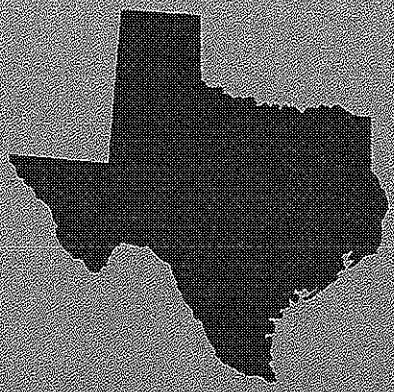

Texas Mathematics Teacher

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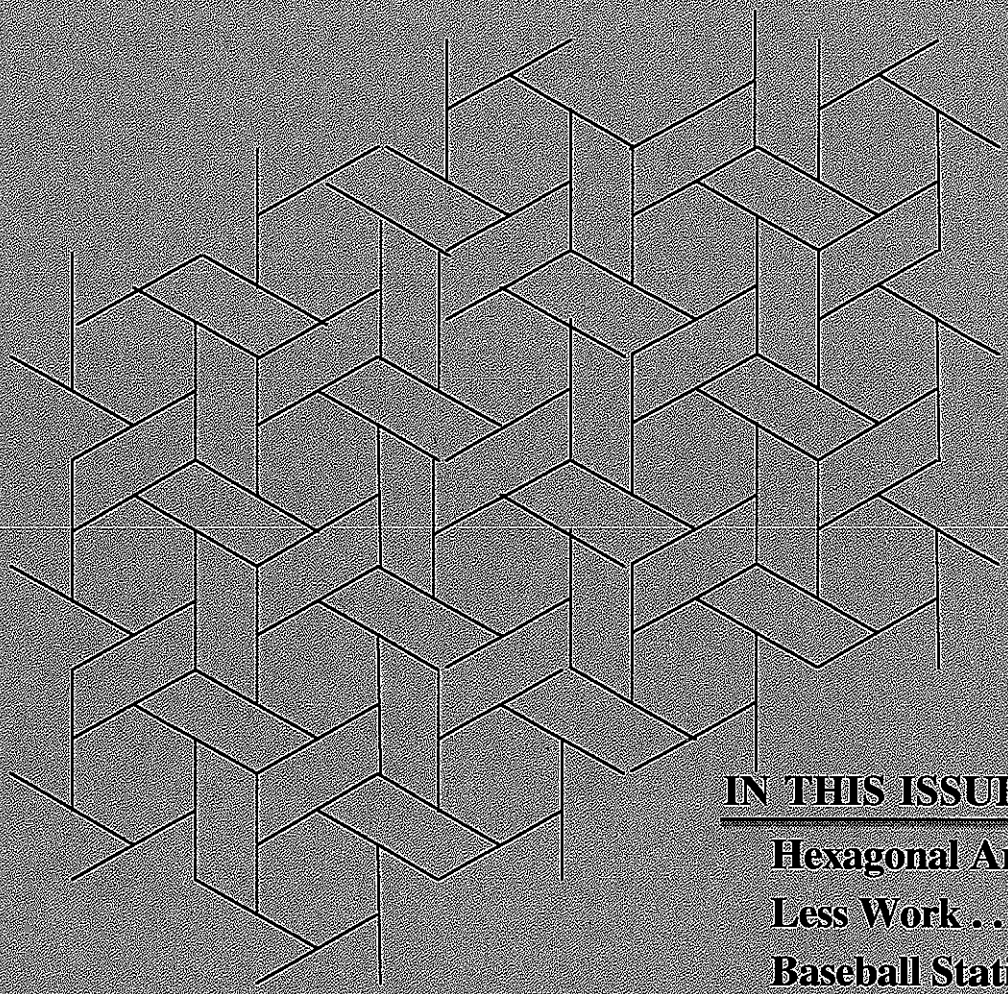

IN THIS ISSUE

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Editor

Dr. Paul A. Kennedy
Department of Mathematics
Southwest Texas State University
601 University Drive
San Marcos, TX 78666

Phone: (512) 245-3741
Fax: (512) 245-3425
E-mail: pk03@swt.edu

Associate Editor and Director of Publication

Diane McGowan
4511 Langtry
Austin, TX 78749
dmcgowan@tenet.edu

Assistant Editor

Michelle Morvant
Department of Mathematics
Southwest Texas State University
601 University Drive
San Marcos, TX 78666
mm40370@swt.edu

The ***Texas Mathematics Teacher***, the official journal of the Texas Council of Teachers of Mathematics, is published in the fall and spring. Authors are encouraged to submit articles that deal with the teaching and learning of mathematics at all levels. Editorial correspondence and manuscripts should be mailed or e-mailed to the editor, Paul Kennedy.

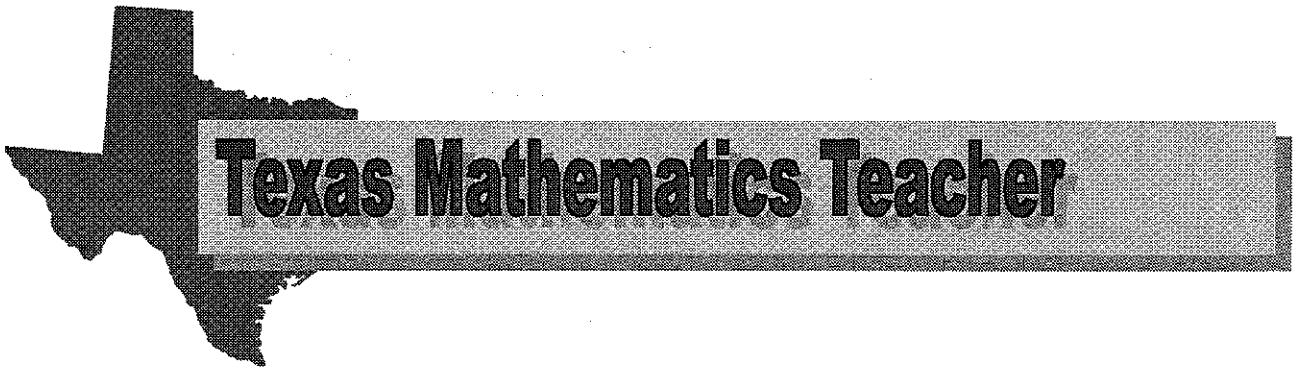
Potential authors should adhere to the following guidelines.

1. Manuscripts should be word-processed meeting APA guidelines. Tables and figures should likewise be computer generated. No author identification should appear on the manuscript.
2. Submit four copies. Include a Macintosh or IBM 3 1/2 inch diskette containing the manuscript indicating the word-processing program used on the label or send as an attachment on e-mail to pk03@swt.edu.
3. Include a cover letter containing author's name, address, affiliations, phone and fax numbers, e-mail address, and the article's intended level.
4. Articles for *Voices From the Classroom* should be relatively short and contain a description of the activities sufficient in detail to allow readers to incorporate them into their teaching. A discussion of appropriate grade level and prerequisites for the lesson should be included.

After refereeing, authors will be notified of a publication decision. Two copies of the issue in which an author's manuscript appears will be sent to the author automatically.

Items for *Lone Star News* include reports, TCTM affiliated group announcements, advertisements of upcoming professional meetings, and any other appropriate news postings.

Advertisements support the publication of this educational journal. Businesses interested in placing an advertisement for mathematics materials should contact Paul Kennedy.



Do you have a great, new idea?

An original activity that your students enjoy?

Send it to us!

See inside cover for details.

Deadline for:

Fall journal – June 15

Spring journal – December 1

Letter from the President

Hi, Everyone!

The Board had a very successful meeting in January where several important items were discussed and acted on. The one that most directly affects you is an increase in annual dues. The price of the CAMT breakfast will remain the same for members, but increased costs for our publications and the actual cost of the breakfast necessitated the dues increase. To maintain the quality of the journal and newsletter is a goal of the Board as well as improving communication with the membership. We, therefore, want to encourage each of you to attend the breakfast, Wednesday, July 14. Since it is our only regular meeting, please plan to join us and bring a friend!

Another topic among mathematics educators currently is the release of the draft version of the *Standards 2000 – Principals and Standards for School Mathematics.* If you have not had a chance to read them yet, you may download the electronic version from the NCTM web site at www.nctm.org or order a printed copy to be mailed to you. This formidable document is well-organized and is consistent with earlier Standards publications of NCTM in challenging "the assumption that mathematics is only for the select few with a persuasive argument that everyone needs to understand mathematics and that there should be no conflict between equity and excellence." Consider adding this document to your professional reading list and plan also to attend the session TCTM will host at CAMT as a discussion forum of *Standards 2000.*

Last, as you may know, ticketed sessions will be available this year at CAMT, and our volunteer members will assist with distributing the unclaimed tickets as well as the NCTM materials booth and the Registration Desk. This will require lots of volunteers! Please consider giving a couple of hours of your time in one of these areas; the volunteer form will be in the newsletter. Look for it with your request form for breakfast tickets!

Have a great end of the school year, and I look forward to seeing each of you in Dallas!

Pam

Pam Alexander. TCTM President

Headquarter's Update

NCTM Released "Do Math" Campaign

NCTM's "Do Math . . . and you can do anything" campaign reaches out to children as well as parents and highlights the uses of mathematics in various careers. The campaign focuses on young children's desires to "do" something—like going into space, reporting the weather, or creating special effects. In the PSAs, mathematics drives the action—for instance, the data analysis in the space program, the process of tracking storm patterns to predict snowfall, and the measurement and scaling necessary for creating special effects.

The campaign is designed to reach people wherever they are: the living room, the highway, the waiting room, and even the airport. To do this, the PSA campaign includes broadcast spots for television and radio, and print components for newspapers, magazines, billboards, and airport back-lit boards.

To make it easier for television stations to air the PSAs within their various formats, 60-, 30-, 15-, and 10-second video PSAs have been developed. The video PSAs are aimed at young children, but the message transcends all ages. To strengthen and complement that message, five 30-second radio PSAs encourage adult audiences to remind children that the key to a future of possibilities is mathematics. Print PSAs for newspapers, magazines, billboards and airport back-lit boards are also available.

For more information or to request a copy of specific PSA components, please contact the NCTM communications Department at (703) 620-9840, ext. 2107, or send an e-mail to jrussell@nctm.org.

"Do Math" Website Launched

In an effort to help students and parents see the excitement in "doing math," NCTM has launched a companion Web site (**www.domath.org**) to the "Do Math . . . and you can do anything" public service announcement campaign. The site features activities that children can do at home with their families or on their own. The mathematics activities are divided into four grade bands: Pre-K–2, 3–5, 6–8, and 9–12 (mirroring the grade-band delineations in NCTM's updated *Standards* draft document, *Principles and Standards for School Mathematics).*

***Standards 2000* Discussion Draft Released**

The discussion draft of *Principles and Standards for School Mathematics* has been released—both in printed and electronic versions. After a seven-month public review and comment period, the writers will revise the draft on the basis of feedback. The final document will be released in the spring of 2000.

So what should you look for that is different in the discussion draft of *Principles and Standards*? First, it will be one document, not three. The classroom portions of the three original *Standards* have been synthesized into one document. This brings content, teaching, and assessment together and makes it easier to see what students should know and be able to do.

Second, *Principles and Standards* starts with a set of principles that are the foundations on which to build high-quality mathematics instruction. These principles look at such issues as equity, curriculum, and technology.

Third, the grade bands are different. In *Principles and Standards*, there are four grade bands: pre-K-2, 3-5, 6-8, and 9-12. This allows more specific guidance in examining mathematics programs across all levels.

Fourth, the writers have developed overviews of the mathematical strands in the *Standards* that show the growth of fundamental ideas in that strand over time. These overviews have the potential to help a teacher, parent, or administrator see how ideas fit together and what students can do across the grades.

Another way in which *Principles and Standards* is different is that it uses the power of technology to help increase access to the document itself. An electronic version that can be viewed with a Web browser is available to complement the print version. The Web version will allow for examples that are dynamic, that let us see and try out ideas talked about in the *Standards* and understand how the ideas fit together. Ten years ago this was not possible. Today it is essential.

