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The ***Texas Mathematics Teacher***, the official journal of the Texas Council of Teachers of Mathematics, is published in the fall and spring. Authors are encouraged to submit articles that deal with the teaching and learning of mathematics at all levels. Editorial correspondence and manuscripts should be mailed or e-mailed to the editor, Paul Kennedy.

Potential authors should adhere to the following guidelines.

1. Manuscripts should be word-processed meeting APA guidelines. Tables and figures should likewise be computer generated. No author identification should appear on the manuscript.
2. Submit four copies. Include a Macintosh or IBM 3 1/2 inch diskette containing the manuscript indicating the word-processing program used on the label or send as an attachment on e-mail to pk03@swt.edu.
3. Include a cover letter containing author's name, address, affiliations, phone and fax numbers, e-mail address, and the article's intended level.
4. Articles for *Voices From the Classroom* should be relatively short and contain a description of the activities sufficient in detail to allow readers to incorporate them into their teaching. A discussion of appropriate grade level and prerequisites for the lesson should be included.

After refereeing, authors will be notified of a publication decision. Two copies of the issue in which an author's manuscript appears will be sent to the author automatically.

Items for *Lone Star News* include reports, TCTM affiliated group announcements, advertisements of upcoming professional meetings, and any other appropriate news postings.

Advertisements support the publication of this educational journal. Businesses interested in placing an advertisement for mathematics materials should contact Paul Kennedy.

TEXAS MATHEMATICS TEACHER

A Publication of the Texas Council of Teachers of Mathematics

ARTICLES

VOICES FROM THE CLASSROOM

LONE STAR NEWS

Letter From the President

Hi, Everyone!

Just a few thoughts as this year gets off and running for each of you. It's finally raining in Nacogdoches - only for a short while, but we will take anything!

Opportunities are "raining down" on mathematics educators this year as well. As we choose instructional materials for grades K - 8, focus will be on learning first about the new Texas Essential Knowledge and Skills, or the TEKS. For those of you who are not clear, TEKS is pronounced phonetically correct (the "e" is short since it lies between two consonants); therefore, TEKS can be thought of as part Texas, or TEKSas! Please take the opportunity to study the TEKS, so that mathematically-sound decisions can be made in the best interest of children about the materials we will be using in the foreseeable future. Even materials on the conforming list need to be examined in depth for the thoroughness with which the activities address the needs of children. The Texas Statewide Systemic Initiative has published a document to assist with instructional materials selections. Contact your ESC if you do not have access to a copy.

There are new offerings from TEXTEAMS, both in mathematics and science. The Algebra2/PreCalculus Institute will arrive in the spring following the widespread utilization of the Mathematical Models with Applications course and its accompanying Clarifying Activities. Institutes released in science include the TEKS Overview Module. In the spring, Modules for Systems will be available in all grades, K - 8. We should also see the newest mathematics institute, *Geometry for All*, available this fall. Look for all these great opportunities for your own professional development at www-tenet.cc.utexas.edu/ssi/.

We will see some welcome modifications at CAMT in Dallas next summer! First of all, it will take place at the Adams Mark Hotel on Monday, Tuesday, and Wednesday and will be followed by Post-Conference sessions, instead of the traditional Pre-Conference. Matharama will be offered on Monday and Tuesday followed by a new event, *Poster Sessions* , on Wednesday. These will display in poster fashion interesting classroom ideas with a time in the program that the presenter will be available to answer questions about the display. Poster sessions are a great way for your best idea to be shared! Each of you should consider completing a proposal. TEXTEAMS Institutes will be offered concurrently with CAMT which will allow institute participants to attend some CAMT functions (i.e. the luncheon). Each pre-registrant will have the opportunity to receive two tickets to the new *ticketed sessions*, limited to the smaller activity sessions that are often popular and so crowded. Lots of changes at CAMT.

Last, and definitely not least, our goal as a TCTM Board is to improve the lines of communication within the organization. For that reason, I have included contact information for myself and each of the Regional Directors. Please feel free to contact any of us about something exciting going on in your area professionally or individually in your own classroom. It may be something that can be shared in an upcoming newsletter or journal. In turn, we will try to share with you any upcoming events that might be of interest or impact each of us. Texas is a huge state with many diverse sectors; however, we all are involved with the teaching and learning of mathematics for the children of this state. With technology, we can all be connected, so to speak. Let's figure out ways to share with each other - *together everyone achieves more!*

Have a great year!

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INTERESTING ACTIVITIES AND ARTICLES NEEDED!!

Please send your submissions by January 1, 1999 to:

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Standards 2000 Draft

NCTM is in the process of updating, clarifying, and synthesizing the three existing *Standards* documents. The Standards 2000 Project will release the first draft - both a hard copy and an electronic version - in October 1998, followed by a period of dialogue through May 1999. The final document will be released in Spring 2000.

Role of Affiliated Groups

The Standards 2000 Project is requesting that NCTM Affiliated Groups (AGs) play an active role in providing feedback on the draft. Two specific opportunities exist.

First, AGs are being encouraged to hold reaction sessions to the draft or to plan special focus groups on particular topics.

Second, AGs are being urged to reach local teachers who are not members of AGs or NCTM by forming discussion or focus groups at the local level to involve these educators.

Detailed materials for Standards 2000 Project sessions will be sent to AGs in late August. Feedback received will be used by the writers in preparing the final Standards 2000 Project version.

Getting the Draft

Getting the draft will be easy. The draft will be posted on the NCTM web site. NCTM members only will be able to order a copy using a postcard to be included in the October *News Bulletin*. Members and nonmembers will be able to obtain a copy by phone at (703) 620-9840 ext. 191; e-mail at future@nctm.org; and through the web at www.nctm.org/standards2000.

A draft of the Standards 2000 Project electronic format will also be posted on the Web for reaction.

Standards 2000 Project Timeline

Summer 1998	Writing Group meets to prepare draft
Mid-October 1998	**Print and electronic drafts released**
1998-1999	Year of Dialogue Responses collected and analyzed throughout the year until May 1999
Summer 1999	Writing Group meets to finalize the document
Spring 2000	Standards 2000 released

NCTM News

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Changing My Instruction of Middle School Mathematics One Lesson at a Time

Dr. Jacqueline Leonard

"The only constant in life is change," one teacher-supervisor said to me when I began my teaching career in 1981. However, very little has changed in middle school mathematics classrooms. The National Council of Teachers of Mathematics published the *Standards* — a series of three documents — from 1989 to 1995. The *Standards* were written to reform the mathematics education of K-12 students in the areas of curriculum, pedagogy, and assessment. Yet, the transition has been especially slow in secondary classrooms.

According to Sanchez and O'Harrow (1997), the traditional approach to teaching mathematics — dependency upon textbooks, drills, and problem-solving repetitions — is under attack. Yet this type of teaching remains the most common in the majority of mathematics classrooms in the United States. For many students, learning mathematics the traditional way is boring and causes them to dislike the subject. Moreover, this type of teaching has failed at preparing many U. S. students to achieve in mathematics (Campbell & Johnson, 1995). The alternative is to implement the curriculum reforms recommended by the *Standards* in mathematics classrooms (NCTM, 1989).

Changing Teacher Pedagogy

Some believe reform needs to take place at a grassroots level among individual teachers if measurable change is to occur (Leonard, 1997). Teaching reform mathematics requires teachers to provide opportunities for students to take more active roles in their own learning (Bush & Kincer, 1993; Longo, 1993). If mathematics instruction is to become more student-centered, individual teachers must change their pedagogy and provide activities for students to engage in critical thinking skills and classroom discourse.

Teachers can change their pedagogy by limiting their use of the direct-teaching model. Direct teaching places the teacher at the center of instruction as the sole possessor of mathematical knowledge. The use of student-centered instructional strategies allows the responsibility for learning mathematics to be shared by teachers and students. One instructional strategy is the use of teaching questioning in whole-group instruction. Proper questioning techniques encourage students to think for themselves and to suggest solutions to problems that may not otherwise be considered (Campbell & Johnson, 1995). Another instructional strategy is the use of complex tasks with small groups (Cohen & Lotan, 1995). Small group tasks allow students to share mathematical knowledge with their peers and to learn by teaching others.

While many teachers may agree with the philosophical tenets of reform, they may lack the knowledge to implement these changes in their classrooms. In order to promote change, it is important that teachers who have reformed their instruction of mathematics share ideas that have worked for them with other teachers. The purpose of this paper is to share the experience of teaching one geometry lesson on surface area.

The Surface Area Lesson

During the 1996-1997 school year, I asked my suburban Maryland sixth grade students to participate in a classroom research study. As part of a pilot study, they participated in a series of integrated mathematics and science lessons relating geometry to the Earth and the Sun. The students were asked to share their background knowledge about the Earth and the Sun while they learned some of the "big ideas" in geometry. These ideas were diameter, circumference, and surface area.

I learned through informal student interviews that the lesson on surface area was of great interest to the students. In this lesson, I asked the students to find the surface area of a small ball and extended that concept to a discussion about the surface area of the Sun. The students had previously completed a lesson on finding the relationship between the circumference and the diameter of objects. Both lessons, *Discovering Pi* (Eckley, 1994) and *Surface*

Area of a Sphere (Jewell, 1994) were found on the Internet. The lesson on surface area was videotaped and transcribed. The dialogue below provides an example of the teacher questioning I used during whole-group instruction. Pseudonyms are used for student anonymity.

Text 1

Teacher:	When I say pi, do I mean apple pie?
Class:	No!
Teacher:	What do I mean when I say pi? [Pause] Herman?
Herman:	3.141592 . . .
Teacher:	What?
Herman:	That's what pi is!
Teacher:	I hate to ask you this, but could you repeat that slowly please?
Herman:	3.141592653589793238462643383179502884 1901639.
Teacher:	Well, that is pretty far.
	That goes off my chart past the hundred billionths place.
	I am impressed! You get a floating A.
	How is pi related to what we did yesterday? [Pause] Joyce?
Joyce:	It's the diameter divided by the circumference.
Teacher:	Is it the diameter divided by the circumference?
	You got us off on a roll, Joyce.
David:	It's the circumference divided by the diameter.

