# Texas Mathematics Teacher

A PUBLICATION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

http://www.tenet.edu/tctm/

**Volume L Issue 2** **Fall 2003**

## Developing Understanding of Fraction Concepts: Lessons from Two Teachers

## A Whole Lotto Education!

## Building on Children's Thinking to Develop Proportional Reasoning

## Mathematics, the TEKS, and Literature Is it a good fit?

## So, Who Invented the Order of Operations?

**TCTM Nominations**
Due January 1, 2004

**Awards Recipients**
Scholarships, leadership, and more!

**Lone Star News**
Check for upcoming local conferences

**Check the Back Cover**
for your membership card
and renewal date

## Texas Council of Teachers of Mathematics
## 2003 Mission and Goals Statement

### MISSION

To promote mathematics education in Texas.

### GOALS

**Administration**

- Investigate online membership registration through CAMT and/or the TCTM website

**Publications**

- Survey membership to identify what they want in the Texas Mathematics Teacher (TMT)
- Review and redesign the TMT journal and the TCTM website based on above findings

**Service**

- Increase the number of Mathematics Specialist College Scholarships
- Increase the donations toward Mathematics Specialist College Scholarships
- Staff CAMT registration with volunteers and other volunteers as needed
- Advertise affiliated group conferences on the TCTM website and in the TMT

Communication

- Create and maintain an e-mail list of members for timely announcements
- Communicate with affiliated groups in a timely manner
- Improve communication with NCTM consignment services

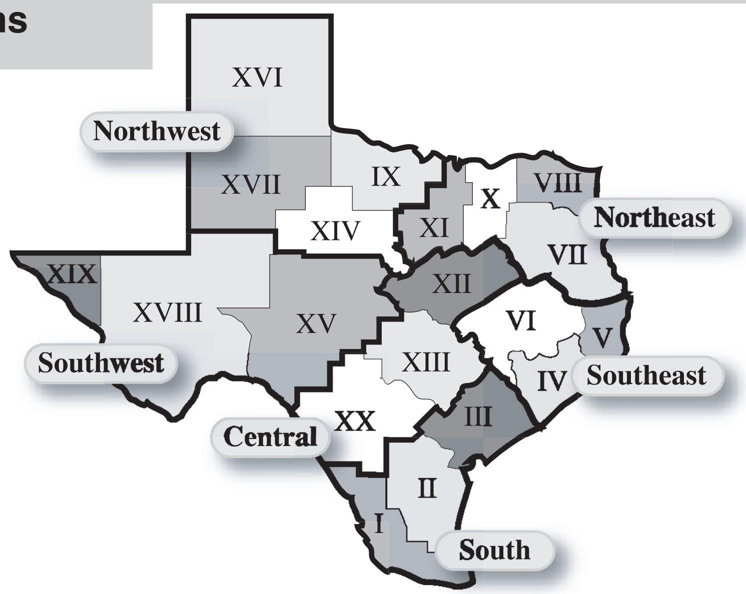
Membership

- Based on information gathered by TCTM board members as to advisability, advocate at CAMT Board meetings for TCTM membership to be required for all CAMT participants
- Encourage affiliated groups to include TCTM registration on their membership forms

**Public Relations**

- Staff and sponsor the NCTM/TCTM booth at CAMT
- Follow NCTM *Communication Guidelines* (1993) for increased media coverage of TCTM membership and issues relevant to our mission

## TCTM Regions




## TCTM Past-Presidents

| | | | | | |
|---|---|---|---|---|---|
| 1970-1972 | James E. Carson | 1982-1984 | Betty Travis | 1994-1996 | Diane McGowan |
| 1972-1974 | Shirley Ray | 1984-1986 | Ralph Cain | 1996-1998 | Basia Hall |
| 1974-1976 | W. A. Ashworth, Jr. | 1986-1988 | Maggie Dement | 1998-2000 | Pam Alexander |
| 1976-1978 | Shirley Cousins | 1988-1990 | Otto Bielss | 2000-2002 | Kathy Mittag |
| 1978-1980 | Anita Priest | 1990-1992 | Karen Hall | | |
| 1980-1982 | Patsy Johnson | 1992-1994 | Susan Thomas | | |

# Texas Mathematics Teacher

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**Volume L Issue 2** **Fall 2003**


## Articles



## Features



## Departments



## Applications




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***Texas Mathematics Teacher***, the official journal of the Texas Council of Teachers of Mathematics, is published in the fall and spring. Editorial correspondence should be mailed or e-mailed to the editor.

**Call For Articles**

The ***Texas Mathematics Teacher*** seeks articles on issues of interest to mathematics educators, especially K-12 classroom teachers in Texas. All readers are encouraged to contribute articles and opinions for any section of the journal.

Manuscripts, including tables and figures, should be typed in Microsoft Word and submitted electronically as an e-mail attachment to the editor with a copy to the director. No author identification should appear on or in the manuscript. A cover letter containing author's name, address, affiliations, phone, e-mail address, and the article's intended audiance should be included. After refereeing, authors will be notified of a publication decision.

Teachers are encouraged to submit articles for ***Voices From the Classroom,*** including inspirational stories, exemplary lessons, or management tools. If submitting a lesson, it should include identification of the appropriate grade level and any prerequisites.

Items for ***Lone Star News*** include, but are not limited to, NCTM affiliated group announcements, advertisements of upcoming professional meetings, and member updates

Businesses interested in placing an **advertisement** for mathematics materials should contact Mary Alice Hatchett.

Deadline for submissions: Fall, July 1 Spring, January 1

# Letter from the President

Dear TCTM Members,

Well, it's been one year in office and I've learned a lot. I want to begin by thanking all the volunteers that worked at CAMT, both in the registration area and at the NCTM/TCTM booth. We had a lot of walk-up help that was invaluable. In particular, I would like to thank Joanna Weilmuenster and Jim Wohlgehagen; they manned the registration area for days at a time. Thanks for all your support.

Another group of volunteers helped review the many articles we received for this edition of the journal. We had over forty members involved in the review process and I want to thank Mary Alice Hatchett for the great job she did in coordinating it all. We already have members lined up to review for the spring journal. If you would like to serve in this way, please contact Mary Alice, or me, our contact information is listed on the inside back cover of this journal.

I continue to be challenged by the writing of this letter. Should my comments be inspirational, informational, or content-focused? Does anyone actually read it? In the hopes that some do, I would like to share my synthesis on the articles that we have in this journal. Whether we talk about understanding fractions, developing proportional reasoning or the order of operations, the common theme that jumped out at me was the need to work from concrete to abstract reasoning. It is such an over-whelming issue in much of our work in mathematics education. We certainly talk a lot about it. We know we need to do it. However, it is clear that we fall short in laying the concrete foundation.

It occurred to me that perhaps this is because we all think that the goal – abstract reasoning – is so important we need to just get there. Could it be that simple? Or is it because most of us are still not comfortable with laying out the concrete foundation, making the connections, and having the patience to let it work its magic? I cannot determine from my observations and work with teachers if it is a lack of pedagogical content knowledge or a belief that the foundation has already been laid, so it's time to get on down the road. Do we believe that a few days work on concrete representations is enough? Is it a belief that the concrete representation is a crutch, and needs to be removed as soon as possible? I hope these beliefs continue to be challenged.

Of course, I could refer back to all the literature that says we teach the way we were taught. Since many of us were not taught moving from concrete to abstract reasoning, we don't do it with our students. I can certainly believe it's the way we were taught, but at what point will this no longer hold true? When are we going to provide thorough use of this developmental process and provide our students with the opportunity to learn mathematics in a deep and meaningful way that leads to success in school mathematics and beyond?

I close with the recommendation that you read this current journal and reflect on your practice. What can you change today about instruction that will incorporate the lessons learned herein?

Sincerely,

Cynthia L. Schneider
TCTM President 2002-2006

## September 27, 2003

**Panhandle Area Math and Science Conference**
"Building Highly Qualified Educators"
Contact Gilbert Anunez, *<gantunez@mail.wtamu.edu>*
WTAMU, Box 60208
Canyon, TX 79016-0001
ph# (806) 651-2610
*http://www.wtamu.edu/academic/ess/edu/*
or
*http://www.texasrsi.org/PMSC/register.htm*

## October 6-10, 2003

**National Metric Week**
"Ideas for a school's celebrating National Metric Week"
The week that includes 10/10 is a great time to focus on the importance of this measurement system. The U.S. Metric Association offers on-line information, resources, and suggestions for celebrating.
*http://lamar.colostate.edu/~hillger/ideas.htm*

## October 18, 2003

**Austin Area Council of Teachers of Mathematics**
"Annual Making Every Minute Count Conference"
9:00 a.m. to 3:30 p.m. at Dessau Middle School
Pflugerville ISD
12900 Dessau Rd.
Austin, TX 78754
Contact Linda Shaub, *<linda.shaub@esc13.txed.net>*
1111 Highland Hills
Marble Falls, TX 78654
ph# (512) 919-5305

## October 18, 2003

**Greater El Paso Council of Teachers of Mathematics (GEPCTM)**
"Fall Conference"
Transmountain Campus of El Paso Community College
El Paso, TX
Contact Robert Kimball *<k2rc@email.com>*

## October 23, 2003

**Fort Bend Council Teachers of Mathematics, Houston, TX**
"Fall Meeting" 4:30 p.m. to 6:00 p.m. at
Alief Taylor High School
7555 Howell Sugar Land Drive
Houston, TX 77083
Contact Marla Cortes, *<mkcortes@alief.isd.tenet.edu>*
Alief Taylor High School
Mathematics Department
ph# (281) 988-3757

## October 25, 2003

**DFW Regional National Engineers Week Future City Competition**
Contact Jean Eason, *<j.eason@ieee.org>*
Future City FW-D Regional Coordinator
ph# (817) 923-1032
To help students better understand the practical applications of math and science principles, the National Engineers Week Committee sponsors the annual Future City Competition. Teams of seventh and eighth grade students, with the guidance of a teacher and an engineer, design a city using SimCity software. Winning teams compete in the finals in Washington, D.C. Registration continues through October 25.
*http://www.dfwfuturecity.org*

## November 15, 2003

**Rio Grande Valley Council of Teachers of Mathematics**
"No Child Left Behind - Mathematics is the Key"
Contact Nancy Trapp,
Rural Route 2, Box 312
Raymondville, TX 78580
ph# (956) 347-3521 Fax: (956) 347-5034
*http://www.rgvctm.org*

## April 21-24, 2004

**National Council of Teachers of Mathematics**
"Defining Mathematics for All"
Philadelphia, PA
*http://www.nctm.org/*

# Developing Understanding of Fraction Concepts: Lessons from Two Teachers

• *Ye Sun*
• *Gerald Kulm*

Teaching for student understanding is an important goal in mathematics instruction. Project 2061 of the American Association for the Advancement of Science developed textbook evaluation criteria that are based on research on mathematics teaching and learning (AAAS, 2000). We were interested in seeing how the AAAS criteria can be used to characterize qualitative differences in the way teachers present the mathematics concepts and skills of understanding and using equivalent fractions. How do teachers use representations in the presentation of these important ideas about fractions?