NCTM individual members can request a single printed copy of the discussion draft by submitting the Web request form at www.nctm.org/standards2000; by e-mailing their name, address, and membership number to standards2000-draft@nctm.org; by calling toll free (888) 220-7952. View the electronic version at www.nctm.org/ standards2000.

U.S. Department of Education Launches "America Counts"

On November 12,1998, Vice President Gore unveiled a U.S. Department of Education initiative, AMERICA COUNTS, designed to help all students master challenging mathematics, including the foundations of algebra and geometry, by the end of 8th or 9th grade. During a visit to the Dana Middle School in Los Angeles, CA, Gore announced a five point strategy to:

* Provide help, personal attention and additional learning time for students who need extra assistance in mastering the fundamentals of mathematics in elementary and middle school.
* Equip teachers to teach challenging mathematics by ensuring that they enter the profession with a solid understanding of mathematics and the best ways to teach it and provide ongoing opportunities for teachers to upgrade and expand their knowledge and skills.
* Provide a more challenging and engaging curriculum for all students based on rigorous standards that meet national and international benchmarks of excellence.
* Ensure that local, state and federal resources are coordinated in support of high-quality and coherent mathematics programs that hold high expectations for all students.
* Build public understanding of the mathematics our students must master to ensure their and our nation's prosperity and growth.

Following the announcement Gore discussed with students, parents, teachers, tutors, university presidents, business leaders, and a mathematics educator, the national mobilization of mathematics tutors and the newly expanded Federal Work-Study Waiver portion of America Counts. These efforts are designed to help America's elementary through ninth grade students improve their achievement in mathematics. Adults with an affinity for mathematics from many walks of life—including engineers, health care professionals, accountants, mathematicians, scientists, and college students—will be called upon to provide extra assistance and personal attention to elementary and middle-grades students.

The vast majority of colleges and universities nationwide participate in the Federal Work-Study program and no special steps are needed to implement the new waiver for mathematics tutoring. This effort will build upon the success of the America Reads Challenge, in which over 1000 colleges and universities have college students tutoring elementary school students in reading. With the 1999 budget increase, more than 930,000 college students will be supported through the work study initiative.

The national mobilization of tutors has great potential to provide students with the extra help they need to master the fundamentals of mathematics. The success of the America Counts' tutoring campaign, however, depends upon the active involvement and expertise of mathematics educators in the field. The creation and implementation of high-quality tutoring programs and the effective coordination of efforts at the state, district, and local levels can build bridges between universities, schools, and community-based organizations.

Success of America Counts in your community depends on your active participation. For more information, or to get involved, please contact Linda P. Rosen, Special Advisor to the Deputy Secretary of the U.S. Department of Education, at Linda_Rosen@ed.gov or (202) 401-3389.

You can visit the America Counts Web site at www.ed.gov.inits/Math. The America Reads website is located at www.ed.gov/inits/americareads. To find out more about the U.S. Department of Education's federal work study program visit their Web site at www.ed.gov/offices/OPE/pubs/WorkStudy. And to view the "Yes, You Can," guide for establishing mentoring programs with a focus on mathematics and science online, go to www.ed.gov/pubs/YesYouCan.

The Concept of Owing and the Early Childhood Curriculum

Dr. Charles E. Mitchell and Elisabeth Riggs,
Department of Mathematics & Physics, Tarleton State University

Investigations spanning the past two decades have helped educators become increasingly aware of the power of pre-school children's intellect and the mathematical knowledge possessed by these children (Ginsburg, 1977; Carpenter and Moser, 1982; Nunes, Schliemann, and Carraher, 1993). Ginsburg (1975) has described pre-school children's informal knowledge of mathematics as being a powerful system. Nunes, Schliemann, and Carraher (1993) further stated that if educators are to fully explore the depths of a child's knowledge of mathematics, then the child should be placed in familiar contexts and language should be used with which the child is familiar. The purpose of this manuscript is to report the results of an investigation in which pre-school children were presented with items involving the concept of owing. Children's ability to correctly solve the items was measured as well as the impact of the availability of manipulatives on the children's performance. Furthermore, children's performance on the owing items was compared to selected items, which were similar in mathematical structure to the owing items.

The Owing Item

One kind of owing situation develops when a child is promised a specified quantity of an object, but insufficient numbers or amounts of the object are on-hand to fulfill the promise. For example, a child might be informed that s/he can have five cookies after completing a task, such as cleaning a room or taking a nap, but when the supply of cookies is checked after fulfilling the initial conditions, it is determined that only three cookies are available. Thus, two more cookies would be owed to the child.

The structure of the owing item seems similar to what is often referred to as the missing addend item, but key differences arise once manipulatives are used to solve the item. A traditional missing addend context might involve a bus that initially carried three passengers. Informed that some more people boarded the bus and now a total of five people are on the bus, the child would be asked to determine that two people must have boarded the bus. Using objects, the child would set out three objects to model the number of people who were originally said to be aboard the bus. Mitchell (1987) identified several methods that pre-school age children employed

to solve this item. Some children could solve the item mentally, but many of those who needed manipulatives would add objects to the original set of three objects until five objects were set aside. Some of these children counted continuously as they added objects to the original set, while others stopped to recount after each new object was added. The owing item is distinct in that the five cookies that are promised only represent an idea. Children are handed three objects and asked to determine the shortage, if they realize that they have been shorted. The action of the bus item, more people boarding, is also an additional clue as to the solution method. In the owing item, children could be asked to identify how many more cookies they should be *given* without being shown additional cookies from which to choose. In real-life, in fact, the ones still owed may not be present, yet the additional people who boarded the bus must have been present to have boarded the bus. Mitchell hypothesized that the owing item would be a more difficult item to solve for some of these children. A purpose of the current investigation was to examine this hypothesis as well as replicate previous findings regarding children's performance on the items involving owing.

The Current Investigation

The purpose of the current investigation was in part to replicate the results obtained by Mitchell (1987) but then extend the focus on children's understanding of the owing item by comparing children's performance on it to selected items thought to have a similar mathematical structure. The current investigation involved twenty children from a day-care center. These children ranged from 3.5 years of age to nearly six years of age, and none of the children had started first grade. However, all could correctly determine the number of objects in a set of up to five objects.

Individual interviews with the twenty children involved the presentation of four items randomly ordered for each child to solve. Two of the items represented traditional examples of missing addend items. For example, in one item, three toy people were placed on a toy bus. The child was informed that the bus had stopped to pick up more people. The bus was removed from the child's view, and when it was returned, it held five people. In another item, the child was shown two sets of objects differing in number by two objects, and the child was asked to determine the number of objects to place in the smaller set in order to equalize the number of objects in both sets. The two owing items presented to the children were similar to ones presented in the Mitchell study. If a child was unable to solve the items involving a shortage without objects, a tray of similar objects was presented to the child for a second solution attempt. The children's performance on the two traditional items did not differ substantially, so these two items were reported together, as were the data on the owing

items. Table 1 summarizes children's performance on the items.

Table 1: Traditional Items Versus owing Items: A Comparison of Performance

	Initial Item	With Tray
Traditional	35%*	90%**
Owing	30%*	60%**

*Percent of the children who responded correctly initially

**Percent of the children who responded correctly initially or when presented with a tray of objects

The data collected in the current investigation on the owing items supported the findings reported by Mitchell. Of the twenty children interviewed, approximately one-third of the children correctly solved the item without the aid of a tray of additional manipulatives. Most of these children were amongst the oldest in the group. The data in Table 1 further suggests that children's performance on the owing items was similar to performance on the traditional items until the tray holding more objects was introduced. Children's performance on both types of items improved but much more so in the case of the traditional items.

Differences in children's ability to correctly solve the traditional and owing items did not surface until objects were introduced. This does not necessarily lead to the conclusion that children used the same solution method initially. It may be necessary to ask children to show how they solved the items initially before differences between the two kinds of items can be more clearly established. The action and context of the owing item may not be as suggestive as some of the more traditional contexts. The fact that five candies have been promised is just an idea. The action in the owing item begins when the child is handed an amount to help satisfy the debt. For sophisticated problem solvers, this distinction may seem inconsequential, but it may have important consequences for the pre-operational child.

Summary

As is the case with all investigations of this type, the implications for curriculum development depend greatly on the generalizability of the statistics. Although an effort was made to choose traditional missing addend contexts which were representative of those with which children would be familiar, in truth, there is a wide range of contexts, and children's performance could vary in each one. Children's performance on the owing items clearly was different from the contexts used and suggests that a focus on these items might be an important supplement to the early childhood curriculum. The owing items may be more complicated for many of the children which suggests that the introduction of these items into the curriculum should be delayed until some level of expertise is achieved on more traditional items.

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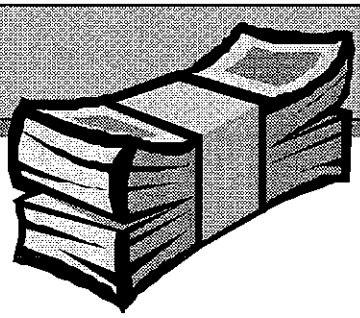
1998-99 NCTM Regional Conference Schedule

27-30 July 1999	Western	Honolulu, Hawaii
7-9 October 1999	Western	Boise, Idaho
13-25 October 19999	Eastern	Pittsburgh, Pennsylvania
11-13 November 1999	Central	Topeka, Kansas
2-4 December 1999	Western	Phoenix, Arizona
27-29 January 2000	Southern	El Paso, Texas
16-18 March 2000	Southern	Mobile, Alabama

A Statistical Analysis of the Texas Cash 5 Lottery After 350 Drawings



John Lamb, Jr., Retired Professor of
Mathematics, Texas A & M-Commerce

In "A Statistical Analysis of the Texas Lottery,"[2] along with three colleagues, I stated that analyzing statistical data has become an important part of the curriculum in our schools beginning with the elementary level. However, learning about means, modes and medians along with standard deviations and other statistics can be rather dry even when using a computer for all the tedious calculations. One way to get students interested in statistical analysis is to apply statistical methods to current and exciting situations. Texas teachers have four such situations from which to choose: the Texas Million, LOTTO, PICK 3, and CASH 5 lotteries. This article concerns the Texas CASH 5 Lottery.

The Rules of the Game

Twice a week, on Tuesday and Friday, from October 13, 1995 to November 22, 1996, a machine in Austin, Texas was used to select five balls numbered from 1 to 39. Due to the popularity of the game, the number of drawings per week increased to four on November 25, 1996. The machine now selects five balls on Mondays, Tuesdays, Thursdays and Fridays.

Lottery players attempt to pre-select the winning numbers to be awarded various amounts of money. Each Lotto play slip has five play boards. Each play board contains the numbers one through thirty-nine. Five numbers can be selected on any or all the play boards. Provision is made for these numbers to be entered into more than one drawing by marking a multi-draw number from two to ten. Players can win in the following ways, according to the play slip:

- Match all five of the numbers drawn -- odds 1 in 575,757,
- Match four of the five numbers drawn -- odds 1 in 3,387, or
- Match three of the five numbers drawn -- odds 1 in 103.

The overall odds of winning for each play board played are 1 in 100.

Probability of Winning or Losing

The probabilities of the preceding events occurring are calculated as follows. The probability of selecting all five numbers correctly is $\frac{1}{C(39,5)} \bullet C(5,5) \bullet C(34,0) = \frac{1}{575,757} \approx .00000174$. The probability of selecting four correctly is $\frac{1}{C(39,5)} \bullet C(5,4) \bullet C(34,1) = \frac{170}{575,757} \approx .0002952$. Finally, the probability of selecting three correctly is $\frac{1}{C(39,5)} \bullet C(5,3) \bullet C(34,2) = \frac{5,610}{575,757} \approx .0097087$. The sum of these probabilities is approximately $.0100213 \approx \frac{1}{100}$, the probability of winning anything.

The probability of losing is also interesting to calculate. The probability that none of the numbers will be chosen is $\frac{1}{C(39,5)} \bullet C(5,0) \bullet C(34,5) \approx .4832872$. The probability that exactly one number will be

chosen is $\frac{1}{C(39,5)} \bullet C(5,1) \bullet C(34,4) \approx .4027394$. Finally, the probability that exactly two numbers will be chosen is $\frac{1}{C(39,5)} \bullet C(5,2) \bullet C(34,3) \approx .1039327$. The probability of losing is the sum of these numbers, approximately 0.9899593. Of course, this number is 1 - .0100407, one minus the probability of winning anything at all.