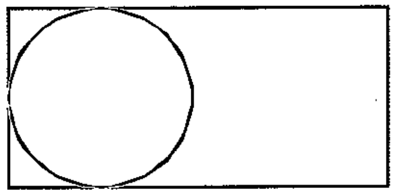
Analysis of Text 1

The preceding dialogue shows the depth of the students' background knowledge about pi. The class and I determined that pi was close to 3.14 the day before. Herman memorized the value of pi up to 43 decimal places on his own overnight. Herman's number provided us with an example of a non-repeating decimal (Line 8). Thus, the students were able to think about pi as a non-terminating, non-repeating and not just an approximation. Herman's response also provided an opportunity for me to review the concept of pi and, along with the students' prior knowledge about diameter and circumference, to complete the surface area task.

On the previous day, the students discovered that a relationship between the diameter and the circumference of round objects was the number pi. Joyce stated the relationship as a division problem (Line 14), and David remembered how to set the problem up (Line 16). I could have told the students what they needed to know, but I believed they would remember the information better if the answers came from them. I wanted the students to use their prior knowledge about diameter and circumference to solve the surface area problem.

Procedures

The instructional strategy used to teach the surface area task was cooperative learning. Eight teams of three to four students each were assigned jobs after they chose one of four colored chips. Students were given the jobs of recorder, measurer, cutter, and wrapper based on the color they chose. Their overall task was to wrap a ball with enough wrapping paper to cover the surface area and to find the surface area of the ball based on the dimensions of the paper used to wrap it (see Figure 1).



Note: Paper is the height of the ball's diameter and the length of the ball's circumference (Jewell, 1994).

Figure 1: Ball and Wrapping Paper

Research has shown that groups composed of equal numbers of boys and girls with compatible mathematical abilities were most successful (Webb, 1991). To determine the students' level of mathematics achievement, I examined their fifth grade report cards and considered their scores on standardized tests. Using this information, I ranked all of the sixth grade students as low, average, or high in mathematics achievement and placed them into groups by low-middle ability, middle ability, or middle-high ability.

To collect the data, two vide cameras were set up on opposite ends of the classroom. In order to minimize threats to validity, the school counselor selected which students would be videotaped. Only the cameramen and the counselor knew the identity of the target students. I learned which students were subjects by reviewing and analyzing the videotapes when the pilot study was completed.

The student participants were Susan, Joyce, Herman, and Tony (pseudonyms). These students were ranked middle-high in mathematics achievement and were racially diverse. Susan is African American, Herman is Asian, and Joyce and Tony are Caucasian. It is coincidental that Joyce and Herman also shared their ideas in Text 1. The following text reveals the thinking of all four students as they engaged in the surface area task.

Text 2

Joyce: We need centimeters.
Tony: 50. Now, we are going to do this. [Lays string on meter stick.]
50 inches . . . centimeters.
Susan: I get to measure it. [Measure the string, too].
Tony: 50 centimeters and about 3 centimeters.
Susan: We've got to measure the paper first.
Herman: No, we are not wrapping it yet; we got to . . .
Joyce: I know, but . . . No, we got to measure the paper.
Tony: [Measures the height of the ball.] 16.
Joyce: 16.
Susan: Where's our card, so we can write it down?
Tony: Look what I can do! I can twirl this pencil between my fingers.
Joyce: What was the height of it? 16 right!
Tony: Yeah.
Joyce: Write down your measurements.
Susan: 16 centimeters wide or long?
Joyce: Where's the string?
Susan: [Holding the string.] We got to measure this.
Joyce: No, we've already measured it.
We can put on the wrapping paper now.
Tony: 50.
Joyce: We already got 50.
Now, we need to measure the wrapping paper with that string.
Herman: Make a line.
Tony: Aren't you going to trace it?

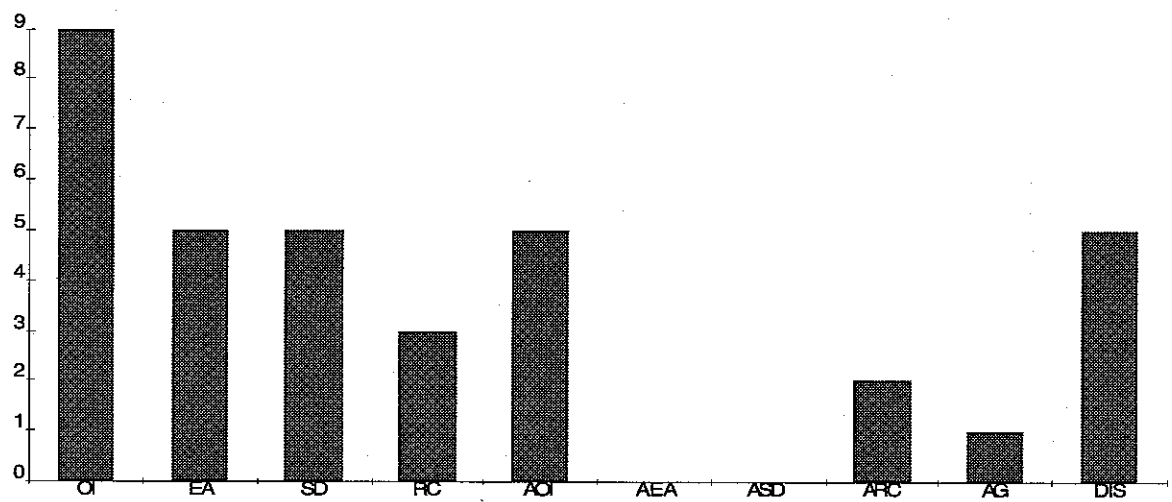
26 Susan: [Draws the line and takes the scissors.] I'm the cutter.
27 Joyce: No, I'm the cutter.
28 Susan: No, I'm the cutter. [She takes the scissors and cuts the paper.]
29 Tony: Now, how tall was it? 16 centimeters. Hold it out again.
30 Joyce: 16 centimeters.
31 Tony: Let's check the fitting. [Puts ball in the center of the paper.]
32 Herman: Wrap it.

Analysis of Text 2

The surface area task generated quite a bit of discussion from the group. An analysis of the discourse reveals how the students were thinking about the surface area task and how they used their knowledge about measurement to complete it. To gain an understanding of how the students were thinking, the discourse was analyzed by type in Figure 2 (Mulryan, 1995). The types of discourse were categorized in the following manner: 1) gives opinion or information; 2) gives evaluation or analysis; 3) gives suggestion or direction; 4) repeats or confirms information; 5) asks for opinion or information; 6) asks for evaluation or analysis; 7) asks for suggestions or directions; 8) asks for repetition or confirmation; 9) agrees; and 10) disagrees.

The frequencies of the types of student responses are shown in Figure 2. The data shows that the students made 22 "giving" statements and seven "seeking" statements in Text 2. These data suggest that the students had a high level of understanding about the task itself. Susan, Joyce, Herman, and Tony were able to articulate that understanding and proceed without a great deal of assistance from me. These four students, diverse in culture and ethnicity, were self-motivated to initiate and complete the task.

Moreover, Text 2 reveals that the students had a high level of on-task behavior. Transcripts of the 15-minute activity show that only one off-task remark was made during the entire lesson (Line 12). However, because the other members of the



OI	Gives opinion or information	AEA	Asks for evaluation or analysis
EA	Gives evaluation or analysis	ASD	Asks for suggestions or directions
SD	Gives suggestions or directions	ARC	Asks for repetition or confirmation
RC	Repeats or confirms	AG	Agrees
AOI	Asks for opinion or information	DIS	Disagrees

Figure 2: Types of Student Disclosure

group were very focused, Tony's statement was totally ignored. The group was able to find the height of the ball, which was 16 centimeters, and the circumference of the ball which was about 53 centimeters. Their measurements had to be precise in order to cut enough paper to cover the ball's surface area.

Furthermore, Text 2 shows that all of the students were actively engaged in the task. Although Joyce and Tony assumed leadership roles at the outset, Susan and Herman were not afraid to share their thoughts about how the task should be done (Lines 15 and 16). Because of their input, the group was able to obtain the measurements of the ball quickly. These measurements were recorded and used to calculate the area of the paper.

Although the students understood the task, Text 2 also reveals the students' lack of confidence in their measuring. The students asked for confirmation of the measurements and checked them more than once. When Tony measured the string, he found that it was a little longer than 50 centimeters (Line 5). However, the group estimated the length to be 50 centimeters instead. When Susan wanted to know whether the number 16 was the height or the length, the group ignored her question (Line 16), but this information was critical to the success of the task. In order to find the answer for herself, Susan wanted to measure the string again. However, Joyce decided to use the string as a guide to measure the length of the wrapping paper. This was not what I expected the students to do, but it was good thinking. If the students' measurement was incorrect, Susan would have cut the paper too short. Using the string guaranteed that the length would be long enough.

The discourse in Text 2 also shows that disagreement over procedures and roles occurred during the group task. Perhaps setting the task up as a collaborative group activity with unassigned roles would have prevented the friction. Nevertheless, the group was able to resolve their disagreements and focus on completing the task. These data informed me that additional classroom research is needed on cooperative group learning.

Conclusion

Lessons that integrate mathematics and science have been found to improve student outcomes in both content areas (McBride & Silverman, 1991). Furthermore, complex activities with small groups have been found to capitalize on multiple abilities and to improve the level of students' understanding (Cohen & Lotan, 1995). The group had a high-level of cohesiveness, and every member of the group made a significant contribution to the overall task. This was possible because the task was of interest to the students and an attempt was made to form group compositions that would optimize student interactions. Reforming my instruction did not occur overnight. I learned through trial and error by making changes and adjustments to one lesson at a time. As a result, I have found that students enjoyed mathematics more and developed a deeper understanding of mathematical concepts. This paper should convince other mathematics teachers that students can learn by investigating, discussing, and collaborating on meaningful tasks. They, too, can reform their pedagogy one lesson at a time.