In this article, we will present examples of mathematics lessons from two teachers to illustrate one of the key categories of AAAS criteria: Developing Mathematical Ideas. The following summary is drawn from *Middle Grades Mathematics Textbooks: A Benchmarks-based Evaluation* (AAAS, 2000) and outlines the research base for the importance of developing mathematical ideas.

## Research Base

The introduction of mathematical terms, symbols, and procedures is critical to developing skill and understanding. Students at all grade levels need to work consistently at developing their understanding of the ideas captured in conventional mathematics representations and symbols (Greeno & Hall, 1997). However, Wagner and Parker (1993) found that extensive work with symbolic manipulation before developing solid understanding results in inability to progress beyond mechanical manipulations.

Because representations of mathematical ideas are so important to conceptual development (Ball, 1988; Hiebert & Wearne, 1986), these representations should be carefully developed, and should connect with earlier informal and concrete experiences. According to Resnick (1982), making explicit the connections between concrete representations and their associated symbols helps students construct necessary relationships.

Student understanding of concepts leads to the ability to generate new connections (Mayer, 1989) and promotes remembering ideas so concepts and procedures can be applied to solving problems and in learning more advanced concepts (Bruner, 1960). In addition, strong connections between concepts enhance transfer to other contexts (Carpenter & Moser, 1984; Kieren, 1988).

For learning to become formalized and ready to use, appropriate and meaningful practice in a variety of contexts and applications is necessary (Peterson, 1988). Hiebert et al. (1997) found that excessive practice before attaining understanding can lead to difficulty in making sense of the procedures later. (AAAS, 2000, pp. 169).

## The Examples

Both sixth-grade teachers had over 10 years of teaching experience and had taught in Texas for all of these years. They used different textbooks. Ms. Able taught from a popular commercial textbook that she had used for the past five years. Ms. Baker used a textbook that was developed to address math reform goals. She was using the book for the first time. Both textbooks

This article is based on a paper presented at the Educational Research Exchange, College of Education and Human Development, Texas A&M University, Friday, February 7, 2003. The work is funded by a grant from the Interagency Educational Research Initiative (No. REC-0129398). Special thanks to Dr. Robert Capraro, for his support in the preparation of this paper. The findings and opinions are those of the authors.

had been evaluated using the AAAS criteria. Ms. Able's book received low ratings by AAAS; Ms. Baker's book received moderate ratings. The teachers were videotaped while they taught a lesson on equivalent forms of fractions.

The videotapes were analyzed using the AAAS instructional quality category, Developing Mathematical Ideas, which consists of six criteria:

1. Justifying the importance of the mathematical ideas
2. Introducing terms and procedures
3. Representing ideas accurately
4. Connecting mathematical ideas with previously learned concepts
5. Demonstrating and modeling procedures and skills
6. Providing appropriate and sufficient practice

Each of the six criteria is defined by several indicators, which are scored as being "met" or "not met" by the instruction. We viewed both videotapes, noting the presence or absence of each of the indicators for the criteria. When an indicator was present, notes were made of the quality or extent to which the indicator was met. In addition, notes were made of the teaching activity to illustrate the quality of the indicator.

## Interpretations of the Lessons

Neither teacher addressed the first criterion, justifying importance of equivalent fractions. At the beginning, the teachers simply started the lesson without explaining or justifying the reasons for learning about equivalent fractions or learning the skills that were the focus of the lesson. Students were not provided an opportunity to discuss or consider why the concepts or skills were important to learn.

On criterion two, both teachers limited the use of the math vocabulary to those that were absolutely necessary. They both introduced mathematical vocabulary or procedures in conjunction with experiences, rather than having students simply memorize definitions or procedures. For example, Ms. Able introduced the conversion of improper fractions to a mixed number $\frac{7}{6} = 1\frac{1}{6}$ using a hexagon. She did not introduce the term "mixed number" until the students had hands-on experience with the idea.

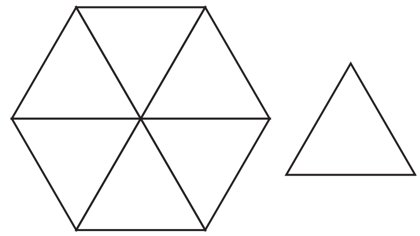

**Figure 1: Showing that** $\frac{7}{6} = 1\frac{1}{6}$

Ms. Baker used trapezoids and triangles to illustrate that $\frac{4}{6}$ and $\frac{12}{18}$ are equivalent. She only introduced the vocabulary "equivalent fraction" after this experience.

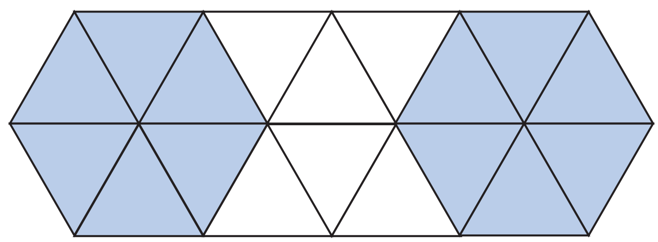

**Figure 2: Showing that** $\frac{4}{6} = \frac{12}{18}$

Both teachers provided appropriate examples of the terms and procedures. The use of hexagon, trapezoid, and triangle manipulatives made the equivalent fractions concrete and visual for the sixth-grade children.

Both Ms. Able and Ms. Baker represented the ideas accurately (criterion three). The trapezoids and triangles used by Ms. Able and the triangles and hexagons used by Ms. Baker were clear and comprehensible representations for equivalent fractions. Ms. Able showed three demonstrations and Ms. Baker gave two demonstrations of the concepts, which is an appropriate number of representations. However, in this lesson only one context was used to represent equivalent fractions. Other models or applications would help students to generalize and apply the ideas and skills.

Ms. Baker made connections (criterion four) between the fraction ideas. For example, she pointed out that the fraction bar meant division, and the units get larger when you simplify. She demonstrated the procedure of finding equivalent fractions by illustrating $\frac{4}{6} = \frac{12}{18}$ with trapezoids and triangles. She pointed out that one trapezoid is equal to three triangles, so four out of six trapezoids meant four multiplied by three was twelve, which is the numerator, and six multiplied by three equals eighteen, which is the denominator. That is, four out of six trapezoids is equal to twelve out of eighteen triangles, which is numerically represented as $\frac{4}{6} = \frac{12}{18}$. However, she did not fully explain the connections between the manipulatives and the symbolic representations of the fractions. Neither teacher engaged the students in making and/or explaining the identified connections. Students simply did the exercises with the manipulatives without being asked to explain the connections between the representations and the fractions symbols.

Ms. Able demonstrated the procedure (criterion five) of changing from an improper fraction to a mixed number by illustrating $\frac{7}{6} = 1\frac{1}{6}$ with a hexagon and triangles. However, she simply showed the procedure of changing seven triangles to one hexagon and one triangle with no justification or explanation for each step and she didn't mention the connection between the picture and $\frac{7}{6}$ and the reason for dividing 7 by 6.

Both teachers provided appropriate amounts and variety of practice (criterion six) since they both connected the exercises with the procedure. For example, Ms. Baker used practice exercises such as $\frac{1}{2} = \frac{?}{4}$, and Ms. Able included exercises such as $2\frac{1}{5} = \frac{11}{5}$. Ms. Able included seven exercises in a 60-minute class period and Ms. Baker included sixteen exercises in an 80-minute class period, so both of the numbers of exercises were appropriate. Neither of the two teachers included a variety of contexts, such as real world contexts.

## Conclusions

The opportunity to characterize qualitative differences in the way teachers presented fraction concepts and procedures was limited in looking at a single lesson. On the other hand, even in a single lesson, it was possible to observe several instances of most of the criteria.

Both teachers had some common characteristics in developing ideas of equivalent fractions and mixed numbers. They did quite well on introducing terms and procedures and representing ideas accurately and clearly. However, neither teacher justified the importance of the fraction ideas and skills, nor did they use a variety of representations. Few of the students were given an opportunity to explain or make connections for themselves between the ideas and the manipulative.

The teachers' implementation of the lessons tended to mirror the AAAS analysis of the textbooks themselves. That is, we found the same indicators to be present or missing in the teachers' lessons that the AAAS analysis did. Even though the teachers used hands-on materials effectively to demonstrate the concepts, student understanding of the concepts and connections suffered, perhaps because the emphasis of the textbooks seemed to be on using the manipulatives without clear guidance to the teachers or students on thinking about the relationships between the activity and the symbolic fractions. Textbooks should, like some of those that received top ratings by AAAS, provide more activities in which students are asked to explain the connections between the concrete and symbolic representations. Textbooks should also provide teachers with more examples of probing questions they can use to determine whether students understand these connections.

In summary, we were able to use the criteria to identify and describe the teaching of specific fraction concepts and procedures. Hiebert and Stigler have supported the idea of "lesson study"(1999). The AAAS criteria offer a useful tool for teachers to study their own lessons and identify their strengths and areas that need improvement. In the coming year, we will use this approach to help teachers examine their teaching through the analysis of videotaped lessons.

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*Curtis D. Robert Professor of Mathematics • Texas A&M University*

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# CAMT 2004

**Strengthening Texas Through Mathematics**
**July 15 – 17, 2004**

CAMT 2004, Strengthening Texas Through Mathematics, will be held July 15-17, 2004, at the Henry B. Gonzalez Convention Center in San Antonio. Program Co-Chairs are Dr. Elizabeth Kreston and David Eschberger. For more information on speaker proposals visit the website at :

*http://www.tenet.edu/camt/*

Registration and program information will be posted on this website in the spring.

# TEA Talks

This column will provide TCTM members with the latest information from the agency about TEKS, educational policy, and of course the TAKS test.

## Hot News

Refer to the websites listed for additional information on each topic

- The Texas Adequate Yearly Progress (AYP) Guide is now available and can be downloaded from the TEA AYP website. This Guide provides a description and explanation of the AYP evaluation process for 2003. The data tables with the preliminary 2003 AYP status of districts and campuses will be available online at noon on September 10, 2003. These reports will not be mailed to districts and campuses, but will be available to be printed directly from the web. Districts and campuses will need to meet performance goals for all student groups previously included on the TEA accountability standards, as well as within the LEP and Special Education student groups. The AYP system is independent of the TEA Accountability system that will be finalized in December of 2003. Many of the measures will be similar; however, the final plan may include measures not included in AYP.