Odds versus Probability

On the play slips, it states the odds of winning. However, as we have shown above, the numbers actually printed on the play slips are the probabilities of winning. These numbers are usually quite different. If p is the probability of winning an event, then 1 - p is the probability of losing that event. The odds of winning the same event are $\frac{\text{probability of winning}}{\text{probability of losing}} = \frac{p}{1-p}$. Suppose the probability of winning an event were $\frac{1}{3}$. Then the probability of losing the event would be $\frac{2}{3}$, so the odds of winning that event would be $\left(\frac{1}{3}\right) \div \left(\frac{2}{3}\right) = \frac{1}{2}$, which is quite different from the probability of winning. Fortunately, when the probabilities for winning an event are very small, the probabilities and odds are very close to the same number. In the case of the CASH 5 Lottery, we have the following:

ODDS VERSUS PROBABILITIES			
	Odds	Probability	Difference
Matching 5	0.000001736846	0.000001736843	0.000000000003
Matching 4	0.000295350659	0.000295263453	0.000000087206
Matching 3	0.009839567957	0.009743694216	0.000095873741

The difference column would seem to indicate that there would be no problem using the terms *odds* and *probabilities* interchangeably when discussing the CASH 5 Lottery.

Randomness of the Lottery

The remainder of this article concerns the statistical behavior of the numbers chosen so far to determine if the numbers were selected randomly. The methods considered in this paper are the expected frequency, the mean, the standard deviation, and the Chi square test. GW Basic computer programs were written and used to tabulate the frequency of each number's occurrence and to calculate the probability of its recurrence. The mean, standard deviation and Chi square test results were computed to help determine whether the numbers were chosen at random.

Frequency of Numbers Chosen

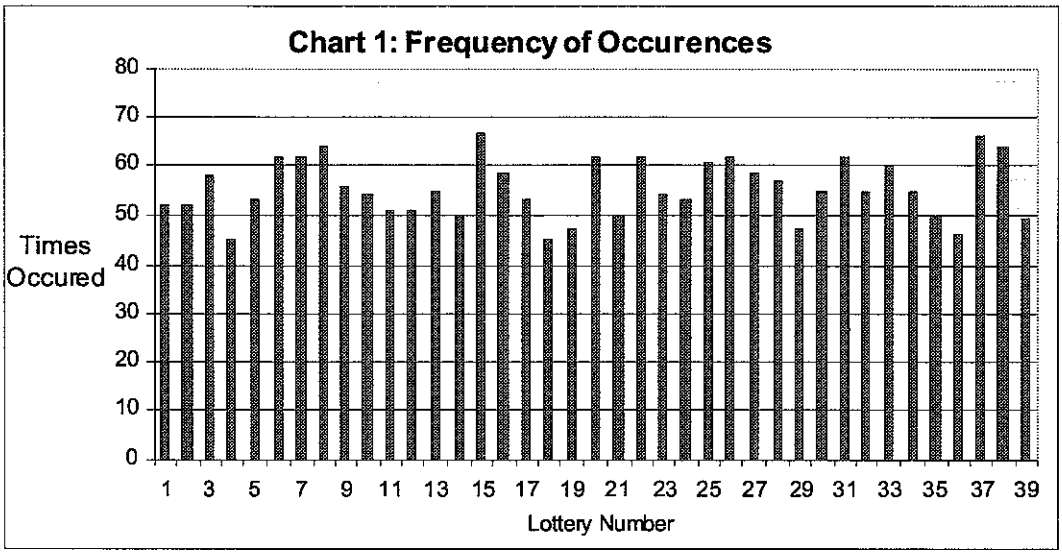
Theoretically, the probability P(x) that any given number x will be one of the five drawn is:

$P(x) = \frac{C(1,1) \bullet C(38,4)}{C(39,5)} = \frac{5}{39} \approx .1282051$, the hyper-geometric probability formula. Thus, the number of times we expect x to occur in n drawings is $n \bullet .1282$. Since there were 350 drawings at the time of this

writing, x should have occurred $350 \bullet .1282 \approx 44.87350$ times. Compare this theoretical frequency with the actual frequencies given in Table I and Chart I.

Table 1: Frequency of the Cash 5 Lottery Numbers

Number	Frequency	Number	Frequency	Number	Occurrences
1	52	14	50	27	59
2	52	15	67	28	57
3	58	16	59	29	47
4	45	17	53	30	55
5	53	18	45	31	62
6	62	19	47	32	55
7	62	20	62	33	60
8	64	21	50	34	55
9	56	22	62	35	50
10	54	23	54	36	46
11	51	24	53	37	66
12	51	25	61	38	64
13	55	26	62	39	49



Mean, Standard Deviation and Distribution of Numbers Chosen

If the machine is choosing the numbers randomly, the average number chosen from the numbers 1 to 39 should be 20, and the standard deviation should be 11.11. The actual average number chosen by the Texas Cash 5 Lottery machine in 350 drawings is 20.05714, and the actual standard deviation is 11.18222.

For the Chi-square test, the 39 CASH 5 numbers were grouped into 13 intervals containing three numbers each as follows: 1, 2 and 3; . . . ; 37, 38 and 39. The following shows the frequency of the numbers per interval:

Table 2: Frequency Over Intervals

Interval	Frequency
1-3	162
4-6	160
7-9	182
10-12	156
13-15	172
16-18	157
19-21	159
22-24	169
25-27	182
28-30	159
31-33	177
34-36	151
37-39	179

Because 1750 numbers have been chosen so far and there are thirteen intervals, the average amount of numbers in each interval is $\frac{1,750}{13} = 134.6$. Note that this number is also three times the expected occurrence of each number found earlier, 44.87. The Chi-square test can now be run on the data in the intervals for 350 drawings as follows:

$$\chi^2 = \frac{(123-134.6)^2}{134.6} + \frac{(131-134.6)^2}{134.6} + \frac{(154-134.6)^2}{134.6} + \frac{(127-134.6)^2}{134.6} + \frac{(139-134.6)^2}{134.6} + \frac{(129-134.6)^2}{134.6} + \frac{(131-134.6)^2}{134.6} + \frac{(142-134.6)^2}{134.6} + \frac{(144-134.6)^2}{134.6} + \frac{(118-134.6)^2}{134.6} + \frac{(146-134.6)^2}{134.6} + \frac{(127-134.6)^2}{134.6} + \frac{(139-134.6)^2}{134.6} \approx 9.44.$$

According to a table of critical values of Chi-square[1], the Chi square value needs to be at least 18.55 to indicate non-randomness with a probability of at least .9. Thus, it cannot be concluded after 350 drawings that the number selections are non-random with an error of 10% or less.

Mathematical Expectation

The CASH 5 prizes are awarded by dividing the total ticket sales for each drawing by two. Half goes to the state and half goes to the winners. The latter half, call it T, is divided among the winners as follows: 20% of T is distributed among those who matched 5 numbers, 30% of T is distributed among those who matched 4 numbers, and 50% of T is distributed among those who matched 3 numbers. If X people matched all 5 numbers, then each of them wins $\frac{.2T}{X}$ *dollars*. If Y people matched 4, then they each win $\frac{.3T}{Y}$ *dollars*. Finally, if Z people matched 3, then they win $\frac{.5T}{Z}$ *dollars*. There is one exception to these rules: if no one matches 5 numbers, then 50% of T is distributed among those matching 4.

The empirical probabilities of matching 5, 4, and 3 numbers are $\frac{X}{2T}$, $\frac{Y}{2T}$, and $\frac{Z}{2T}$, where each is the number of winners divided by the total number of players. Since it costs \$1 to play the game, it takes 2T players to generate T dollars for prizes since the state takes 50% off the top.

The mathematical expectation for an event is the product of the probability of that event times the value of the prize for winning the event. For the CASH 5 Lottery mathematical expectation, we have the following table:

Probability	Prize	Expectation (Product)
$\frac{X}{2T}$	$\frac{.2T}{X}$	$\frac{.2T}{2T}$
$\frac{Y}{2T}$	$\frac{.3T}{Y}$	$\frac{.3T}{2T}$
$\frac{Z}{2T}$	$\frac{.5T}{Z}$	$\frac{.5T}{2T}$

The sum of the expectations is $\frac{T}{2T} = \frac{1}{2}$. Thus, a player can expect to win 50 cents for every dollar spent on a ticket.

Actual data has been kept on how many people have won how much money during the first 150 CASH 5 drawings. It is summarized as follows:

Average prize for matching	3 numbers	$26
	4 numbers	$571
	5 numbers	$66,390
Average number of people matching	3 numbers	19,819
	4 numbers	605
	5 numbers	3
Estimated expectation for matching	3 numbers	.25
	4 numbers	.17
	5 numbers	.12
Sum of estimated expectations	.54	

Note that the expectation is pretty close to the predicted expectation of 50 cents.

Classroom Applications

Teachers and their students can use the Texas CASH 5 lottery and other Texas lotteries to provide them with daily data for statistical analysis. This data may be obtained from television, most Texas newspapers , or the web (http://www.cpa.state.tx.us/txgovinf/lottery/).

The intensity of study can be depending on the game studied. Hopefully, teachers and their students will enjoy studying an ongoing situation as it happens instead of something static in a textbook.

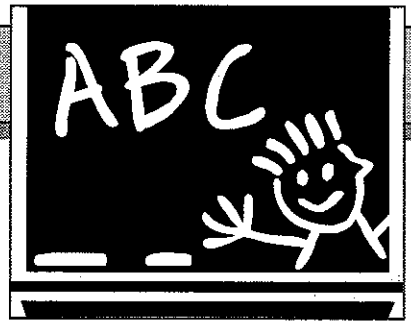
Conclusions:

It has been interesting to keep track of the number behavior for the Texas CASH 5 Lottery since it began. There seems to be no indication of non-randomness in the number selection process, and players can expect to win about 50 cents on the dollar since the state takes 50% off the top before awarding prizes. As someone once said about this and other lotteries, it is a fun game for those people who enjoy donating their money to the state.

References

1. Mendenhall, W. and Beaver, R. J. Introduction to probability and statistics. PWS-Kent Publishing Co., Boston, 1991. 670-671.
2. Lamb, Jr., J. F., Huffstutler, R., Brock, A. and Aslan, F. "A statistical analysis of the Texas lottery." Texas Mathematics Teacher, Vol. XLI (1) January, 1994.

Less Work . . . More TAAS



Kelle K. Clark, Pease Elementary, Austin, Texas

I have implemented a math TAAS strategy in my classroom I would like to introduce to you. "Wall TAAS" is a great way to get your students (written for second graders) to love and be excited about math. "Wall TAAS" consists of several activities on a wall covering TAAS skills including:

- ✓ date
- ✓ time
- ✓ money
- ✓ greater than/less than
- ✓ rounding by 10 and 100
- ✓ place value
- ✓ patterns
- ✓ addition and subtraction
- ✓ fractions
- ✓ charts

Everyday, a different "Person of the Day" completes every section of the wall first in the morning, while the other students are busy on another task.

Later in the day--after lunch is a good time--the Person of the Day presents the "Wall" to the class. During this time, everyone in the class listens. The Person of the Day needs to use his/her best presentation skills: face the audience, point to the items with only one finger as a weather man would, speak clearly and loudly, make eye contact with the audience, stand where everyone can see, and answer questions if hands are raised. These are my favorite skills that Wall TAAS develops. The Person of the Day explains the activities on the "Wall," not just telling the answer.

To prepare for the "Wall," the teacher will create problems dealing with money, addition, subtraction, and greater than/less than. As times goes by, the students will catch on, and they can begin to create problems for the Person of the Day.

On the following pages, you will find a chart that describes everything you and your students need to do to make "Wall TAAS" work. Look at the pictures found after the charts while you are going through the following chart pages. All parts of "Wall TAAS" must be laminated. Students will need erasable markers to write on your laminated materials.

This process at the beginning of the year (the first month) takes about 30-45 minutes. I stop the students and model when I see a slouchy job. After the first month, they improve and become quicker. Toward the end of the year, they are completing "Wall TAAS" in 15-20 minutes.

