Texas Mathematics Teacher

A PUBLICATION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

http://www.tenet.edu/tctm/

Volume L Issue 1 **Spring 2003**

Volunteer for CAMT
sign up by May 9, 2003

TCTM Elections
vote by June 1, 2003

TCTM Annual Breakfast Meeting at CAMT
sign up by June 1, 2003

Texas Council of Teachers of Mathematics
2002-2003 Mission and Goals Statement

MISSION

To promote mathematics education in Texas.

GOALS

Administration
- Investigate online membership registration through CAMT and/or the TCTM website

Publications
- Survey membership to identify what they want in the Texas Mathematics Teacher (TMT)
- Review and redesign the TMT journal and the TCTM website based on above findings

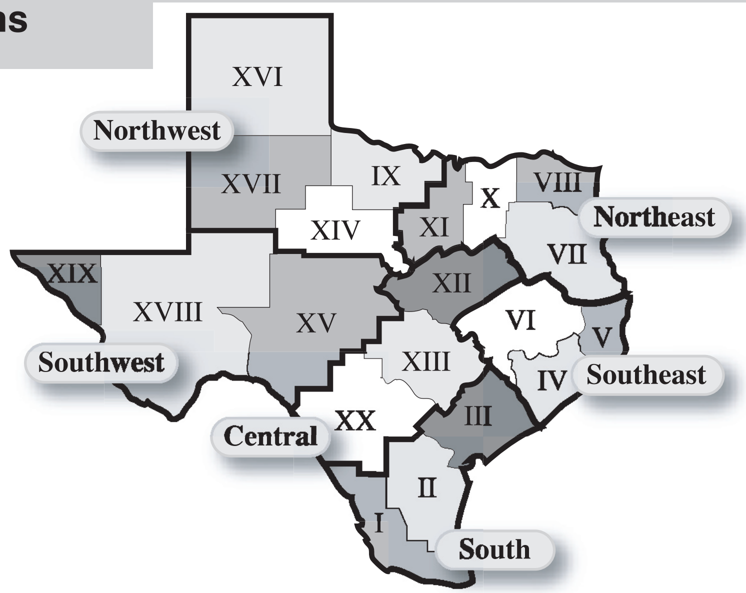
Service
- Increase the number of Mathematics Specialist College Scholarships
- Increase the donations toward Mathematics Specialist College Scholarships
- Staff CAMT registration with volunteers and other volunteers as needed
- Advertise affiliated group conferences on the TCTM website and in the TMT

Communication
- Create and maintain an e-mail list of members for timely announcements
- Communicate with affiliated groups in a timely manner
- Improve communication with NCTM consignment services

Membership
- Based on information gathered by TCTM board members as to advisability, advocate at CAMT Board meetings for TCTM membership to be required for all CAMT participants
- Encourage affiliated groups to include TCTM registration on their membership forms

Public Relations
- Staff and sponsor the NCTM/TCTM booth at CAMT
- Follow NCTM *Communication Guidelines* (1993) for increased media coverage of TCTM membership and issues relevant to our mission

TCTM Regions



TCTM Past-Presidents

1970-1972	James E. Carson	1982-1984	Betty Travis	1994-1996	Diane McGowan
1972-1974	Shirley Ray	1984-1986	Ralph Cain	1996-1998	Basia Hall
1974-1976	W. A. Ashworth, Jr.	1986-1988	Maggie Dement	1998-2000	Pam Alexander
1976-1978	Shirley Cousins	1988-1990	Otto Bielss	2000-2002	Kathy Mittag
1978-1980	Anita Priest	1990-1992	Karen Hall		
1980-1982	Patsy Johnson	1992-1994	Susan Thomas		

Texas Mathematics Teacher

A PUBLICATION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Volume L Issue 1 **Spring 2003**

Articles

Features

Departments

Applications

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Texas Mathematics Teacher, the official journal of the Texas Council of Teachers of Mathematics, is published in the fall and spring. Editorial correspondence should be mailed or e-mailed to the editor.

Call For Articles

The ***Texas Mathematics Teacher*** seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Texas. All readers are encouraged to contribute articles and opinions for any section of the journal.

Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to the editor with a copy to the director. No author identification should appear on or in the manuscript. A cover letter containing author's name, address, affiliations, phone, e-mail address, and the article's intended audiance should be included. After refereeing, authors will be notified of a publication decision.

Teachers are encouraged to submit articles for ***Voices From the Classroom,*** including inspirational stories, exemplary lessons, or management tools. If submitting a lesson, it should include identification of the appropriate grade level and any prerequisites.

Items for ***Lone Star News*** include, but are not limited to, NCTM affiliated group announcements, advertisements of upcoming professional meetings, and member updates

Businesses interested in placing an **advertisement** for mathematics materials should contact Mary Alice Hatchett.

Deadline for submissions: Fall, July 1 Spring, January 1

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Letter from the President

Dear TCTM Members,

With the stress in our daily lives due to the war, the economy and the new accountability system, I'm sure many of us feel like we can't take much more. If you are feeling overwhelmed, reach out to others for support. If you see someone that needs help – fellow teachers or students – don't wait for them to ask, offer your support now. Our family and friends are always there, but colleagues may need to build new relationships in order to handle all the changes and demands that these times bring. Additional support comes from belonging to and communicating through professional organizations such as the Texas Council of Teachers of Mathematics. Through this publication, we hope to bring you the most up-to-date information from the state and exemplary materials to sustain you in the classroom.

April is Mathematics Education Month. The March 2003 Bulletin from NCTM recommends the following website for free resources to celebrate this event with the connection between mathematics and art: mathforum.org/mam/o3. Other recommended websites include:

Elementary level: MegaMath at

http://www.cs.uidaho.edu/~casey931/mega-math

High School: Algebra I for students at

*http://www.bonita.k12.ca.us/schools/ramonda/teachers/carlton/index.htm*l

Introductory Statistics: Exploring Data at

http://exploringdata.cqu.edu.au

All levels: Classroom Calendar at

http://enc.org/thisweek/calendar/

Remember that speaker proposals for the 2004 NCTM annual meeting are due May 1, 2003. The conference next year will be in Philadelphia. Look for the CAMT 2004 speaker proposal forms soon on the website at *http://www.tenet.edu/camt/*. I encourage all members to consider becoming speakers. You all have something worthwhile to share.

The Texas Legislature is in Austin and working hard on budget issues. Considerable funding cuts in education are under review. News from the various education committees may be found online, for example, at

http://www.tasanet.org/depserv/govrelations/78Legislation/capitolwatch/capitolwatch.html

As concerned educators, I recommend that you keep abreast of the news from the Legislature. When teachers express concerns, they should express them proactively rather than reactively. Your opinions do matter to your representatives. A phone call or e-mail is your opportunity to be heard on the issues under discussion.

I recently heard the inspirational show tune "The Impossible Dream" (lyrics by Joe Darion) from *Man of La Mancha*. While listening to the song, I knew now was the time to remind my fellow members that every day teachers dream the impossible dream, fight the unbeatable foe, and right un-right-able wrongs. However hard it may be, under whatever circumstances we must work, we are all trying to build a safe and happy future for our children. Keep up the good work!

Sincerely,

Cynthia L. Schneider
TCTM President 2002-2004

Volunteer by May 9, 2003 to work at CAMT 2003 in Houston!

All members of TCTM should take an active role to help make CAMT successful. Come work 'behind the scenes' to facilitate the experience for all participants. Please examine the times below and volunteer to serve. There are two locations for volunteers to work. We need you! Identify the time slot(s) when you can help and e-mail the date and time(s) on or before May 9, 2003 to Cynthia Schneider by a method listed below. We will confirm your assigned time via e-mail or phone on or before June 30.

Volunteer Information

Name: ____
Last First Middle

Address: ____
Number and street Apt. number

____
City Zip Code

(___) ______________ (___) ______________ < ______________ >
Home Phone Work Phone Email Address

Distrcit or Professional Afffiliation ______________ ESC ______

Registration Area

Wednesday July 16

1:30 p.m. – 3:30 p.m.	3:30 p.m. – 5:00 p.m.

Thursday July 17

6:45 a.m. – 9:00 a.m.	9:00 a.m. – 11:00 a.m	11:00 a.m. – 1:00 p.m.	1:00 p.m. – 3:00 p.m.

Friday July 18

7:15 a.m. – 9:00 a.m.	9:00 a.m. – 11:00 a.m.	11:00 a.m. – 1:00 p.m.	1:00 p.m. – 3:00 p.m.

Saturday July 19

7:15 a.m. – 9:00 a.m.	9:00 a.m. – 11:00 a.m.	11:00 a.m. – 1:00 p.m.	1:00 p.m. – 3:00 p.m.

Exhibits Area –NCTM Materials Booth

Thursday July 17

9:45 a.m. – 12:00 p.m.	12:00 p.m. – 2:00 p.m.	2:00 p.m. – 5:00 p.m.

Friday July 18

7:45 a.m. – 10:00 a.m.	10:00 a.m. – 12:00 p.m.	12:00 p.m. – 2:00 p.m.	2:00 p.m. – 5:00 p.m.

Saturday July 19

7:45 a.m. – 10:00 a.m.	10:00 a.m. – 12:00 p.m.	12:00 p.m. – 2:00 p.m.

Please submit your form

by mail: **Cynthia Schneider,**
234 Preston Hollow,
New Braunfels, TX 78132

by fax: **(512) 232-1855**
ATTN: Cynthia Schneider

by email: ***<cschneider@mail.utexas.edu>***

Letter from Award Recipients

Dear TCTM Members,

Each year, the Texas Council of Teachers of Mathematics recognizes two individuals for their leadership in mathematics education. In 2002, we were honored to be the recipients of these two awards:

Judy Kelley from West Texas A&M University in Canyon and Executive Director of the NSF-funded Rural Systemic Initiatives received the E. Glenadine Gibb Achievement Award for her contributions to the improvement of mathematics education at the state and/or national level; and

Janie Schielack from Texas A&M University in College Station and Director of the NSF-funded Information Technology in Science (ITS) Center for Teaching and Learning received the TCTM Leadership Award for her contributions to the improvement of mathematics education at the local and state level through innovative staff development design.

