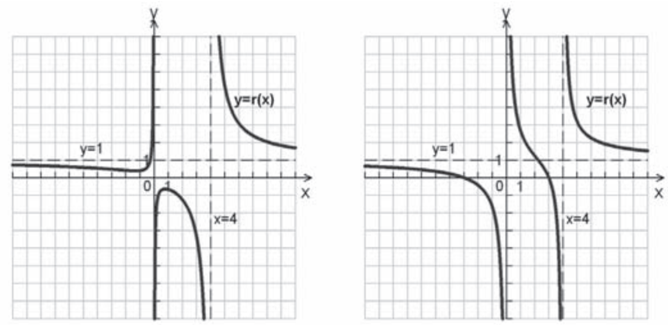
Thus, when generating an example for (A) above, problem-solvers can readily determine that x and $x - 4$ must be factors of the denominator and that in order to have a horizontal asymptote at $y = 1$, the polynomials in the numerator and the denominator of the rational function must grow approximately the same rate as $x \to \infty$ (i.e. in the "long run") requiring that the numerator and denominator have the same degree. In addition, the ratio of the leading coefficients corresponds to the value of the horizontal asymptote. In the case where the horizontal asymptote is $y = 1$ the ratio of the leading coefficients must then be 1.

$$r(x) = \frac{\quad}{x(x-4)}$$

Numerator must have degree 2

Depending upon where the problem-solver wants the zeros of r to occur, any quadratic can be used here as long as the leading coefficient is equal to the leading coefficient of the denominator.

If the problem-solver does not want to introduce any zeros (x-intercepts), a quadratic function may be used that has no real roots, such as $x^2 + 1$. The number of specific examples is infinite because we can generate an infinite number of quadratic functions for the numerator by varying the parameters a, b, and c in $p(x) = ax^2 + bx + c$. We can also change the degree of the numerator and denominator simultaneously by multiplying both by same-degree polynomials with no real roots. The restriction to polynomials with no real roots is necessary so that no additional vertical asymptotes are introduced. Two examples for (A) are given in Figure 5.



$$r(x) = \frac{x^2+1}{x(x-4)} \qquad r(x) = \frac{x^2-9}{x(x-4)}$$

Figure 5

For example (B), an acceptable example would be:

$$r(x) = \frac{x-2}{x-1}$$

Since $x = 2$ is an x-intercept, the numerator must have a factor x - 2.

Since $x = 1$ is a vertical asymptote, the denominator must have a factor x - 1. Also, note that the ratio of the leading coefficients must be 1 since we must ensure that there is a horizontal asymptote at $y = 1$. (See Figure 6.)

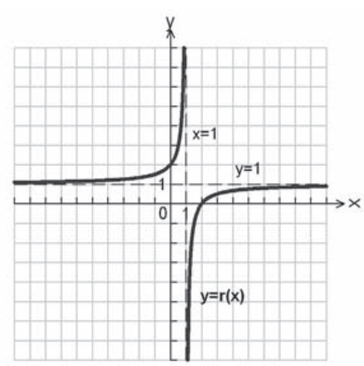


Figure 6

A tabular representation that highlights the behavior of $r(x)=\frac{x-2}{x-1}$ is given in Figure 7 where we calculate the values $r(x)$ for as x grows without bound (i.e. the end behavior) and values of $r(x)$ near its vertical asymptote.

x	y1(x) (x-2)/(x-1)
-10000	1.0001
-1000	1.001
-100	1.0099
-10	1.09091
0	2
10	0.888889
100	0.989899
1000	0.998999
10000	0.9999

Tabular representation to determine horizontal asymptote at $y=1$.

x	y1(x) (x-2)/(x-1)
.9	11
.99	101
.999	1001
.9999	10001
1	undef
1.0001	-9999
1.001	-999
1.01	-99
1.1	-9

Tabular representation to determine vertical asymptote at $x=1$.

Figure 7

The capacity for multiple solutions makes generating examples for (A) and (B) an interesting collaborative task for problem-solvers.

Next, we ask problem-solvers to investigate a simpler case of the original task.

(C) Give an example of a rational function that has a horizontal asymptote and crosses this asymptote once; three times.