  *http://www.tea.state.tx.us/ayp/index.html*

- A High School Initiative was passed during the 78th Legislative Session. This legislation, SB 1108, has many components that will serve to support the educational progress of students that are at risk of failing for the year or dropping out of high school. Personalized graduation plans and IEPs will be created as a requirement of this law, along with mentoring programs for students. The agency has been charged with the responsibility of creating online diagnostic assessments and interventions that are tentatively scheduled for release in the fall of 2004. Optional Extended Year funds will be expanded to include high school students along with the grade 3-8 students already served in prior years. Watch the TEA website for new information on this exciting initiative.

  *http://www.tea.state.tx.us/*

- The Texas Education Agency will not release tests every year due to a change in legislation created by HB 3459. This law requires the reduction of released tests, and the frequency will now be every other year. The SBOE will be meeting in mid-September to determine the next release date. Tests comprising the student assessment program: TAAS, TAKS, SDAA, and RPTE are all subject to the new release policies created as a result of this legislation.

- The Master Mathematics Teacher program received additional funding during this session, the estimated stipend for this SBEC certification is $5,000. Remember that an MMT must first complete an approved program, pass the SBEC exam, and serve on a TEA designated high need campus to be eligible for the stipend. Two additional programs have been approved by SBEC over the summer: Education Service Center, Region 12 in Waco and SMU in Dallas. Check the SBEC website for additional programs as they become available and to gain more information on testing dates.

- The number of students taking college placement exams is on the rise. Fifty-seven percent of Texas graduating seniors took the SAT I, as did 48 percent of graduating seniors nationally. The total number of 2003 Texas graduating seniors who took the SAT I was 124,571, an increase of 8,114 over the previous year. Nationally, about 1.4 million students took this exam. The average math SAT I score for Texas students is 500 and the national average is 519.

- The number of students taking the ACT is also increasing. In 2003, 73,145 Texas high school graduates took the ACT, up from 67,842 in 2002. Texans represented about six percent of the nearly 1.18 million students tested nationwide. Although test scores normally drop as the number of students increases, the average composite score for Texans remained at 20.1, out of a possible score range of one to 36. That is identical to the state's 2002 average score, despite an eight percent increase in the number of Texans taking ACT in 2003. Nationally, the average composite score remained unchanged at 20.8, as reported by ACT, Inc. The average mathematics score for Texas students was 20.0 this year, compared to 20.1 the previous year. On the national front, the average mathematics score was at 20.6. As educators in Texas we should be proud of the strides we are making to ensure that all students have an equitable chance for attending college.

- Information from the College Board, which oversees the AP program, shows that 90,880 students took AP exams last school year. That represents a 13.3 percent increase in participation over 2002 and a 106 percent increase in the number of candidates since 1998. Many students take more than one AP exam in hopes of earning college credit. Texas students took 164,804 in the 2002-2003 school year, an increase of 14.4 percent over the previous year and a 122.1 percent increase since 1998. Of the 164,804 AP exams taken by Texans, students earned scores of three, four or five on 52 percent of them or 85,545 exams. Typically, colleges and universities award course credit for scores of three or higher. According to the College Board, 42 percent of the Texans who take the exams are minorities, compared to 28 percent nationally.

- The TMDS (Texas Mathematics Diagnostic System) is now available for use with students in grades 5-8. This diagnostic assessment is provided by legislative appropriation in HB 1144. The diagnostic will measure students' algebra readiness and can be administered online or in hard copy in both English and Spanish (grades 4-6). As a new component of the system this year teachers will be able to develop their own assessments by accessing an item bank aligned to all TEKS student expectation statements in grades five through eight. Computer adaptive testing will also be available in early October. Districts will need to register and send data to Vantage Learning, even if the district was registered last year, prior to accessing the system. Directions for registration and data file creation may be found on the TMDS website.
  *http://www.accesstmds.com/*

- The State Board of Education has retained the adopted TAKS passing standards for 2003-2005. All students will be measured against the new 1SEM below the panel's recommended standard, with the exception of students in grade 11 this year. These $11^{th}$ grade students will remain at the 2 SEM below level until they graduate, while current $10^{th}$ grade students will remain at the 1 SEM below level until they graduate. Students will need to get between three and four additional items correct on this year's TAKS test to meet the new passing standard. Refer back to phase-in scores sent with the TAKS 2003 campus results to determine the estimated percentage of students that would meet the standard for the spring of 2004 and 2005. Remember if you are standing still in the curriculum renewal process, you are falling behind as we progress to the new passing standards.
  *http://www.tea.state.tx.us/press/taksapprov.html*

- The textbook adoption cycle has been amended due to the state deficit. Proclamation 2004 will be released next summer for publishers to begin the creation of new instructional materials. The State Board of Education will be discussing the cycle during the fall to determine if elementary or secondary mathematics will be adopted first.

- Have you joined the TEA Mathematics Listserve? The latest information on statewide initiatives, policy, and assessment are available to members that have subscribed to this list. Join today and get the scoop on Texas mathematics.
  *http://www.tea.state.tx.us/list/*

## Keep in Mind

- HB 1144 requires incoming freshman in 2004 to graduate under the Recommended High School Program; this means all students will need Algebra II to graduate. It is important to begin evaluating current district and campus policy regarding this course, and the instructional program available. Begin thinking of new opportunities for students as they enter this rigorous course.

- Many activities, lessons and assessments are available free of charge on the Mathematics Toolkit. If you are searching for TEKS-based materials—this is the website you want to visit!
  *http://www.mathtekstoolkit.org/*

- Research shows long-term, intensive staff development is most effective in changing current practice. Texas is fortunate to have the TEXTEAMS staff development program available for all educators. Check with your local ESC mathematics contact to determine when the next institute will be held. Information about the TEXTEAMS Institutes developed can be found at
  *http://www.utdanacenter.org/texteams/institutes/mathematics/*

*Paula Gustafson • Director of Mathematics*
*Texas Education Agency • <pgustafs@tea.state.tx.us>*

# A Whole Lotto Education!

• *Larry Lesser*

The twice-weekly Texas Lotto Lottery drawings began on November 14, 1992, when I was in the mathematics education Ph.D. program at the University of Texas at Austin. After seeing how many people seemed to have misconceptions about the probabilities and process involved, I decided it was my civic duty as a mathematics educator to offer people a more informed basis about how or whether to play. In 1993, I wrote letters-to-the-editor in local newspapers and created and taught a non-credit, adult continuing education course for the University of Texas Informal Classes called "Lotto Luck!"

The "Lotto Luck!" course was fun to prepare, and it felt good that the course made a difference for the two dozen folks who took it. The course attracted extensive media coverage -- starting from a story spanning 37 column inches in the Austin newspaper (Elliot 1993) all the way to the lead "Dollars and Sense" segment throughout that weekend's Cable News Network (CNN) Headline News! Since then, I've been interviewed by media (e.g., Houston's KTRH-AM, Atlanta's WGST-AM, and Austin's KVUE-TV) on this topic when state lotteries begin new games or amass particularly huge jackpots.

During these unexpected "15 minutes of fame," I've realized anew that good classroom teaching and giving good interviews both require offering examples that allow the listeners to relate the idea to something concrete in their lives or surroundings. Many people of all ages have a hard time visualizing the magnitude of very small or very large numbers. So, let's exercise our number sense as we try to grasp, for example, the probability (1/47,784,352) of winning a jackpot under the Texas Lotto configuration initiated on May 7, 2003 (and discussed later in this article):

How about correctly guessing a particular minute from the life of someone almost 91 years old? How about correctly guessing a particular second a student has spent in academic classes by the time she gets to college? How about picking a randomly selected second of music from a radio station's collection of almost 11,000 compact disks? How about correctly guessing a particular inch from the length of the Pecos River? How about dividing the entire state of Texas (264,508 square miles) into pieces of land 3.55 acres each (many individuals or schools or businesses own land this size), and guessing which piece will be selected? How about picking three people at random and seeing if they all were born on April Fools Day? Students and teachers are invited to add to this list by examining their own surroundings -- perhaps they will find that the probability will be similar to that of choosing a particular cubic inch from the volume of the house they live in. Or perhaps they will relate it to choosing a particular letter of a word from a couple of shelves' worth of books!

Because the course was a single two-hour meeting with no mathematical prerequisites, I used familiar low-tech concrete manipulatives such as spinners and dice to illustrate the probability concepts. The spinners were especially simple: we flicked bobby pins or partially unbent paper clips around the points of our upright pencils!

In the full-semester high school and college mathematics courses we teach, however, we can also utilize powerful technology in our explorations as we build up the underlying mathematics step by step. I have discussed (Lesser 1997) how classes can explore lottery probabilities and expected values using spreadsheet technology, but in this article I offer some probability examples with the common TI-83Plus calculator my high school uses. (Go ahead and get yours now -- I'll wait a moment.)

## Explorations with Drawings

Let's start with how many possible drawings of balls there are. When Lotto Texas started in 1992, 6 balls were drawn without replacement (and without order mattering) from balls numbered 1 to 50. The combination coefficient "50 choose 6" can be evaluated by the TI-83 by entering the following sequence: 50; MATH --> PRB --> nCr; 6; ENTER. We obtain 15,890,700 and the probability a ticket would match the jackpot set of numbers was therefore 1/15,890,700. We can simulate a single drawing (i.e., our own "Quick Pick") with the TI-83 by either the sequence: MATH--> PRB--> randInt(1,50,6); ENTER or the sequence APPS --> ProbSim --> Random Numbers; Set (Numbers: 6; Range: 1-50; Repeat: No); Draw. (I have sometimes used this command when I want an absolutely impartial way of selecting students for a particular task or question.) Students can do many 6-ball simulated drawings and summarize the results. Often they will note and focus on whatever number appears the most often (which motivates some explorations using distributions, as described in the next section).

On July 19, 2000, Lotto Texas was made more challenging by choosing 6 balls from a set numbered 1 to 54. Though the number of balls increased only 8%, the number of combinations increased about 62.5%, which shows how fast combinatorial growth can be. In the bonus ball version of Lotto Texas that started in May 2003, 5 balls are chosen without replacement from white balls numbered 1 to 44, and then a ball is chosen from a set of blue balls numbered 1 to 44. Students can verify that the number of combinations is equivalent to 44 times the quantity "44 choose 5", and that this means the jackpot became 3 times harder to win than the 1992 game.