We would like to take this opportunity to thank the membership of TCTM for creating the professional environment that has made it such a pleasure to be involved in mathematics education in Texas. The foundation of effective leadership lies with the people who are directly charged with the task of implementing quality instruction to develop future citizens who are critical thinkers and educated decision makers. We are grateful to be a part of this many-faceted process, and we look forward to having continued opportunities to work together to improve mathematics education.

Sincerely,

Judy Kelley and Janie Schielack

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Shelter Volunteer

Thousands of children pass through homeless shelters each year in Texas. A significant number of these children are behind academically because of mobility, domestic violence, and other trauma in their lives. Current and retired teachers who seek a fulfilling volunteer opportunity can find it by providing tutoring for these children after school, on week-ends, or during the summer. For information about how you can volunteer, please contact the Texas Homeless Education Office at 1-800-446-3142.

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Coming Soon! Problem Solving for Middle School Mathematics Teachers

Teachers who attend this new TEXTEAMS Institute will not only learn how to help their students use and master problem solving, but they will also deepen their own content knowledge and leave with a broader understanding of what problem solving means. One of the special features of the Problem Solving Institute is a set of 30 lessons written by middle school mathematics teachers, mathematics supervisors and university mathematics educators which will provides teachers with extra support as they incorporate problem solving into their daily lesson plans.

This institute in problem solving will be part of the TEXTEAMS Institutes for K-12 mathematics and science teachers, developed, supported and coordinated through the Charles A. Dana Center at the University of Texas at Austin. The training will give teachers ample and varied opportunities to explore how and where problem solving pervades and connects the TEKS, and why and how students must use and master problem solving to be successful in mathematics. Because skillful problem solving ensures success in all content areas of mathematics, teaching students the processes that problem solving demands will better prepare them for higher mathematics courses. It will also better prepare them for the more challenging TAKS.

Middle school mathematics teachers who are looking for meaningful, relevant and timely professional development on problem solving should call your regional Education Service Center, visit the TEXTEAMS website at http://www.texteams.org/ or call Bonnie McNemar (512 232-5997), Program Coordinator, Mathematics Team, Charles A. Dana Center.

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Affiliate Group News

If you are actively involved with your local affiliated group, please send future meeting and conference information to *<cschneider@mail.utexas.edu>* so we may publicize your events. Contact information for each group is available on the NCTM website, *http://www.nctm.org/*.

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Reviewers

The quality of the Texas Mathematics Teacher very much depends on the voluntary efforts of many mathematics educators. Each article that appears in TMT is judged for content and style by at least three referees, who volunteer their time and expertise. TCTM members who are interested in refereeing 2 or 3 manuscripts each year should contact Cynthia L. Schneider, TCTM President, 2002-04; <*cschneider@mail.utexas.edu*> or Mary Alice Hatchett, Director of Publications Texas Mathematics Teacher; <*mahat@earthlink.net*> .

The efforts of all of these referees in maintaining the high quality of the Texas Mathematics Teacher are very much appreciated.

Cindi Beenken	Barbara Montalto
Patti Bridwell	Erika Pierce
Kathy Cook	Ullrich Reichenbach
Lee Holcombe	Linda Shaub
Bill Hopkins	Jim Telese
Bill Jasper	Alicia Torres
Kathleen Mittag	Jacqueline Weilmuenster

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CAMT 2003 & NCTM 2004

The 50th Anniversary of the Conference for the Advancement of Mathematics Teaching (CAMT) is scheduled for July 17-19, 2003 at the George R. Brown Convention Center in Houston. For registration, housing and program information visit the CAMT website:

http://www.tenet.edu/camt/

Expand your opportunities by joining us at the 82nd NCTM Annual Meeting in Philadelphia, Pennsylvania April 21-24, 2004. If you attended the conference in 2002, you should have received a registration booklet in March. For the most up-to-date information or to register online, visit

http://www.nctm.org/meetings/

or call (800) 235-7566.

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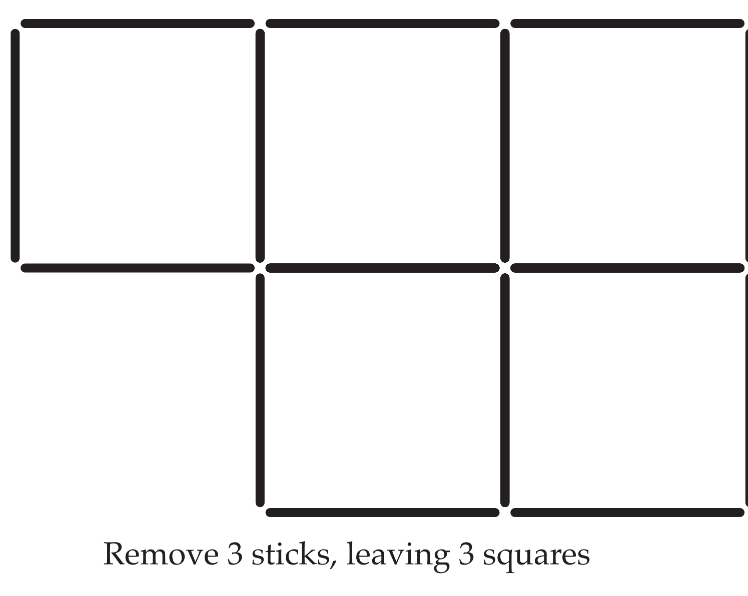
Puzzle Corner

Sticks #1

We are interested in how your students responded to this problem and how they explained or justified their reasoning. Please e-mail copies of your students' work, include your name, grade level, campus name and district name by July 1, 2003 to Mary Alice Hatchett, Director of Publications *Texas Mathematics Teacher*. Selected submissions will be acknowledged and published in subsequent issues.

Please prepare a sketch of your solution

Arrange 15 craft sticks to form 5 squares as shown



Remove 3 sticks, leaving 3 squares

TCTM Leadership Award Application

Eligibility: The TCTM Leadership Award is presented to a TCTM member who is nominated by a TCTM Affiliated Group. This person is to be honored for his/her contributions to the improvement of mathematics education at the local and state level. He/she has designed innovative staff development and has promoted the local TCTM Affiliated mathematics council.

Deadline: May 1, 2003

Information about the **affiliated group nominating a candidate**:

Name: ____
Last First Middle

Address: ____
Number and street Apt. number

____
City Zip Code

(___) ______________ (___) ______________ < ______________ >
Home Phone Work Phone Email Address

Are you a member of TCTM? ☐ yes ☐ no NCTM? ☐ yes ☐ no

Information about the **person being nominated**:

Name: ____
Last First Middle

Address: ____
Number and street Apt. number

____
City Zip Code

(___) ______________ (___) ______________ < ______________ >
Home Phone Work Phone Email Address

Is the nominee a member of TCTM? ☐ yes ☐ no NCTM ☐ yes ☐ no Retired ☐ yes ☐ no

Applications should include 3 pages:

☐ Completed application form

☐ <u>One</u>-page, <u>one</u>-sided, typed biographical sheet including:
- Name of nominee
- Professional activities
- State/local offices or committees
- Activities encouraging involvement/improvement of math education
- Staff Development
- Honors/awards

☐ <u>One</u>-page, <u>one</u>-sided essay indicating why the nominee should be honored for his/her contribution to the improvement of mathematics education at the state/national level.

Submit the completed application, biographical sketch, and essay to

by mail: **Cynthia Schneider,**
234 Preston Hollow,
New Braunfels, TX 78132

by fax: **(512) 232-1855**
ATTN: Cynthia Schneider

by email:
<cschneider@mail.utexas.edu>

Pitfalls of Over-Reliance on Cross Multiplication as a Method to Find Missing Values

• *Dick Stanley, Ph.D.*
• *Diane McGowan*
• *Susan Hudson Hull, Ph.D.*

The following article is one of several that are included in the Texas Mathematics Academy for 7th and 8th-grade teachers. These articles are also available through the resource section of the Mathematics TEKS Toolkit: *http://www.mathtekstoolkit.org/* .

1. If Kobe bought 17 gallons of gasoline for $23.63, how much would he have to pay for 11 gallons?

In a traditional approach used for many centuries to solve problems like this, the problem situation is represented as an equality of two ratios of cost to amount:

$$\frac{c_1}{q_1} = \frac{c_2}{q_2}$$

In school mathematics, such an equation is often called a "proportion". Filling in the 3 numerical values gives this equation:

$$\frac{23.63 \text{ dollars}}{17 \text{ gallons}} = \frac{c_2 \text{ dollars}}{11 \text{ gallons}}$$

To solve this equation for c_2, students are taught to "cross multiply". This means taking each denominator and multiplying it by the numerator of the other side of the equation. (It is equivalent to multiplying each side of the equation by the product of the denominators, and canceling.) Cross-multiplying gives the product 23.63•11 = 259.93. and the equivalent equation:

$$259.93 = c_2 \bullet 17$$

Solving this equation for the cost c_2 gives the correct answer, $15.29.

A difficulty with using this as the standard or preferred approach to solving such a problem is that the number created by cross multiplying (in this case the number 259.93) has no relevance for the problem situation. Even its units ("dollar gallons") are meaningless.

This would not be so bad, perhaps, except that this cross-multiplying approach *prevents* students from seeing a number that *does* have a great deal of relevance for the problem situation, namely the unit price. To show what we mean, let us illustrate an alternative approach.

In this approach, we divide the cost ($23.63) by the amount of gasoline (17 gallons) it buys to give the unit price: $1.39 per gallon. This is an important quantity in the problem situation. It is the constant value that each ratio above is equal to:

$$\frac{23.63 \text{ dollars}}{17 \text{ gallons}} = \$1.39 \text{ per gallon}$$

To find the unknown value c_2 dollars we multiply the unit price $1.39 per gallon by the given number of gallons, 11, to find the price of 11 gallons: $15.29.

$$1.39 \text{ dollars/gallon (11 gallons)} = 15.29 \text{ dollars}$$

Notice that computing the unit price gives us an important and useful piece of information about the problem situation in general. If we were asked a variant of the problem (say, how much would 14 gallons cost?), we could solve it easily: just multiply 14 by the unit price $1.39 per gallon we have already found. On the other hand, if we had solved the original problem by cross multiplying, to solve this variant problem we would have to start from scratch (and in the process, we would create another number that has meaningless units and that is useless in any other version of the problem).

