

Texas Council of Teachers of Mathematics Volume XLIV, No. 1 - Fall 1996

#### **EDITOR**

Dr. Paul A. Kennedy

Department of Mathematics

Southwest Texas State University

San Marcos, Texas 78666

Phone:

(512) 245-3741 (512) 245-3425

Fax: email:

pk03@swt.edu

#### ASSOCIATE EDITORS

ELEMENTARY
Wayne Gable
Karen Lindig
8705 Verona Trail

Austin, TX 78746

SECONDARY Judy Rice 2400 Old South Dr. #707 Richmond, TX 77469 LONE STAR NEWS
Diane McGowan
4511 Langtry
Austin, TX 78749
dmcgowan@tenet.edu

The TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published two times each year, in Fall, and in Spring. Authors are encouraged to submit articles that deal with the teaching and learning of mathematics at all levels. Editorial correspondence and manuscripts should be addressed to the Editor, Paul Kennedy. News bulletins for Lone Star News section should be sent to Associate Editor, Diane McGowan.

Potential authors should adhere to the following guidelines:

- (1) Manuscripts should be word-processed, double-spaced with wide margins on 8<sup>1</sup>/<sub>2</sub> x 11 paper meeting APA guidlelines. Tables and figures should likewise be computer generated. No author identification should appear on the manuscript.
- (2) Submit the original and four copies. Include a Macintosh or IBM 3<sup>1</sup>/2 diskette containing the manuscript. On the disk label indicate the word processing program used.
- (3) Include a cover letter containing the following information: author(s) name, address, affiliations, phone and fax numbers, email address and intended level of the article.

(4) An article for Voices From the Classroom should be relatively short, and contain a description of the activities sufficient in detail to allow readers to incorporate them into their teaching. A discussion of appropriate grade level and prerequisites for the lesson should be included.

As soon as possible after refereeing, authors will be notified of a publication decision. Originals of articles not accepted for publication will be returned to the authors. Two copies of the issue in which an author's manuscript appears will be sent to the author automatically.

We also need items for *Lone Star News*. These include reports, TCTM affiliated group announcements, and any other appropriate news postings. We would especially like to advertise upcoming professional meetings.

**SUBSCRIPTION** and **MEMBERSHIP** information is on the back cover.

# TEXAS MATHEMATICS TEACHER

## TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

e etatigi, la grandi ji gaza ere gija la da

Fall 1996

#### TABLE OF CONTENTS MESSAGE FROM THE PRESIDENT **ARTICLES** A Comparison of Secondary School Mathematics in the United States and Russia/ VOICES FROM THE CLASSROOM Grades 1 and 2 TEXTEAM Institute Grades 3 through 5 TEXTEAM Institute Focus Activity Grades 6 through 8 TEXTEAM Institute Algebra Institute Geometry Institute LONE STAR NEWS Ballot APPLICATIONS INSIDE BACK COVER—LIST OF OFFICERS



## What Does TCTM "Do" For You?

As I worked at the on-site registration desk at CAMT this past summer, I overheard many teachers debating over whether or not to join TCTM at the same time that they registered for the conference. Friends asked their colleagues what the organization "did", and, to my surprise, very few people really knew! I thought my first message to you might be the perfect opportunity to reflect upon what TCTM "does"!

The Texas Council of Teachers of Mathematics serves you through:

Representation TCTM represents mathematics teachers in Texas on key issues such as teacher training and

certification, assessment, curriculum, and mathematics reform. TCTM is your own special

interest group!

Affiliation As an affiliate of the National Council of Teachers of Mathematics, TCTM represents Texas

teachers in the NCTM.

Public Relations TCTM supports programs to improve the public image of mathematics education and math-

ematics teachers.

Conferences TCTM co-sponsors the annual Conference for the Advancement of Mathematics Teaching

(CAMT).

Journals The Texas Mathematics Teacher, a journal published for Texas teachers, focuses on practical

strategies, techniques, and classroom-ready activities that help teachers integrate the Standards in their classrooms. Special focus is being given on implementing the upcoming

TEKS. This is a journal written by teachers, for teachers.

Forum for Ideas TCTM provides a forum for the exchange of teaching philosophies and techniques.

Statewide Support TCTM assists local affiliated NCTM organizations by announcing activities and providing

communication.

Workshops TCTM sponsors regional workshops to form a cooperative effort among the local

affiliated groups to become positive forces in the determining of issues relating to mathe-

matics education.

Recognition TCTM has on-going cooperation with other Texas and national educational organizations.

Annual State Meetings TCTM holds its annual meeting the CAMT conference.

Scholarships TCTM co-sponsors college scholarships for graduating high school seniors who major in

mathematics education.

Leadership TCTM honors those who have contributed to the enrichment of mathematics education in

Texas, and in the nation. The E. Glenadine Gibb Leadership Award and the TCTM

Leadership Award are presented annually at CAMT.