By trial-and-error, problem-solvers may have generated an example of a rational function that crosses its horizontal asymptote once (as is the case in Figure 5). If so, they should be asked if they can identify the characteristics of their previous examples that seem to have played a role in the crossing of the horizontal asymptote.

In practice, it is crucial to allow problem-solvers sufficient time to construct these examples. The facilitator (teacher) can prompt approaches to the tasks by asking problem-solvers to think about the roles of the zeroes of the numerator of a rational function expressed in lowest terms. This may be followed by posing the following question:

(D) If a rational function, q, (expressed in lowest terms) has an asymptote at $y=0$, how can we guarantee that q will cross its asymptote three times?

Assuming that problem-solvers have had prior experience determining end-behavior of rational functions, their thinking leads to generating an example of a function, g, such that the degree of the denominator is at least one degree higher than that of the numerator. Informally, in order for a rational function to have a horizontal asymptote at $y=0$, the denominator must overtake (grow faster than) the numerator in the long-run—forcing the quotient to approach zero in the long run. Again, they should be encouraged to use a graphing calculator for experimentation.

Problem-solvers will eventually realize that a rational function meeting the criteria in (D) will have the form $g(x)=\frac{a(x-r_1)(x-r_2)(x-r_3)}{p(x)}$ where a is a nonzero real number, r_i are distinct real numbers for each $i=1, 2,$ or 3, and p is a fourth degree (or higher) polynomial with zeros distinct from $r_1, r_2,$ and r_3. It should be noted that we can increase the degree of the numerator and denominator by multiplying by a polynomial function with no real zeros as in the previous examples.

A plausible example for such a function g would be

$$g(x)=\frac{(x-1)(x+2)(x-4)}{(x+3)(x+1)(x-2)(x-3)}$$

(see Figure 8).

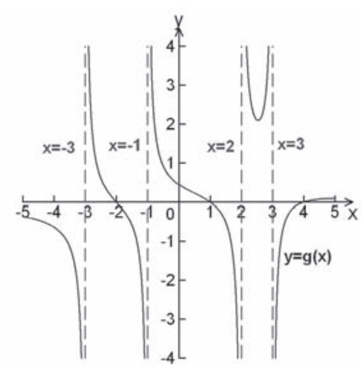


Figure 8

At this point, the facilitator can ask for an example for (D) that has no vertical asymptotes. Problem-solvers must conclude that this entails finding a polynomial function with no real zeros for the denominator of the rational function. Recalling their strategy for increasing the degree of the numerator or denominator without introducing new zeros or vertical asymptotes, they can again be encouraged to think about elementary examples of polynomials that have no real zeros, such as $f(x) = x^2 + 1$. After some experimentation, the following example may be constructed $g(x) = \frac{(x-1)(x+2)(x-3)}{2\left(x^2+1\right)^2}$

(see Figure 9).

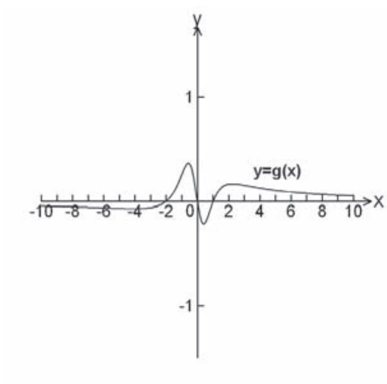


Figure 9

After generating a similar example to the latter, problem-solvers are ready to give an example of a rational function, q, (in lowest terms) that has a horizontal asymptote at $y = 0$, crosses this asymptote n times, and has no vertical asymptotes. Because the horizontal asymptote is at $y = 0$, the zeros of the rational function correspond to the crossings or intersections of the horizontal asymptote. Their example may have the form:

$$q(x) = \frac{a(x-r_1)(x-r_2)\cdots(x-r_n)}{\left(x^2+1\right)^k}$$

where k is an integer greater than $\frac{n}{2}$, r_i are distinct real numbers for each $i = 1, 2, \ldots n$, and a is a nonzero real number. This form ensures that the asymptote will be crossed n times because it has n zeros. Also, the degree of the denominator must be at least one degree higher than the degree of the numerator; thus, we must require that the quadratic $x^2 + 1$ be raised to an integer power greater than $\frac{n}{2}$ to ensure this. A particular example with five crossings could be given as:

$$q(x) = \frac{5x\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)(x+1)(x-1)}{\left(x^2+1\right)^4}$$

(see Figure 10).