It was recently reported (Hughes and Marshall, 2003) that by around early November 2003, Texans would be able to play the Mega Millions multistate lottery in which players select five numbers from 1 to 52 and then a sixth number from 1 to 52, resulting in a jackpot probability of 1 in 135,145,920, which is 8.5 times harder to win than the 1992 Lotto Texas game! By the way, the rationale of having the jackpot odds that increase in difficulty is that jackpots will be won less often, therefore rolling over into larger jackpots more often, presumably tempting more people to play (and generating more money for Texas education!).

## Explorations with Distributions

Recall that for discrete probability distributions, the pdf (probability density function) gives you the probability that a random variable equals a specific value, while the cdf (cumulative distribution function) gives the probability a random variable is less than or equal to a specific value. While these concepts can be very abstract to students in their first statistics course, lottery drawings offer a concrete real-world application for many of the most commonly encountered discrete random variables. (Actually, drawing one number from a set of numbered balls has already illustrated the discrete uniform distribution!) We will keep this section concise, but the distributions that follow are all also in the TI-83Plus calculator manual (with syntax examples, defining formulas, and screen shots) and in standard statistics textbooks:

A binomial random variable is the number (x) of successes when there is a fixed number (n) of independent trials, each of which has a success probability (p). Entering values for n, p, and x (in that order) into the TI-83's binomcdf or binompdf commands (via the 2nd DISTR keys) can be used to investigate how rare it is for a particular lottery number to occur so frequently out of a fixed number of drawings. For example, in the 1992 50-ball game, the probability of the number 17 occurring exactly 3 times in 20 6-ball drawings would be binompdf(20, .12, 3), or about 22.4%.

A geometric random variable is the number x of trials (each of which is independent and has probability p of success) for the first success to occur. The geometpdf and geometcdf commands (inserting numbers for the values of p and x) can therefore be used to explore the probability that it takes a particular number of drawings before a drawing includes a particular number (or, for that matter, to finally win a jackpot). For the 1992 game, the probability of no more than 50 drawings needed for the first occurrence of 17 is: geometcdf(.12, 50), which is about 99.8%.

A Poisson random variable is the number x of occurrences of an event during an interval with a known average (mean) number of occurrences per interval of that length. If players choose number combinations randomly (e.g., Quick Pick), then we can use the Poisson distribution to find the probability of, for example, exactly one person winning the jackpot. If the jackpot probability is p and t tickets are sold, then the number of jackpot winners is a binomial random variable with expected value pt. Using the value of p for a particular lottery game (as discussed in the Exploration with Drawings section), try out various values of t to see when the probability of winning a jackpot all by yourself is maximized. On the TI-83 use this sequence: 2nd DISTR; poissonpdf(your value for p times t; 1).

And to explore the randomness of all the past Lotto numbers chosen (they may be found via the Texas Lottery website), we can do descriptive statistics and a chi-square test, as did Lamb et.al. (1994). For extra credit, the hypergeometric distribution is not in the TI-83Plus calculator, but can be assembled as an arithmetic combination of three uses of the MATH --> PRB --> nCr sequence. The hypergeometric distribution allows students to verify the playslip's printed odds of winning each of the cash prizes, not just the grand jackpot. Students may also be interested to verify the surprising situation of a national lottery in which the fifth-highest cash prize actually turned out to be more probable than the sixth prize (Helman 2003)!

## A Musical Conclusion

Let's close with a very different mode of outreach education I've employed on this topic -- a math song! I first published this parody lyric in the Winter 2002 issue of *STATS*, and performed it most recently at Lesser (2003). The lyric may be sung to the tune of the Don Schlitz song of the same title (which yielded Kenny Rogers a #1 country hit, signature song, and television mini-series). It helps students distinguish the small bit of strategy (e.g., Henze and Riedwyl 1998) from the multitude of misconceptions:

"The Gambler"
(lyrics (c) 2001 Lawrence M. Lesser;
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On a warm summer's evenin', on a train bound for nowhere,
I met up with a gambler -- we were both too tired to sleep.
So he told me how he planned winnin' lottery prizes
'Til, as a math teacher, I just had to speak:

"Son, I've made a life out of readin' students' faces,
Checkin' comprehension by the way they held their eyes.
And I can see your blackboard is erased in some places--
Give me some peanuts and I'll give ya some advice.

First, your instant scratch-off tickets give 1 in 5 chances,
But that don't mean that 1 in 5 will win.
'Cause ev'ry ticket's sep'rate, like a new flip of a coin:
It has no mem'ry how your wallet's gotten thin!

And you track those weekly draws, you say ya got a system--
You call some numbers "hot", you deem others "due";
But I insist, they each have the same chance--
If you're gonna play the game, boy, ya gotta know what's true!
(Chorus)
You gotta know when you pick 'em,
What's superstition,
And where strategy is there to be had,
Or you'll learn why
Lotteries seem like
Tax on folks who don't know much math!

Now all sets of numbers are equally unlikely,
More rare than death by lightning, still there's somethin' you should know;
If you should happen to win that big jackpot,
You'll win more money if you picked it all alone!

So avoid those numbers that more folks are playin':
Like sevens and birthdays and sequences, too.
'Til this song gets famous, you'll have the advantage--
Maybe you'll thank me with a share of your loot!"
(Repeat Chorus)

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Texas Lottery web site: *http://www.txlottery.org/*

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**Deadline:** May 1, 2004

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Last First Middle

Address: ______________________________________________
Number and street Apt. number

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Are you a member of TCTM? ☐ yes ☐ no Have you attended CAMT before? ☐ yes ☐ no

**Note:** If you are not a member of TCTM, you must enclose $13 with this application to apply for membership.

How long have you been teaching? ____________ Describe your primary teaching responsibilities

Send your completed application to:

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by fax: **(512) 232-1855 ATTN: Cynthia Schneider**

by email: ***<cschneider@mail.utexas.edu>***

# Building on Children's Thinking to Develop Proportional Reasoning

• Susan B. Empson
• Jennifer Knudsen

In this article, we show how a student's own reasoning — along with good problem-setting and guidance from the teacher — is used to help a student develop her proportional reasoning abilities. Through analyzing what one student did, we provide insights and strategies teachers can use with all students.

First of all, what is proportional reasoning? We know it is central to learning goals in Texas, particularly at middle school, but starting even earlier. We also know proportional reasoning serves as a foundation for algebra and other higher level mathematics. We focus on three related aspects of proportional reasoning, demonstrated in the student's work: the use of unit rate, equivalent ratios, and patterns of proportionality.

Unit rate is a way of expressing how many $y$ we get for one $x$. Miles per hour and cents per donut are two examples of commonly used unit rates. When students use unit rates to solve problems, they find the number of $y$ for each $x$ given the information in a particular problem or context. Then they use the 1 to $y$ unit rate to construct other rates to solve the problem.

Reasoning with equivalent ratios means students use relationships found among the ratios and terms in the form, $a/b=c/d$. Often three of the terms will be given, and students will need to find a fourth. Flexible reasoning allows students to see that $a$ is related to $c$ by the same multiplier that relates $b$ to $d$, as well as to understand that the quotient $a/b$ is equal to the quotient $c/d$. In elementary aged students' problem solving, however, students will not likely set up equations, but their reasoning – as we show in this paper – still fits these patterns. Notice that this flexible reasoning completely bypasses the problematic cross multiplication strategy (see Stanley, McGowan, & Hull, 2003 for a discussion of the "pitfalls" of cross multiplication).

Finding, extending and using proportional patterns is the third kind of proportional reasoning we consider. Using proportional patterns is simply an extension of reasoning with equivalent ratios that implies the potential for infinite continuation of the pattern. Proportional patterns involve two related and changing quantities —$x$ and $y$— for which every pair (x, y) satisfies the equation $y = kx$. The patterns can be expressed in tables of values. When students make tables of related variables, then use those tables to extend their reasoning and solve problems, they are building a strong foundation for algebra (Kaput, 1989).

Although we have used algebraic symbols to describe three aspects of proportional reasoning, when students first develop and use these strategies they do not use these symbols. Instead they will use the kind of reasoning we describe in detail in the following case of a student solving a series of related problems.

The case involves a fifth-grade girl's thinking as she solved three missing-value proportion problems. Krystal was mathematically proficient in many ways: she was a flexible problem solver, could recall and use multiplication facts, and was confident of her mathematical abilities. She was African American, and attended school in a suburb of a large Texas city. The first author, Empson, worked with Krystal for 45 minutes in a problem-solving session. Empson posed a series of related problems, each a bit more difficult than the last, and asked Krystal questions that helped her extend her thinking and connect ideas.

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| Type of Problem | Problem |
|---|---|
| Missing-value proportion $\frac{2}{3} = \frac{\square}{6}$ | At one table, 3 children are sharing 2 boxes of clay. How many boxes of clay should a table of 6 children get, so that each child has as much clay as a child at the first table? |
| Missing-value proportion $\frac{6}{8} = \frac{\square}{12}$ | At one table, 8 children are sharing 6 liters of soda. How many liters of soda should a table of 12 children get so that each child has as much soda as a child at the first table? |
| Missing-value proportion $\frac{15}{18} = \frac{25}{\square}$ | Joan used exactly 15 cans of paint for 18 chairs. How many chairs can she paint with 25 cans? |

**Table 1. Problems solved by Krystal**

Empson chose the numbers in the first problem to involve the simplest proportional relationship possible: doubling. The problem read: "At one table, 3 children are sharing 2 boxes of clay. How many boxes of clay should a table of 6 children get, so that each child has as much clay as a child at the first table?" First Krystal figured each child at the first table would get two thirds of a box, because three children each sharing one box of clay get one third of the box, and three children sharing the second box of clay get another one third. Then she added this amount six times, once for each child at the second table, to get four boxes total (Fig. 1). Krystal found a unit rate — two thirds of a box of clay per child — though she did not use those words. She then used the unit rate and repeated addition to find her answer of four boxes.

Wondering if Krystal could use the doubling relationship between the numbers to solve the problem, Empson asked if she could think of a way to solve it without using fractions. Krystal thought to herself for a while, then explained, "What I did is, I estimated this at first, but I went back to check and make sure. Three is twice as many-- I knew that three times two was six, and six is twice as many as three, so you should get twice as many boxes of clay. And two times two is four." So Krystal recognized the multiplicative relationship between the quantities in the problem, but she did not seem confident this relationship could be used to solve the problem, as she referred to her method as an "estimate." In fact, Krystal was reasoning with equivalent ratios by using the fact that the ratio two to three is related to four to six by the multiplier two. (Symbolically, $\frac{2}{3} = \frac{4}{6}$, though she of course did not use this form.)