The students love Wall TAAS. They are eager to be the Person of the Day. Without me ever saying anything, they invite their parents to watch them. This is a great way to get those hard to get parents into the classroom. I usually try to get in a quick mini-conference on the side. The students worry about their "Wall TAAS" answers during the day. They sometimes sneak up to the Wall and change an answer that they now see is wrong. What a concept—elementary students caring about and eager to do TAAS.

What the Teacher Needs to Do to Get Started	What the Teacher or a Student Needs to Do in the Morning	What the Person of the Day Needs to Do in the Morning	What the Teacher or Student Needs to Do During Presentation	What the Person of the Day Needs to Do During Presentation
Calendar				
Buy or make a calendar. Leave off the days. Put the dates in a baggie.		The student will put the date on the calendar.		The student will say, "Today is . . ." and spell the day of the week without looking.
The Date				
Write "The Date" on top of a sentence strip.		The student will write the date on the strip using capitalization and punctuation correctly.		The student will say, "Today is . . ." They have to spell the month without looking.
Money				
Cut 3" x 5" tag board labeled "Money." Keep a play change and dollar bills in a baggie.	Write an amount in this section of the wall.		Extension: Have them write the amount of money in words.	The student will count out the amount using coins. For example, for $.40, say, "twenty-five, thirty-five, forty."
Time				
Cut a 20" clock out of tag board. Write the numbers on the clock but no hands. On a fourth of a sentence strip, write "Time" and "_ _ : _ _." On another piece, write "Time in Words."	Write down a time in words.	The student will write the time in numbers. He or she will then draw the hands on the clock.		The student will say, "The time was 9:35. I put the big hand, which is the minute hand, on the 7 because . . ."
Greater Than/Less Than				
Write the title on a third of a sentence strip. Draw 2 lines.	Write down 2 numbers in the blanks.	The student will write the greater or less than sign.		The student will say, "493 is greater than . . ."
Rounding by 10 and 100				
Title a fourth of a piece of poster board.	Write down two numbers (one to round by 10 and the other to round by 100).	The student will number 2 lines, one by 10s and one by 100s; put a dot for the given number, and draw an arrow to where the number should round.		The student will say, "My first number is 58. I put a dot between the 50 and 60. I know 58 rounds up because . . ." Do the same for the other number.

Chart				
On a poster board, write "How's the Weather?" Draw a graph. Going down the side of the graph write: Foggy, Sunny, Clear, Rainy, and Cloudy.		The student will put a dot beside the word describing the weather at that moment.	Ask the Person of the Day questions like, "How many more cloudy days than rainy days are there?" Hint: Never erase their graphing.	The student will say, "This morning the weather was . . ."
Place Value				
Turn 6 sentence strips the long way. At the tip, write one of these: Ones, Tens, Hundreds, Thousands, Ten Thousands, Hundred Thousands.			Tell the Person of the Day a number. Ask the student, "What is the place value of the 4 in the number?" Extension: Write the amount in words.	The student will write each number in its correct space. The student will then say the number, pointing to the strips.
Pattern of the Day				
Cut out a forth of a poster board and write the title on the top.	Write a pattern down with some missing parts.	The student will complete the pattern.		The student will say, "My pattern was 435, 440, 445, 450. I knew this because . . ."
Addition and Subtraction				
Cut out a fourth of a poster board. Draw a line down the middle of it. Write "Addition" on one side and "Subtraction" on the other.	Write an addition and subtraction problem at any level.	The student solves the problems.		The student will take the class through each step of how they solved the problem as if thinking aloud, using terms like sum and difference.
Fraction				
Get one piece of construction paper and put the title on top.	Write a fraction in numbers or words.	The student will draw a representation of that fraction.		The student will tell how and why he or she chose the representation to model the fraction.

October

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
			1	2	3	4
5	6					

The Date

October 6, 1998

Money

$_ _.4 0

How's the Weather?

Foggy													
Sunny	■	■	■	■	■								
Clear	■												
Cloudy	■	■	■										
Rainy	■												

Greater Than/Less Than

234 326

Pattern of the Day

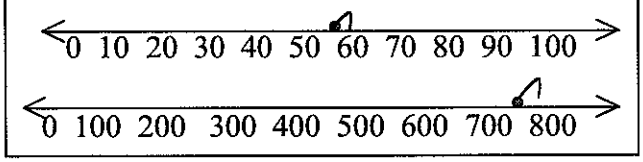
435, 440, ____, 450

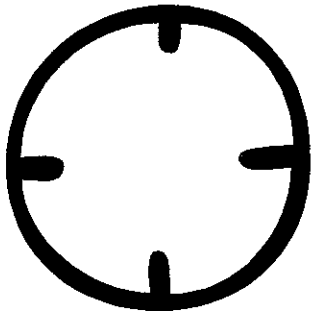
Addition	**Subtraction**
364 +928	796 -128

Fraction

Rounding by 10 and 100

58 and 764





Time

_ _:_ _

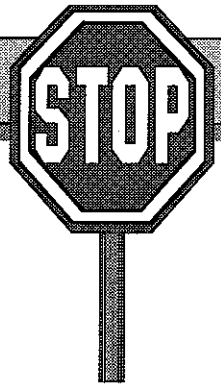
Time in Words

Nine thirty-five

Place Value

Hundred Thousand	Ten Thousand	Thousand	Hundred	Tens	Ones

Hexagonal Art



Dr. John Huber, Professor of Mathematics, Sam Houston State University
Pamela Weber Harris, Southwest Texas State University

Overview: We explore transformational geometry using dynamic geometry software as we create interesting art using hexagons.

Objective: **Mathematical Models with Applications TEKS:** 9A

Terms: Hexagon, transformation, translation, rotation, reflection, and vector

Materials: Dynamic geometry software, a demonstration computer, and a computer lab

Procedures: Start off by talking generally about dynamic geometry software. Compare it to the construction tools of old: the compass and straight edge. You may want to briefly demonstrate how to construct a few things like a point, line, line segment, circle, etc. Show how you can grab points or other objects and drag them or stretch them. Point out that as you drag or stretch objects, they maintain the constraints by which they were constructed. That is, a circle will not stretch to be an ellipse, but a quadrilateral which *looks* like a square will stretch to be any quadrilateral.

Activity: Transformations

1. The instructions given are for the Mac version of Sketchpad. Demonstrate for students how to create the hexagonal design in the student activity, and then let them try it. Circulate and help. If your students are not familiar with dynamic geometry software, let them work in pairs to help each other. Encourage creativity as they create their own designs.

Extensions:

1. Use different transformations (i.e., rotations, translations, reflections, etc.) to transform the original hexagon to tile the plane. Find as many different ways as you can.
2. Create a different polygon with n sides. Can your n-gon tile the plane?
3. Using transformations, what is a different way (rather than rotating a segment 120°) that you could create a regular hexagon?
4. Create your own design, including the original hexagon you use to create it.

Summary: Through the use of dynamic geometry software, participants try different ways to tile the plane by using reflections, rotations, and translations of hexagons.

* This article is part of the TexTEAMS Mathematical Modeling Institute for Secondary Teachers.

Student Page

Geometer's Sketchpad Hexagonal Art

(For Mac)

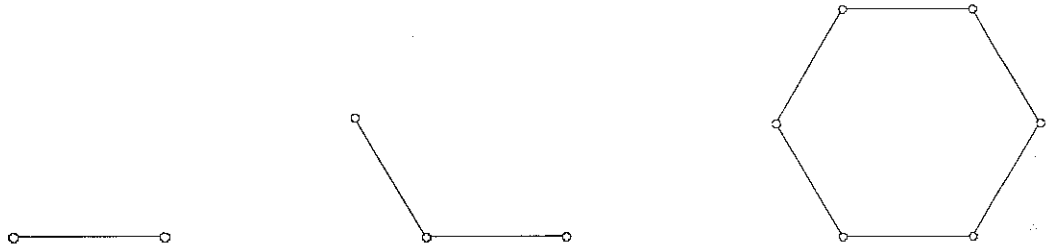
Constructing a Regular Hexagon

Step 1 Construct a segment.

Step 2 Click on the left-hand endpoint and choose Transform and Mark Center to mark the center of the rotation.

Step 3 Select Edit, Select All (or hold down the Shift key and select the segment and its endpoints). Choose Rotate in the Transform menu and rotate 120°.

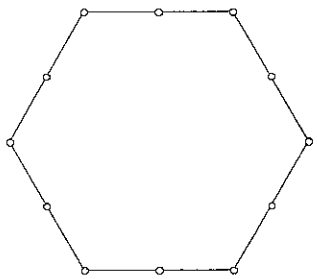
Step 4 Repeat to construct the remaining sides.



Step 1 Step 2 and 3 Step 4

Constructing the Basic Design

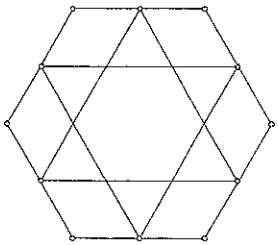
Step 5 Construct the midpoints of each side of the regular hexagon.



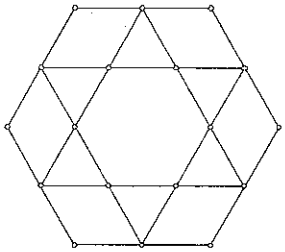
Step 6 Construct the segments joining the midpoints as shown below.

Step 7 Construct the points of intersection of each pair of segments.

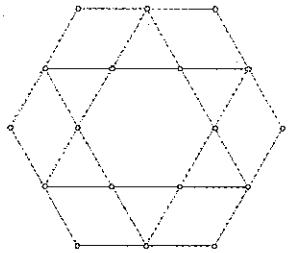
Step 8 Select the entire figure (Edit, Select All), and in the Display menu, choose Line Weight: Dashed.



Step 6



Step 7

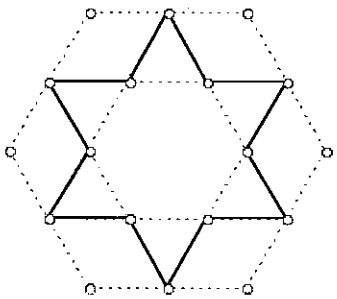


Step 8

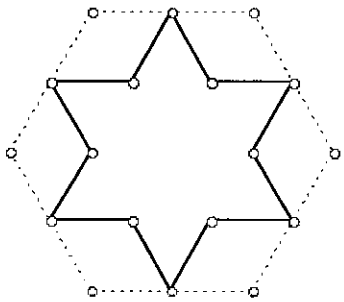
Step 9 Make sure nothing is selected in the sketch. In the Display menu, choose Line Weight: Thick. Nothing should change in your sketch, but the next segment you draw will be thick.

Step 10 Construct the solid segments as shown in the figure below.

Step 11 Click on the inside of each of the dashed segments where they remain visible, hold down the Shift key, and choose Hide Segments in the Display menu.



Step 10



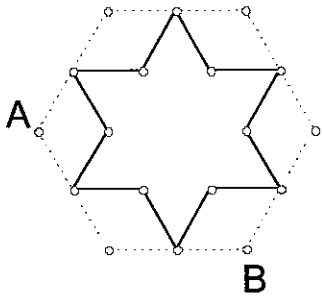
Step 11

Making the Repeating Pattern

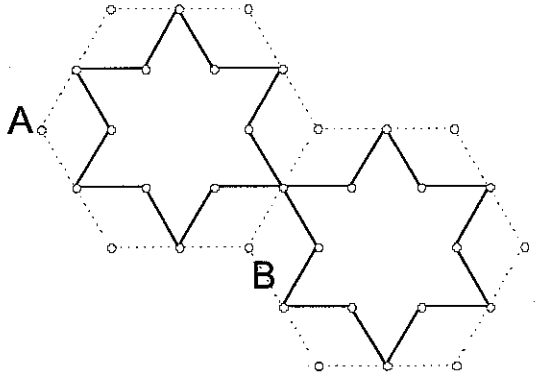
Now that you have the basic design, you are ready to make a repeating pattern found in Islamic art.

Step 12 Holding the Shift key down, select, in order, the points shown with labels A and B. In the Transform menu, choose Mark Vector.

Step 13 Select the entire figure. In the Transform menu, choose Translate to translate your design.