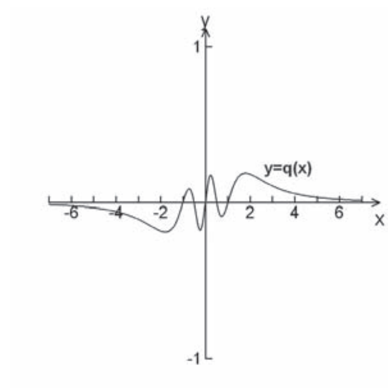


Figure 10

The facilitator must now ask problem-solvers to create an example with a horizontal asymptote at $y = L$ where L is a nonzero real number. The goal here is to have them realize that they can simply take q (given above) and vertically translate this function by L units to get

$$q_L(x) = \frac{a(x-r_1)(x-r_2)\cdots(x-r_n)}{\left(x^2+1\right)^k} + L.$$

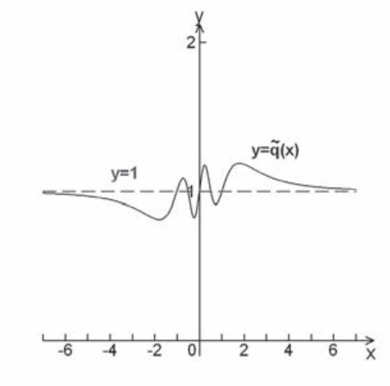
Those who are not convinced that q_L is a rational function should be encouraged to get a common denominator in the expression above and examine the results. Using the example

$$q(x) = \frac{5x\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)(x+1)(x-1)}{\left(x^2+1\right)^4}$$

seen in Figure 10 above, we can create a function, $\tilde{q}$, that has a horizontal asymptote at $y = 1$ and its graph crosses this asymptote five times:

$$\tilde{q}(x) = \frac{5x\left(x-\frac{1}{2}\right)\left(x+\frac{1}{2}\right)(x+1)(x-1)}{\left(x^2+1\right)^4}+1$$

$$= \frac{4x^8 + 16x^6 + 20x^5 + 24x^4 - 25x^3 + 16x^2 + 5x + 4}{4x^8 + 16x^6 + 24x^4 + 16x^2 + 4}$$



The graph of $\tilde{q}$ is shown in Figure 11.

Figure 11

Thus, at this point, problem-solvers will have produced an example of a rational function that has a horizontal asymptote and crosses this asymptote n times.

The scaffolding to build toward a solution to the original task (1) entailed an emphasis on making conceptual connections and integrating mathematical knowledge to construct examples rather than an emphasis on learning procedural steps and applying them to examples constructed by the facilitator or a textbook. It is important to note that the *process* for creating the example is more important than the example generated. Hiebert and Stigler (2004) indicate that U.S. teachers are most challenged in implementing "making connections" problems as compared with "using procedures" problems.

> *… a teacher might transform a making connections problem designed to have students figure out a method for calculating the area of various types of triangles into a using procedures problem by giving students, at the outset, the formula (1/2 Base • Height) and telling students to simply plug in the relevant values.* (Heibert and Stigler, 2004)

In the case of task (1), giving students a function similar to q_L upfront turns this task into a "using procedures" problem rather than a "making connections" problem.

The constructivist approach toward an example that satisfies task (1) provides a very powerful experience for students and teachers to emphasize that horizontal asymptotes can indeed be "crossed" or intersected and that the end-behavior is not affected.

■

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DEPARTMENT

Math Alive! CAMT 2005

Engaging Hearts and Minds
July 11 – 13, 2005

CAMT 2005 will be held July 11-13, 2005, at the Adam's Mark Hotel in Dallas, Texas. Program Co-Chairs are Barbara Holland and Michelle King. Registration and program information are available online at:

http://www.tenet.edu/camt/

Kay Tolliver of the Eddie Files will be a featured speaker at a special ticketed session with lunch on Wednesday.