Encouragement TCTM awards several "CAMTership"s annually to first-year teachers who are members of

TCTM. The award is intended to encourage teachers to attend CAMT.

As you can see, the Texas Council of Teachers of Mathematics is a very active organization! Our success is directly related to YOU! Please share this information with a colleague, and encourage them to become a part of TCTM. Encourage your local affiliated group to communicate and interact with TCTM! Share ideas with your peers by submitting teaching tips and original classroom activities to the journal editors. We need every person and every affiliated group to assume an active role to ensure success in changing mathematics education in Texas.

I am looking forward to a productive, and exciting year! Please join me in making TCTM the best it can be!

Basia Rinesmith Hall TCTM President



## Teaching Statistics at the Middle School Level Using Student Generated Data Sets

Sharon Taylor
Department of Mathematics
Georgia Southern University
Staesboro, GA

Kathleen Cage Mittag
Department of Mathematics
University of Texas at San Antonio

The NCTM recognized the importance of statistics and probability as topics in the K - 12 curriculum when it published the Standards (NCTM, 1989). Singer and Willett (1990) wrote that in order to improve statistics education, artificial data sets should be eliminated from the curriculum and replaced with real data sets. Real data sets are authentic, interesting and relevant. They have background information available. Multiple analyses can be performed on them. Hogg (1992) suggested that to improve statistics education goals should be stated, data should be analyzed, projects should be implemented and lectures should be de-emphasized.

The original idea for a data generating questionnaire was developed for a mini-course, "Teaching the Introductory Statistics Courses," presented at the Mathematics Association of America National Conference in 1993. One adaptation of that questionnaire has been used extensively in courses at other levels. Projects, laboratory activities, and cooperative learning are essential to the success of the instructional approaches involving questionnaires. At the middle school level, students can create graphical displays by making posters to represent their findings. Higher level cognitive thinking can be developed by having students design their own questionnaires and laboratory activities. The following questionnaire was developed by the authors as an example. Note that Questions 1, 2, 6, 8, 10, 11, 12, 14, and 16 all require numerical responses

#### **QUESTIONNAIRE**

1.	What is your height in inches?
2.	What is your pulse rate in beats per minute?
3.	Choose a number in the range 0 to 10.
4.	What time did you go to bed last night?
5.	How did you get to school today? (circle one choice) walk bus car bicycle other
6.	How many hours of television do you watch weekly?
7.	Do you get an allowance? Circle: yes or no
8.	How much allowance do you get?
	(If you do not get one, write \$0).
9.	What is your eye color?
10.	How many letters are in your last name?
11.	How many small paper clips long is your shoe?
12.	How many children live in your home?
	(Include yourself)
13.	Who is your favorite cartoon character?
14.	How many pairs of tennis shoes do you own?
15.	Toss a coin and circle your result. Heads or Tails
16.	How many pencil lengths long is your desk?

#### SUGGESTED INSTRUCTIONAL STRATEGIES

To encourage cooperative learning, students can be organized in groups to summarize the data from items 2, 10, 11, 12, and 14. Values such as mean, median, and range can be calculated by each group and then reported to

the entire class. Discussions can center around inferences about other classrooms as well as the school population.

It is useful to discuss variability on questions 6 and 8 by having students create box-and-whisker plots for the data collected. Students will be interested in knowing how they fall within these groups. This data can be organized by the class as a whole or by having students work together.

Question 1 lends itself to being analyzed by a stem and leaf plot. Students can see all the data in one graphical representation and see where their measurements are in relation to the others. This question also provides a way to look at the dispersion of the entire class. The teacher can have students take the information from the stem and leaf plot and turn it into a histogram.

Question 16 can produce interesting numerical data. While there are many objects in the classroom with standard measurements, such as desk length, the lengths of students' pencils is probably highly variable. This can lead to discussions of non-standard units of measure and why there is so much variability for objects that are supposed to be the same. The data generated actually turns out to be more a measure of pencil length variability than desk length. Questions such as this one pose thought provoking situations that might not have occurred to many students.

Questions 3, 4, 5, 7, 9, 13, and 15 all require categorical responses. Question 3 at first appears to provide numerical data, but the students actually tally how many times a number is chosen. An average of 3.8 would not make sense for this data. For example, the teacher could make the analogy with the situation where students choose a color-these cannot be averaged. After the students tally the data, an excellent way to display their findings is in the form of a bar chart. This works better than a frequency table because the students are inundated with too many numbers.

Even though question 3 is not appropriate for a frequency table, question 4 is nicely displayed in this form. Grouping the data by 30 minute intervals and then tallying the results provides students with a look at how they relate to others in the class. The tallied results can be transferred to a histogram for a graphical representation.