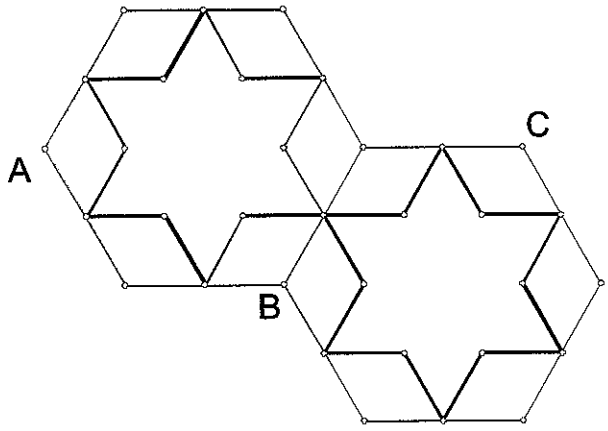


Step 12



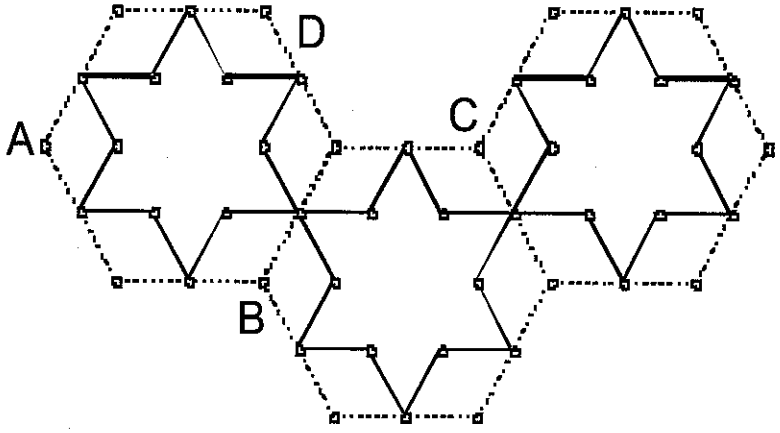
Step 13

Step 14 Hold the Shift key down and select, in order, the points with labels B and C. In the Transform menu, choose Mark Vector.



Step 14

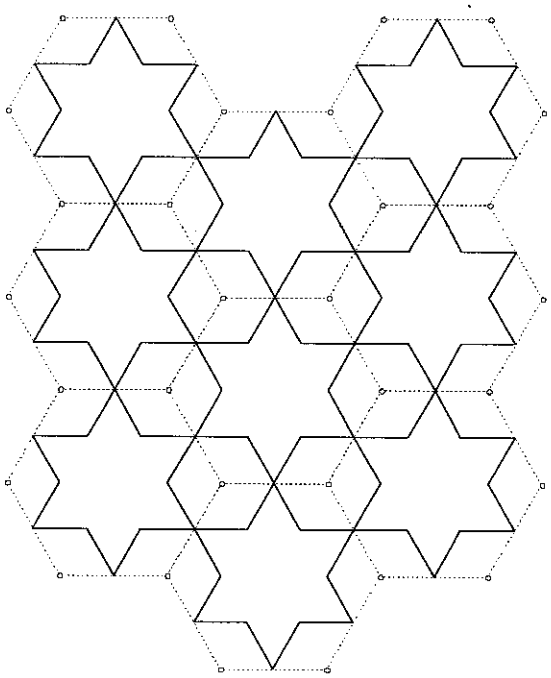
Step 15 Select all of the second hexagon. In the Transform menu, choose Translate to translate your design.



Step 15

Step 16 With the Shift key held down, select, in order, the points with labels D and B. In the Transform menu, choose Mark Vector.

Step 17 Select the entire figure consisting of the three hexagons. In the Transform menu, choose Translate to translate the figure. Repeat the translation so that you have a row of three designs.

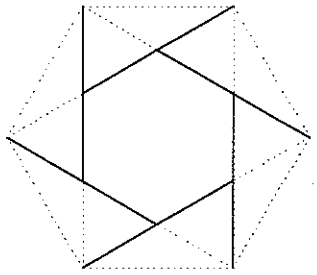


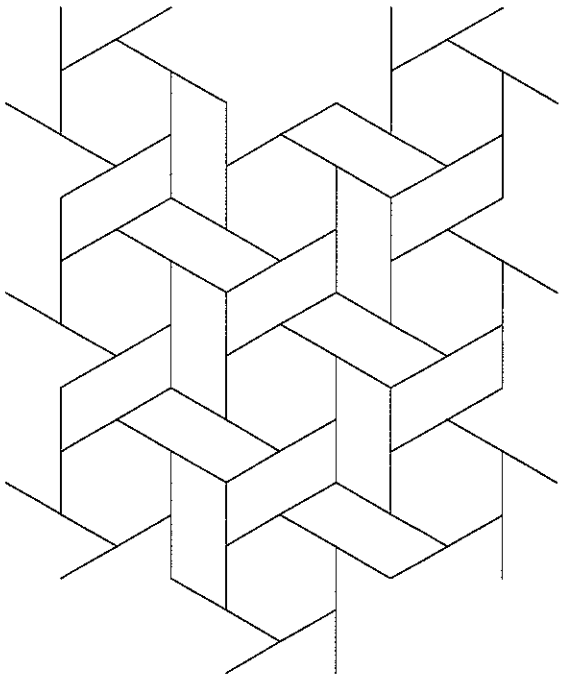
Steps 16 and 17

Constructing Additional Figures

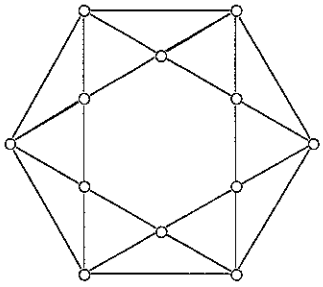
Given the basic hexagon, basic design, and repeating pattern, construct each of the following designs.

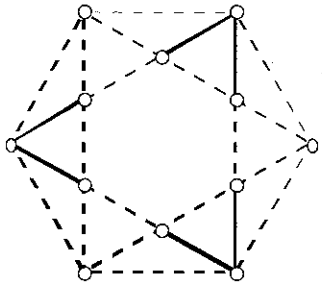
1.

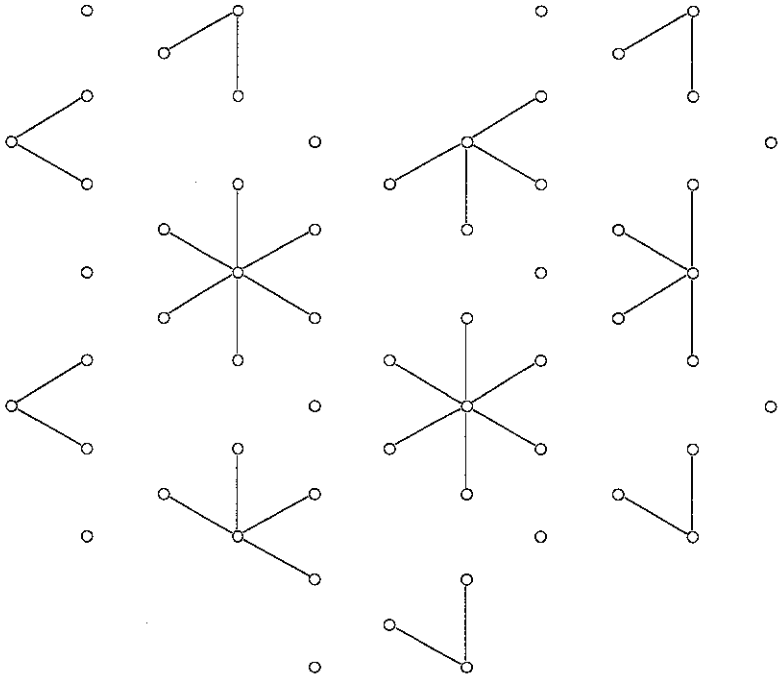




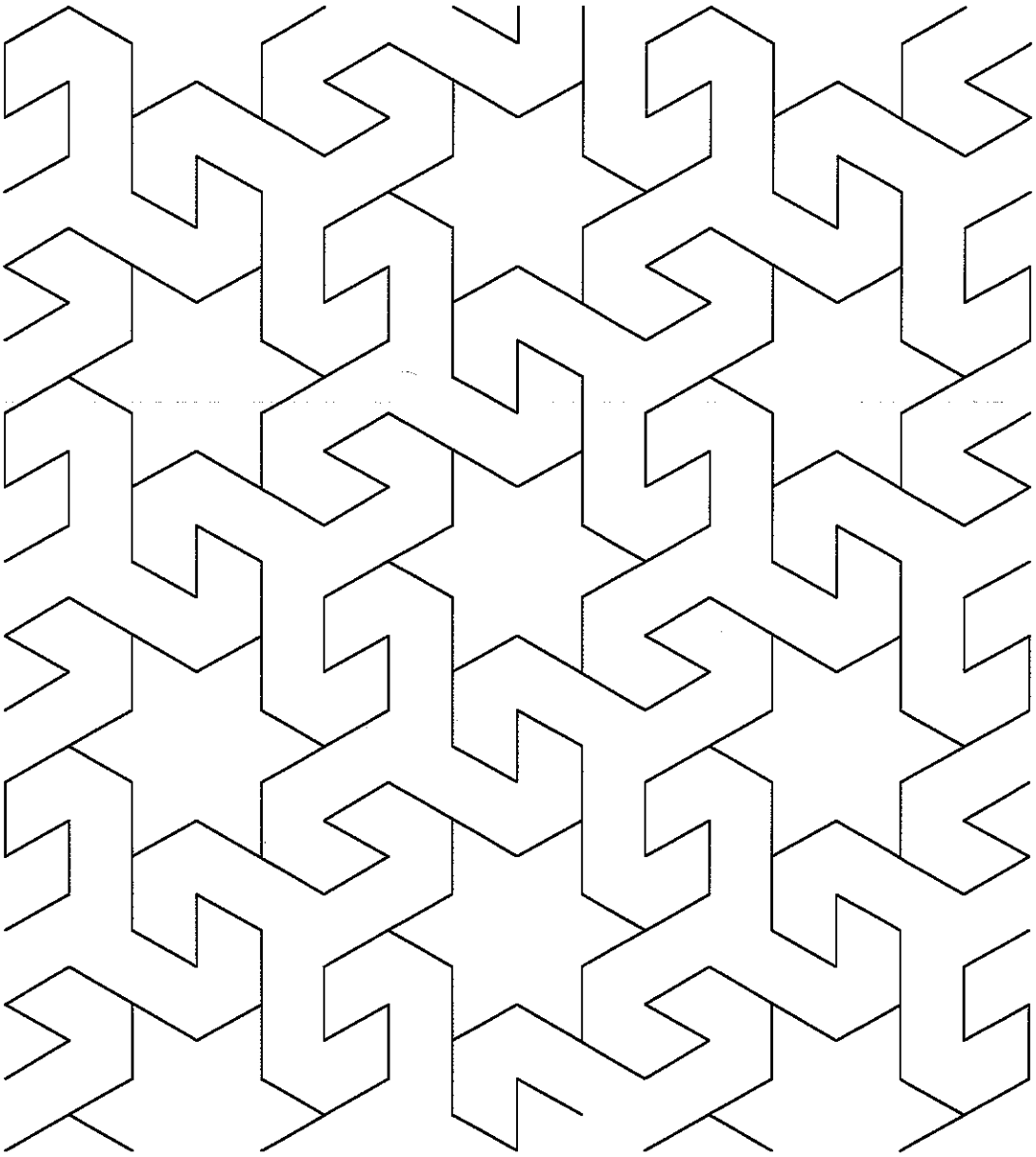
2.





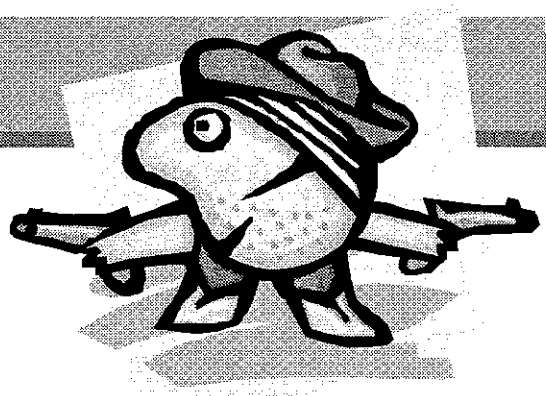


3. Given the Islamic design below, determine the basic hexagon and the basic design, and then construct the design.



COUNTING TEXAS COUNTY POPULATIONS AND AREAS

Dr. Bonnie H. Litwiller and Dr. David R. Duncan,
Professors of Mathematics, University of Northern Iowa



Introduction:

Teachers are always seeking situations in which mathematics can be connected to real world data. The counties of a given state can be used as a setting for these connections.