Another variant of the original problem might give the same information and ask how many gallons Kobe could buy for some amount of money (say $20.00.):

2. If Kobe bought 17 gallons of gasoline for $23.63, how much could he get for $20.00?

For this problem, if we knew the "unit amount" (how much gasoline he could get for 1 dollar), we could easily answer the question. This suggests we set up an equation using the *reciprocals* of the ratios in the equation above.

$$\frac{17 \text{ gallons}}{23.63 \text{ dollars}} = \frac{q_2 \text{ gallons}}{20.00 \text{ dollars}}$$

Here, if we divide 17 gallons by \$23.63 we get the unit amount, 0.719 gallons per dollar. It is the constant value that each ratio above is equal to:

$$\frac{17 \text{ gallons}}{23.63 \text{ dollars}} = 0.719 \text{ gallons per dollar}$$

To find the unknown value q_2 gallons we multiply the unit amount by the given number of dollars, 20, to find the amount that \$20.00 will buy: about 14.38 gallons.

$$0.719 \text{ gallons/dollars (20 dollars)} = 14.38 \text{ gallons}$$

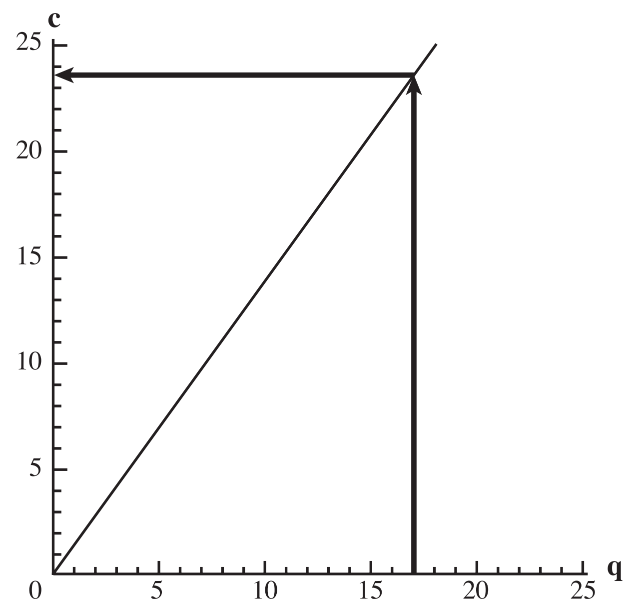
This second problem could have also been solved by cross-multiplying, but using this approach would have involved the same sort of drawbacks as we saw using cross multiplying in the first problem. Further, this approach would not have shown any relationship between the two problems. On the other hand, using the unit price and the unit amount, as we illustrated above, allows us to see clearly the connection between the two problems: namely, these unit amounts are reciprocals of each other:

$$\frac{1}{0.719} \approx 1.39 \text{ dollars per gallon}$$

$$\frac{1}{1.39} \approx 0.719 \text{ gallons per dollar}$$

That this is necessarily true follows from the very meaning of each of these quantities.

As a final remark we note that all problems of the type illustrated above can also be solved with a graph. In the case of problem 1 we would mark the point (q, c) = (17, 23.63), and then draw a line through this point and (0, 0). A line drawn up from q = 17 to the graph and then over to the vertical axis locates the answer c = \$23.63.



Note that the slope of this graph is just the unit price, \$1.39 per gallon, found above. This graph is the graph of a linear function $c = 1.39 \bullet q$ that represents the general proportional relationship between cost and amount. The unit price method thus connects nicely with the work with algebra and functions that students will soon be immersed in. Another disadvantage of the cross multiplying method is that it is essentially a dead end that does not lead to representations in terms of graphs and functions.

For centuries the above sorts of problems have been called "rule of three" problems: three values are given and a fourth is sought. Cross-multiplying has been a method for solving such problems learned by many generations of school children. It arose in a context that valued rote methods applied without thought to solve certain standard problem types, and in an era before functions and their graphs were used at all in mathematics. It is surprising how persistent this method is even today given the change in times, the fundamental drawbacks of the method, and the arrival on the scene of functions and graphs as key algebraic concepts.

In addition, research (Hull, 2000) shows that although teachers can set up problems similar to those above and find an answer by cross multiplying, many have no idea how to interpret their answers because units were left out of the problem solving process. Moreover, units cannot be included since units found in the intermediate step in cross multiplying are meaningless. Another research study (Nunes, Schliemann, Carraher, 1993) showed that students can be taught cross multiplication, but then quickly forget the method and instead rely on other methods that make more sense to them when asked to solve problems in the real world.

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Susan Hudson Hull, Ph.D. • Charles A Dana Center.
<shhull@mail.utexas.edu>

References

Hull, S.H. (2000). Teachers' mathematical understanding of proportionality: Links to curriculum, professional development, and support. Unpublished doctoral dissertation, University of Texas, Austin.

Nunes, T., Schliemann, A.D., & Carraher, D.W. (1993). Street mathematics and school mathematics (pp. 77-126). New York: Cambridge University Press.

CAMTership Application

Eligibility: Five $200 CAMTerships will be awarded to teachers with five or fewer years teachig experience who are members of TCTM and have not attended CAMT before. CAMTerships will be awarded to teachers in each of the following grade levels: K - 4, 5 - 8, and 9 - 12. Winners will be determined by random drawing of names and will be notified by June 1, 2003. Winners will be asked to work for two hours at registration or the NCTM material sales booth and will be TCTM's guest at our breakfast, where the checks will be presented. Good luck!

Deadline: May 1, 2003

Name: ______________________________
Last First Middle

Address: ______________________________
Number and street Apt. number

____
City Zip Code

(___) ____________ (___) ____________ < ____________ >
Home Phone Work Phone Email Address

Are you a member of TCTM? ☐ yes ☐ no Have you attended CAMT before? ☐ yes ☐ no

Note: If you are not a member of TCTM, you must enclose $13 with this application to apply for membership.

How long have you been teaching? _____________ Describe your primary teaching responsibilities

Send your completed application to:

by mail: **Cynthia Schneider,**
234 Preston Hollow,
New Braunfels, TX 78132

by fax: **(512) 232-1855**
ATTN: Cynthia Schneider

by email: ***<cschneider@mail.utexas.edu>***

Patterns and the Power of the Variable

• *Murray H. Siegel*

Students in middle school and high school mathematics classes spend a significant amount of time manipulating variables. While adding, subtracting, multiplying and factoring polynomials, or while solving equations, they ask why do we have to know about variables? The response often heard is that these skills are needed in a future unit or a future course. In today's classroom, teachers require a better "motivator" than "the skills are necessary in future study." Students are fascinated by patterns, especially ones that they discover. This article provides examples of patterns that can be used to demonstrate the importance of being able to work with variable expressions.

The sum of the first ten terms of a Fibonacci-like sequence

This investigation can begin with a look at the Fibonacci sequence. The first two numbers are both one. All other numbers in the sequence are found by adding the previous two numbers (symbolically $a_n = a_{n-1} + a_{n-2}$). Thus the third number in the sequence is 2 (1 + 1), the fourth number is 3 (2 + 1) and the fifth number is 5 (3 + 2). Depending on the level of the class, the teacher might want to spend time talking about Leonardo of Pisa, who is known as Fibonacci, or about applications of the sequence, but neither discussion is necessary. We want to introduce the students to the Fibonacci method for obtaining the next number in a sequence.

What if we chose numbers other than one and one to start this sequence? Now we would have a "Fibonacci-like" sequence. Allow each student to choose two one-digit numbers. These will form the first two numbers in a ten-number sequence. The third number in the sequence is found by adding the first two numbers. Each succeeding number is calculated by finding the sum of the two preceding numbers in the sequence. When the tenth number in the sequence is determined, the sequence is complete. For example, if the first two numbers were 2 and 6, the sequence would be 2, 6, 8, 14, 22, 36, 58, 94, 152, 246. Each student should then find the sum of the ten numbers in his/her sequence. The sum of the sequence 2, 6, 8, 14, 22, 36, 58, 94, 152, 246 is 638.

Students should then compare their own sequences and sums with those of other students to see if a pattern emerges. With a little guidance, the class can discover that the sum is eleven times the seventh number. Will the sum always be eleven times the seventh number? In mathematics, examples are not proof of a conjecture. If we use a and b as our first two numbers, the sequence is a, b, a + b, a + 2 b, 2 a + 3 b, 3 a + 5 b, 5 a + 8 b, 8 a + 13 b, 13 a + 21 b, 21 a + 34 b. The sum of this sequence is 55 a + 88 b. Eleven times the seventh term (5 a + 8 b) is 55 a + 88 b. Therefore we have proven that no matter what the first two numbers are, the sum of the first ten terms of <u>any</u> Fibonacci-like sequence will always equal the product of eleven and the seventh term. Without the use of variables we could not have determined this.

The triangular numbers

The sequence 1, 3, 6, 10, 15, 21… is called the "triangular numbers." Figure 1 demonstrates why this sequence is called the triangular numbers. The sequence is formed as follows:

1
1 + 2 = 3
1 + 2 + 3 = 6
1 + 2 + 3 + 4 = 10
1 + 2 + 3 + 4 + 5 = 15
1 + 2 + 3 + 4 + 5 + 6 = 21
…And so on.

It is apparent that the next triangular number is 21 + 7 = 28, but what about the 100th triangular number or the 250th triangular number? It would require a great deal of work to complete all of that adding. The n th triangular number can be computed using 1 + 2 + 3 + 4 + …+ n, an arithmetic sequence whose first term is 1 and whose last term is n. Students who have not yet seen the method for finding the sum of an arithmetic sequence can certainly discover it here.

The sum of the first and the last term is n + 1. The sum of the second and the next-to-last term is 2 + (n – 1) = n + 1. In fact, there are n/2 pairs of numbers whose sum is n + 1. Thus the sum of those n numbers must be $\frac{n}{2}(n+1)$. The 100th triangular number is $\frac{100}{2}(101) = 5050$. The 250th triangular number is $\frac{250}{2}(251) = 31,375$. Once again, the use of a variable has permitted us to find answers with a minimum amount of effort.