■

DEPARTMENT

TCTM E-mail Communications

Timely announcements are sent to our membership using e-mail.

If you have an e-mail address, please be sure it is on file and up-to-date with TCTM. If you do not have an e-mail address, please let us know by indicating this on your membership application. We will attempt to contact you via postcard if there is a crucial issue at hand. TCTM members that have e-mail and have not received e-mail messages from the president, Cynthia Schneider, in the last six months, should contact her immediately at *<cschneider@satx.rr.com>* or by phone at 512-475-9713. Also note, if your server is not accepting our messages due to security, we would like to work with you on this issue.

■

DEPARTMENT

Errata

Texas Mathematics Teacher,
Volume LI, Issue 2, Fall 2004

On page 8, the symbol for pi appears as an inequality sign throughout this page. The integral symbol appears as three horizontal lines. The footer of this issue should be Fall 2004 rather than Spring 2004, the correct date is on the cover. The editor extends her sincere apology for these errors.

■

VOTE

TCTM Candidates

for President

Garland Linkenhoger

Garland Linkenhoger has been an educator since 1980. She has taught students from Kindergarten through Junior College. She has experienced rural, suburban, and urban school districts as a teacher and a coordinator, most recently with McAllen ISD as their Mathematics Coordinator, PK-12. Her experiences at Texas A&M, Sul Ross, and Northern Arizona University have contributed to her knowledge and understanding of how students learn and she avidly seeks research to keep abreast of the newest breakthroughs in education. She has pursued and been awarded several grants to allow her district to explore lab settings during summer for math, the development of teacher leaders and the opportunity for teachers to return to school and continue to develop their mathematics education. She is a TEXTEAMS leader and certified in several Navigations institutes as well as an Investigations, Connected Math Program, and Agile Mind consultant. She also served on the advisory committees for several of the Dana Center's Assessment projects. Her district professional development plan for the integration of technology for math and science teachers was published on the TI website and by District Administrator magazine.

Jo Ann Wheeler

Jo Ann Wheeler is a Managing Director for Education Services with the Region 4 Education Service Center (ESC) in Houston, Texas. She is responsible for providing strategic vision and leadership in the development, implementation, and delivery of Region 4 ESC's comprehensive educational services to its customers in support of the ESC's mission and strategies. Prior to her role as managing director, she served as Director for Mathematic and Science Services with Region 4 ESC and taught secondary mathematics for many years. Jo Ann continues to serve as a principal writer for numerous instructional resources to assist districts in raising standards and student achievement in mathematics. Jo Ann developed and maintains a statewide website, www.mathbenchmarks.org, that contains TEKS aligned performance assessments for grades 3–8, Algebra I, Geometry, and Algebra II.

for Vice-President Elementary

Abigayle Barton

Abigayle Barton is currently a Mathematics Specialist for the Alliance for the Improvement for Mathematics Skills PreK-16, a MSP grant funded by the National Science Foundation. The AIMS project serves nine local school districts, Del Mar College, and Texas A&M University-Kingsville. The overarching goal of the AIMS project is to prepare all students for college level mathematics courses by graduation from high school through vertical alignment, professional development, challenging curriculum, the use of information technology, and research on strategies and interventions. Abigayle also teaches a Master's Level Early Childhood Mathematics Methods course at Texas A&M University-Kingsville. Prior to working for AIMS, Abigayle taught third grade and PreK-5 special education in Corpus Christi, Texas and taught second grade and was a second grade mathematics specialist in Slaton, Texas.

Abigayle's education includes a B.S. in Multidisciplinary Studies with an emphasis in Mathematics from Texas Tech University and an M.S. in Educational Administration from Texas A&M University-Corpus Christi. She is currently working on a doctoral degree in Educational Leadership at Texas A&M University-Kingsville to be completed in December 2005.

Angela Murski

Angela Murski is now in her 20th year of education. During that time, she has taught mathematics in almost every grade level from Pre-Kindergarten to 9th grade. Angela was a Professional Development Specialist for Elementary Mathematics for several years. She has developed many math workshops and has presented many more, including the Texas Math Academy and TEXTEAMS. She has been a contributor for numerous publications. Currently, Angela is the Coordinator of Mathematics in Pflugerville, Texas.