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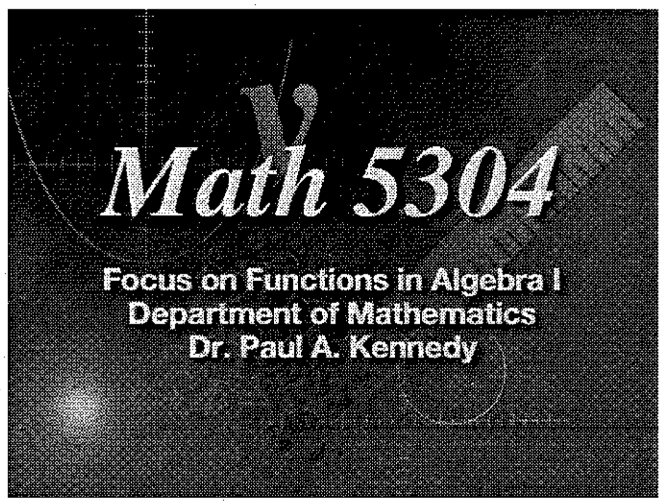
Topics in Mathematics for the Secondary Teacher



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Using a Scoring Rubric to Evaluate Statistics Problems

Dr. Sharon Taylor and Dr. Kathleen Cage Mittag

According to the National Council of Teachers of Mathematics (NCTM) *Assessment Standards* (1995), it is important for teachers to develop new assessment strategies that are valid tools to measure students' performance. These strategies should also reflect the Texas Essential Knowledge and Skills (TEKS), technology usage, and NCTM *Curriculum and Evaluation Standards* (1989). Presented in this paper are actual classroom assessment items, a scoring rubric, and a discussion of how to implement the rubric for statistical competencies appropriate for the Grades 3-5, 6-8, and 9-12. Measures of central tendency, dispersion, and exploratory data analysis are the common concepts used for each item. Statistics was chosen because it is being included more in the elementary through secondary curriculum, yet many teachers have had very little training in statistics.

The six assessment standards were considered when developing the rubric. The Mathematics Standard was addressed by including statistical topics referred to in the NCTM *Curriculum and Evaluation Standards* (1989). The assessment items included were designed to enhance learning, discourse, and higher order thinking skills which are mentioned in the Learning and Equity Standards. The rubric used to score the instrument was designed with consideration of the Openness, Inferences, and Coherence Standards. Teachers have found the rubric quick and easy to use yet also justifiable and explainable to students, parents, and administrators.

RUBRIC

The rubric was developed and patterned after other scoring rubrics in the literature (Stenmark, 1989). Lester and Kroll (1990) discussed focused holistic scoring as a consistent and expeditious assessment technique. It is holistic because the entire solution is scored, not just the steps or the answer. The technique is focused because the scorer looks for particular characteristics in the student's work. Focused holistic scoring assigns a single number according to specific criteria. Each question should be scored in this manner; then, an overall score can be determined.

The specific criteria for the rubric consisted of a scale with Levels 0 to 5 where Level 0 indicated no attempt was made by the student. A Level 1 score was given when the student began the problem but in the wrong direction. The student may or may not have completed the problem. Level 2 meant that the student began the problem correctly but did not finish and had major difficulties. A Level 3 response was when the student began the problem correctly and neared completion but had flaws. Level 4 scoring meant that overall the problem was acceptable with only a few minor flaws, but the student did not seem to have conceptual understanding of the problem. A Level 5 score occurred when the student gave an appropriate response with correct reasoning.

SAMPLE PROBLEM

A common strand throughout the Standards is that students collect, organize, and describe their own data. When students collect data about themselves, it is much more meaningful to them. Whenever possible, any kind of student related data should be collected. The questions at each level were based on common ideas that can be found in textbooks and standardized tests at all levels. The following problem situation and subsequent questions serve as examples for data that can be collected and analyzed by students.

Problem Situation

Ten students are asked to count the number of times they can snap their fingers in one minute. (Data of any type could be collected, depending on the make-up of the classroom.)

Questions for Grades 3-5

1. Put the data set in order from smallest to largest.
2. What is the range?
3. What is the mean?
4. What is the median?
5. What is the mode?
6. Write a sentence that describes why you think there is a difference between the lowest and highest number of snaps.

Questions for Grades 6-8

1. Calculate and compare the mean, median, and mode of the number of finger snaps.
2. Explain whether the mean, median, or mode best describes the average number of finger snaps and why.
3. Draw a stem-and-leaf plot of the finger snapping data.
4. Draw a box-and-whisker plot of the finger snapping data.
5. Looking at the box-and-whisker plot, what can you tell me about the data?

Questions for Grades 9-12.

1. Construct a frequency table using 6 intervals starting with the first interval being 40-49.
2. Construct a stem-and-leaf plot.
3. Why is the stem-and-leaf plot better than the frequency table? Does it tell you something that the frequency table does not?
4. Are there any outliers and why?
5. What do you think would be the most likely number of times you would snap your fingers in three minutes and why?
6. Find the standard deviation of the finger snapping data using a graphing calculator. Explain the standard deviation in terms of this problem.
7. If you multiplied each data point by 5, what would happen to the mean and standard deviation?

EXAMPLE AND DISCUSSION

The following data is used to illustrate the scoring rubric. It is strongly suggested that teachers have their students collect their own data.

48 52 80 60 54 40 65 56 50 96

Sample responses to questions are given below. Examine each response and assign a score based on the rubric. After all the sample responses, the authors' scores and rationale are given.

Sample Responses

Grades 3-5:

Question 1:

40 48 50 52 56 54 60 65 80 96

Question 3: $601 \div 9 = 66.8$

Question 4: $(56 + 54) \div 2 = 55$

Grades 6-8:

Question 2: Mode. Since there is no mode for this data, no two people snap the same number of times.

Question 5: The right whisker is really long because 96 is far away from the other points. The left whisker is really short because 40 is close to the other points.

Grades 9-12:

Question 3: The stem and leaf is better because it shows all the data. The frequency table only shows how many data points are in an interval.

Question 4: Yes, 96 is an outlier because it is big.

Question 7: Both the mean and the standard deviation would also be multiplied by 5.

Scoring and Rationale

Grades 3-5:

Question 1: The student interchanged the 56 and 54. The student would receive a "4" because the ordering was nearly correct.

Question 3: The student added the data correctly but divided by 9 instead of 10. The student would receive a "3" because the student began the question correctly and neared completion but had flaws.

Question 4: Ordering did not affect the median since the mean of 56 and 54 is the same as the mean of 54 and 56. The student would receive a "5" for question 4 provided he/she showed work with correct reasoning.

Grades 4-6

Question 2: This student would receive a "2". Although the student provided a reason, it did not make any sense.

Question 5: This student would receive a "3" because there was no mention of quartiles or median. The student did discuss the minimum and maximum which is why the score was "3" instead of "2".

Question 3: This student would receive a "5" since the reasoning is correct and complete.

Grades 9-12

Question 4: This student would receive a "2" because the student has a vague idea of an outlier but does not have any statistical understanding of how to determine an outlier.

Question 7: This student would receive a "3" because the student did understand what would happen to the mean but had flawed reasoning about the standard deviation.

CONCLUSION

Statistics is more than mathematical computation; therefore, holistic scoring is an excellent assessment method. Holistic scoring can be applied to any area of mathematics as long as the scorer is consistent throughout the grading. It is especially useful where reasoning is to be evaluated. It should be the goal of all assessment items to evaluate conceptual understanding and not just numerical answers.

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Shapes on the Sidewalk

Jennifer McDonald

Imagine a classroom where all of the students are interested in what you are teaching. All of the children are actively involved in the learning process. They are problem solvers and use their journals as tools. Their reasoning comes through validating their own thoughts and actions, and connections are constantly made to other areas.

During this activity, that is what the children in my classroom look like. The activity is fun. It reinforces all of the important characteristics of a great mathematics lesson. Perimeter and measurement will make a lasting impression when the children participate in this activity that utilizes the outdoors, which all children love. Try it and find out just how much your students will learn simply by using their hands and investigative skills.

Suggested Grade Levels: 2-5

Objective: Students will work in cooperative groups to create polygons and measure perimeter.

Materials: yardsticks or meter sticks, transparency, perimeter chart, large sidewalk area or blacktop, sidewalk chalk, perimeter charts (1 per student, see fig. 1), calculators, class perimeter chart (Make a larger version of figure 1 on a poster.)

Standards: The children will be using problem solving skills in order to decide as a group what kind of shape they will be creating, how they will measure it, and how they will go about finding the perimeter for other shapes. The children will be communicating within their own group by talking and listening. Also, they will be writing on their chart and in their journals about the activity and their experience with the group. The students will use reasoning skills by validating their thoughts about perimeter throughout the activity.

Overview: You may need to provide guidelines for this lesson. Students will be designing a closed shape with no curved sides using chalk and meter sticks or yardsticks, in addition to measuring the perimeter. The children get excited about drawing shapes with the chalk, so you may want to limit the number of sides the shape can have to help cut down on very large perimeter measurements.

The units of measure could vary. If you use meter sticks, the children could measure their shapes in meters, decimeters, centimeters, etc. Using yardsticks gives the children the choice of yards, feet, inches, etc. You could also use a combination of these to give your students more freedom in choosing a unit of measure. In some cases, it may be beneficial to use smaller units, but not too small. However, you may want to allow the students to challenge themselves.

The lesson length could range anywhere from 2 to 3 hours. However, it can be easily broken into sections for use on consecutive days.

Before Going Outside:

Model: Model the activity on the overhead. Draw any figure. Measure the sides with a ruler and label the lengths. On your overhead chart, fill in the column titled "number of sides." Next, write out the addition sentence under the column heading "length of sides." The addition sentence indicates how to arrive at the perimeter. You will leave the perimeter column blank for now.

Grouping: Explain that the children will be working together to design a shape for which they and other groups will be calculating the perimeter. Put students into groups of four or five, and assign each group a letter: A, B, C, D, or E. Give each group a meter stick (or yardstick) and a piece of chalk. The children will need to decide the following: who will draw, who will hold the stick, what kind of shape they will be designing, and how they will measure. One student from the group should draw their shape on construction paper for the mathematics bulletin board. When all students are in a group, go outside.

Outside:

Lesson: Once outside on a large sidewalk area, instruct the children to decide how they are going to draw their shapes, and let them go! Give them about 25-30 minutes to discuss and draw their shapes and label the sides. Remind them not to forget their unit of measure. It can be written inside the shape or on each side of the shape. The children should also label their charts with the information from their own group during this time. The children will wait to solve for perimeter until you come back together as a class. (See fig. 2 for a bird's eye view of some shapes and completed chart.)

When the children get the information for their group, they will rotate groups. Group A should go to B, B to C, C to D, etc. Continue this rotation until all groups have observed each shape that was drawn. During each 10-15 minute period the children are at a new shape, they should count and record the number of sides and write the addition sentence in order to find the perimeter. The children may also sketch the shape with the labels on the back of their chart to refer to later. After all of the groups have investigated all of the shapes, return to the classroom.