Table I is a listing of the population, area in square miles, and "density" (population ÷ area) for each of the 254 Texas counties. The population and area data were found in *The World Almanac* (St. Martin's Press, 1995); the density was computed from the other data.

Following Table I is a worksheet for students and an answer key.

Challenges for the reader and his/her students:

A. Perform the same analysis for other states, the United States and its 50 states, or other countries and their subdivisions.

B. Find other situations in which data can be manipulated and characterized.

Editors' Note:

Because of the amount of data in Table I, the editors' suggest dividing the data among groups, entering the data, and then linking calculators to share data. This could also be done with a spreadsheet program, which would make rank ordering the data easy. An Excel file of Table I is available by emailing a request to mm40370@swt.edu by August 1, 1999.

Table I

County	Population	Area (m^2)	Density
Anderson	48,024	1,077	44.6
Andrews	14,338	1,501	9.6
Angelina	69,884	807	86.6
Aransas	17,892	280	63.9
Archer	7,973	907	8.8
Armstrong	2,021	909	2.2
Atascosa	30,533	1,218	25.1
Austin	19,832	656	30.2
Bailey	7.064	826	8.6
Bandera	10,562	793	13.3
Bastrop	38,263	895	42.8
Baylor	4,385	862	5.1
Bee	25,135	880	28.6
Bell	191,073	1,055	181.1
Bexar	1,185,394	1,248	949.8
Blanco	5,972	714	8.4
Borden	799	900	0.9
Bosque	15,125	989	15.3
Bowie	81,665	891	91.7
Brazoria	191,707	1,407	136.3
Brazos	121,862	589	206.9
Brewster	8,653	6,169	1.4
Briscoe	1,971	887	2.2
Brooks	8,204	942	8.7
Brown	34,371	936	36.7
Burleson	13,625	669	20.4
Burnet	22,677	994	22.8
Caldwell	26,392	546	48.3
Calhoun	19,053	540	35.3
Callahan	11,859	899	13.2
Cameron	260,120	906	287.1
Camp	9,904	203	48.8
Carson	6,576	924	7.1
Cass	29,982	937	32
Castro	9,070	899	10.1
Chambers	20,088	616	32.6
Cherokee	41,049	1,052	39
Childress	5,953	707	8.4
Clay	10,024	1,086	9.2
Cochran	4,377	775	5.6
Coke	3,424	908	3.8
Coleman	9,710	1,277	7.6
Collin	264,036	851	310.3
Collingsworth	3,573	909	3.9
Colorado	18,383	965	19
Comal	51,832	555	93.4
Comanche	13,381	930	14.4
Concho	3,044	992	3.1
Cooke	30,777	893	34.5
Coryell	64,226	1,057	60.8
Cottle	2,247	895	2.5
Crane	4,652	782	5.9
Crockett	4,078	2,806	1.5
Crosby	7,304	899	8.1
Culberson	3,407	3,815	0.9
Dallam	5,461	1,505	3.6
Dallas	1,852,810	880	2,105.4
Dawson	14,349	903	15.9
Deaf Smith	19,153	1,497	12.8
Delta	4,857	278	17.5
Denton	273,525	911	300.2
DeWitt	18,840	910	20.7
Dickens	2,571	907	2.8
Dimmit	10,433	1,307	8
Donley	3,696	929	4
Duval	12,918	1,795	7.2
Eastland	18,488	924	20
Ector	118,934	903	131.7
Edwards	2,266	2,121	1.1
Ellis	85,167	939	90.7
El Paso	591,610	1,014	583.4
Erath	27,991	1,080	25.9
Falls	17,712	770	23
Fannin	24,804	895	27.7
Fayette	20,095	950	21.2
Fisher	4,842	897	5.4
Floyd	8,497	992	8.6
Foard	1,794	703	2.6
Fort Bend	225,421	876	257.3
Franklin	7,802	294	26.5
Freestone	15,818	888	17.8
Frio	13,472	1,133	11.9
Gaines	14,123	1,504	9.4
Galveston	217,396	399	544.9
Garza	5,143	895	5.7
Gillespie	17,204	1,061	16.2
Glasscock	1,447	900	1.6
Goliad	5,980	859	7
Gonzales	17,205	1,068	16.1
Gray	23,967	921	26
Grayson	95,019	934	101.7
Gregg	104,948	273	384.4
Grimes	18,828	799	23.6
Guadalupe	64,873	713	91
Hale	34,671	1,005	34.5
Hall	3,905	877	4.5
Hamilton	7,733	836	9.2
Hansford	5,848	921	6.3
Hardeman	5,283	688	7.7
Hardin	41,320	898	46

County	Population	Area (m^2)	Density
Harris	2,818,101	1,734	1,625.2
Harrison	57,483	908	63.3
Hartley	3,634	1,462	2.5
Haskell	6,820	901	7.6
Hays	65,614	678	96.8
Hemphill	3,720	903	4.1
Henderson	58,543	888	65.9
Hidalgo	383,545	1,569	244.5
Hill	27,146	968	28
Hockley	24,199	908	26.7
Hood	28,981	425	68.2
Hopkins	28,833	789	36.5
Houston	21,375	1,234	17.3
Howard	32,343	901	35.9
Hudspeth	2,915	4,567	0.6
Hunt	64,343	840	76.6
Hutchinson	25,689	872	29.5
Irion	1,629	1,052	1.5
Jack	6,987	920	7.6
Jackson	13,039	844	15.4
Jasper	31,102	921	33.8
Jeff Davis	1,946	2,257	0.9
Jefferson	239,389	937	255.5
Jim Hogg	5,109	1,136	4.5
Jim Wells	37,679	867	43.5
Johnson	97,165	730	133.1
Jones	16,490	931	17.7
Karnes	12,455	753	16.5
Kaufman	52,220	788	66.3
Kendall	14,589	663	22
Kenedy	460	1,389	0.3
Kent	1,010	878	1.2
Kerr	36,304	1,107	32.8
Kimble	4,122	1,250	3.3
King	354	914	0.4
Kinney	3,119	1,359	2.3
Kleberg	30,274	853	35.5
Knox	4,837	845	5.7
Lamar	43,949	919	47.8
Lamb	15,072	1,013	14.9
Lampasas	13,521	714	18.9
La Salle	5,254	1,517	3.5
Lavaca	18,690	971	19.2
Lee	12,854	631	20.4
Leon	12,665	1,079	11.7
Liberty	52,726	1,174	44.9
Limestone	20,946	930	22.5
Lipscomb	3,143	933	3.4
Live Oak	9,556	1,057	9
Llano	11,631	939	12.4
Loving	107	670	0.2
Lubbock	222,636	900	247.4

County	Population	Area (m^2)	Density
Lynn	6,758	888	7.6
Madison	10,931	472	23.2
Marion	9,984	385	25.9
Martin	4,956	914	5.4
Mason	3,423	934	3.7
Matagorda	36,928	1,127	32.8
Maverick	36,378	1,287	28.3
McCulloch	8,778	1,071	8.2
McLennan	189,123	1,031	183.4
McMullen	817	1,163	0.7
Medina	27,312	1,331	20.5
Menard	2,252	902	2.5
Midland	106,611	902	118.2
Milam	22,946	1,019	22.5
Mills	4,531	748	6.1
Mitchell	8,016	912	8.8
Montague	17,274	928	18.6
Montgomery	182,201	1,047	174
Moore	17,865	905	19.7
Morris	13,200	256	51.6
Motley	1,532	959	1.6
Nacogdoches	54,753	939	58.3
Navarro	39,926	1,068	37.4
Newton	13,569	935	14.5
Nolan	16,594	915	18.1
Nueces	291,145	847	343.7
Ochiltree	9,128	919	9.9
Oldham	2,278	1,485	1.5
Orange	80,509	362	222.4
Palo Pinto	25,055	949	26.4
Panola	22,035	812	27.1
Parker	64,785	902	71.8
Parmer	9,863	885	11.1
Pecos	14,675	4,777	3.1
Polk	30,687	1,061	28.9
Potter	97,841	902	108.5
Presidio	6,637	3,857	1.7
Rains	6,715	243	27.6
Randall	89,673	917	97.8
Reagan	4,514	1,173	3.8
Real	2,412	697	3.5
Red River	14,317	1,054	13.6
Reeves	15,852	2,626	6
Refugio	7,976	771	10.3
Roberts	1,025	915	11.2
Robertson	15,511	864	18
Rockwell	25,604	128	200
Runnels	11,294	1,056	10.7
Rusk	43,735	932	46.9
Sabine	9,586	486	19.7
San Augustine	7,999	524	15.3
San Jacinto	16,372	572	28.6

County	Population	Area (m^2)	Density
San Patricio	58,749	693	84.8
San Saba	5,401	1,136	4.8
Schleicher	2,990	1,309	2.3
Scurry	18,634	900	20.7
Shackelford	3,316	915	3.6
Shelby	22,034	791	27.9
Sherman	2,858	923	3.1
Smith	151,309	932	162.3
Somervell	5,360	188	28.5
Starr	40,518	1,226	33
Stephens	9,010	894	10.1
Sterling	1,438	923	1.6
Stonewall	2,013	925	2.2
Sutton	4,135	1,455	2.8
Swisher	8,133	902	9
Tarrant	1,170,103	868	1,348.0
Taylor	119,655	917	130.5
Terrell	1,410	2,357	0.6
Terry	13,218	887	14.9
Throckmorton	1,880	912	2.1
Titus	24,009	412	58.3
Tom Green	98,458	1,515	65
Travis	576,407	989	582.8
Trinity	11,445	692	16.5
Tyler	16,646	922	18.1
Upshur	31,370	587	53.4
Upton	4,447	1,243	3.6
Uvalde	23,340	1,564	14.9
Val Verde	38,721	3,150	12.3
Van Zandt	37,944	855	44.4
Victoria	74,361	887	83.8
Walker	50,917	786	64.8
Waller	23,389	514	45.5
Ward	13,115	836	15.7
Washington	26,154	610	42.9
Webb	133,239	3,362	39.6
Wharton	39,955	1,086	36.8
Wheeler	5,879	904	6.5
Wichita	122,378	606	201.9
Wilbarger	15,121	947	16
Willacy	17,705	589	30.1
Williamson	139,551	1,137	122.7
Wilson	22,650	807	28.1
Winkler	8,626	840	10.3
Wise	34,679	902	38.4
Wood	29,380	689	42.6
Yoakum	8,786	800	11
Young	18,126	919	19.7
Zapata	9,279	999	9.3
Zavala	12,162	1,298	9.4
Total	16,986,335	262,019	

Student Page

Answer the following questions using Table I.

A. Rank order the 254 counties by population.
   1. Find the mean and median of the 254 county populations. What does the inequality of these two measures tell you about the skewness of the data?

   2. What is the smallest number of counties needed to account for one-half of Texas' population of 16,986,355?

   3. Fill in the blank with the largest possible number: Harris county has a larger population than the combined total of the _____ smallest counties.

   4. Redo problem 3 for the top two counties, Harris and Dallas.

B. Rank order the 254 counties by area.
   1. Find the mean and median of the 254 county areas. What does the inequality of the two measures say about the skewness of data?

   2. What is the smallest number of counties needed to account for one-half of Texas' total area of 262,019 square miles?

   3. Fill in the blank with the largest possible number: Brewster has a larger area than the combined areas of the _____ smallest counties.

   4. Redo problem 3 for the top two counties in area, Brewster and Pecos.

C. Rank order the 254 counties by density.
   1. Find the total population and area of the three most densely populated counties. What are they?

   2. What is the density of the three-county group?

   3. If the entire state of Texas were as densely populated as this three-county group, what would be the state's population?