**Figure 1. Using a part-whole representation of two thirds, Krystal added two thirds of a box six times to get four boxes.**

Next, Empson gave Krystal this problem to solve: "At one table, 8 children are sharing 6 liters of soda. How many liters of soda should a table of 12 children get so that each child has as much soda as a child at the first table?" This combination of numbers is harder, because the proportional relationship between the two tables is not a whole-number multiple. Krystal started out as before, finding the unit rate showing how many liters per child. She figured each child at the first table got $\frac{3}{4}$ of a liter of soda, and used that quantity to build the total number of liters needed at the second table. This time she created, essentially, a table of values (Fig. 2). She said, "Well I knew that everybody at this table (the first table) was going to get three fourths. Well I found out that one is three fourths (i.e., one person gets three fourths of a liter of soda), two people would be one and a half, two people would be two and one fourth, and four people would be three whole (liters). And when I got to three whole I knew it would be four people, and four goes into 12 three times, and three times three is nine." Krystal's build-up strategy demonstrates the emergence of proportional reasoning: she identified the multiplicative relationship between the number of children at each table, and used it to determine the number of liters for the second table. (In other words, $\frac{3}{4} = \frac{9}{12}$, because each quantity in the second ratio is three times bigger than each corresponding quantity in the first ratio.) This connection was facilitated as she created the table of values by finding an intermediate ratio in which the number of children was a factor of the number of children at the second table. (Notice we are applying proportional vocabulary to Krystal's thinking. She is not explicitly using $a/b = c/d$ or $y = kx$. But these algebraic forms allow us to see the proportionality involved in her solutions.)

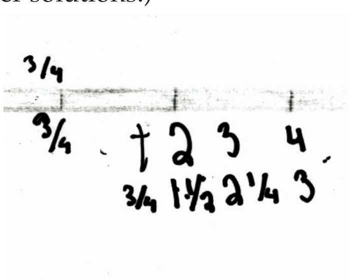


**Figure 2. Krystal built a table of ratio quantities using the unit rate**

Intrigued by her use of ratios, Empson again asked Krystal if she could solve the problem without using fractions. She thought for a long time. Then, she said, "Oooh. I found a pattern. …. Well what I see is, half of eight is four. …So, eight plus eight divided by two, or eight plus half of eight – which is four – equals 12. And six plus half of six – which is three – equals nine. But I just noticed that. So really I don't know how I could solve it without fractions." Krystal described the proportional relationship between the quantities at the two tables using a combination of additive and multiplicative concepts. She used halving, another simple form of multiplicative reasoning to generate a ratio, four to three, equivalent to eight to six, and then combined these two ratios, adding the component quantities of the ratios. (In other words, $\frac{8}{6} = \frac{4}{3} = \frac{12}{9}$.)

Wanting to follow up on how Krystal used ratios in her first strategy, Empson made a suggestion: "Could you solve this problem using cubes?" Krystal counted out six Unifix cubes for liters of soda, and eight Unifix cubes for people, and, thinking aloud, said, "I knew they get three fourths. That's the same as saying for every two-- no. For every four children, there are three liters of soda." As she talked, she paired up four cubes with three cubes to illustrate the ratio: "So for this four there's three, and for this four there's three (Fig. 3a). She partitioned the first table into two identical ratios. Counting out 12 cubes for people at the second table, she said, "So, if I did that over here, there's 12 kids … if for every four children there are three liters of soda over here (at the first table), I'm going to try that here (at the second table), so they get the same." Krystal partitioned the 12 cubes into groups of four, then placed three cubes, for liters of soda, with every group of four people (Fig. 3b). Counting "three, six, nine" for the total number of liters of soda at the second table, Empson remarked, "So that works too," and Krystal agreed. As in her previous solutions, she built up related values with repeated addition, which is a simple form of multiplication.

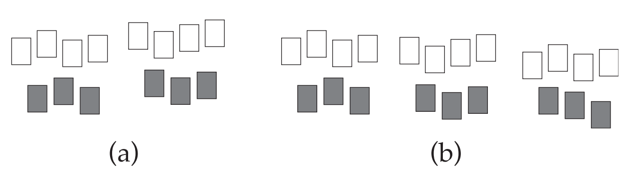


**Figure 3. (a) Krystal figured eight children sharing six liters could be broken down into two equivalent ratios of four children sharing three liters; and (b) used the ratio of four children sharing three liters to build a table of 12 children sharing nine liters.**

Empson asked Krysal to solve one more

problem: "Joan used exactly 15 cans of paint for 18 chairs. How many chairs can she paint with 25 cans?" This problem is difficult for two reasons: 1) the multiplier between the like terms (i.e., cans and cans) is a not a whole number, and does not involve halving or doubling; and 2) the context of painting chairs has no natural unit rate (one could think either about how many cans per chair, or how many chairs can be painted per can). In a study of 115 sixth graders, Kaput and West (1994) found only about 17% solved this problem.

Krystal thought for a long time, wrote some fractions on paper, then exclaimed, "I have no idea how to do this!" Empson asked her if using the cubes could help her and she said no. She wrote some more fractions on paper (Fig. 4), then said, "I don't know. I've totally lost myself. I can't even tell you what I was doing!" If Krystal was trying to use her earlier unit rate strategy, she may have run into trouble because the unit rate she needed for this problem was chairs per can, rather than cans per chair, which follows the order given in the problem. Additionally, the required fractions are more difficult than in the previous problems.

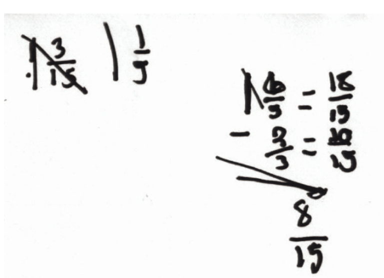

**Figure 4. Krystal's first attempts to solve the problem.**

Empson suggested she try solving it with the cubes, thinking that she might be able to draw on her previous use of ratios. Krystal replied, "Maybe," then, "Yes it will help!" She abandoned the fractions she was writing and began to think out loud in terms of ratios to solve the problem: "For every 15 cans (counts out 15 cubes), you paint 18 chairs (counts out 18 cubes), so, that means for every (pulls out a group of three cubes), say, three chairs..." She partitioned the 15 cans into five groups of three cans each. She started to partition the 18 chairs into groups of three too, then stopped herself with an insight. "Oh! For every five cans she spray painted (pauses, thinking) six chairs!" In other words, Krystal recognized not only that three was a common factor, but that the groups of cans and chairs had to be partitioned into *three groups*, instead of *groups of three*. Then she used the ratio unit of five cans for six every chairs to build the situation where 25 cans of paint were used. Referring to the cubes she had already assembled, she said, "Well, if for every five cans she spray paints six chairs, that's only 15 cans. If you add another five cans (pulls out five more cubes), that'll only make 20 cans. That would be another six chairs (pulls out six cubes to pair with the five). So 6 plus 18 is 24… So 24 chairs. But it's only 20 cans. So if I add another five cans (pulls out another five cubes), that would be 25, then you add six more (for the chairs). So it's 30 chairs."

After Krystal had solved the problem, Empson asked how the cubes had helped her. She replied, "Well, what I had to do was, I had to visualize how many you could do for every, say for every, in this case, five… and go on and keep adding five till I got to 25."

Empson also asked her why, at first, she put the 15 into groups of 3. She explained, "I was trying to find a multiple (sic) of both 18 and 15, that went into both 15 and 18. But then I figured out that 18 can't go into-- If I did it with threes, it turned out to be five groups, and five can't go into 18…. Because each person had to have the same amount for the strategy to work. For every group, they had to have the same amount [i.e., same number of groups] for the strategy to work."

What can teachers take to their classrooms from this one-on-one session? In the discussion above, it is important to note that the new problem solving strategies came from Krystal herself, not the teacher. Empson used three teaching strategies to elicit these:

1) She posed related problems that built in difficulty. 2) She asked Krystal for alternate strategies even after she had found the right answer. 3) She encouraged Krystal to reuse strategies that had yielded success and related to proportional reasoning. All of these teaching strategies can be applied in a classroom full of students.

In selecting problems, when the problem type is new, familiar number relationships can help students develop solution strategies. Students understand doubling and halving earlier and with more confidence than other multiplicative relationships, so the first problem Empson used relied on doubling to help Krystal reason with equivalent ratios. Teachers can build or select a series of word problems with similar mathematical structure, but using increasingly complex number combinations, to help students develop understanding and confidence.

In a classroom setting, students can be called upon to provide a variety of solutions via the chalkboard.[1] Students can then explore and compare these different solution methods. Interestingly, Krystal's first stated strategy depended on finding the unit rate (though she did not call it that), then building up a solution using repeated addition, which involved finding and then adding fractions. On further probing, though, Krystal revealed she had used doubling but did not have confidence in this strategy. She was able to see, through Empson's questioning, that each solution gave the same answer, building her confidence in reasoning on the basis of equivalent ratios

Finally, Empson encouraged the use of ratio strategies, which relied on multiplication or skip counting. She made these strategies easy to model by making the cubes available to Krystal and asking if she could use them to solve the problem at hand. Teachers can provide students with tools such as these that help them build models that support their reasoning. Cubes, strips of construction paper, and Cuisinnaire rods are among the possibilities. Encouraging students to draw diagrams to support their reasoning is another. As a next step in supporting the type of reasoning Krystal displayed in this session, teachers might introduce the T-table as a tool to keep track of the related values Krystal generated (e.g., Fig. 5).

| children | liters |
|---|---|
| 4 | 3 |
| 8 | 6 |
| 12 | 9 |
| ? | 12 |
| 28 | ? |
| ? | ? |

**Figure 5. Example of a T-table, using values from Krystal's second problem**

Full development of students' proportional reasoning takes time. In this one session, significant time was devoted to solving just three problems, and using them to elicit and solidify strategies. But this is only one lesson in a series that will build across the years. Krystal demonstrated the beginnings of solid reasoning on which teachers can build, enabling students' success in higher and higher levels of mathematics. This kind of reasoning at fifth grade will support students' writing and using proportions in symbolic form in seventh grade, and the development of reasoning with algebraic symbols and about functions in eighth grade and beyond.

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[1] Use the chalk board, or poster paper, rather than the overhead projector, so that students may see all of the strategies at once for comparison purposes.