Back in the Classroom:

Re-Group: When you arrive in the classroom, cluster together new groups, so the children may review the activity with different peers. The children should discuss with each other any problems they encountered, and they should compare the addition sentences they gathered from each shape. If there are any significant disagreements, discuss as a class and come to a conclusion. At this time, the children should write in their journals about their experiences. While they are writing, prepare the class chart and place it at the front of the room. When the children are finished with their journals, get together as a class and record the conclusions on the large class chart. The perimeter column should still be blank.

Homework: The activity will be completed either at home or in class the next day. The children should add up the lengths of the sides of each shape and write their answers in the perimeter column of their charts. If some of the lengths get too large, the children may get lost in their adding. If this happens, allow them to use calculators.

Extensions: In addition to being a wonderful and entertaining mathematics lesson, this activity can provide opportunities for further learning.

- The class chart, drawings of shapes, and sample student charts will make an excellent mathematics bulletin board or backdrop for the mathematics center. It will make an excellent reference.
- Cut out miscellaneous shapes and place in a center with measurement tools. The children could use rulers, paper clips, erasers, or crayons to measure the shapes. Make a chart similar to the one used in the previous activity to let the children show how they arrived at the perimeter.

- Allow your students to construct shapes with straws and pipe cleaner by inserting the pipe cleaner into the straw and connecting it to another straw. Once the shapes have been constructed, have them measure the perimeter with any of the units mentioned above.
- Cut out some shapes and label each with the total perimeter. Do not label the unit of measure you used, only the number of units. The children are to determine which unit you used to measure the shape, by using paper clips, rulers, erasers, etc. In their journals, have them write about why they chose that unit of measure.
- A group of high achievers would enjoy going back outside to draw a new shape. They should use a different unit of measure the second time. Afterwards, they could compare in their mathematics journals which unit was easier to use and why.

Options: Depending on the focus of the lesson, this activity can be modified many different ways.

- Instead of letting the children choose which measurement to use, assign each group a different unit. When they rotate from group to group, they will measure every shape with their specific unit. Before beginning, have them hypothesize which unit of measure will be the easiest to work with, which one will have the largest or smallest number for perimeter, and why. A great communication activity involves having the children discuss their journals and compare their predictions.
- On a rainy day, students could design shapes with string or twine inside the classroom or gym.
- For younger students, the activity could be used by having regular shapes previously drawn. They could still use the meter sticks or yardsticks to measure the shapes.

Closure: This activity can be altered for a variety of lessons. You could wait to use this as a re-teaching tool for measurement, perimeter, or almost anything in geometry. Your children will have a blast while carrying out this exciting hands-on activity. They won't even realize they are using all of these skills and learning.

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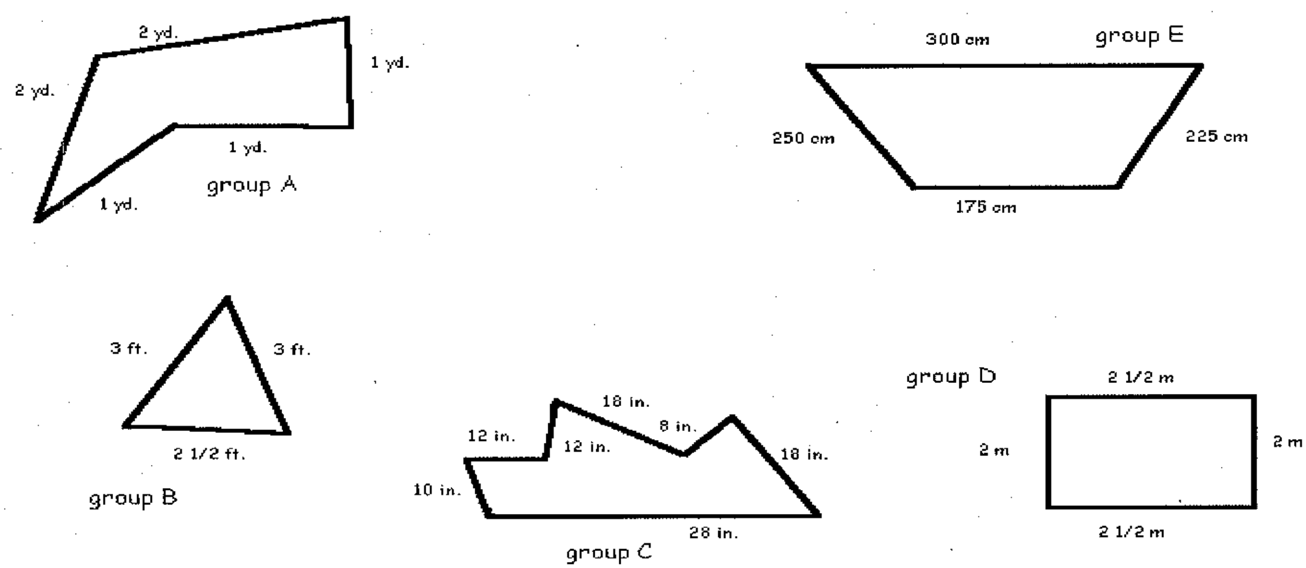
Jennifer McDonald teaches third grade at Kiker Elementary in Austin.

Group Letter	How Many Sides	Length of Sides (side 1 + side 2 + . . .)	Perimeter (sum of all sides)
A			
B			
C			
D			
E			

figure 1: Perimeter Chart



Group Letter	How Many Sides	Length of Sides (side 1 + side 2 + . . .)	Perimeter (sum of all sides)
A	5	2+2+1+1+1	7 yd.
B	3	$3+3+2\frac{1}{2}$	$8\frac{1}{2}$ ft.
C	7	10+12+12+18+8+18+28	96 in.
D	4	$2+2\frac{1}{2}+2+2\frac{1}{2}$	9 m
E	4	250+300+225+175	950 cm

Figure 2: Sample shapes and chart

Best Kept Secret in Austin

Stephanie Bahnmiller

The marquee sits at the front of the school. I happened to glance up expecting to see, "Have a great summer!" but found my heart racing as I read, "Best Kept Secret in South Austin!" It was then I knew our campus had done an awesome job on the Texas Assessment of Academic Skills (TAAS). I smiled to myself thinking that our year of hard work with the "I CAN" after-school tutoring program had given us the desired result — we had become a "recognized" school!

In the 1996-1997 academic year, our campus saw a need to help students who were bordering on failing the TAAS. We quickly pulled together a six week after-school program that year. The results encouraged us to institute a year long program for the 1997-1998 school year.

Getting Started

In October 1997, the third through fifth grade students took a pre-assessment test. The results helped to form a list of students who barely passed the test and all those who failed. This list was compared to the lists of students who had failed or barely passed the May 1997 TAAS. Students who appeared on both lists became candidates for the after-school program. Those who passed the pre-assessment but not the TAAS, or vice versa, were looked at later in the year. Our school assesses student progress throughout the year, so the list for the tutoring program was adjusted as needed. The advice of the classroom teacher was given great weight. The P.T.A. volunteered to provide daily snacks for the participants and stipends for the teachers.

Grouping

The recruited teachers met to form the student groups with five students per teacher. Small groups had the advantage of reaching all students with as much attention as needed, and it held discipline problems to a minimum. A mix of boys and girls also helped. If siblings were on the list, we put them in the same session during the year.

Informing Parents

Letters were mailed to parents two weeks before their child's session began. The letter gave dates and times of the session and let the parents know they were responsible for picking up their child. A consent form had to be returned. Follow-up calls were made to parents from whom we did not hear. If some students could not attend, we returned to the list to fill openings. Many times a different session worked better for a family, so we tried to accommodate all qualified participants. We had three eight week sessions. Each session met three days a week from 3:00-4:00 p.m.

The Tutoring Session

The after-school program extended the child's day at school, so it was important to make the hour fun. Students would come to their tutor's class right after dismissal and have fifteen minutes to visit and eat the snack provided by the P.T.A.

A typical hour would begin with a review of math vocabulary and introduction of new words. We found that reading difficulties and lack of math vocabulary hurt students' success.

Fun hands-on activities enabled tutors to motivate their group for the major portion of the hour. Although test objectives were reviewed, teachers focused on the needs of their group. Activities were pulled from a variety of resources in an attempt to choose those that had not been used in the regular classroom.

The hour wrapped-up by practicing three or four problems that focused on the day's lesson. These problems followed the format of the standardized test. The group used a series of steps that helped them to understand and solve the problems. The steps were:

1. Highlight the question.
2. Cross out numbers you do not need. Circle those numbers you will use.
3. Decide on a math strategy that will help you solve the problem. These

might include drawings, graphs, maps, charts, or tables.
4. Decide what operation you will use to solve the problem.
5. Make a good estimate.
6. Write an equation.
7. Solve.
8. Check your work for reasonableness and computation errors Be sure you answered the question.

At the end of the eight week session, we held a celebration that included an ice cream reward.

Results

During the 1997-1998 school year, 37 third graders, 22 fourth graders, and 25 fifth graders participated in the program. Of those involved in the tutoring program, 86.5% of the third graders, 91% of the fourth graders, and 88% of the fifth graders passed the TAAS. Even children who did not pass the test showed significant gains in their scores. The five third graders missed passing by one question! Three of the six fourth graders who did not pass more than doubled their score from the previous year. Only five of the 84 participants showed little progress.

Looking Forward

We will continue to work and refine our program. In looking forward, we plan to put a lesson plan book together for each grade level involved in the after-school program. Many of the teachers involved do not teach the grade level they are tutoring. Pulling together a variety of hands-on activities that can be used for each objective the test covers will help ease the teacher workload during the test.

Conclusion

Cunningham Elementary's "I Can" tutor program was a success in 1997-1998. Many factors contributed to the difference this program made on our campus. The program received strong support from the teaching staff and our principal. (She was one of the tutors!) The P.T.A. worked tirelessly during the year to find volunteer tutors for students during the regular day. These volunteers came from the community and parents. The life blood of such a program is an uninterrupted flow of dedicated individuals, whether volunteer or willing staff member. Our commitment to refine the "I Can" program will insure its continued success.