Adding adjacent triangular numbers

Given the triangular numbers, 1, 3, 6, 10, 15, 21, 28… adding two adjacent numbers (3 + 6, 15 + 21, 21 + 28) will provide a square number. A square number is a whole number that is the product of a number and itself (the number squared). Will any two adjacent triangular numbers sum to be a square number? Earlier in this article we determined that the formula for the n th triangular number is $\frac{n}{2}(n+1)$. Thus the k th triangular number is $\frac{k}{2}(k+1)$ and the (k+1) th triangular number is $\frac{k+1}{2}((k+1)+1) = \frac{k+1}{2}(k+2)$.

Adding these adjacent triangular numbers:

$$\frac{k}{2}(k+1) + \frac{k+1}{2}(k+2)$$
$$\frac{1}{2}k^2 + \frac{1}{2}k + \frac{1}{2}k^2 + \frac{3}{2}k + 1$$
$$k^2 + 2k + 1 = (k+1)^2$$

Thus, the sum of any two adjacent triangular numbers will be a perfect square.

While the author was conducting this activity some years ago at a summer honors program, an eighth grade student raised her hand and noted that it appeared that the difference between two adjacent triangular numbers was the square root of the square number formed by their sum. To prove that conjecture the class looked at the difference between the k th and (k+1)th triangular numbers. That difference is $\left(\frac{1}{2}k^2 + \frac{3}{2}k + 1\right) - \left(\frac{1}{2}k^2 + \frac{1}{2}k\right) = k + 1$. We have now shown that the difference between two adjacent triangular numbers is the square root of the sum of those two numbers. We have obtained two generalizations through the use of a single variable, k. In this particular activity we have added, subtracted, multiplied and factored variable expressions in an effort to discover if our two observations are always true.

Polygonal numbers

Polygonal numbers are sequences of numbers whose geometric representations form a sequence of polygons (NCTM 1993). The triangular numbers form a polygonal set (see Figure 1). The square numbers, 1, 4, 9, 16, 25, 36…, another set of polygonal numbers (see Figure 2), can be written as n^2. There are some interesting patterns when we examine first and second differences for triangular and square numbers. The results for the first six triangular and square numbers are shown in Table 1.

Figure 1
the first four triangular numbers

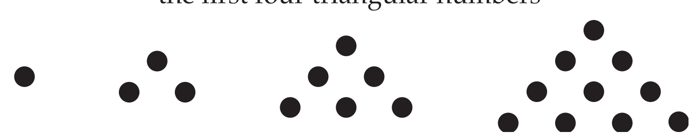

Figure 2
the first four square numbers

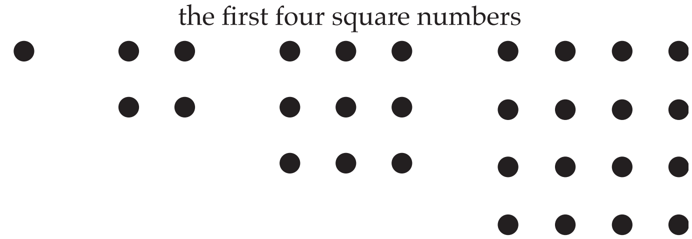

Table 1
Differences and 2nd Differences
For Triangular and Square Numbers

Triangular

n	1	2	3	4	5	6
n^{th} term	1	3	6	10	15	21
1^{st} difference	2	3	4	5	6	
2^{nd} difference		1	1	1	1	

Square

n	1	2	3	4	5	6
n^{th} term	1	4	9	16	25	36
1^{st} difference	3	5	7	9	11	
2^{nd} difference		2	2	2	2	

Both sequences have a constant second difference, which is two less than the number of sides (triangular is 3 – 2 = 1, square is 4 – 2 = 2). In the first century, Nichomachus identified the first six pentagonal numbers as 1, 5, 12, 22, 35 and 51 (NCTM 1993). Just as the triangular numbers formed triangles and the square numbers formed squares, the pentagonal numbers form pentagons. Note that the common second difference is three. A sequence with a common second difference can be written as a quadratic function where the n^{th} term = $an^2 + bn + c$. By using the first three terms of such a sequence, we can solve three simultaneous equations for a, b and c. The first three terms of the pentagonal numbers are used to create three ordered pairs in the form (n, n^{th} term). Those ordered pairs are (1, 1), (2, 5), (3, 12). The three equations are written:

$$n^{th}\text{ term} = an^2 + bn + c$$
$$1 = a + b + c$$
$$5 = 4a + 2b + c$$
$$12 = 9a + 3b + c$$

The solution to the three simultaneous equations is $a = \frac{3}{2}$, $b = -\frac{1}{2}$, $c = 0$. Thus, the n^{th} pentagonal number is $\frac{3}{2}n^2 - \frac{1}{2}n$. We can check this form because the fourth pentagonal term is 22. Using the general term for a pentagonal number we compute the fourth term to be $\frac{3}{2}(16) - \frac{1}{2}(4) = 24 - 2 = 22$, which is correct. In this activity we have used four variables to confirm a pattern. We have also worked with quadratic equations and solved systems of linear equations.

Table 2, which displays the first six numbers in the triangular, square, pentagonal, hexagonal and heptagonal sequences, reveals another pattern among the various polygonal numbers. Each vertical column appears to have a unique common difference. Table 3 displays the common vertical differences. It appears that the common vertical difference for the n^{th} column is the $(n-1)^{th}$ triangular number. The proof of this conjecture is left to the reader or to your students.

Table 2
An Array of Polygonal Numbers

Number of sides \ n	1	2	3	4	5	6
3	1	3	6	10	15	21
4	1	4	9	16	25	36
5	1	5	12	22	35	51
6	1	6	15	28	45	66
7	1	7	18	34	55	81

Table 3
Vertical Differences

n	common vertical difference
1	0
2	1
3	3
4	6
5	10
6	15

Students at all grade levels are fascinated by patterns. Patterns are seen but are they always true? Through the use of variables, students are able to demonstrate the truth of their observations about patterns. Along the way students have manipulated polynomials, worked with quadratic equations and solved simultaneous equations. We have looked at a few interesting patterns. The sum of Fibonacci-like sequences could be utilized in a pre-algebra class. The patterns involving triangular numbers would be useful in an Algebra I class. The investigation of figurate number sequences should be helpful in an Algebra II or Pre-calculus course.

Of course, the patterns described in this article are not the only ones that could be used. They are the ones that the author has used successfully in his classes. Variables are no longer "stuff" we learn in mathematics class. They are significant tools that can be of assistance in answering the questions "Does this pattern always work?" and "Why does this pattern work?" Fewer students ask, "When will we ever use this?" once they have discovered the power of the variable in generalizing conjectures.

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References

National Council of Teachers of Mathematics (NCTM), Historical Topics for the Mathematics Classroom. Reston, Va.: NCTM, 1993.

TCTM Mathematics Specialist Scholarship

Eligibility: Any student who will graduate in 2003 from a Texas high school - public or private - and who plans to enroll in college in Fall 2003 to pursue a career in mathematics teaching either as a mathematics specialist in elementary school or as a secondary school teacher with certification in mathematics.

Deadline: May 1, 2003 **Amount:** $1000

Name: ____
Last First Middle

Address: ____
Number and street Apt. number

____
City Zip Code

(___) ______________ (___) ______________ < ______________ >
Home Phone Work Phone Email Address

Social Security #: ______________ Birth date: ______________ High school(s) attended: ______________

What college or university do you plan to attend? If you are awarded this scholarship, TCTM's treasurer will send a check directly to the business office of the college. We need the college's complete address.

Enclose the completed application with each of the following in the same envelope and mail to the address listed below. YOU MUST INCLUDE 3 COPIES OF ALL REQUIRED MATERIALS.

1. On a separate sheet, list high school activities including any leadership positions.
2. Official high school transcript
3. Letter of recommendation from a TCTM member
4. An essay describing your early experiences learning mathematics and any experiences explaining mathematics to your classmates or friends. This essay must be no more than two pages, double-spaced.
5. An essay telling why you want to be a mathematics specialist in elementary school or a mathematics teacher in middle or high school. This essay must be no more than one page, double-spaced.

Please submit all materials in one envelope to:

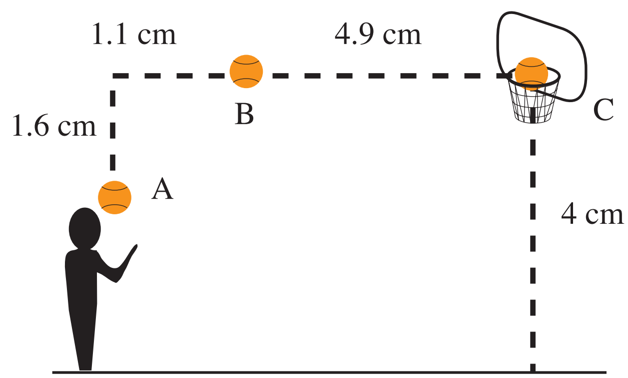
by mail: **Cynthia Schneider**
234 Preston Hollow
New Braunfels, TX 78132

by fax: **(512) 232-1855**
ATTN: Cynthia Schneider

The following assessment sample "Basketball Throw" is from the upcoming publication *Algebra II Assessments* by the Charles A. Dana Center supported by funding, in part, from TEA. The complete document will be available in print form, on CD, and on the Dana Center web site at *http://www.utdanacenter.org/* in August of 2003.

Basketball Throw

A student, Foe Tagrafer, was taking yearbook pictures at the district championship basketball game. While Ray Bounder was taking the game-winning free throw, Foe took three pictures. The ball was athe position A, B, and C on the diagram below when the pictures were taken. He knew that the distance from Ray's feet to the base of the goal was actually 15 feet. He created a diagram from the three photographs. The diagram shows the measurements he was able to take from the photographs.