TCTM Candidates

for Treasurer

Kathy Hale

Kathy Hale, mathematics consultant for Region 14 Education Service Center in Abilene, currently serves as the treasurer of the TCTM board of directors. Kathy has been involved in public school education for over 30 years, teaching both middle school and high school students and has served as the president of her local affiliate of NCTM and the treasurer for the Texas Math and Science Coaches Association.

Prudence Cain

Prudence Cain has taught math and science to secondary students for 20 years. She has served as a math specialist for four years in both the Texas Rural Systemic Initiative and the UTeach: Dell Center for New Teacher Success. She has a Masters in Mathematics from West Texas A&M University. She is a member of NCTM, AACTM and TASM. Prudence is a certified TFAS (Technology for All Students) trainer. She has previously served as treasurer of the Amarillo Local TSTA chapter as well as a Dumas Local Girl Scout Troop.

for Southeast Regional Director

vote only if you live in Service Center Region 4, 5, or 6

Paul Gray

Paul Gray currently serves as a Secondary Mathematics Specialist for Region IV Education Service Center. His responsibilities include curriculum development as well as the development and implementation of professional development for secondary teachers of mathematics. He is also an author of instructional materials that provide students with the opportunity to develop a conceptual understanding of mathematics. He is instrumental in the development and evaluation of assessment items that are written at the rigor demanded by TEKS and TAKS.

Paul holds a bachelors' degree from the University of Oklahoma and a masters' degree in Mathematics Education from the University of Houston, where he is currently enrolled in the doctoral program. Paul is also a certified 8-12 Master Mathematics Teacher. During his eight years in the classroom, Paul taught junior high mathematics and science in Chickasha (Oklahoma) Public Schools, and taught high school mathematics and science in Aldine ISD. He has presented professionally at CAMT, NCTM national and regional conferences, and at American Educational Research Association annual conferences, and is currently a member of TCTM, TASM and NCTM.

Gayle Stahl

Gayle Stahl has served as an educator of Texas students for twenty-two years, a total of twenty-nine years in education. She has taught every grade level, K-5; was a principal of a private elementary school for a year; and was an Elementary Mathematics Specialist at Region IV Education Service Center for four years. Currently she is an Elementary Mathematics Specialist for Galena Park ISD.

Gayle is passionate about student achievement in mathematics. In working with students, teachers and administrators, she emphasizes the importance of manipulatives, good questioning strategies and on-going assessment to develop deep, conceptual understanding. Gayle has a strong knowledge base that she conveys in an encouraging manner. Training teachers to deliver conceptual, procedural and factual mathematics is the key, Gayle believes, to a new generation of students who enjoy mathematics and who thrive on life-long problem solving skills.

Lenore Walker

Lenore Walker has 32 years of experience in the field of education as a Texas middle school and high school mathematics teacher, department chair, Testing Coordinator, Secondary Mathematics Instructional Specialist and School Improvement Facilitator. She currently is the School Improvement Facilitator for the North Central Administrative District of the Houston Independent School District. As a School Improvement Facilitator, Lenore works closely with district and campus administrators and teachers to analyze data, provide instructional assistance, training, and support in the development and implementation of innovative teaching techniques and practices that enhance student achievement in the area of secondary mathematics.

Lenore was named the Pasadena Independent School District Secondary Teacher of the Year in 2000. She is a member of TCTM, NCTM, TASM, and NCSM and is currently serving as the 2005 Exhibits Co-Chair for the Conference for the Advancement of Mathematics Teaching (CAMT).

TCTM Candidates

for Southwest Regional Director

vote only if you live in Service Center Region 15, 18, or 19

Rebecca Ontiveros

Rebecca Ontiveros is currently the Director for the Math/Science Partnership, a grant with the El Paso Collaborative for Academic Excellence. The Math/Science Partnership, located at Region 19 ESC, works primarily with secondary mathematics and science teachers in nine rural school districts. She has also been involved with the Texas Math Academies, where she served as a Master Trainer. As part of her responsibilities, she coordinated 22 sessions that served around one thousand 5th and 6th grade mathematics teachers in the Region 19 area. She has also been a K-12 Mathematics Instructional Coordinator at Clint ISD (the largest rural school district in the area). Additionally, she was an Urban Systemic Initiative/Program Mentor for 7 years. The last 12 years of her educational experience have involved, in some capacity, the work of systemic reform in the area of mathematics and science by providing staff development in the area of mathematics. She is currently a member of NCTM, TASM and the Greater El Paso Council of Teachers of Mathematics. She was involved in the coordination of the NCTM Regional conference in El Paso, and is involved with CAMT 2005.