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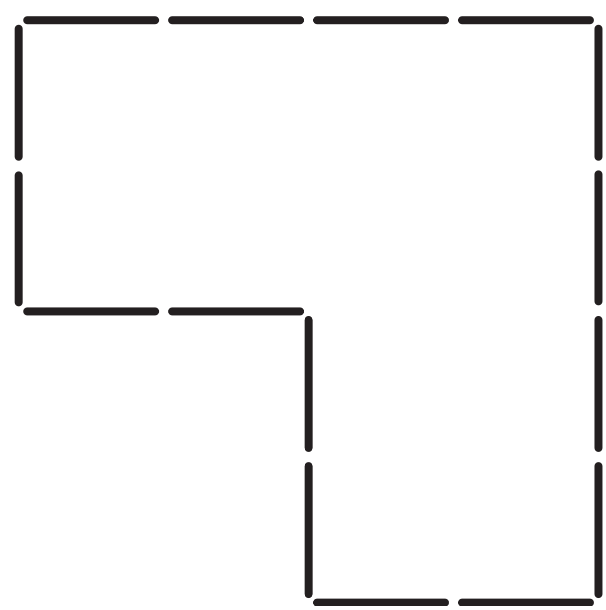
# Puzzle Corner

We are interested in how your students responded to this problem and how they explained or justified their reasoning. Please e-mail copies of your students' work, include your name, grade level, campus name and district name to Mary Alice Hatchett, Director of Publications *Texas Mathematics Teacher*. Selected submissions will be acknowledged and published in subsequent issues.

## Sticks #2

Please prepare a sketch of your solution

Arrange 16 craft sticks to form the following figure



Add 8 more sticks to divide the area into four congruent figures without disturbing any of the original sticks.

# TCTM E. Glenadine Gibb Achievement Award Application

**Eligibility:** The E. Glenadine Gibb Achievement Award is presented to someone nominated by a TCTM member to be honored for his/her contribution to the improvement of mathematics education at the state and/or national level.

**Deadline:** May 1, 2004

Information about the **TCTM member** nominating a candidate:

Name: ______________________________________________
Last First Middle

Address: ______________________________________________
Number and street Apt. number

______________________________________________
City Zip Code

(___) ____________________ (___) ____________________ < ____________________ >
Home Phone Work Phone Email Address

Are you a member of TCTM? ☐ yes ☐ no NCTM? ☐ yes ☐ no

Information about the **nominee**:

Name: ______________________________________________
Last First Middle

Address: ______________________________________________
Number and street Apt. number

______________________________________________
City Zip Code

(___) ____________________ (___) ____________________ < ____________________ >
Home Phone Work Phone Email Address

Is the nominee a member of TCTM? ☐ yes ☐ no NCTM? ☐ yes ☐ no Retired ☐ yes ☐ no

Applications should include 3 pages:

- ☐ Completed application form
- ☐ <u>One</u>-page, <u>one</u>-sided, typed biographical sheet including:
  - Name of nominee
  - Professional activities
  - National offices or committees
  - State TCTM offices held
  - Local TCTM-Affiliated Group offices held
  - Staff Development
  - Honors/awards
- ☐ <u>One</u>-page, <u>one</u>-sided essay indicating why the nominee should be honored for his/her contribution to the improvement of mathematics education at the state/national level

Please submit the completed application, biographical sketch, and essay

by mail: **Cynthia Schneider, 234 Preston Hollow, New Braunfels, TX 78132**

by fax: **(512) 232-1855 ATTN: Cynthia Schneider**

by email: ***<cschneider@mail.utexas.edu>***

# Mathematics, the TEKS, and Literature
## Is it a good fit?

• *Caren Sorrells*

The Texas Essential Knowledge and Skills have been the state's adopted curriculum since 1997, but we still seem to struggle with implementing them in our Texas classrooms. Teachers have difficulties remembering them and embracing them, and they struggle with just exactly HOW to teach the TEKS to the students.

To help teachers in Texas, I would like to offer one scaffold support beam. All elementary children enjoy picture books. We encourage them to read books, and have books, and we urge parents to read to their children as an important foundation of literacy. We even read books to our classes at school. . . . But do we use literature in the math classroom on a regular basis? I would like to encourage teachers to do just that.

Literature connections to math can be easy, fun and stimulating to the students, especially in a problem solving setting. Third grade students can do some very in-depth problem solving after reading *17 Kings and 42 Elephants* (Mahy, 1987). The teacher poses the question, "How do 17 kings take care of 42 elephants?" The students must decide how many elephants each king would have to care for, and then they must justify the "fairness" of their answer. If you have just grabbed your TEKS and discovered that third grade students do NOT do any division with two-digit numbers, I would agree. These students are using manipulatives to discover their solutions (which can all be different). The students are problem solving and communicating mathematically. They need to use strategies that they have developed in the past to solve today's problem. This not only supports the TEKS, but also is supported by brain research.

Fifth grade students learn about area and perimeter as they read *Spaghetti and Meatballs for All* (Burns, 1997). The students use manipulatives to move the tables and decide seating capacity while they record their actions and mathematical expressions for what is happening. Next they need to apply their learning as they create a banquet for thirty-six people. These students easily cover six different student expectations, including multiplication and division, during this lesson.

Fourth graders use transformational geometry as they read *When a Line Bends. . . A Shape Begins* (Greene, 1997). They read the book and discuss the differences between the shapes in the book and polygons in geometry. The lesson exploration has them build dominoes, tetrominoes and finally pentominoes. As they build these polygons, they use geometric vocabulary, including translations, reflections, rotations, angles, congruence and more.

Literature connections are abundant and great tie-ins to the mathematics TEKS. Teachers enjoy using the books as much as the students enjoy having the books be a springboard to their learning. An abundance of publications provide educators with literature lessons connected to mathematics. The Internet is also teeming with resources.

So please, go forth and read…mathematically.

*Caren Sorrells • <caren_sorrells@birdville.k12.tx.us>*
*Math Consultant • Birdville ISD*

References

Burns, M. (1997). *Spaghetti and Meatballs for All*. NY: Scholastic Press.

Greene, R. G. (1997). *When a Line Bends…A Shape Begins*. NY: Houghton Mifflin Company.

Mahy, M. (1987). *17 Kings and 42 Elephants*. Puffin NY: Pied Piper.

# Awards Recipients

## TCTM Leadership Award

Honored for her service in mathematics education in Texas to improve professional development and empower teachers to provide the best teaching environment, **Bonnie McNemar** of the University of Texas at Austin received the 2003 **TCTM Leadership Award**. Bonnie has been instrumental in the design and delivery of innovative staff development such as the Texas Mathematics Modules and TEXTEAMS institutes. She possesses a tenacious desire to serve the mathematics teachers of Texas. She was recognized for her contributions to the improvement of mathematics education in Texas at the 2003 CAMT luncheon in Houston.


McNemar

## TCTM E. Glenadine Gibb Achievement Award

Honored for her service in mathematics education at the state and national level to empower teachers to provide the best teaching environment, **Dinah Chancellor** of Carroll ISD received the 2003 **E. Glenadine Gibb Award** from the Texas Council of Teachers of Mathematics. Dinah has served as CAMT board president, initiated Math-A-Rama at CAMT, and is an author for hand-held technology publications. Her dignity and respect for others is a model for all teachers and leaders. She was recognized for her contributions to the improvement of mathematics education in Texas at the 2003 CAMT luncheon in Houston.


Chancellor

## 2003 CAMTership Awardees

Two $200.00 CAMTerships were awarded this past summer by TCTM. We would like to extend our congratulations to **Nakendra Matthews** of Nacogdoches and **Kim Wheeler** of Friendswood. Kim has been teaching five years and Nakendra has been teaching for two years. Both recipients volunteered two hours of their time at CAMT and attended the annual TCTM Business Meeting and Breakfast as guests of TCTM. If you are a member of TCTM, have not attended CAMT before, and have been teaching for five or fewer years, look for the CAMTership application in the *Texas Mathematics Teacher*. The CAMTership is intended to encourage beginning teachers to attend CAMT by helping cover part of the expenses associated with attending the conference.

## 2003 TCTM Mathematics Specialist Scholarship Awardees

Two $1,000.00 scholarships were awarded this past summer by TCTM. We would like to extend our congratulations to **Hugo M. Perez** of El Paso and **Glenn E. Lahodny, Jr.** of Schulenburg. Mr. Perez is attending the University of Texas at El Paso, and Mr. Lahodny is attending Texas Tech University. As part of his application, Mr. Perez stated, "I would love to teach high school students. I believe that the information will stay fresh on their memory, right before college." Mr. Lahodny wrote, "I look forward to the opportunity to teach math to students at the middle and high school level. People use math every day of their lives. The greatest contribution I can give future generations of students is a solid foundation in math." Congratulations and good luck to these college freshman!


Lahodny


Perez

## *Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST)*

The Presidential Awards for Excellence in Mathematics and Science Teaching (PAEMST) identifies outstanding mathematics and science teachers, kindergarten through 12th grade, in each state and the four U.S. jurisdictions. These teachers serve as models for their colleagues and leaders in the improvement of science and mathematics education. The 2003 Texas nominees are :

- Nancy Margarita Arroyo, Riverside High School, Ysleta ISD, El Paso, Texas;
- Edy Lu Gaston, B.F. Terry High School, Lamar Consolidated ISD, Rosenberg, Texas;
- Susan Carole Green, Colleyville Middle School, Grapevine-Colleyville ISD, Colleyville, Texas.

The 2003 PAEMST Awardees will be announced at the beginning of March 2004. Each Presidential Awardee will receive a $10,000 award from the National Science Foundation. Each award recipient will also be invited to attend, along with a guest, recognition events in Washington, D.C. during the week of March 15-20, 2004. These events will include an award ceremony, a Presidential Citation, meetings with leaders in government and education, sessions to share ideas and teaching experiences, and receptions and banquets to honor recipients.

The competition alternates each year between teachers of grades K-6 and teachers of grades 7-12. The nomination form for 2004 (K-6 teachers) can be downloaded at :

*http://www.ehr.nsf.gov/pres_awards/2004_Nominations_form.doc*

or see

*http://nsf.gov/pa/*

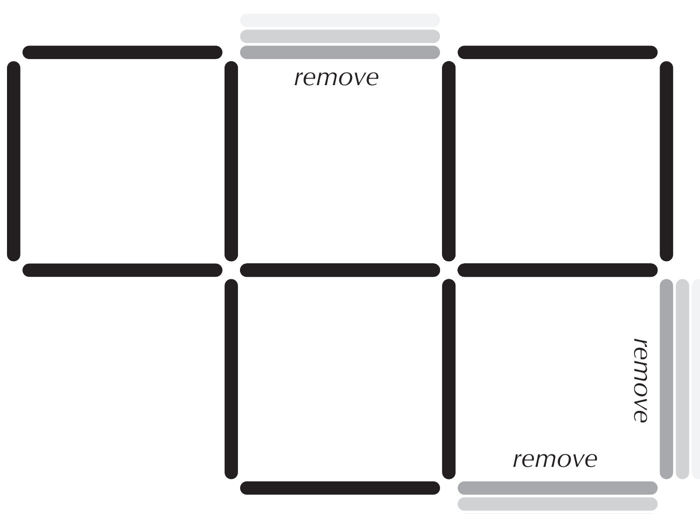
Nomination forms must be submitted to Paula Gustafson at the Texas Education Agency prior to the application being sent to qualified candidates. Email her at <*pgustafs@tea.state.tx.us*> if you would like to nominate a colleague.