Stephanie Bahnmiller is a fifth grade teacher at Cunningham Elementary in Austin. She has been teaching for 13 years. For more information on the program, contact Ms. Bahnmiller or Lynda Horne, principal of Cunningham, at (512)414-2067.

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Walk This Way

Pamela Weber Harris

Focus: Introduction to graphs of motion data, specifically linear data.

Objective: **Mathematical Models with Applications TEKS**:
2. The student uses graphical and numerical techniques to study patterns and analyze data.
D. The student is expected to use regression methods available through technology to describe various models for data such as linear, quadratic, exponential, etc.; select the most appropriate model; and use the model to interpret information.

Terms: Linear model, rate of change.

Set-Up: The classroom should be set up with an aisle down the middle. Set up a motion detector pointing down the aisle, hooked to a viewscreen calculator, so the class can see both the students walking down the aisle and the data projected from the calculator on a screen if front of the room.

Materials: Copies of Activity 1, copies of Assessment 1, a motion detector connected to a viewscreen calculator, a motion detector with graphing calculator for each group of 3-4 students.

Prerequisites: None.

Procedures: Explain that the motion detector sends out an ultrasonic pulse. The pulse bounces off the walker, and the motion detector records the distance at that time. The calculator displays the data as a graph with the distance measured in meters and the time measured in seconds.

Run the CBR Ranger program or the equivalent. Use the following screens to set up the experiment and then follow the instructions on the screen.

Ask a few students to walk one at a time in front of the motion detector. Encourage students to walk differently - slowly, quickly, standing still, toward the motion detector, away from the motion detector, etc.

After each walk, discuss the following:

- What does the starting point represent?
- What does a fast walk look like?
- What does a slow walk look like?
- What does a pause look like?
- What does it look like when you walk away from the motion detector or toward the motion detector?

- Discuss a "straight" verses a "curved" graph. Point out that for constant rates of change, we use a linear model to model the data.

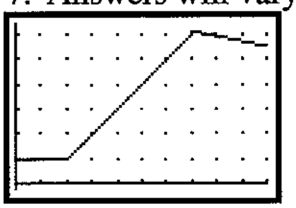
Activity 1: Practice Walking Linear Graphs.
Each group of 3-4 students practices walking different linear graphs using a motion detector and a graphing calculator.

First Time: Hold the motion detector and the calculator. Point the motion detector at the wall and practice walking the graphs.

Second Time: Have the group hold the motion detector/calculator combination. Point the motion detector at one person in the group. As a group, instruct the walker how to walk the graph.

Assessment 1: Linear Motion.
Now your students should be able to complete Assessment 1.

Notes on Assessment 1:
1. B
2. A
3. C
4 - 7. Answers will vary.



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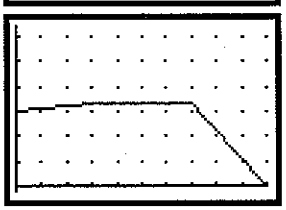

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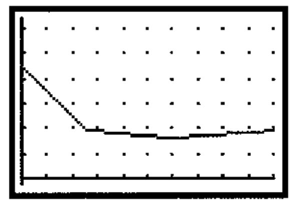

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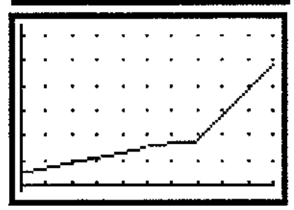
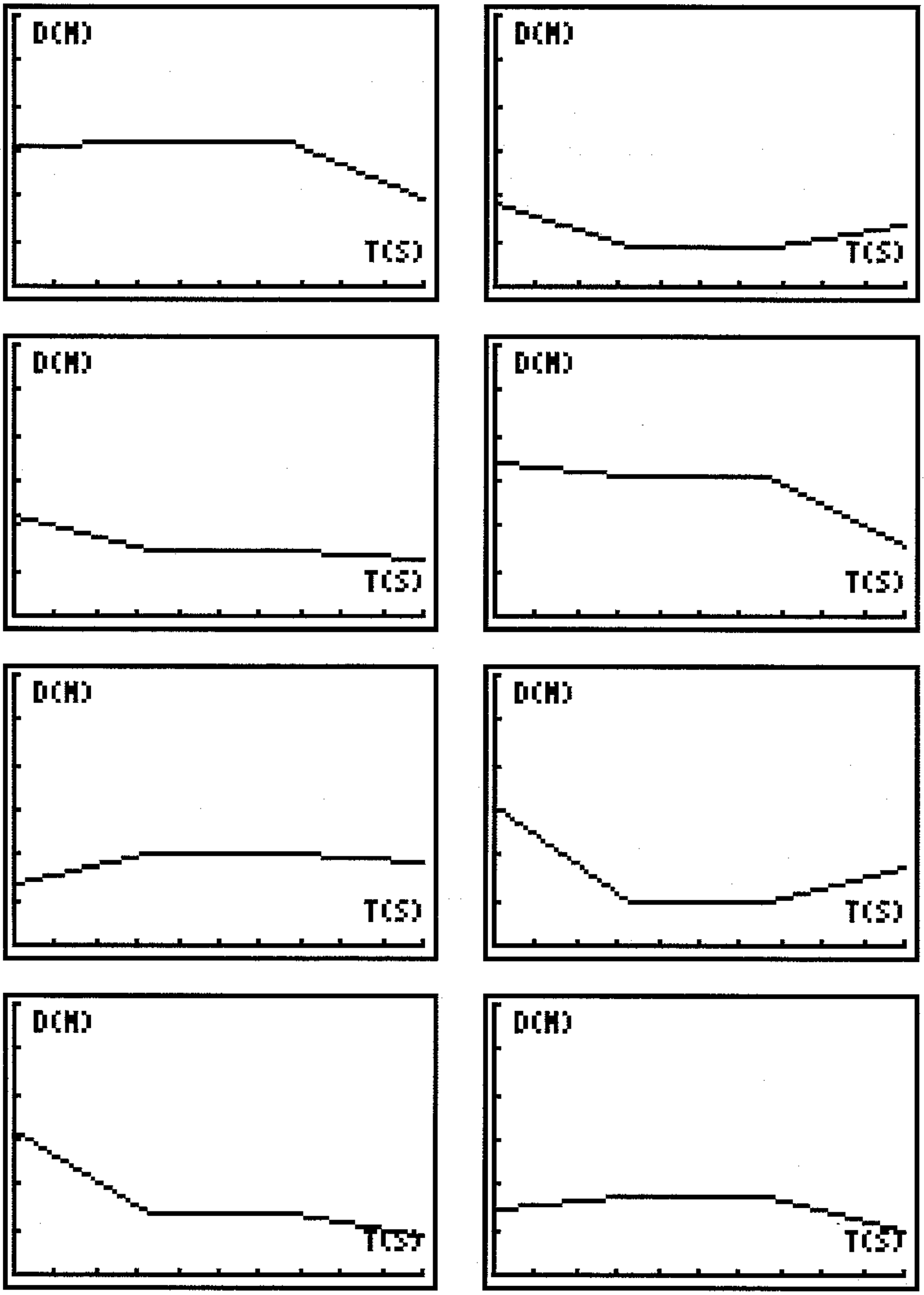


11.

Pamela Weber Harris is part of a team writing the Mathematical Modeling Institute for Secondary Teachers. "Walk This Way" is a precursor to "Rates of Change," which appeared in the *Texas Mathematics Teacher*, Spring 1998.

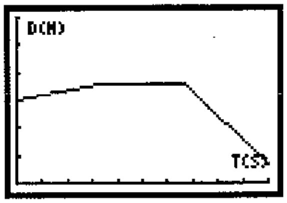
Activity 1: Practice Walking Linear Graphs

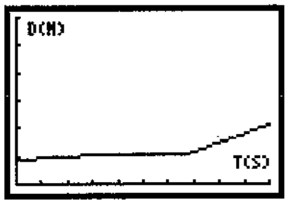
Practice walking the following linear graphs using a motion detector and a graphing calculator.

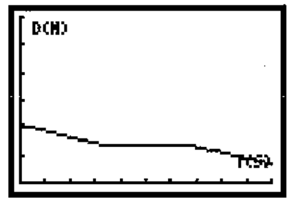


Assessment 1: Linear Motion

Match the description with the graph.

A. 

B. 

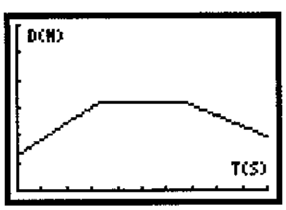
C. 

______ 1. Start one meter away from the motion detector. Walk slowly away from the motion detector for about 3 seconds, stand still for about 4 seconds, and then walk quickly away from the motion detector for about 3 seconds.

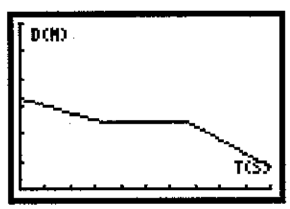
______ 2. Start 3 meters away from the motion detector and walk away from the motion detector at a moderate rate for about 3 seconds. Stand still for about 4 seconds and then walk quickly toward the motion detector for 3 seconds.

______ 3. Start 2 meters away from the motion detector and walk toward the motion detector at a moderate rate for about 3 seconds. Then stand still for about 4 seconds, and then walk toward the motion detector at about the same moderate rate as earlier for about 3 seconds.

Write a description for a walk which could produce each of these graphs.



4. ______________________________



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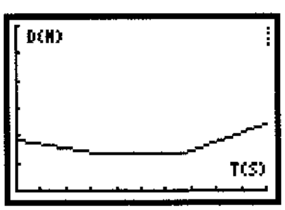

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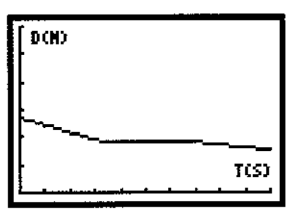
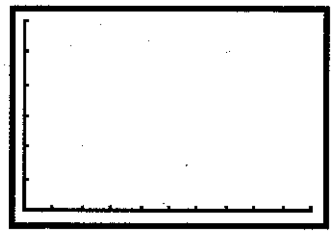


7. ______________________________

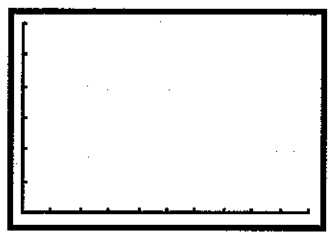
Assessment 1: Linear Motion (cont.)