1. Using the scale diagram, determine the actual measurements for each situation:

Picture	Horizontal Distance before (-) or after (+) the front of the goal	Vertical Distance above (+) or below (-) the goal
A		
B		
C		

2. Find a quadratic function to model the relationship between these two quantities:
    - Horizontal distance before or after the goal
    - Vertical distance above or below the goal

3. Graph the function. What windows did you use? Justify your choice.

4. For these values, find how high off the ground the ball gets at its maximum height.

Teacher Notes:

Materials:
Graphing calculator

Connections to Algebra II TEKS:
d1A, d1B, d3A, d3D

Texas Assessment of Knowledge and Skills:
1, 5

Scaffolding Questions:

- Describe how to find actual measurements from the picture measurements.
- How would you position a coordinate system on the picture?
- What kind of function would describe the trace of the ball in the picture?
- If your function does not measure height of the ball from the ground, how can you use your function to determine this height?

Sample Solution:

1. Because the diagram is drawn to scale the ratio of the distances in the diagram to the actual distances is 6 to 15.

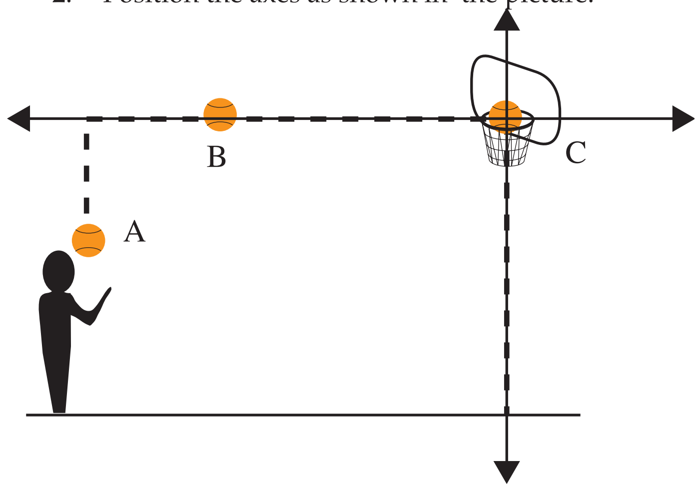
Point A:

$$\frac{6}{15} = \frac{1.6}{a}$$
$$a = 4$$

Point B:

$$\frac{6}{15} = \frac{4.9}{b}$$
$$b = 12.25$$

2. Position the axes as shown in the picture.



The parabola will pass through three points: (-15,-4), (-12.25,0), and (0,0).

The equation of the parabola is of the form
$y = ax^2 + bx + c$.

Substituting point (0,0):
$0 = a(0) + b(0) + c$
$c = 0$

Use the other two points to get a system of equations.

$$-4 = a(-15)^2 + b(-15)$$
$$0 = a(-12.25)^2 + b(-12.25)$$

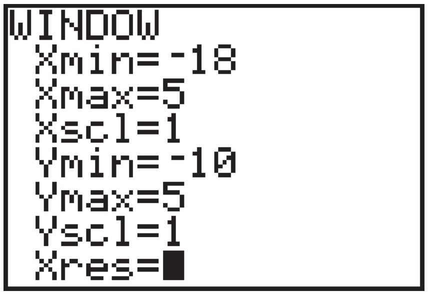
Simplify the two equations and solve for a and b.

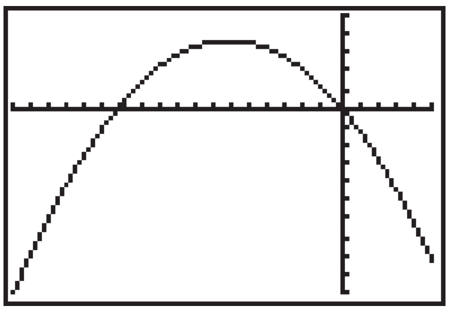
$$-4 = 225a - 15b$$
$$0 = 150.0625a - 12.25b$$
$$b = \frac{150.0625}{12.25}a = 12.25a$$
$$-4 = 225a - 15(12.25a)$$
$$-4 = 225a - 183.75a$$
$$-4 = 41.25a$$
$$a = -0.097$$
$$b = 12.25a = 12.25(-0.097) = -1.188$$

The equation is $y = -0.097x^2 - 1.188x$.

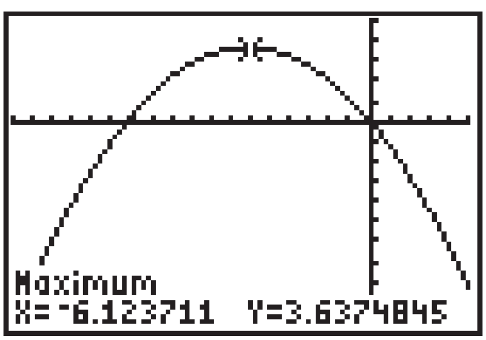
Picture	Horizontal Distance before (-) or after (+) the front of the goal	Vertical Distance above (+) or below (-) the goal
A	-15	-4
B	-12.25	0
C	0	0

3. The graphing window was selected based on the values in the table. The range of values for x is from -15 to 0 and the range of values for y is from -4 to about 5.





4. The high point may be obtained by finding the maximum point on the graph. The maximum y-value occurs at x = -6.123 or when the horizontal distance of the ball from the goal is is 6.123 feet. The y-value of this point must be added to the height of the basketball to determine the distance from the ground.
   3.637 + 10 or 13.637 feet above the ground



Extension Questions:

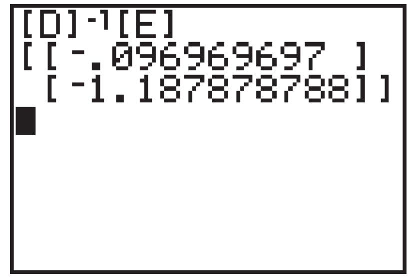
- Describe another approach to solving the system of equations.

The system could be solved using matrices.

$$D = \begin{bmatrix} (-15)^2 & -15 \\ (-12.25)^2 & -12.25 \end{bmatrix} \qquad E = \begin{bmatrix} -4 \\ 0 \end{bmatrix} \qquad X = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$DX = E$$

$$X = D^{-1}E$$

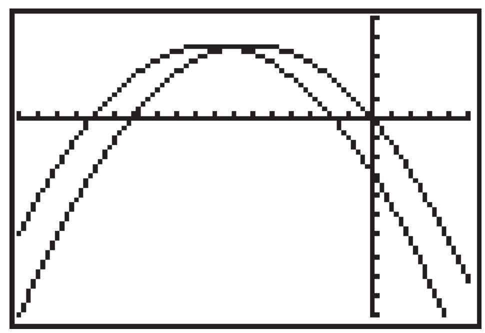


a= -0.097 and b = -1.188

- If Ray Bounder had stepped two feet back (or away from the goal) and thrown the ball in exactly the same manner would the ball land in the basket?

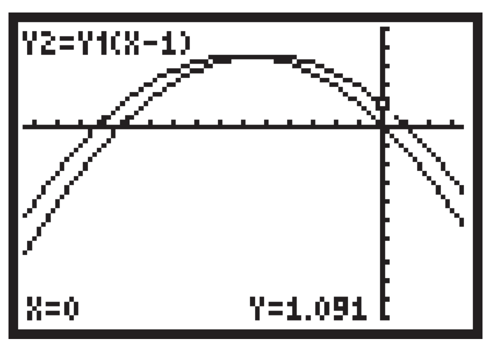
The graph is shifted horizontally two units to the left. The equation is $y = -0.097(x+2)^2 - 1.188(x+2)$.

The new graph and the orignal graph are shown below. The ball would travel below the goal.



- What if instead of stepping two feet to the left Ray had stepped one foot closer to the basket and thrown the ball in the same manner.

The equation is $y = -0.097(x - 1)^2 - 1.188(x - 1)$. It appears that the ball would be 1.091 feet above the goal.



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Mathematical Mysteries: Grab Bag Activities for Estimation, Measurement and Problem Solving

• *Sharon Taylor*
• *Kathy Mittag*

Introduction

Children and adults of all ages enjoy a good mystery. The simple act of putting items into a brown paper lunch bag ignites the minds of those looking into the bags. By using common, everyday items, teachers can create excitement in the classroom. Each of the activities described involves an aspect of the unknown. Each one of the three activities can be used as a stand-alone classroom activity or in conjunction to carry the "unknown" theme across longer time periods.

Activity 1: "It's In The Bag" Mystery

Before class begins, place ten to 15 items in a paper lunch bag for each student (or for every two students if you prefer they work in pairs). To begin, have students pull out objects from the bag one at a time. They are to estimate the length of each object in centimeters and then record their estimate on the "It's In The Bag" Mystery worksheet, in ink so they are not tempted to change their estimates. They continue this process until the bag is empty. At that point, you give them a ruler and let them find the actual measures and record those values. Students should then find the difference for each item and then find their total difference. All differences should be reported as positive, so underestimated and overestimated differences are both reported in the same way. Prizes can be given for the lowest, median, or highest total difference. Suggested items for the bags: unsharpened pencils, straws, paper clips, notepad paper, make up sponges, twist ties, plastic spoons, pennies, tootsie rolls, and crayons. If the metric measurement system is being studied, having the students estimate and measure in centimeters is wonderful practice.

The "mystery" in this activity comes in two ways. First, students enjoy putting their hand in the bag and <u>then</u> pulling out a mystery item. Because the students cannot see the items, you must be careful in your selection of items to go in the bags. We have used toothpicks with in-service teachers, but do not use them with students. Obviously, soft, non-harmful items are most desirable.

The second mystery comes in the estimation process. It seems that no matter how well we teach the ideas of measurement, many students' (and some teachers') estimation skills are far off. Students enjoy the measurement portion of the activity since they are anxious to see how well they estimated. The discovery of how close they are solves their mystery.

Activity 2: "The Snack-Attack" Mystery

First, show the students a small snack-size package of some food. We like to use fruit snacks since they have a semblance of nutrition. Ask the students to estimate the number of snacks in the package and write the estimate on "The Snack-Attack Mystery" worksheet in ink so they are not tempted to change their estimate. After guesses are made, students are put into groups and each group then calculates the mean, median, mode, minimum, maximum and range of the guesses for their group. Group results are then reported to the class. The class must then decide what number to use as the best estimate for the number of fruit snacks in the pouch. Depending on grade level, students can draw stem-and-leaf plots and box-and-whisker plots for the group data. Each student is then given a package of fruit snacks and is told to count the number of snacks in their own package. These data can be recorded on the worksheet and descriptive statistics can then be calculated for the actual values. Data on the package can be used for other mathematical activities.