# Puzzle Corner

## Sticks #1 Answer

Arrange 15 craft sticks to form 5 squares.
Remove 3 sticks, leaving 3 squares

Shown is a diagram of *a* solution



# So, Who Invented the Order of Operations?

• *Concepcion Molina*

As part of their long-term professional development for mathematics teachers, the Eisenhower Southwest Consortium for the Improvement of Mathematics and Science Teaching (SCIMAST) polled math faculties at selected campuses in the southwest. Teachers indicated that the order of operations was a topic in mathematics that was difficult for their students. Our research turned up no training model focused strictly on the order of operations, and no staff member could recall ever seeing a presentation or session on the topic at any mathematics conference. Perhaps researchers see the topic as so simplistic that it does not warrant special attention or focus. After all, in the typical lesson on the order of operations, students memorize a simple list of rules and the only justification students are given is the need for everyone to arrive at the same answer for a given computation. For use in a training model for professional development for mathematics teachers, the justification "to get the same answer" is shallow at best. Any instructor developing a lesson should be prepared for that one student who is always asking why. With the order of operations however, there has been no problem with that inquisitive student because the teacher is ready with the tried and true "to get the same answer" justification.

If these rules were invented for the purpose of reaching consistent solutions, then some astute mathematician from years past would have received recognition for his or her contribution. The research yielded no such inventor. If no one "invented" the order of operations, what is the origin and foundation of those rules? This question was a driving force behind the search for understanding of the mathematics content and the reflective thinking needed to develop an effective model for teaching the order of operations.

## The Order of Operations Training Module

To develop a two-hour training module addressing "Please Excuse My Dear Aunt Sally" was a daunting task, but after considerable thought regarding the mathematical foundation of the rules, the following goals were used to develop this module :

I. To investigate what difference order makes in real life and in mathematics.

II. To discover and connect the prerequisite knowledge needed to understand the order of operations.

III. To understand the order of operations and the basic properties that serve as its foundation

IV. To understand possible negative implications of the order of operations.

To address Goal I, participants were asked to consider everyday activities or actions that involve several steps in which the order of the steps matters and in which the order of the steps does not matter. This is nothing new to teachers. Goal II is patterned after the idea of "knowledge packages" (Ma, 1999, pp. 17-19) which focus on identification of the critical prerequisite mathematics knowledge necessary for a student to learn and understand a new mathematics topic. The result for this training is usually a list that consists primarily of facts and procedural skills. Up to this point nothing in the training is different or unexpected. However, the experience with the use of this module indicates that in Goal III participants will hit a brick wall because of the following questions:

A. Why are addition and subtraction done last?

B. Why must multiplication and division be done before addition and subtraction, and what is the role or purpose of the multiplication and division, assuming one or both are part of the computation?

C. What is the connection between the distributive property and the order of operations?

D. What is the role or purpose of grouping symbols?

E. What can you conclude regarding the rationale used for the establishment of the "rules" in the order of operations?

The key to the training is the problem 4 + 6 • 5. Participants are asked if a middle school student could compute it correctly without any formal instruction on order of operations. All teachers to date answered "no" to the possibility of middle school students consistently computing correctly a problem such as 4 + 6 • 5 without being taught the order of operations. Participants are then asked if students could answer the question "How much money would I have if I had 4 pennies and 6 nickels?" Almost all agree that students can do this because of the contextual clues not given in the original expression 4 + 6 • 5. An investigation as to why this is so leads us to the answers for the Goal III questions.

Elementary students first learn the process of addition with concrete objects and always in some context such as 2 apples plus 3 apples is 5 apples, 4 cows and 2 more cows is 6 cows, etc. Further down the math education road we drop the context and use only the numeric symbols 2 + 3 = 5, 4 + 2 = 6, etc. Students all too often work with numerals with no context or knowledge of what each number represents. Before long students lose sight of the very simple fact that we can only add apples with apples and cows with cows. Students and adults alike forget that the expression 2 + 3 *assumes* that 2 and 3 both represent cows and can therefore be combined. Thus the idea that only like items can be added or subtracted is critical.

There is a second important mathematical concept that plays a key role in this situation. If students knew the true meaning of multiplication, they would realize, even without context, that in the expression 4 + 6 • 5, the 4 and 6 are not both cows and thus cannot be combined. The 6 • 5 represents 6 sets or groups of 5 items each. This investigation leads us to conclude that 4 + 6 x 5 could easily be done in the context of 4 pennies plus 6 nickels without formal training in the order of operations. However, without that context, most students will answer with 50 because they are accustomed to reading from left to right. Students have forgotten that you can only add or subtract like items, and they might also not understand the meaning of multiplication and thus do not envision what 6 • 5 represents. If both the aforementioned concepts are firmly entrenched, then it makes it possible to answer and understand questions A and B. Whether given a context or not, the fundamental foundation is that *quantities can only be added or subtracted if they are like items* – cows with cows, etc. The multiplication or division must be done prior to the addition or subtraction because it is necessary to determine how many of those like items you have in each expression (such as 6 • 5), *then* you can add or subtract whatever those like items are.

The answer to C is best understood by looking at the standard multiplication algorithm. If we use the example 23 • 45, what is the process that is followed? The multiplication is done first, then the addition. The order of operations is exemplified by the standard multiplication algorithm, which in turn is a direct example of the distributive property. What we tend to forget is that the proper name is not the "distributive property", but rather the distributive property of *multiplication over addition*. It is important to note that the standard multiplication algorithm also illustrates the principle that only like items can be combined. If you consider the 23 x 45 example, we do the multiplication first, but we line up the partial products in such a way so as to insure that we are combining like items… ones and ones, tens and tens, hundreds with hundreds, etc. Like items do not have to necessarily appear in overt forms such as cows.

Math teachers know the answer to question D is that the grouping symbols are used as a means to perform the operations in a different or specified manner. For example, without context or symbols, in the expression $20 - 5 \bullet 1.08$, we would multiply before subtracting. But if we needed to take a $5.00 discount from a $20.00 item prior to assessing an 8% sales tax, some man-made interjection is needed to allow the appropriate computation. This imposition is the introduction of grouping symbols that allow that to happen. In answering question D, a teacher in one of the training sessions stated that the grouping symbols serve the purpose of allowing us to perform operations differently than they would have *naturally* been done.

This thought leads directly to Question E which asks what one can conclude regarding the rationale used for establishment of the "rules" in the order of operations. Did someone sit down and deliberately make up these rules, or were they just "naturally" based on the simple principle that only like items can be combined through addition or subtraction, and that the role of multiplication and division are simply to convert sets or groups into quantities of like items that can then be combined? It is apparent that no one "invented" the order of operations. Perhaps these rules should be called the *Natural* Order of Operations. With the exception of the man-made injection of grouping symbols such as parentheses, do these "rules" actually exist as a separate math concept? If students truly understood the math concepts discussed, the contention of the SCIMAST staff is that students could compute an expression such as $4 + 6 \bullet 5$ without formal training in what we call the order of operations. Rather than have students memorize a list of rules that supposedly exist only to insure that we all get the same answer, shouldn't we have them look deeply into the fundamental mathematics that serve as their roots instead?

The training on this topic to this point is enlightening to participants, but additional reflection on the topic led staff to believe that there is more to consider. Do the order of operations have any negative impacts? In particular, do these rules give students the impression that they have no choices or alternatives? To investigate this, activities for Goal IV were developed so that participants are led through several questions involving computations that students may confront.

$$25 \bullet 13 \bullet 4$$

$$4\frac{2}{3} + 3\frac{1}{7} + 6\frac{1}{3}$$

For these computation problems, the order of operations is obviously not the best approach. However, the rule says literally that the above computations must be done from left to right, period! Because of how the order of operations is stated, do students feel they have no options and thus multiply and divide, or add and subtract by going strictly from left to right? Students should be taught that the rule should state to multiply/divide (or add/subtract) from left to right *unless* … Students should know that properties such as the commutative property can supersede the order of operations and be used to make the computation much easier.

Participants were also asked to compute the following: $16 \div 3 \div 2$. We have all been ingrained with the fact that division is not commutative: $A \div B \neq B \div A$. Because of this, participants were much more hesitant to compute these in an alternative fashion. However, when pressed by the staff facilitator, participants discovered that they could change the order with division. To investigate this the training turns to a more abstract and algebraic perspective. Participating teachers were asked the following:

Given: $a \div b \div c = a \div c \div b$ Is this statement true or false? (assuming none of the variables are zero) Justify your response.

The initial reaction is to conjecture that it is a false statement because $b \div c$ and $c \div b$ are not equivalent. However, the numeric example solved previously showed otherwise. If you lock in on the idea that division is the same as multiplying by the reciprocal, the answer and proof of the above follow almost immediately:

$a \div b \div c = a \div c \div b$ converts to:

$$a \bullet \frac{1}{b} \bullet \frac{1}{c} = a \bullet \frac{1}{c} \bullet \frac{1}{b}$$

It can now be seen that it is a true statement because the $\frac{1}{b} \bullet \frac{1}{c}$ can be commuted to read $\frac{1}{c} \bullet \frac{1}{b}$. The essential concept of inverse operations enables a more simple and straightforward solution to this problem. For teachers to better understand this idea, the old reliable pizza can model the situation. If we let "a" be one whole pizza, $b = 2$ and $c = 3$, the left side of the equation says to cut the pizza into 2 equal pieces, then each of those into 3 pieces each, while the right side of the equation says to cut the whole into 3 equal size pieces first, then each of those 3 into 2 pieces each. Either way you get one slice out of the total of 6 pieces, which is 1/6 of the whole pizza. This also enables one to realize that the related equation $a \div b \div c = a \div (b \bullet c)$ is simply saying that I could have cut it into 6 equal pieces in the first place! Algebraically, it can be clarified by conversion of division to multiplication by the reciprocal. Teachers have also been quick to see that the statement $a \div b \bullet c = a \bullet c \div b$ is true and can be proven in a similar fashion.