Sketch a graph which would match the description.

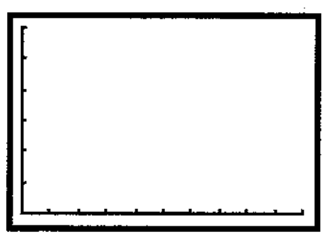
8. Start one meter from the motion detector and stand still for 2 seconds. Walk away from the motion detector quickly for 5 seconds. Walk back slowly toward the motion detector for 3 seconds.



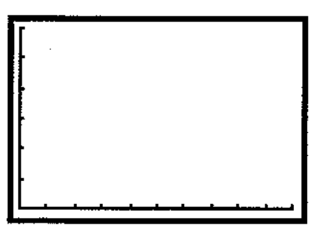
9. Start 3 meters from the motion detector and walk slowly away from the motion detector for 3 seconds. Stand still for 4 seconds. Walk quickly toward the motion detector for 3 seconds.



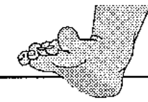
10. Start 4.5 meters from the motion detector and walk quickly toward the motion detector for 2.5 seconds. Walk slowly toward the motion detector for 3 seconds. Walk even more slowly away from the motion detector for 4.5 seconds.



11. Start 0.5 meters from the motion detector. Walk slowly away from the motion detector for 5 seconds. Walk extremely slowly away from the motion detector for 2 seconds. Walk quickly away from the motion detector for 3 seconds.



Let Your Feet Do the Walking



This a motion activity that does not use a motion detector.

Overview: Students naturally solve problems recursively. In this activity, we build linear patterns from repeated addition as students walk at different rates.

Objective: **Mathematical with Applications TEKS: 2A**
This activity could also be used in middle school mathematics and pre-algebra.

Terms: Recursion, linear model

Materials: Graphing calculator, grid paper, foot tiles on the floor or tape on the floor at foot intervals for 20 feet, metronome or a student with a stopwatch or watch with seconds easily readable.

Procedures: Mark a 20 foot walking course in your room so that students can readily see at which foot mark they are walking. Turn on the metronome or stopwatch, and have a student count out each second. Choose a Walker, and have him/her practice walking at a constant rate of 2 feet per second.

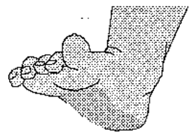
1a. Do a 2 feet per second walk, and fill in the table. Look at the pattern.
- What's happening here? [repeated addition]

1b. Graph the ordered pairs.
- What does repeated addition look like graphically? [a line]
- Is it increasing or decreasing? [increasing - The distance from the beginning of the course is increasing.]

2a. Now practice walking a constant rate of 4 feet per second. Once your Walker is consistent, have him/her start at the 20 foot mark, and walk toward the beginning at 4 feet per second. Fill in the table, and look at the pattern.
- What is happening here? [repeated subtraction or repeated addition of a negative number]

2b. Graph the ordered pairs.
- What does repeated subtraction look like graphically? [a line]
- Is it increasing or decreasing? [decreasing - The distance from the beginning of the course is decreasing.]

The idea you want your students to understand is that walking at a constant rate is repeated addition, a linear process.

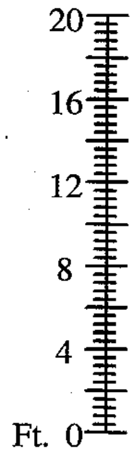


Let Your Feet Do the Walking

For this activity, you will need a Walker, a Time Keeper, and a Recorder. The Time Keeper claps and calls out the seconds as they go by. The Walker walks at a given rate. The Recorder records the distance in the table below.

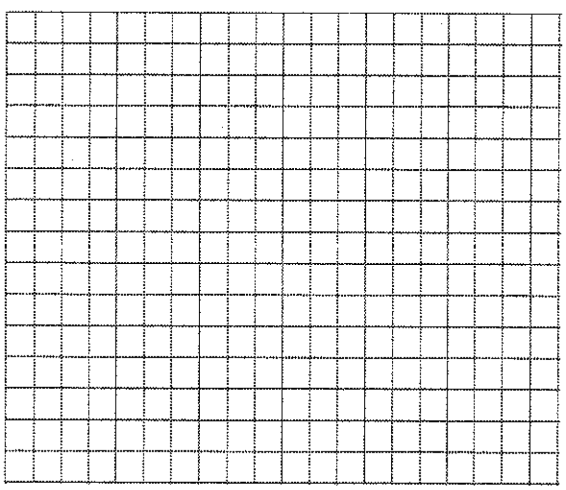
1a. The Walker should walk at the rate of two feet per second. Plot the information on the number line.

Seconds	Distance from beginning
0	0



1b. Graph the information in the table above on the grid below.

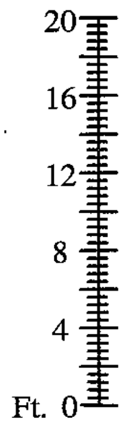
ft.



sec.

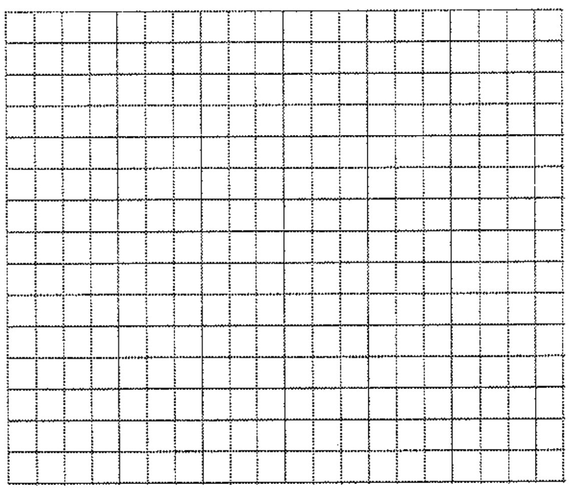
2a. The Walker should start at the 20 feet mark and walk 4 feet per second toward the beginning of the course.

Seconds	Distance from beginning
0	20



2b. Graph the information in the table above on the grid and number line below.

ft.



sec.

Assessment

1. Plot the graph from questions1b on the grid above.

2. What does the intersection point mean?

Geometric Modeling: Trees and Branches

G. T. Springer

Focus: This unit is an introduction to geometric recursion and fractals. Just as recursive sequence building provides an intuitive bridge to functions, so recursive geometric constructions provide a visual introduction to the concepts of rate of change and limit. These are basic concepts that need not wait until a formal course in calculus just to be introduced and discussed at the conceptual level.

Objectives: The student will be able to extend a given recursive construction through its next stage(s). The student will also be able to extend a sequence of numbers based on such a construction (number of segments, number of vertices, perimeter, area, etc.). Finally, the student will be able to give a numerical and/or geometrical arguments as to whether or not the sequence has a limit.

Level: Mathematical modeling, Algebra II, pre-calculus, calculus; Parts of this activity may be useful in Algebra I and geometry classes.

Terms: Recursion, self-similarity

Materials: Pencils, calculators

Prerequisites: None, although experience working with recursively building sequences is preferred.

Teacher Notes for Sheet 1

This sheet introduces a simple recursive geometric construction. If you use the Geometer's Sketchpad, the file GTREE.GSS is provided to you free of charge for your classroom use. If you add it to the C:\Sketch directory and set that directory as the script tool directory (under Display, Preferences, More), then this construction can be carried out on a computer to whatever level the machine is capable.

1. The tree, as defined, is neither self-intersecting nor self-tangent.

2. The completed table appears below.

Number of Growing Seasons	Length of New Growth	Height of Tree (meters)
1	1	1
2	1/2	1 + 1/2 = 1.5
3	1/4	1.5 + **1/4** = **1.75**
4	1/8	**1.75 + 1/8 = 1.875**
5	**1/16**	**1.875 + 1/16 = 1.9375**
6	**1/32**	**1.9375 + 1/32 = 1.96875**

The tree doesn't look like it will ever reach 2 meters in height.

3. Trees definitely grow in time and have a finite life-span, so in that sense the model is unrealistic. This question is posed simply to get students thinking about limits.

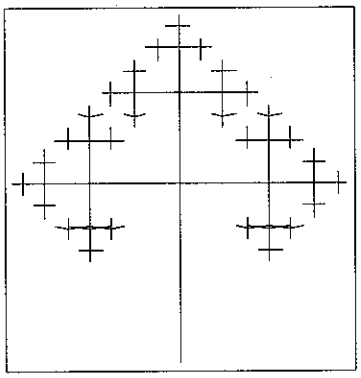
4. Everyone should read and research. Use this opportunity to make yourself get on the world wide web and use it as a research tool, if you haven't already.

5. See #4. Search on "tallest wooden structure," or visit Guiness Book of World Records web site.

6. The lack of intersection points is a desirable trait. As for realistic traits, growing tips branching three ways is common. There are others; listen carefully to what the students think!

7. So many branches growing straight down is one undesirable trait, due to lack of sunlight under all the other branches. An unrealistic trait is having branches all at right angles to each other.

8. Many students will come up with Figure A; others will come up with the smaller square in Figure B.



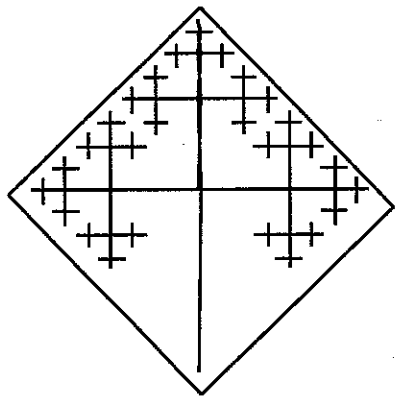


Figure A: 2 m x 2 m square   Figure B: $\sqrt{2}$ m x $\sqrt{2}$ m square

Teacher Notes, Sheet 2

This activity brings up all sorts of sequences, from the simple arithmetic sequence of the procession of growing seasons (1, 2, 3, 4, ...) through the geometric sequence of the number of new branches each season (1, 3, 9, 27, ...) to the final sequence of the total length of all the branches. The focus here is on seeing that the total length of all the segments has no limit, yet the region in which these segments are contained has a limited area. This concept of unlimited growth may be dramatically contrasted with global population considerations. At the very least, this activity again causes the student to focus on the ideas of rate of change and limit.