While this activity does not make use of the lunch bags, it does allow the students another opportunity to solve a mystery. Mystery #1: When you look at the side of the box containing the fruit snack pouches, you see the approximate number of snacks per pouch. How did somebody come up with that number? Mystery #2: How good are your estimation skills when you have no visual guidance?

This activity not only further explores estimation skills; it provides an interesting way to look at descriptive statistics. Many times students are not interested in calculating statistics for meaningless data. However, snack data is certainly meaningful in their world. It also provides some insight about the use of mathematics in the workplace. After all, someone had to come up with that approximate number of fruit snacks to put on the outside of the box.

Activity 3: "What's In The Bag" Mystery

As a new concept is introduced, put an object related to that concept in a lunch bag and have students guess the object. For example, different color chips can be put in the bag as you are about to teach integers, probability, or circles.

This is a very short activity that gets the students' attention. They not only are trying to solve the mystery of what's in the bag, but once they figure it out, they begin to wonder how that object ties into the mathematics lesson. The best thing for you as the teacher is that each time you use an object, students will offer a multitude of ways to use the object in a mathematics lesson and you end up being able to use a single object for numerous concepts. Some examples are: number cubes for probability and three-dimensional geometry; pattern blocks for fractions and geometry; pennies for circles and decimals; and candy bars for non-standard units of measure and fractions. Another example is to put a Cuisenaire rod in a bag and have students guess its color only by touching it through the bag.

Conclusion

Each of these activities has been used with middle school students, pre-service middle grade teachers and in-service middle grade teachers. No matter who participates in these grab bag activities, the response is always overwhelmingly positive. Used alone or as a theme, these activities provide your students with valuable estimation and problem-solving skills as they explore the mysteries in mathematics.

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"What's In The Bag" Mystery

ITEM	ESTIMATE	ACTUAL MEASURE	DIFFERENCE
		Total Difference	

The Snack Attack Mystery

NAME	ESTIMATE	ACTUAL

Group:

Range ________________________

Mean ________________________

Mode ________________________

Median ________________________

Class:

Range ________________________

Mean ________________________

Mode ________________________

Median ________________________

TCTM E. Glenadine Gibb Achievement Award Application

Eligibility: The E. Glenadine Gibb Achievement Award is presented to someone nominated by a TCTM member to be honored for his/her contribution to the improvement of mathematics education at the state and/or national level.

Deadline: May 1, 2003

Information about the **TCTM member** nominating a candidate:

Name: ____
Last First Middle

Address: ____
Number and street Apt. number

____
City Zip Code

(___) ____________________ (___) ____________________ < ____________________ >
Home Phone Work Phone Email Address

Are you a member of TCTM? ☐ yes ☐ no NCTM? ☐ yes ☐ no

Information about the **nominee**:

Name: ____
Last First Middle

Address: ____
Number and street Apt. number

____
City Zip Code

(___) ____________________ (___) ____________________ < ____________________ >
Home Phone Work Phone Email Address

Is the nominee a member of TCTM? ☐ yes ☐ no NCTM? ☐ yes ☐ no Retired ☐ yes ☐ no

Applications should include 3 pages:

☐ Completed application form

☐ One-page, one-sided, typed biographical sheet including:
- Name of nominee
- Professional activities
- National offices or committees
- State TCTM offices held
- Local TCTM-Affiliated Group offices held
- Staff Development
- Honors/awards

☐ One-page, one-sided essay indicating why the nominee should be honored for his/her contribution to the improvement of mathematics education at the state/national level

Please submit the completed application, biographical sketch, and essay

by mail: **Cynthia Schneider, 234 Preston Hollow, New Braunfels, TX 78132**

by fax: **(512) 232-1855 ATTN: Cynthia Schneider**

by email: ***<cschneider@mail.utexas.edu>***

Functions? • *JoAnn Wheeler*

Section I: Building the concept

The following activity is one of the four vertical team activities included in *Foundations of Functions: A Resource for Mathematics Vertical Teams* published by The Charles A. Dana Center at the University of Texas at Austin December 2002. Although the activity (*Getting Started*) was designed for the professional development of vertical teams of mathematics teachers in grades 5-8, the activity is very appropriate as curriculum material for secondary students. *Functions?* builds the concept of functions using examples and non-examples from the venue of purchasing postage stamps from a vending machine. Sample student responses follow the question sets.

Functions play a major role in describing relationships mathematically to predict behavior. A function can be described as an algebraic representation that relates a geometric pattern to an algebraic expression. In Appendix A: Functions of the *Advanced Placement Program®, Mathematical Vertical Teams Toolkit*, a function is described as:

> A relationship between two quantities such that one quantity is associated with a unique value of the other quantity. The latter quantity, often denoted y, is said to depend on the former quantity, often denoted x.

A vending machine for postage stamps is an example of an association that represents a function. This represents a function because the type of stamp we purchase depends upon the selection code we key into the machine. Another way to say this is that the type of stamp we receive from the machine "is a function of " the selection code we input. Abbreviating this phase, we might say something like:

type is a *function* of *code*

Abbreviating still further, we could say

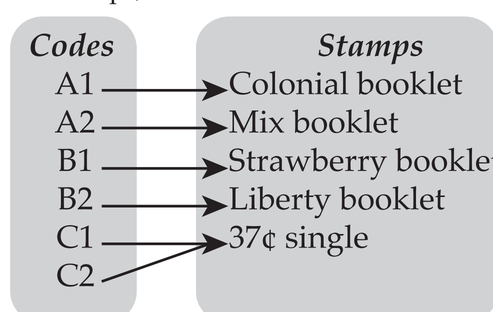
type = function(code)

Finally, we could move to a symbolic representation of this relationship:

type = t(c) or *t(c) = type*

This is an illustration of "function notation." (Although we could have used *f* or any other letter to represent our function, we chose to use *t* to emphasize that the function produces a *type* of stamp.)

This particular machine allows the user to select one of six codes: **A1, A2, B1, B2, C1, or C2.**
Each code corresponds or maps to a unique stamp or booklet of stamps, as shown below.



For example, when code A1 is input, a colonial booklet is output. Using the notation described above, we could say that *t(A1) = colonial booklet.*

1. Write the relationship between code A2 and the type of stamp the machine outputs for that code by completing the equation below.
   t(A2) =

2. Using function notation as in question 1, above, write the relationship between code C1 and the type of stamp the machine outputs for that code.

3. Using function notation, write the relationship between each of the other three codes and the type of stamp the machine outputs for each code.

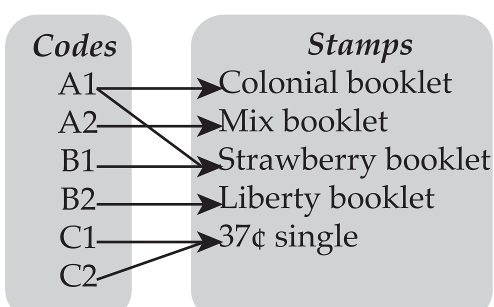
4. The designer of this stamp machine views the set of codes (A1, A2, B1, B2, C1, and C2) as the set of independent variables. Why?

5. Based on the design of this stamp machine, we say that the different types of stamps (colonial, mix, strawberry, liberty, 37¢ single) make up the set of dependent variables. Why?

6. Assuming the machine is loaded according to this design, what will happen every time I input B2?

Note to teacher: The purpose of the mapping listed below and question 7-10 is to provide the student with an example of a stamp vending machine that does not represent a function. This provides the non-example element of building student's conceptual knowledge of a function.

Now, consider the postage stamp vending machine represented below.



7. Assuming the machine is loaded according to this design, what will happen every time I input code C2?

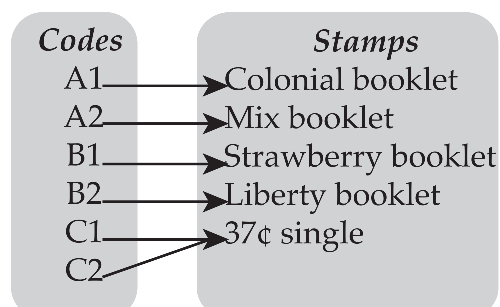
8. Assuming the machine is loaded according to this design, what will happen every time I input code A1?

9. How is the stamp machine represented here similar to the first stamp machine? How is it different?

10. The first stamp machine represented a function. Does the second stamp machine represent a function? Explain your answer, using the definition of a function given at the beginning of this activity.

Note to teacher: Questions 1 – 6 represented the "Doing" in the Doing-Undoing algebraic habit-of-mind as indicated in Mark Driscoll's book *Fostering Algebraic Thinking*. The purpose of continuing the questions with regard to the original mapping is to provide the student an opportunity to engage in the "Undoing" element of algebraic thinking.

Consider our original, function-like stamp machine (represented in the next collum).



11. I used this stamp machine and received a colonial booklet. What code did I input?

12. I used this stamp machine and received a 37¢ single stamp. What code did I input?

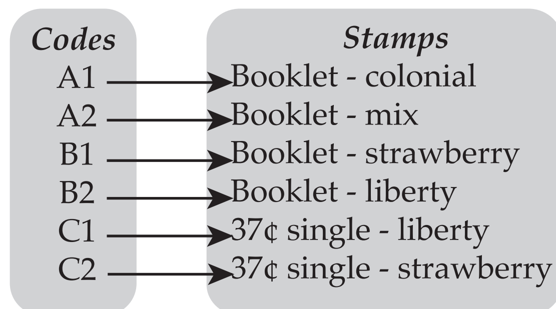
13. Assuming this stamp machine is loaded properly, what can I always predict, based on the code I input?

14. Can I always tell what code I input based on what stamp I receive? What would have to be true about our stamp machine if I wanted to be able to determine my input based on my output?