Rita Tellez

Rita Tellez spent ten years working in the world of engineering; she has combined that experience and education for the last fifteen years. Her thirteen years as a high school and middle school math teacher gave her a wealth of experience to share in her present position, district math Facilitator for the El Paso Independent School District. For the last two years she has planned and implemented staff development for high school math teachers. She has been blessed with the opportunity to experience the real world of mathematic applications through her short career as an electrical engineer, and now she is able to make those connections in her career as an educator. She has a B.S. in Electrical Engineering and a Masters in Mid-management from the University of Texas at El Paso. She is a member of NCTM, NCSM, TASM, the Greater El Paso Council of Teachers of Mathematics, Texas Council of Teachers of Mathematics, and the Association for Supervisors of Curriculum and Development (ASCD).

for South Regional Director

vote only if you live in Service Center Region 1, 2, or 3

David McReynolds

David McReynolds is a mathematics specialist with the AIMS PreK-16 grant. AIMS is a National Science Foundation funded MSP grant that works with nine districts in the Corpus Christi area. He worked for two years as a mathematics specialist with the Texas Rural Systemic Initiative in the San Angelo area. He coached and taught for seven years in the Texas panhandle before moving to Boise City, Oklahoma. He coached for three years before moving to the classroom for another thirteen years. He has enjoyed the opportunity to work all over the state of Texas. It has enabled him to see more of what is happening in education across our great state. He has a bachelor's of science degree from West Texas State University in Canyon, Texas and is currently working on a master's degree in educational administration at Texas A&M University-Corpus Christi. He is married to Charlotte, and they have two children; Ryan just graduated from West Texas A&M University and Lauren is currently enrolled in the nursing program at Texas Tech University in Lubbock.

Barba Patton

Barba Patton is an assistant professor at University of Houston-Victoria, where she teaches pre-service and graduate math classes, math and science for the young child, graduate curriculum and undergrad and graduate child development. She is the coordinator for the Center of Academic Excellence for the School of Education and Human Development. As coordinator, she works closely with pre-service teachers to be sure that each master the TExES tests. Barba earned her Doctorate in mathematics education from the University of Houston. Her interest is working with pre-service and in-service teachers, dyscalculia, and visual representations. She also finds it very rewarding to teach math to special needs children. She teaches with manipulatives and savors helping others to learn to use manipulatives in their teaching. She is certified in both elementary and secondary mathematics as well as holding seven other certifications. In addition, she has several others special training certificates. During her tenure in PK-12 teaching, she taught from kindergarten to eighth grade. She has also taught at the community college, technical college and university level. Barba has presented at numerous local, state, regional, national and international math and research conferences. She has published several math articles as well as a study guide to accompany a child development text.

VOTE !

TCTM Ballot

Circle your choices below. Write-in candidate names are acceptable. Copy and mail your ballot to Linda Shaub at the address below. The voting deadline is June 1, 2005.

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☐ Garland Linkenhoger ☐ Jo Ann Wheeler ☐ ____________________
write in candidates

for Vice-President Elementary

☐ Abigayle Barton ☐ Angela Murski ☐ ____________________
write in candidates

for Treasurer

☐ Kathy Hale ☐ Prudence Cain ☐ ____________________
write in candidates

for Southeast Regional Director

vote only if you live in Service Center Region 4, 5, or 6

☐ Paul Gray ☐ Gayle Stahl ☐ Lenore Walker

☐ ____________________
write in candidates

for Southwest Regional Director

vote only if you live in Service Center Region 15, 18, or 19

☐ Rebecca Ontiveros ☐ Rita Tellez ☐ ____________________
write in candidates

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write in candidates

Mail in your ballot

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