   4. If the entire state of Texas were as densely populated as its three least densely populated counties, what would be the state's population?

   5. If the United States were as densely populated as Texas' three most densely populated counties, what would the population of the United States be? The total area of the United States is 3,787,319 square miles.

   6. If the United States were as densely populated as Texas' three least densely populated counties, what would the population of the United States be?

Answer Page

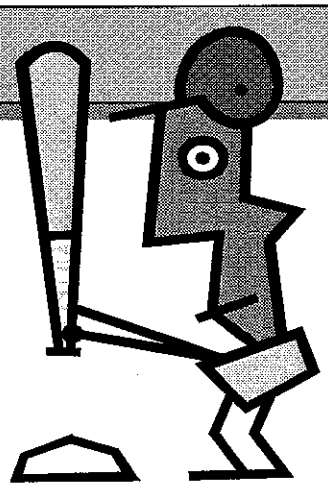
A. Questions about the population data
1. Mean = 66,875.3; median = 15,664.5; skewed to the right.
2. Seven counties: Harris, Dallas, Bexar, Tarrant, El Paso, Travis, and Hidalg.
3. The sum of the populations of the 200 smallest counties is 2,775,501; this is just less than Harris county's population of 2,818,101.
4. The smallest 227 counties are needed.

B. Questions about the area data
1. Mean = 1,031.6; median = 909.5; skewed slightly to the right.
2. 88 counties are needed, from Brewster to Ellis.
3. The sum of the areas of the 18 smallest counties is 6,122 square miles, just less than the 6,169 square mile area of Brewster.
4. 26.

C. Questions about the density data
1. Dallas, Harris, and Tarrant have a total population of 5,841,014 and a total area of 3,482 square miles.
2. $\frac{5,841,014}{3,482} = 1,677.5.$
3. $\frac{5,841,014}{3,482} = \frac{x}{262,019}$, x = 439,533,000. This is almost twice the population of the entire United States.
4. Total population = 921, total area = 2,973 square miles; $\frac{921}{2,973} = \frac{x}{262,019}$, x = 81,170. This is just less than the population of McAllen, Texas, alone.
5. We must solve the following equation: $\frac{5,841,014}{3,482} = \frac{x}{3,787,319}$, so x = 6,353,180,000. This is approximately the population of the entire world.
6. $\frac{921}{2,973} = \frac{x}{3,787,319}$, x = 1,173,266. This is approximately the population of the state of Maine alone.

Professional Baseball Statistics

Carol Smith, Kingwood High School, Kingwood, Texas



"When am I ever gonna use this?" Sound familiar? This is an exercise in a real-life application of statistics. Baseball is said to be America's favorite sport, so let's capitalize on the interest.

Editors' Note: This article uses Houston Astros batting averages and a calculator to find measures of central tendencies and other one-variable statistics.

Answer Key

1. a. mean = .235.
   b. Σx = 7.1041.
   c. n = 30.
   d. Σx / n = .2347.
   e. the mean before rounding.
2. a. median = .257.
   b. mode = 0.
   c. Q1 = .139.
   d. Q3 = .304.
   e. minimum = 0.
   f. maximum = 1.
   g. range = 1.
3. See calculator.
4. IQR = .165.
5. yes; Magnante.
6. See calculator.
7. a. 4
   b. 2
   c. 2
   d. 3
   e. 8
   f. 9
8. a. 4
   b. 1
   c. 2
   d. 6
   e. 2+5=7
9. a. make Ymax greater than13.
   b. 8
   c. 13
   d. 8
10. .056
11. a. .316
    b. .203
    c. 14
    d. 78%
    e. not really
    f. 372
    g. .147
    h. 17
    i. 94%
    j. yes

Student Page

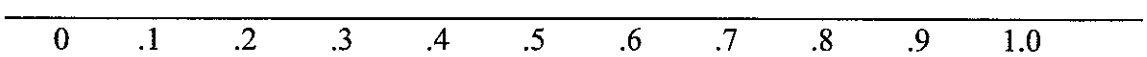
PROFESSIONAL BASEBALL STATISTICS

Using the Astros' 1998 Regular Season Official Statistics, enter the data for batting averages (AVG) into your graphing calculator. Enter only the combined statistics (not both Right and Left) for Everett and Meluskey. Enter only the Houston statistics for Powell. There should be 30 entries altogether, one for each player.

1. Calculate the one-variable statistics for the batting averages on your calculator.
   a. What is the mean to the nearest thousandth? _____
   b. What does Σx equal? _____
   c. What does n equal? _____
   d. What does Σx divided by n equal? _____
   e. Where have you seen that number before? _____

2. Find the following statistics using your graphing calculator after you order the data from least to greatest.
   a. What is the median? _____
   b. What is the mode? _____
   c. What is the first quartile? _____
   d. What is the third quartile? _____
   e. What is the minimum? _____
   f. What is the maximum? _____
   g. What is the range? _____

3. Use the median, the first and third quartiles, the minimum, and the maximum to draw a box and whisker plot below.



4. Subtract the first quartile from the third quartile to find the inter-quartile range. What is the inter-quartile range for the batting averages? _____

5. An outlier is a data point that is more than 1.5 times the inter-quartile range above the third quartile or below the first quartile. Are there any outliers in this list of data? If so, who is the player? _____

6. Use your graphing calculator to draw two statistical plots and sketch the plots below:
   a. a box and whisker plot using all the data

   b. a box and whisker plot eliminating the outliers.

7. Use your graphing calculator to draw a histogram using a .050 interval. (Suggested WINDOW: Xmin=0, Xmax=1.05, Xscl=.05, Ymin=(-1), Ymax=10, Yscl=1)
   a. How many players have a batting average between .000 and .050? _____
   b. How many players bat between .050 and .100? _____
   c. How many players bat between .100 and .150? _____
   d. How many players bat between .150 and .200? _____
   e. How many players bat between .250 and .300? _____
   f. How many players bat over .300? _____

8. Change the interval on your histogram to .025.
   a. How many players bat between .000 and .025? _____
   b. How many players bat between .075 and .100? _____
   c. How many bat between .150 and .175? _____
   d. How many bat between .250 and .275? _____
   e. How many bat between .275 and .325? _____

9. Change the interval to .150.
   a. What other changes must be made for this histogram to be completely seen? _____
   b. How many players bat between .000 and .150? _____
   c. How many bat between .150 and .300? _____
   d. How many bat between .300 and .450? _____

10. Before working with the standard deviation, eliminate the pitchers' statistics since batting is not their primary concern. Delete data from Bergman and players after him in the list, leaving 18 data entries. What is the standard deviation to the nearest thousandth? _____

11. If the Astros' (excluding pitchers) batting average statistics follows a normal distribution, 68% of the players will be within one standard deviation of the mean, and 95% of the players will be within two standard deviations of the mean. Determine whether these conditions are met by answering the following. Round to the nearest thousandth where appropriate.
   a. What is the sum of the mean and the standard deviation? _____
   b. What is the difference of the mean and the standard deviation? _____
   c. How many players have an AVG between the answers to *a* and *b*? _____
   d. What *percent* of the eighteen players have an AVG between the answers to *a* and *b*? _____
   e. Is that close to 68%? _____
   f. What is the sum of the mean and twice the standard deviation ? _____
   g. What is the difference of the mean and twice the standard deviation? _____
   h. How many players have an AVG between the answers to *c* and *d*? _____
   i. What *percent* of the eighteen players have an AVG between the answers to *c* and *d*? _____
   j. Is that close to 95%? _____

Houston Astros Official Statistics

Regular Season Record (102-60) -- Thru 9/27/98

DATE: 9/29/98 HOUSTON ASTROS TEAM BATTING ID: TEAMBPR
TIME: 4:08:21 Compiled by the MLB Baseball Information System THRU GAMES OF 9/28/98

* = D/L, + = ROOKIE, # = MULT. TEAMS

PLAYER	AVG	G	AB	R	H	TB	2B	3B	HR	RBI	SO	E
Alou, M	.312	159	584	104	182	340	34	5	38	124	87	5
Ausmus, B	.269	128	412	62	111	147	10	4	6	45	60	7
Bagwell, J	.304	147	540	124	164	301	33	1	34	111	90	7
Bell, D	.314	156	630	111	198	309	41	2	22	108	126	8
Berry, S	.314	102	299	48	94	152	17	1	13	52	50	10
Biggio, C	.325	160	646	123	210	325	51	2	20	88	113	15
Bogar, T	.154	79	156	12	24	33	4	1	1	8	36	3
Clark, D	.206	93	131	12	27	34	7	0	0	4	45	3
Eusebio, T	.253	66	182	13	46	57	6	1	1	36	31	3
Everett, C	.296	133	467	72	138	225	34	4	15	76	102	4
RIGHT	.268		82		22	35	4	0	3	15	17	
LEFT	.301		385		116	190	30	4	12	61	85	
Gutierrez, R	.261	141	491	55	128	164	24	3	2	46	84	15
+Hidalgo, R	.303	74	211	31	64	100	15	0	7	35	37	3
*Howell Jr, J	.289	24	38	4	11	19	5	0	1	7	12	0
Incaviglia, P	.125	13	16	0	2	3	1	0	0	2	4	0
+Johnson, R	.231	8	13	2	3	4	1	0	0	0	5	0
+Meluskey, M	.250	8	8	1	2	3	1	0	0	0	4	0
RIGHT	.333		3		1	1	0	0	0	0	1	
LEFT	.200		5		1	2	1	0	0	0	3	
Phillips, J	.190	36	58	4	11	17	0	0	2	9	22	3
Spiers, B	.273	123	384	66	105	152	27	4	4	43	62	9
Bergman, S	.083	31	60	3	5	7	2	0	0	4	30	1
+Elarton, S	.000	28	7	0	0	0	0	0	0	0	4	0
Hampton, M	.262	32	61	3	16	20	4	0	0	2	12	4
Henry, D	.000	59	4	0	0	0	0	0	0	0	2	0
Johnson, R	.063	11	32	1	2	3	1	0	0	2	17	1
Lima, J	.139	33	79	6	11	13	2	0	0	4	27	3
Magnante, M	1.000	48	2	0	2	2	0	0	0	1	0	0
+Miller, T	.333	37	3	1	1	2	1	0	0	0	0	0
Nitkowski, C	.000	43	4	0	0	0	0	0	0	0	3	0
Powell, J #	.000	62	1	0	0	0	0	0	0	0	1	0
HOU	.000	29	1	0	0	0	0	0	0	0	1	0
Reynolds, S	.159	35	82	10	13	17	4	0	0	9	39	2
Wagner, B	.333	58	3	0	1	1	0	0	0	0	2	1

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

INDIVIDUAL MEMBERSHIP APPLICATION

Name: _____

Home Mailing Address: _____

City: _____ State: _____ Zip: _____

(Be sure to include a street address or PO box.)

E-mail address: _____

Circle area(s) of interest: K-2 3-5 6-8 9-12 College

Check One: Renewal _____ New Member _____ Change of Address _____

Individual Membership Dues: $13.00

PROFESSIONAL MEMBERSHIP FORM

To enroll school, institution, or affiliated group :

School District or University: _____

Campus: _____

School Mailing Address: _____

City: _____ State: _____ Zip: _____

Check One: New _____ Renewal _____

Professional Membership Dues: $40.00 (includes 3 journals)

Total Amount due to TCTM: $ _____

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS MEMBERSHIP

_____ New

_____ Renewal

_____ One Journal (regular) $57.00

_____ Two Journals (regular) $75.00

_____ Three Journals (regular) $99.00

___Teaching Children Mathematics
___ Mathematics Teacher

___Mathematics Teaching in the Middle School

___Journal for Research in Mathematics Education

_____ Institutional Membership for Campus $62.00 (Please indicate the one journal you wish to receive. Each additional journal is $18.)

Total Amount Due to NCTM: $ _____

Make check payable to TCTM and mail to:
TCTM Treasurer
38 Bradford Circle
Sugar Land, TX 774

Total Amount Due: $ _____

Lone Star News: Changes to the Constitution and By-Laws

The TCTM Executive Board proposes the following changes to the Constitution and By-Laws. Amendments are underlined and deletions are marked through. After reviewing the changes, please use the ballot included to vote on the changes.