15. What is the advantage of using a one-to-one function as a model for a relationship between quantities?

Answers to *Section I: Building the concept* questions 1-15:

1. *t(A2) = mixed booklet*
2. *t(C1) = 37¢ single*
3. *t(B1) = strawberry booklet*
   t(B2) = liberty booklet
   t(C2) = 37¢ single
4. *Answers will vary. The codes (A1, A2, B1, B2, C1, and C2) represent the independent variables because we have "control" over the selection we choose. We must input a specific code (the independent quantity) before the vending machine will dispense the stamp we want. The code we input determines the type of stamp we receive.*
5. *Answers will vary. The stamps are delivered according to which code we select. The type of stamp we receive depends on the code we select.*
6. *I will receive a liberty booklet of stamps.*
7. *I will receive a 37¢ single stamp.*
8. *Answers will vary. You don't know with certainty. You might get a colonial booklet or you might get a strawberry booklet.*
9. *Answers will vary. This stamp machine is similar to the first stamp machine in that they are both built on some relationship between the code you input and the type of stamp you receive. The first machine produces only one type of stamp for each code. However, the second machine produces two different types of stamps for code A1. No matter what code we input in the first machine, we know the type of stamp we will receive. However, this isn't always the case with the second machine. Code A1 produces two types of stamps, and we cannot predict which type of stamp we will receive when we input that code.*
10. *No, this machine does not represent a function. There is a relationship that associates the type of stamp with the code that is input; however, each input (the code) is not associated with only one unique output (the type of stamp), so the relationship is <u>not</u> a function.*
11. *A1*
12. *C1 or C2; we know it's one of these, but we don't know which one.*
13. *We can predict the type of stamp we'll receive.*
14. *Answers will vary. The main problem with our original stamp machine was that codes C1 and C2 both produced the same type of stamp. We would need to change the machine so that those two codes produced different types of stamps-from each other and from other types of stamps produced by the other four codes. One example is shown below.*



This stamp machine represents a one-to-one function. For the function to be a one-to-one function, every code would have to deliver a different stamp. Since C1 and C2 both delivered the same stamp in our original machine, our original function was not one-to-one.

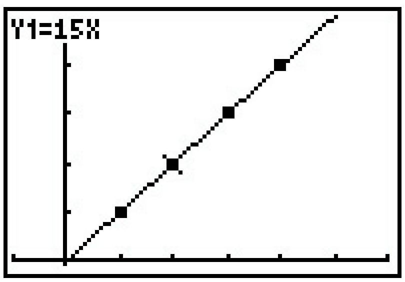
15. *Not only will you be able to predict any output value given an input value, you will also be able to recover the unique input value that produced any output value you might be given. This can be useful in situations in which you already know the result your model must produce and you must determine what input will produce that result. For example, you may have a sheet of plywood of which you are going to build a cubic crate, and you would like to use as much of the lumber as possible. You could model the situation with a one-to-one surface area function and then determine the dimensions that produce a surface area equal to the area of your sheet of plywood.*

Section II: Bridging the concept

Suppose you could rent a surfboard for $15 per hour at your favorite family vacation resort. The total cost of the rental depends on the number of hours you surf. The number of hours you surf is the independent variable (input or stamp machine code) and total cost is the dependent variable (output or type of stamp). So total cost is a function of the number of hours you surf. Data that relates the hours you surf to the total cost of the rental can be represented as ordered pairs (hours, cost), symbolically (cost = $15 times number of hours), in a table (shown below), and graphically (shown below).

Hours you surf	Total cost of rental
1	15
2	30
3	45
4	60

(1,15)
(2,30)
(3,45)
(4,60)



1. Describe the relationship between the number of hours you surf and the total cost of the rental using function notation.

2. Using function notation as in question 1, above, write the relationship between surfing for 4 hours and the total cost of the rental.

3. The designer of this relationship views the set of hours surfed (1, 2, 3, 4) as the set of independent variables. Why?

4. Based on the design of this relationship, we say that the total cost of the rental ($15, $30, $45, $60) makes up the set of dependent variables. Why?

5. If you could only see the graph of this relationship, how could you determine that the relationship represented a function?

6. If you could only see the table of data for this relationship, how could you determine that the relationship represented a function?

7. If you could only see the set of ordered pairs for this relationship, how could you determine that the relationship represented a function?

Determine if the following data tables represent functions. Explain why or why not.

8.

x	y
-1	-8
4	7
-6	-23
1	-2

9.

x	y
-1	-8
4	7
-6	-23
-1	-2

10.

x	y
-1	-8
4	7
-6	-23
1	-8

11. Sketch a graph that would not represent a function. Explain why your graph does not represent a function.

Answers to *Section II: Bridging the concept* questions 1-11:
1. *Answers will vary. C(h) = 15h*
2. *C(4) = $60*
3. *Answers will vary. The hours we surf (1, 2, 3, and 4) represent the independent variables because we have "control" over the amount of time we choose to surf. We must determine the number of hours we expect to*

surf (the independent quantity) before the company can estimate or determine the total cost of the rental. The number of hours we surf determines the total cost of the rental.

4. *Answers will vary. The cost of the rental is determined according to the number of hours we select to surf. The cost of the rental depends on the number of hours we surf.*
5. *Answers will vary. I can tell by the graph if the relationship is a function by looking at the x-values. If every x-value represented by the graph is mapped to one and only one y-value, the relationship is a function.*
6. *The data contained in the table will represent a function if exactly one y-value is assigned to each x-value.*
7. *The data contained in the set of ordered pairs will represent a function if exactly one y-value is assigned to each x-value.*
8. *The relationship is a function because exactly one y-value is assigned to each x-value.*
9. *The relationship is NOT a function because exactly one y-value is NOT assigned to each x-value. The x-value of –1 is mapped to both –8 and –2.*
10. *The relationship is a function because exactly one y-value is assigned to each x-value. The y-value of –8 corresponds to two x-values, -1 and 1. According to the definition of a function, if every x-value is mapped to one unique y-value then the relationship is a function. It does not matter that two different x-values correspond to the same y-value.*
11. *Answers will vary. The graph should contain multiple y-values for the same x-value.* Ω

JoAnn Wheeler • Region IV Education Service Center
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TEA Talks

Has TAKS anxiety arrived for you and your campus? For the many third and fourth grade teachers across the state who have already administered one section of the exam their stress level should be on a downward slide. When the TAKS Administration Manuals arrived on campuses prior to the reading or writing exams, the agency logged numerous calls. Many teachers were "leafing" through the manuals only to find sample items without carefully reading the associated text. The agency was accused of developing items that were not aligned to grade specific TEKS. Please note a disclaimer is included in each manual that explains the samples included were developed for a range of grade levels.

Many high school teachers are currently working to learn the TI Testguard program prior to the grade nine through eleven TAKS administration. Refer to page 18 and 19 in the TAKS 2003 District Coordinator's Manual for specific information on the models that are allowable during assessment. Please note the calculators must be cleared to factory default. Remember to call your calculator vendor if you are struggling with the memory and application clearing procedure. Listed below are two helpful websites that include step-by-step instructions for clearing calculator memory.

The passing standards have been set at reachable levels for our students throughout the state, and the agency expects favorable results. As we approach the new challenges in assessment this spring, remember that students are bright and highly capable of success. TAAS mathematics scores were at an all time high last year, have confidence that you have prepared your students to do well on the TAKS test this year.

Hot News

Refer to the websites listed for additional information on each topic

- The TMDS (Texas Mathematics Diagnostic System) is now available for use with students in grades 5-8. This diagnostic assessment is provided by legislative appropriation in HB1144. The diagnostic will measure students' algebra readiness and can be administered online or in hard copy. During later phases of development, teachers will be able to develop their own assessments by accessing an item bank aligned to all TEKS student expectation statements in grades five through eight. Districts will need to register and send data to Vantage Learning prior to accessing the system.
  h*http://www.accesstmds.com/*
- TI has now placed the Test Guard Program in VIP accounts. This move will allow educators to download the program quickly without some of the difficulties previously encountered. This program allows teachers in grades 9-11 to quickly delete all applications and data stored on graphing calculators prior to TAKS testing.

http://education.ti.com/us/product/apps/83p/testguard.html

- Region IV has developed a helpful calculator tutorial. To "walk through" the memory clearing process, visit this site at:

  http://www.esc4.net/students/tutorials/calc/

- Nomination forms for the Presidential Awards for Excellence in Mathematics and Science Teaching are now available. The award for this year will be available for math and science teachers in grades 7-12. Nomination forms must be submitted to me at the agency prior to the application being sent to qualified candidates. Email me at *<pgustaf@tea.state.tx.us>* if you would like to nominate a colleague.

Keep in Mind

- A TAKS study guide will be provided for students that fail any portion of the test. The format will be similar to previous TAAS study guides.

- The spring 2003 TAKS test will be released on the TEA website in late May with hard-copy versions sent to districts by the end of June.

- Many activities, lessons and assessments are available free of charge on the Mathematics Toolkit. If you are searching for TEKS-based materials-this is the website you want to visit!

  http://www.mathtekstoolkit.org/

- Research shows long-term, intensive staff development is the most effective way to change current practice. Texas is fortunate to have the TEXTEAMS staff development program available for all educators. Check with your local ESC mathematics contact to determine when the next institute will be held. Information about the TEXTEAMS Institutes can be found at

  http://www.texteams.org/

- A list serve is provided for educators interested in receiving email updates about mathematics events. To join the list serve visit the following website and follow the prompts.

  http://www.tea.state.tx.us/list/

- CAMT 2003 will be held in Houston on July 17-19, 2003. A 3-hour TAKS session has been created to assist educators in preparing students for the demands of objective 6 in grades 3-8 and objective 10 in grades 9-11. For additional information or to register for the conference, refer to the following website:

  http://www.tenet.edu/camt/ Ω

Paula Gustafson • Director of Mathematics,
Texas Education Agency <pgustafs@tea.state.tx.us>

TCTM Breakfast and Business Meeting at CAMT

Saturday July 19, 2003, 7:00 a.m. –8:30 a.m.