1. The completed table appears below.

Growing Season	# Of New Branches	Length of Each New Branch	Total New Growth	Total All Growth
1	1	1	1 x 1 = 1	1
2	3	1/2	3 x 1/2 = 1.5	1 + 1.5 = 2.5
3	9	1/4	9 x **1/4 = 2.25**	2.5 + **2.25 = 4.75**
4	27	**1/8**	**27 x 1/8 = 3.375**	**4.75 + 3.375 = 8.125**
5	**81**	**1/16**	**81x1/16 = 5.0625**	**8.125+5.0625=13.1875**
6	**243**	**1/32**	**243x1/32=7.59375**	**13.1875+7.59375=20.78125**
7	**729**	**1/64**	**729/64=11.390625**	**20.78125+11.390625=32.171875**

2. The tree adds more and more growth each year.

3. Since we're adding more and more growth each year, there doesn't appear to be any limit to the sum of the lengths of the branches.

4. It is certainly impractical with real branches. For many students, it will also be odd conceptually. Many of us continue to think of segments as two-dimensional objects, since that is how we represent them to each other.

5. Broccoli and cauliflower are two good examples from the plant kingdom; animals tend to be poor examples (why?). Others may point out the common concept of the structure of the solar system is like the common concept of the structure of the atoms out of which the solar system is constructed. Still others may go out on a limb and speculate that the structure of our society is a larger version of the social structure of our towns and neighborhoods, which in turn is based on a similar family structure.

   Most of the analogies in these examples break down at some point. We don't see strict self-similarity in nature any more than we see perfect circles and lines there. The fact of the matter is that how we structure our perception of space into a coherent and consistent whole through geometry remains pretty much a mystery!

   By the way, our tree doesn't have strict self-similarity, either. To have strict self-similarity, one must be able to circle any part of the figure and have it be identical with the whole on a smaller scale. If you zoom in on the base of our tree, it's just a segment, not a smaller tree.

G.T. Springer is part of a team writing the Mathematical Modeling Institute for Secondary Teachers. He is currently on leave from Alamo Heights High School in San Antonio and is working in Aulstralia for Hewlett Packert.

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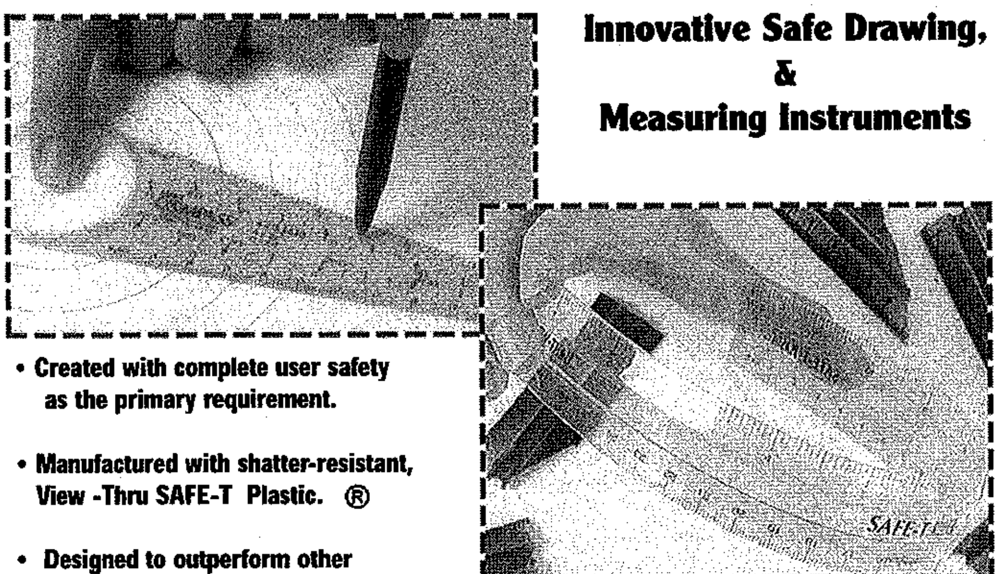

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Geometric Modeling: Trees and Branches (Sheet #1)

In this exercise, we shall simulate the growth of plants using a geometric model. Our geometric model will be recursive; by this, we mean that the growth at any stage will be dependent on the growth at the previous stage. Also, the rules for growing will remain constant.

Suppose a seed sends up a single shoot that grows to 1 meter in length after the first growing season. We will model this growth with a single segment, as shown in Figure 1.

Figure 1: The 1st Season

In the second growing season, the plant sprouts three new branches from its growing tip. Each growing tip is half as long as the branch from which it sprouted; in this case, they are each 0.5 meters long. The center branch continues in the same direction as the parent branch, while the other two are perpendicular to this central branch. The result after the second growing season appears in Figure 2.

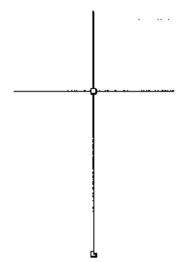


Figure 2: The 2nd Season

In the third and subsequent growing seasons, this growth pattern continues from each of the ever-increasing number of growing tips. In Figure 3, each of the new branches is 0.25 meters long. Figure 4 shows the tree after 4 growing seasons.

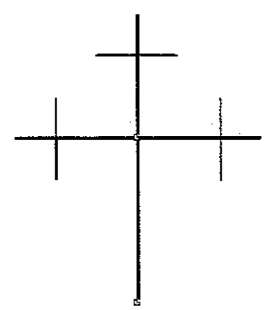


Figure 3: The 3rd Season

1. Trees are often pruned, so the limbs don't interfere with or rub against each other. Will the branches of our tree meet or intersect at any points, such as points A or B in Figure 4? Before you answer this question, use Figure 4 to sketch the growth that will be added in the fifth season.

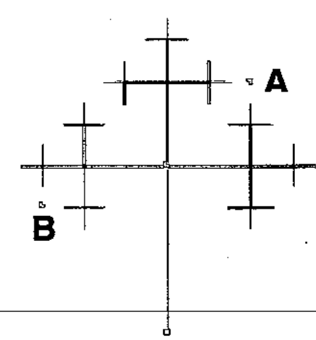


Figure 4: The 4th Season

2. How tall can our tree get? Fill in the table below before making an educated guess.

Number of Growing Seasons	Length of New Growth	Height of Tree (meters)
1	1	1
2	1/2	1 + 1/2 = 1.5
3	1/4	1.5 + =
4	1/8	
5		
6		

Educated guess for the greatest height the tree can attain: ____________________

3. We have built a model for the growth of trees, but how realistic is our model? In general, do you think real trees grow without limit, or does each species have its own limit? Explain your reasoning.

4. **Read and research:** What species of tree grows the tallest, how tall do they get, and where do they grow?

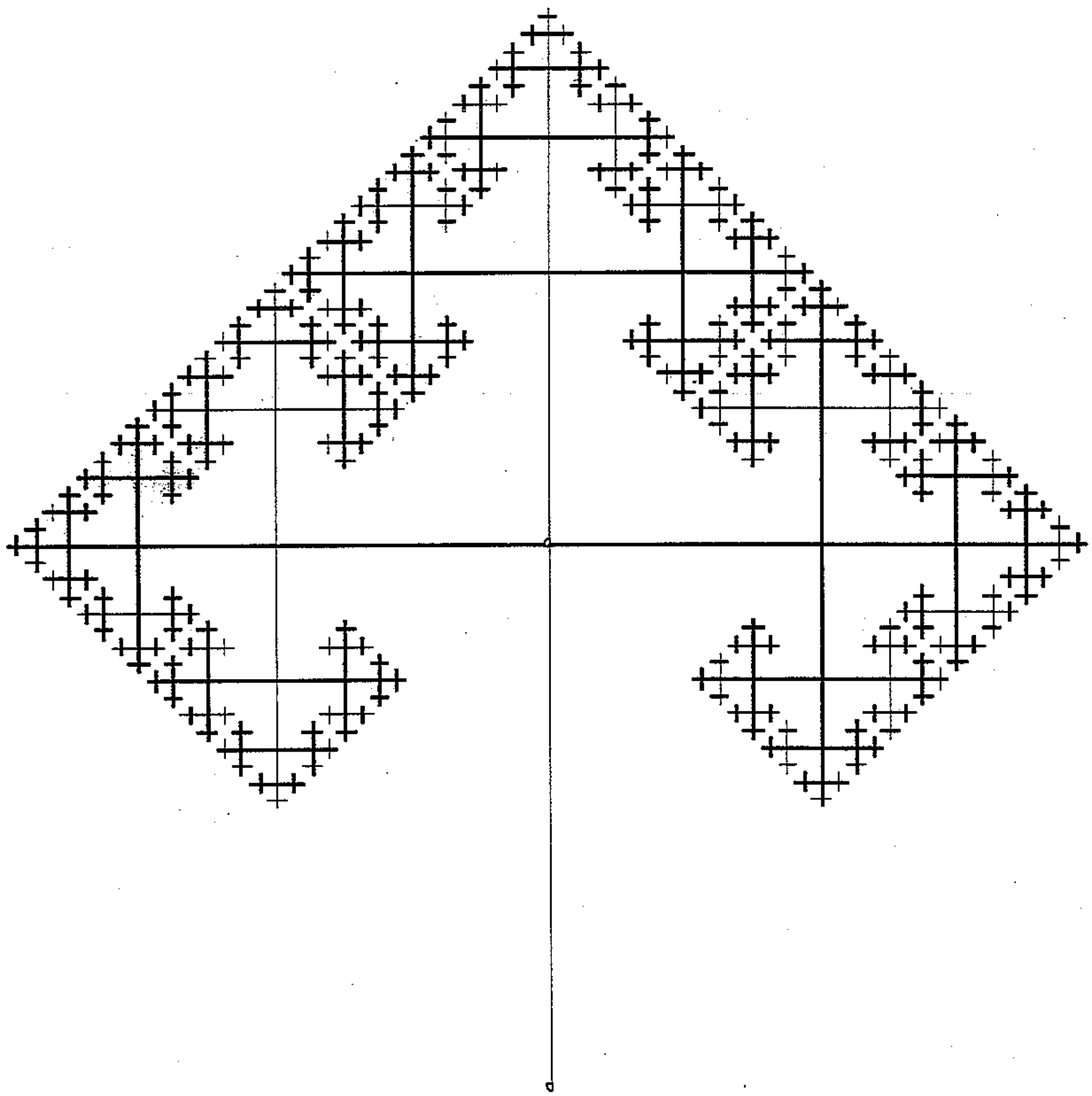
5. A question that is related to the heights of trees is how tall can we make a wooden building? Can a 10-story log cabin exist? Why or why not?