The commutative property of addition and the commutative property for multiplication involve only a single operation, and it is the two *terms* in that *single* operation that are commuted. This should not be confused with the type of order reversal involved in this exercise. There were two operations in each expression, and what was commuted was the operation together with the following number. In essence the exercise proved that any two operations are commutative as long as those two operations are either the same operation or inverse operations. This also shows commutativity in a different light than the commutative property that involves commuting of terms rather than operations. Another key item is that it must be stressed to students that in an expression such as $a \div b \div c$, the initial quantity of "a" is exactly that —the initial quantity— and moving it is not allowed and nonnegotiable, a fact not often stressed by mathematics teachers.

Middle school students enrolled in algebra could grasp the meaning and justification of the previous conversation and effective teachers could find ways to actively engage students in discovering or constructing that knowledge. But what about the struggling math student? One of the complaints often heard in mathematics is that students are not prepared to learn the new mathematics for that grade level. Consequently, time is used to go back and re-teach or review basic skills, leaving teachers less time to teach the material for which they are responsible. There are ways to teach a new concept under the guise of practicing previously covered skills. Rather than assign computation with just one operation, why not use computations with two or more operations that will not only give students the practice desired, but which may also result in student discovery through pattern recognition? For example, students could be given computations such as $16 \div 3 \div 2$ followed immediately by $16 \div 2 \div 3$. Enough of these pairs will eventually lead to students discovering what the teachers in this training have experienced.

Participants in this training on the order of operations now see this topic as much more than a list of rules needed for consistency in computation. It is a rich topic that is rooted in some very basic mathematics principles and can serve as a model for illustrating the connectivity of mathematics concepts. Needless to say, when revisiting the "knowledge packages" that were constructed initially, participants found that extensive revamping was necessary. Unfortunately, their statements of prerequisite knowledge for learning the order of operations illustrated that much of what we teach in mathematics centers on memorization of facts and on knowing *how* to do something as opposed to a focus on deep understanding of a concept, the connection of that concept to others, and knowing the whys behind that concept. This training seeks to address the latter by following the simple motto "know how and also know why" (Ma, 1999, p. 108).

*Concepcion Molina • <cmolina@sedl.org>*
*Program Specialist • Southwest Educational Development Laboratory*

References

Ma, L. (1999). *Knowing and Teaching Elementary Mathematics.* Mahwah, NJ: Lawrence Erlbaum Associates.

# Recommended Readings and Resources

## *The Essential 55: An Award-Winning Educator's Rules for Discovering the Successful Student in Every Child*

Ron Clark, a North Carolina native, and the winner of the 2001 Disney Teacher of the Year Award presents some revolutionary ideas for the classroom: manners, industriousness and accountability. Close examination shows Clark's rules go beyond simple politeness: they promote respect for self and others, and help foster a mature and responsible way of living in the world. As Clark explains each rule, he weaves in anecdotes of student projects, class trips and instances in which the particular rule proved invaluable. Teachers will have to be determined to succeed before any set of guidelines will have an effect in the classroom, he warns—and indeed, Clark's tireless dedication might be daunting to some. And while the content of his lessons is presented only vaguely, for inspiration, this book is a definite winner. Clark's slim but valuable volume, will make a welcome addition to any teacher's library. Published by Hyperion Press, ISBN 1401300014.

(Available on-line through most bookstore websites.)

## *Lessons Learned from Research*

An excellent eye-opener that brings research to K-12 mathematics teachers in an easy-to-use, readable format. Features 29 research articles from the Journal for Research in Mathematics Education rewritten specifically to reach the teacher audience. Provides commentary and guidelines to help teachers maneuver their way through original research and take what will be of value for their own classrooms. Convinces teachers that research contains a wealth of information for improving mathematics teaching. *Edited by Judith Sowder and Bonnie Schappelle.*

(May be ordered at *http://www.nctm.org/* )

## *TEKS to TAKS: Applying the Formulas* (Chart)

This chart provides examples of the application of TEKS formulas that are found on the Grade 10 and Grade 11 science TAKS. This tool may be used in both mathematics and science classrooms to coordinate instruction, understanding and use of the formulas.

(May be ordered at *http://www.utdanacenter.org/* )

# TCTM Mathematics Specialist Scholarship

**Eligibility:** Any student attending a Texas college or university – public or private – and who plans on student teaching during the 2004-05 school year in order to pursue teacher certification at the elementary, middle or secondary level with a specialization or teaching field in mathematics is eligible to apply. A GPA of 3.0 overall and 3.25 in all courses that apply to the degree (or certification) is required.

**Deadline:** May 1, 2004 **Amount:** $1000

Name: ______________________________________________
Last First Middle

Address: ______________________________________________
Number and street Apt. number

______________________________________________
City Zip Code

(___) ____________________ (___) ____________________ < ____________________ >
Home Phone Work Phone Email Address

Social Security #: __________________ Birth date: ___________________

What college or university do you plan to attend? If you are awarded this scholarship, TCTM's treasurer will send a check directly to the business office of the college. We need the college's complete address.

**You must submit three (3) copies of each of the following documents:**

1. Completed application form.
2. One official college transcript and two copies.
3. Two letters of recommendation:
   - One from either a mathematics or mathematics education professor you have taken coursework from and is not related to you.
   - One from a K-12 classroom teacher of mathematics you have worked with recently or that was a former teacher of yours and is not related to you.
   - It required that at least one of these recommendations come from a current member of TCTM, it is preferred that both recommendations come from current members of TCTM.
4. An essay of 1,500 words or more that describes your philosophy of teaching mathematics and how you will implement this philosophy with your future students. Specific examples of how you will teach a mathematics concept are required to illustrate your teaching philosophy. Or you may write an essay that explains a specific mathematics topic or concept, for example, a paper on proportionality.

Please submit all materials in one envelope to:

by mail: **Cynthia Schneider**
**234 Preston Hollow**
**New Braunfels, TX 78132**

by fax: **(512) 232-1855**
**ATTN: Cynthia Schneider**

# TCTM Leadership Award Application

**Eligibility:** The TCTM Leadership Award is presented to a TCTM member who is nominated by a TCTM Affiliated Group. This person is to be honored for his/her contributions to the improvement of mathematics education at the local and state level. He/she has designed innovative staff development and has promoted the local TCTM Affiliated mathematics council.

**Deadline**: May 1, 2004

Information about the **Affiliated group nominating a candidate**:

Name: ______________________________________________
Last First Middle

Address: ______________________________________________
Number and street Apt. number

______________________________________________
City Zip Code

(___) ____________________ (___) ____________________ < ____________________ >
Home Phone Work Phone Email Address

Are you a member of TCTM? ☐ yes ☐ no NCTM? ☐ yes ☐ no

Information about the **person being nominated**:

Name: ______________________________________________
Last First Middle

Address: ______________________________________________
Number and street Apt. number

______________________________________________
City Zip Code

(___) ____________________ (___) ____________________ < ____________________ >
Home Phone Work Phone Email Address

Is the nominee a member of TCTM? ☐ yes ☐ no NCTM ☐ yes ☐ no Retired ☐ yes ☐ no

Applications should include 3 pages:

☐ Completed application form

☐ One-page, one-sided, typed biographical sheet including:
- Name of nominee
- Professional activities
- State/local offices or committees
- Activities encouraging involvement/improvement of math education
- Staff Development
- Honors/awards

☐ One-page, one-sided essay indicating why the nominee should be honored for his/her contribution to the improvement of mathematics education at the state/national level.

Send the completed application, biographical sketch, and essay to

by mail: **Cynthia Schneider,**
**234 Preston Hollow,**
**New Braunfels, TX 78132**

by fax: **(512) 232-1855**
**ATTN: Cynthia Schneider**

by email:
***<cschneider@mail.utexas.edu>***

# Nominations sought for officers of the Texas Council of Teachers of Mathematics!

## Deadline to submit nominations: January 1, 2004

The Texas Council of Teachers of Mathematics is seeking nominations for the board offices listed below. Elections will be held in Spring 2004 with results announced in June. Those elected will serve a two-year term beginning after the CAMT 2004 TCTM board meeting. Therefore, they will serve on the board for the 2004-05 and 2005-06 school years. All officers are expected to attend two board meetings each year, one at CAMT in July and one in January. Travel expenses to the January board meeting are covered by TCTM. The position descriptions are as follows:

- The **Secretary** shall keep all records and minutes of the Council and of the executive board; and shall preserve the annual reports and historical records of the council.
- The **Vice-President Secondary** shall represent the secondary interests, promoting membership and providing publicity. This officer also serves as the Nominations and Elections chairperson every other year, alternating with the Vice-President Elementary. Specifically, this officer will serve as Nominations and Election Committee chairperson in 2004-2005, seeking at least two nominees for each vacant office for that election year (with board and member assistance).
- Regional Directors shall promote the organization and maintenance of the local councils and solicit from the region nominations for TCTM offices. The regional director may organize leadership workshops for officers of local affiliated groups and may organize TCTM sponsored regional conferences, or any other activity which may benefit the local affiliated groups. You must live in one of the regions represented by the regional director position you are seeking.

**NE Regional Director,** to represent ESC Regions 6, 7, 8, 10, and 11
**Central Regional Director**, to represent ESC Regions 12, 13, and 20
**NW Regional Director**, to represent ESC Regions 9, 14, 16, and 17

Nominations should include identification of position sought and a short autobiography explaining the nominee's interests and qualifications. Self-nominations are welcome. If you are nominating someone other than yourself, be sure they have agreed to run for the office for which you are nominating them. Please include current and complete contact information. Submit your documents on or before **January 1, 2004** to the current chairperson of the Nominations and Elections committee, **Wilma Cook** at *<cook4wilma@aol.com>*, or 6821 Norma St. Fort Worth, TX 76112.

If you have additional questions, you may contact Wilma at work at 817-871-2832 or Cynthia Schneider at 512-475-9713. Thank you for considering serving our community.

# Texas Council of Teachers of Mathematics

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Circle area(s) of interest: K-2 3-5 6-8 9-12 College

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☐ Mathematics Teacher ($72)
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234 Preston Hollow
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cschneider@mail.utexas.edu

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**TEA Consultant**
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Austin, TX 78701
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( *End Year* ) – Elected Office
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**Note the expiration date on your mailing label.**
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**Texas Council of**
**Teachers of Mathematics**

**Member 2003-2004**

NAME ___________________________

Texas Mathematics Teacher
234 Preston Hollow
New Braunfels, TX 78132

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