THE CONSTITUTION OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

ARTICLE I - NAME

This council shall be known as the Texas Council of Teachers of Mathematics, hereafter referred to as the Council, or TCTM.

ARTICLE II - AFFILIATION

This council shall be affiliated with the National Council of Teachers of Mathematics (NCTM).

ARTICLE III - PURPOSES

Section 1. To encourage an active interest in mathematics.

Section 2. To provide the opportunity to study and keep abreast of any new trends in the teaching of mathematics.

Section 3. To improve teacher training programs for Texas mathematics teachers.

Section 4. To assist Texas mathematics teachers in obtaining the benefits from the NCTM and the Conference for the Advancement of Mathematics Teaching (CAMT).

Section 5. To promote and sustain active local affiliated councils throughout the state.

ARTICLE IV - MEMBERSHIP

Section 1. Individual membership is available upon payment of dues as provided in the by-laws.

Section 2. Associate membership is available to any full-time student upon payment of dues as provided in the by-laws.

Section 3. Honorary membership may be granted to any person who has rendered service which this council may desire to recognize with a life-time honorary membership, upon the recommendation of the Executive Board. An honorary member shall be exempt from paying dues and shall enjoy all privileges of individual membership.

Section 4. Institutional membership is available to any school, college, or organization, upon payment of dues as provided in the by-laws.

Section 5. Special interest groups, whose goals are consistent with TCTM, may be recognized by the Executive Board.

ARTICLE V - OFFICERS AND EXECUTIVE BOARD

Section 1. The officers of the council shall be a president, two vice-presidents, a secretary, a treasurer, and six regional directors elected by a plurality vote of the membership.

Section 2. There shall be an NCTM representative, a director of publications, two editors, ~~a business manager, and~~ a parliamentarian, a CAMT Board representative, and a Government Relations representative appointed by the president with the approval of the Executive Board.

Section 3. The voting members of the Executive Board shall consist of the elected officers.

Section 4. The non-voting member of the Executive Board shall be the NCTM representative, the director of publications, the editors, ~~the business manager,~~ the parliamentarian, the president-elect or past president, the CAMT Board representative, the Government Relations representative, and the TEA Director of Mathematics.

ARTICLE VI - MEETINGS

Section 1. There shall be at least one regular meeting of the Council during the fiscal year.

Section 2. There shall be at least one meeting of the Executive Board during the fiscal year.

ARTICLE VII - AMENDMENTS

This constitution may be amended in the following manner:

A. The proposed amendment or amendments shall be sent to the president or to the secretary at least ninety days prior to the annual meeting.

B. An announcement of the proposed change(s) and a mailed ballot shall be included in the general announcement of the annual meeting.

C. To become effective, any change(s) in the constitution shall be approved by a two-thirds majority of members who respond to the mailed balloting.

ARTICLE VIII - DISSOLUTION

If, at any time, the Texas Council of Teachers of Mathematics shall cease to carry out the purposes that are herein stated, all assets and property held by it, whether in trust of otherwise, shall, after payment of its liabilities, be paid over to the National Council of Teachers of Mathematics to be used exclusively for any project, or educational purpose, as determined by the Board of Directors of that organization.

THE BY-LAWS OF THE TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

ARTICLE I - RULES OF ORDER

Robert's Rules of Order or *Greggs Parliamentary Law* shall be the authority on all questions of procedure not specifically stated in this constitution and by-laws.

ARTICLE II - EXECUTIVE BOARD

The executive board shall serve as the governing body of the Council. It shall have power to transact the business of the council; initiate, develop, and determine the policies of the council; establish the budget; and appoint officers as provided in the constitution and by-laws.

ARTICLE III - SPECIAL INTEREST GROUPS

Special Interest Groups currently recognized by TCTM are:
State of Texas elementary Association of Mathematics (STEAM).

ARTICLE IV - QUALIFICATIONS, TERMS, AND DUTIES OF OFFICERS AND OTHER MEMBERS OF THE EXECUTIVE BOARD

Section 1. Qualifications of Officers

a. Officers of this council shall be individual members of the Council and the NCTM.

b. One vice-president shall be a member with elementary mathematics responsibilities, and one vice-president shall be a member with secondary mathematics responsibilities.

Section 2. Terms of Office

a. All elected officers shall assume their duties at the beginning of the fiscal year.

b. The president shall serve for a period of two fiscal years, succeeding to office from the office of president-elect. This office, if vacated in the second fiscal year, shall be filled by the president-elect. This office, if vacated in the first fiscal year, shall be filled by the secretary.

c. The president-elect shall be elected to serve for the fiscal year corresponding to the second fiscal year of the president's term of office. This office, if vacated, shall be filled by an election either by mailed ballot or at the annual meeting of the Council.

d. The vice-presidents shall be elected to serve for the period of two years, one elected each year. This office, if vacated, shall be filled by an election either by mailed ballot or at the annual meeting of the Council.

e. The secretary shall be elected to serve for a period of two years and shall be subject to reelection. This office, if vacated, shall be filled by an election either by mailed ballot or at the annual meeting of the Council.

f. The treasurer shall be elected to serve for a period of two years and shall be subject to reelection. This office, if vacated, shall be filled by the executive board, or by an election, either by mailed ballot or at the annual meeting of the Council.

g. The regional directors shall be elected by members in each of the ~~four~~ six geographical regions of the State of Texas, designated as Southeast, Northeast, Northwest, ~~and~~ Southwest, South and Central, for a period of two years and shall be subject for reelection. This office, if vacated, shall be filled by an election either by mailed ballot or at the annual meeting of the Council.

h. The representative to the National Council of Teachers of Mathematics shall be appointed by the president, with the approval of the executive board, for a period of one year, and shall be eligible for reappointment.

i. The director of publications shall be appointed by the president, with the approval of the executive board, for a period of two years and shall be eligible for reapointment.

j. The editors shall be appointed by the president, with the approval of the executive board, for a minimum of two years, and shall be eligible for reappointment.

~~k. The business manager shall be appointed by the president, with the approval of the executive board, for a period of one year and shall be eligible for reappointment.~~

~~l~~k. The parliamentarian shall be appointed by the president, with the approval of the executive board, for a period of one year and shall be eligible for reappointment.

l. The CAMT Board representative shall be appointed by the president, with the approval of the executive board, for a period of one year and shall be eligible for reappointment.

m. The government relations representative shall be appointed by the president with the approval of the executive board, for a period of one year and shall be eligible for reappointment.

Section 3. Duties of Officers

a. The president shall preside at all meetings of the Council and the executive board; shall administer the affairs of the council; shall appoint all committees not otherwise provided for; and shall be an ex-officio member of all committees. The president shall prepare an annual report to be given at the annual meeting of the Council and filed as part of the permanent record. The president, or designated delegate, shall serve as the official delegate to the NCTM Annual meeting.

b. The president-elect shall perform duties as assigned by the president and shall be an ex-officio member of all committees.

c. The immediate past president shall assist the president and executive board.

d. The vice-president with elementary responsibilities shall serve as a board member of STEAM, promoting membership and providing publicity. The vice-president with secondary responsibilities shall represent the secondary interests, promoting membership and providing publicity.

e. The secretary shall keep all records and minutes of the Council and of the executive board; and shall preserve the annual reports and historical records of the council.

f. The treasurer shall collect all dues and other income of the Council; shall pay all routine bills provided for by the annual budget and such other bills as approved by the executive board or the president; shall maintain a current membership list; shall keep all financial records, and shall make an annual financial report to the executive board and the council. A copy of this report shall be filed as part of the permanent records.

g. Each regional director shall promote the organization and maintenance of the local councils and solicit from the region nominations for TCTM offices. The regional director may organize leadership workshops for officers of local affiliated groups and may organize TCTM sponsored regional conferences, or any other activity which may benefit the local affiliated groups.

h. The NCTM representative shall serve as the liaison between NCTM and TCTM and shall be responsible for the NCTM booth at CAMT.

i. The director of publications shall print and distribute council publications, such as the journal, the newsletter, and ballots.

j. The editor(s) of the journal shall solicit articles, submit them for review, and prepare all materials for publication. The editor(s) of the newsletter shall prepare the newsletter for publication.

~~k. The business manager shall be responsible for solicitation and collection of the advertising fees and donations, and shall prepare an annual budget for the board.~~

~~l~~k. The parliamentarian shall advise the president on matters of parliamentary procedure.

l. The CAMT Board representative shall represent TCTM on the CAMT board which meets semi-annually and shall prepare a report from such meetings for the executive board.

m. The Government Relations representative shall represent TCTM in business and public affairs where requested by the executive board or the president.

ARTICLE V - GEOGRAPHICAL REGIONS

The State of Texas shall be divided into ~~four~~ six regions comprised of areas which coincide with designated ESC regions.

Section 1. The NORTHEAST Region consists of ESC regions ~~6~~, 7, 8, 10, and 11 ~~and 12.~~

Section 2. The SOUTHEAST Region consists of ESC regions ~~2. 3.~~.4. 5. and 6 ~~and 13.~~

Section 3. The NORTHWEST Region consists of ESC regions 9, ~~11,~~ 14, 16, and 17.

Section 4. The SOUTHWEST Region consists of ESC regions ~~1,~~ 15, 18, and 19. ~~and 20.~~

Section 5. The SOUTH TEXAS Region consists of ESC regions 1, 2, and 3.

Section 6. The CENTRAL TEXAS Region consists of ESC regions 12, 13, and 20.

ARTICLE VI - COMMITTEES

Section 1. Standing Committees

a. Publications Committee

This committee shall consist of the director of publications, the editors, and two members appointed by the president. The chair shall be the director of publications. The committee shall be responsible for the procurement of materials for publication and distribution of all bulletins of the council and shall submit announcements and other pertinent materials to state and national publications.

Section 2. Special Committees

a. Nominations and Elections Committee
This committee shall consist of the vice-president who is not standing for election, and one member ~~(not serving on the executive board)~~ from each region. The vice-president shall serve as chair. The committee shall serve for regular elections. The committee shall make every effort to secure at least two nominees for each vacant office. The nominees shall be presented to the members by way of a printed ballot. Ballots shall be returned to the chair of the committee, who shall, with one other officer, count the ballots and certify the results to the committee.

b. The Auditing Committee
This committee shall audit the financial records before the annual meeting. This committee shall consist of the president, treasurer, and one other member of the executive board.

c. Other Special Committees
These committees shall assume duties as outlined when appointed.

ARTICLE VII - FISCAL YEAR AND MEMBERSHIP DUES

Section 1. The fiscal year ~~and the membership year~~ for the Council shall be from September 1 extending through August 31.

Section 2. Membership dues

a. Annual dues for an individual or associate member shall be ~~ten~~ thirteen dollars.

~~b. Annual dues for an associate member shall be five dollars.~~

~~c.~~b. There shall be no annual dues for honorary members.

~~d.~~c. Annual dues for an institutional member shall be ~~thirty~~ forty dollars. Institutional members receive ~~either one copy of all publications and one paid CAMT registration, or~~ three copies of all publications.

ARTICLE VIII - MEETINGS AND QUORUMS

Section 1. One regular meeting of the Council shall be held during the annual meeting of the CAMT. A quorum shall consist of twenty-five members.

Section 2. At least one meeting of the executive board shall precede the annual council meeting at the CAMT meeting. A quorum shall consist of ~~five~~ six members.

ARTICLE IX - AMENDMENTS

These by-laws may be amended by a two-thirds majority of the voting members of the executive board present.

BALLOT

1. Circle your choice below.

2. Fold ballot in thirds with the address on the next page showing, staple, and mail. You also may place it in an envelope and mail to the address on the next page.

Do you accept the changes made to the Constitution and By-Laws of the Texas Council of Teachers of Mathematics?

Yes **No**

place
stamp
here

Pam Alexander
Rt. 4 Box 5212
Nacogdoches, TX 75964

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the
National Council of Teachers of Mathematics

1998-1999 Officers

Cut out your membership card.
Note the expiration date on your mailing label.
Renew your membership before that date.
Use the form on the last page.

Texas Council

of Teachers of Mathematics

Member 1998-1999

NAME _____

Southwest Texas State University
Texas Mathematics Teacher
Dr. Paul Kennedy
Department of Mathematics
601 University Dr.
San Marcos, TX 78666

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