6. Name two characteristics of our model that are either realistic or desirable traits for real trees.

7. Name two characteristics of our model that are either unrealistic or undesirable traits for real trees.

8. Use the table above and Figures 1-4 to find the dimensions of the smallest rectangle that will always contain our tree, even if it lives forever! Be careful; there's a trick to this one!

Our Tree After Six Growing Seasons



Geometric Modeling: Trees and Branches (Sheet #2)

In this exercise, we shall continue to examine our tree model, but now we will focus on the sum of the lengths of all the branches.

1. As you recall, our tree model started with a single stem 1 meter in length, which sprouted into 3 branches 1/2 meter in length, each of which sprouted into 3 branches 1/4 meter in length, etc. Use this model to fill in the following table.

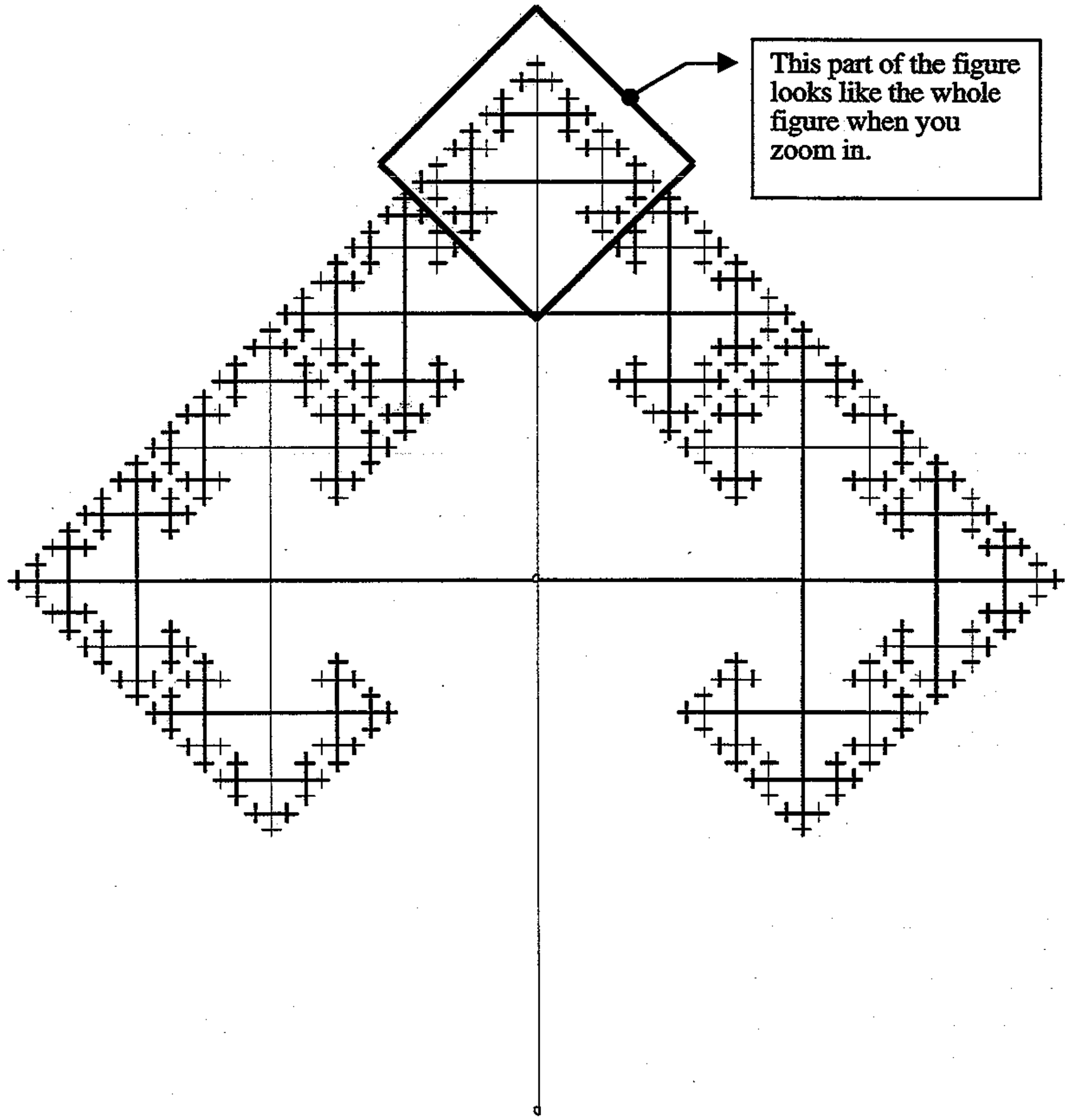
Growing Season	# Of New Branches	Length of Each New Branch	Total New Growth	Total All Growth
1	1	1	1 x 1 = 1	1
2	3	1/2	3 x 1/2 = 1.5	1 + 1.5 = 2.5
3	9	1/4	9 x =	2.5 + =
4	27			
5				
6				
7				

2. Look at the fourth column, which shows you how much new growth is added each season. Does our tree add more and more, or less and less, new growth each year?

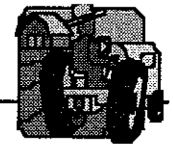
3. In the last sheet, we found out that the tree will never reach 2 meters in height. In other words, there is a limit to the height of the tree. Is there a limit to the sum of the lengths of the branches?

4. In the last sheet, you also found a rectangle that will always contain our tree. It may seem odd that all those branches can fit into this rectangle. Consider it this way: at some point, the new growth would stretch from here to the sun but would still neatly fit into our little box. That's a neat trick! Is it odd because it is impractical with real branches, or is it odd from a purely geometric point of view?

5. Our tree model has one more interesting property, self-similarity. Some parts of the figure resembles the whole figure. For example, if you zoom in on the top of the tree, you will see a figure that looks just like the whole tree. See the box in the figure below. Find something familiar that has self-similarity. Note that the self-similarity might be limited in physical objects, like real trees. Why is this?


This part of the figure looks like the whole figure when you zoom in.

Determining the Value of Hay Silage: An Algebraic Connection



Dr. David R. Duncan and Dr. Bonnie H. Litwiller

Contemporary standards urge that algebra be connected to real world situations. Teachers are constantly looking for settings in which this may be accomplished. We shall present one such situation.

A farmer in our area stored 145 tons of hay silage (hay which is cut and stored without being dried) in his silo. A fire subsequently destroyed his silage, and the insurance company was called to adjust the loss.

A problem arose because there is not an established market price for hay silage; it is commonly used on the farm and is usually not sold. To find the value of the silage, we need to compare it to dried hay (hay cut and dried before being stored). We know:

- One ton (2000) pounds of dried hay has 18% moisture (water content) and costs $100.
- One ton of hay silage has 55% moisture.

What should be the price of silage? Encourage your students to find different methods for solving the problem. Three possible methods are given here.

Method 1: Comparing by Dry Matter

Two thousand pounds of dried hay contain 82% of 2000 or 1640 pounds of dry matter. The dry matter price is then $\frac{\$100}{1640}$ or $0.061 per pound of dry matter.

On the other hand, 2000 pounds of silage contains 45% of 2000 or 900 pounds of dry matter. Consequently, 145 tons of silage contain 145(900) = 130,500 pounds of dry matter. The value of this dry matter contained in the silage is 130,500($0.061) = $7960.50.

Method 2: Converting Silage to "Dried Hay"

Some farmers may argue that Method 1 is theoretically true but is unrealistic since it is impossible to remove all the moisture from hay or hay silage. The dry matter used in Method 1 is an artificial construct.

Consequently, let us calculate how many pounds of dried hay (18% moisture) could be attained by removing the excess water from 145 tons of silage (55% moisture). One-hundred forty-five tons of silage has 130,500 pounds of dry matter and 159,500 pounds of water. Since normal dried hay has 82% dry matter and 18% moisture, $\frac{\text{dry matter}}{\text{water}} = \frac{82}{18} = \frac{41}{9}$. Thus, the allowable pounds of water, x, for the 130,500 pounds of dry matter in the silage is determined by

$$\frac{130{,}500}{x} = \frac{41}{9}$$

$$41x = 9\,(130{,}500)$$

$$x = 28{,}646 \text{ pounds of water}$$

The number of pounds of normal dried hay contained in 145 tons of hay silage is then, 130,500 (dry matter) + 28,646 (water) = 159,146 (dry hay) = 79.57 tons. Its value is then (79.57)(100) = $7957.00.

Method 3: Comparing Silage to Dried Hay Directly

For normal dried hay,

$$\frac{\text{total weight}}{\text{water weight}} = \frac{100}{18} = \frac{50}{9}.$$

Let x be the number of pounds of water to be removed from a ton of silage to produce normal dried hay. Thus,

$$\frac{2000 - x}{1100 - x} = \frac{50}{9}$$

$$9\,(2000 - x) = 50\,(1100 - x)$$

$$18000 - 9x = 55000 - 50x$$

$$41x = 37000$$

$$x = 902 \text{ pounds}$$

Thus, one ton of silage reduces to 1,098 pounds of dried hay, or 54.9% of a ton of dry hay. Consequently, one ton of silage should be worth 54.9% of the value of a ton of hay, or $54.90. The value of 145 tons of silage would be 145 (54.90) = $7960.50.

The difference in the value found by Method 2 as compared to Methods 1 and 3 is caused by the number of digits used in rounding.

Some farmers may feel that Method 3 is the best approach since it avoids the artificial construct of dry matter altogether.

Challenges For the Reader and His/Her Students

1. Find other approaches to solving the problem presented in this article.
2. Find other situations in which algebra can be exploited.

Dr. Duncan and Dr. Littwiller are both professors of mathematics at the University of Northern Iowa.

New Math Magazines for Students

Dr. Max Warshauer

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The council plans a fall meeting with a focus of enrichment activities for the mathematics classroom and a spring conference with a focus on assessment.

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The council's 1998-99 plans include a math fair, UIL practice meet, mini-conference, and two meetings.

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