Hyatt Regency Hotel, Window Box Room

Come join us for the annual TCTM breakfast and business meeting. This is your opportunity to meet the current and incoming board members and be a part of TCTM decisions. Plus, there will be great door prizes from the vendors you see at CAMT. This breakfast is for members only (no children or other guests). Breakfast tickets must be reserved in advance; there will be no tickets available at CAMT. Please mail your contact information, along with your check for the breakfast, to the address below no later than June 1, 2003. Your ticket for the breakfast will be mailed to you no later than June 30.

Member Information

Name: ____
Last First Middle

Address: ____
Number and street Apt. number

____
City Zip Code

(___) ____________________ (___) ____________________ < ____________________ >
Home Phone Work Phone Email Address

☐ Enclosed please find my $10.00 check, payable to TCTM, for the breakfast.

Distrcit or Professional Affiliation ____________________ ESC __________

Please mail your check and form to

Cynthia Schneider,
234 Preston Hollow,
New Braunfels, TX 78132

Texas Council of Teachers of Mathematics

INDIVIDUAL MEMBERSHIP **($13 per year)**

Name: ____

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City: ____

State: ____________________ Zip: ________________________

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Circle area(s) of interest: K-2 3-5 6-8 9-12 College

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State: ____________________ Zip: ________________________

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NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS MEMBERSHIP

Check One:
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One Journal
- ☐ Teaching Children Mathematics ($68)
- ☐ Mathematics Teaching in the Middle School ($68)
- ☐ Mathematics Teacher ($68)
- ☐ Journal for Research in Mathematics Education ($90)

Additional Journal
- ☐ Teaching Children Mathematics ($28)
- ☐ Mathematics Teaching in the Middle School ($28)
- ☐ Mathematics Teacher ($28)
- ☐ Journal for Research in Mathematics Education ($50)

Amount Due to NCTM: $ ______________

Scholarship Donations: $ ______________

TCTM awards scholarships to high school seniors planning to pursue a career in mathematics teaching either as a mathematics specialist in elementary school or as a secondary school teacher with certification in mathematics. Your contributions in any amount are greatly appreciated. Please write a separate check for scholarship donations.

Make check(s) payable to TCTM and mail to:
TCTM Treasurer
1209 Tesoro Ave
Rancho Viejo, TX 78575

Total Amount Due: $ ____________

VOTE!

TCTM Candidates

President

Cynthia Schneider

Cynthia has served on the TCTM Board since 1997, as both Central Regional Director and President. She is seeking a second term as President of TCTM in order to pursue goals such as increasing the number of scholarships and electronic communication among TCTM members. As a representative for TCTM on the CAMT Board, she has worked hard to represent the best interests of TCTM and its members in the planning of this conference and its administrative organization. Cynthia has spoken at many conferences across the state in her capacity as an educational researcher for the Charles A. Dana Center at UT-Austin. She taught mathematics for a number of years at various colleges in central Texas and completed her Ph.D. in mathematics education in 2000.

Carol G. Williams

Carol received her Ph.D. in mathematics education from the University of California at Santa Barbara in 1994. She has 12 years of teaching experience at the secondary level and 16 years at the university level. She has been working with the preservice teachers at Abilene Christian University since 1995. Carol served as secretary of TCTM from 1996-2000. More recently she has been a member of the Advanced Mathematics Educational Support Leadership Team sponsored by the Charles A. Dana Center.

Vice-President Elementary

Wilma Cook

Wilma was employed by Fort Worth ISD in 1983 and taught grades 3, 4, and 5 in the inner city through 1994. Under the direction of Betty Forte she became an Elementary Math Specialist teaching mathematics to all 5th graders in a departmentalized setting from 1995 –1998. From 1999 to 2001 she left the classroom and became an Instructional Support Team member. Her duties included but were not limited to: supporting classroom teachers, providing in-service, demonstrating lessons, executing formative evaluations (for purposes of improving instruction only), and attending conferences to stay abreast of current trends in mathematics on both state and national levels. In 2001, she began work in the central office as a Math Curriculum Specialist, a position she currently holds.

Wilma holds a M.Ed. in Educational Technology. She received Outstanding Teacher in 1996-1997 and in 1998-1999. She received Who's Who Among America's Teachers in 1996 and again in 2000.

Michelle A King

In 1992, Michelle A King earned a Bachelor's Degree in Mathematics with a minor in Education from the University of Iowa, graduating with a K-12 certification in mathematics. Her teaching career began with the Birdville Independent School District. In 1997, she was recruited to teach in a laboratory school in North Oak Cliff, Texas. Teaching grades 6-8 at J. Erik Jonsson Community School. In addition to teaching in the classroom, Michelle was the Mathematics Building Level Specialist for grades PreK through 6 and served as a Change Facilitator for their Assessment Innovation.

Since April 2001, Michelle has been employed by the Denton Independent School District as the Mathematics Curriculum and Staff Development Coordinator, K-12.

Michelle believes that "Dreams are the Seeds to Success." Because of her experience, love of mathematics and technology, enthusiasm, and ability to work with people, she continually desires to fulfill her dream of being a mathematics education leader.

Treasurer

Jo Ann Wheeler

Jo Ann Wheeler is knowledgeable in the area of fiscal management and reporting. Maintaining fiscal records is but one of the many responsibilities associated with her position as Director of Mathematics/Science/Social Studies Services for Region IV Education Service Center. Jo Ann has represented mathematics educators for the past two years as the Southeast Regional Director (ESC 4,5,6); currently serves on the CAMT Board as the TASM representative; and continually designs professional development to facilitate student success in mathematics. If elected Treasurer, she commits to serving the mathematics educators of Texas, supporting the TCTM President, and responding appropriately to the fiscal needs of the organization.

Kathy Hale

Kathy Hale, Math Consultant for Region 14 ESC in Abilene, taught middle school and high school mathematics for over 25 years. She uses her background teaching in schools from 1A to 5A to assist teachers in the development of new methods to teach mathematics to all students. She has served as the treasurer for the Texas Math and Science Coaches Association (TMSCA) and has served as President of the Big Country Council of Teachers of Mathematics, the local NCTM affiliate.

Southwest Regional Director

Vote only if you live in one of these Service Center Regions: ***15, 18, 19***

Alicia R. Torres

My professional experience includes high school mathematics teacher, district staff developer and district math facilitator. I have recently been named the El Paso ISD USP Director for math and science. Because of my different job positions and involvement in math organizations, I have varied experiences in mathematics education. Other experiences include serving as Regional Conference Program Chair for NCTM 2000, Conference Program Committee Member for the National NCTM 2001 and CAMT 2002, and Southwest Region Representative for TCTM.

Southeast Regional Director

Vote only if you live in one of these Service Center Regions: ***4, 5, 6***

Judy Rice

Judy Rice is an Education Specialist in Mathematics, Grades 6-8 at Region IV Education Service Center. Not only is she responsible for the professional development of mathematics teachers; but she also works with mathematics coordinators/ instructional specialists in developing trainings that meet a district's individual needs. Judy coordinates Math Academy trainings with the 54 school districts that are served by Region IV. She was a classroom teacher for 26 years.

Judy has held the offices of NCTM representative and Vice-President, Secondary for TCTM. She has also served as a reviewer for articles submitted for the Texas Mathematics Teacher.

South Regional Director

Vote only if you live in one of these Service Center Regions: ***1, 2, 3***

Jeanne Womack

Jeanne has been a classroom teacher for 24 years in Mesquite, Texas and Harlingen, Texas. She has been the Secondary Math Specialist for Brownsville ISD for 5 years. She is currently Educational Specialist for Region I ESC. She is a member of NCTM, TCTM, TASM, and RGVCTM. She is also a member of Advisory Committees for TEXTEAMS Geometry and Algebra II/Precalculus Institutes and Algebra Assessments.

Sheryl Roehl

Sheryl has a BS degree in Mathematics and a MS in Education. She taught high school math, science and computer programming for twelve years and taught mathematics at the junior college level for 3 years. While teaching in Victoria, she was selected as a Danforth Leadership Fellow. She then served as the math/science, TAAS and curriculum specialist for Region III Education Service Center in Victoria for nine years. She joined the staff of the South Texas Rural Systemic Initiative in April of 2002 as the Assistant Project Director. In her new position, she is currently working with 29 rural school districts in South Texas to improve student performance in math and science. Sheryl served as co-chair of the CAMT exhibits for two years and also served as the Government Relations Representative for the Texas Association of Supervisors of Mathematics (TASM). Sheryl has served as the South Regional Director for TCTM for the past three years. Sheryl is currently pursuing a doctorate degree at Texas A & M Corpus Christi.

VOTE!

TCTM Ballot

Circle your choices below. Write-in candidate names are acceptable. Copy and mail your ballot to Linda Shaub at the address below, or vote online. The voting deadline is June 1, 2003.

President

☐ Cynthia Schneider ☐ Carol Williams ☐ ____________________
write-in candidate

Vice-President Elementary

☐ Wilma Cook ☐ Michelle King ☐ ____________________
write-in candidate

Treasurer

☐ Jo Ann Wheeler ☐ Kathy Hale ☐ ____________________
write-in candidate

Southwest Regional Director
Vote only if you live in one of these Service Center Regions: ***15, 18, 19***

☐ Alicia Torres ☐ ____________________
write-in candidate

Southeast Regional Director
Vote only if you live in one of these Service Center Regions: ***4, 5, 6***

☐ Judy Rice ☐ ____________________
write-in candidate

South Regional Director
Vote only if you live in one of these Service Center Regions: ***1, 2, 3***

☐ Jeanne Womack ☐ Sheryl Roehl ☐ ____________________
write-in candidate

Mail in your ballot

Linda Shaub
Vice-President TCTM
1111 Highland Hills
Marble Falls, TX 78654

OR

Vote online!

Ballots are now available on the internet.
Visit our web site, located at

http://www.tenet.edu/tctm/

and look for instructions to vote online!

Texas Council of Teachers of Mathematics
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*Voting Members
() End Year – Elected Office
[] End year – Appointed Office

When does YOUR membership expire?

Note the expiration date on your mailing label.
Use the membership form inside to renew before that date.

Texas Council of
Teachers of Mathematics

Member 2002-2003

NAME ______________________________

Texas Mathematics Teacher
234 Preston Hollow
New Braunfels, TX 78132