Results from questions 5, 9, and 13 are displayed well in circle graphs. Inferences from these questions provide lively discourse. Circle graphs lend themselves nicely to discussions of percents and ratios and can then lead to an introduction to probability concepts.

Question 15 can be used to facilitate an introduction to probability. The entire classroom set should be used as the data. You can record the results and display it to the class, then ask groups of students to answer questions such as: how many heads; how many tails; how many tosses; calculate the ratio of heads to total tosses and calculate the ratio of tails to total tosses then compare these ratios.

Question 7 is tricky in that it results in yes/no data. Often this type of data set is not displayed, but a circle graph can be used effectively. Since the responses are binomial, probability discussions involving complements can be introduced.

Statistical knowledge is of paramount importance in society today. It is vital that the curriculum for statistics be relevant. This activity provides a variety of learning experiences that can be used throughout the year to make statistics meaningful for your students.

#### REFERENCES

Bentley, D., Lock, R., Moore, T., Parker, M., & Witmer, J. (1993) *Teaching the introductory statistics course* [MAA Mini-Course]. Unpublished manuscript.

Hogg, R. V. (1992) "Towards lean and lively courses in statistics." In F. Gordon & S. Gordon (Eds.), Statistics in the twenty-first century [MAA Notes No. 26], pp.3-13. Washington, DC: Mathematical Association of America.

National Council of Teachers of Mathematics. (1985) Curriculum and evaluation standards of school mathematics. Reston, VA: National Council of Teachers of Mathematics.

Singer, J. D. & Willett, J. B. (1990) "Improving the teaching of applied statistics: Putting the data back into data analysis." *The American Statistician*, 44, 223 - 230.

# Sieve of Six: A Pattern for Primes!

Mark Klespis

Department of Mathematics and Information Sciences Sam Houston State University Huntsville, TX 77341

Susan Beal

Department of Mathematics and Computer Science Saint Xavier University Chicago, IL 60655

#### INTRODUCTION

In Everybody Counts, mathematics is beautifully described as "a science of patterns and order" (National Research Council, 1989, p. 31). Depending on how a problem is presented and/or written, patterns sometimes can be hard to see. To illustrate this point, we are presenting an activity that modifies an existing technique for finding prime numbers, one that is commonly used in elementary and middle school. This modification will present a striking pattern of the distribution of prime numbers, from which additional opportunities can arise that will allow students to generate and test conjectures. In short, students will be doing "real" mathematics!!

#### SIEVE OF ERATOSTHENES

A typical method used to help elementary school children find prime numbers, especially primes less than 100, is the Sieve of Eratosthenes.

This method consists of scratching off 1 (which is not prime by definition); leaving the next unscratched number, 2, and then scratching off all multiples of 2; leaving the next unscratched number, 3, and then scratching off all multiples of 3, and so forth. The process continues indefinitely, and could theoretically be used to find as many primes as desired. Figure 1 illustrates the results of applying the process to the first 100 numbers, displayed in ten columns.

Figure 1. Sieve of Eratosthenes.

4-	-2	3–	4	5	-6-	<u></u> 7-	-8	9	<del>-10</del>
11-	12-	-13-	14	<del>-15</del> -	-16-	<del>-</del> 17-	18	<b>-</b> 19	<del>-20</del>
21	22	<b>-23</b> -	24	25	26	27	28	<b>-29</b> -	-30
31-	-32	-33	34	35	-36-	<b>-37</b> -	38	<del>-39</del>	40
41-	42	<b>-43</b> -	-44-	45	46	<b>-4</b> 7-	48	<del>-49</del> -	<del>-50</del>
<del>51</del>	<del>-52</del> -	<b>-53</b> -	54	<del>-55</del> -	<del>-56</del> -	-57	-58-	<b>-</b> 59-	<del>-60</del>
61-	62	<del>-63</del> -	64	<del>-65</del> -	<del>-66</del> -	<b>-67</b> -	<del>-68</del> -	-69	<del>-70</del>
71-	72	<b>-73</b> -	74	75	76	77	78	<del>-</del> 79-	<del>-80</del>
81	82	-83-	84	<u>85</u>	-86-	-87-	-88	<del>-</del> 89-	90
91	92	-93-	- 94	95	96	<b>-97</b> -	-98	99	100

If you look at the bold numbers in Figure 1, there does not appear to be anything striking about the way the primes are distributed. We have instances where two primes appear to be consecutive odd numbers, but that is about the extent of our findings. However, if the figure is modified, another pattern emerges.

#### SIEVE OF SIX

One of the difficulties in mathematics is looking at something in the "right" way, that is, a way in which a pattern becomes illuminated. Revising Figure 1 so that the 100 numbers are displayed in six columns instead of ten, Figure 2 displays the result using the same process as the original Sieve of Eratosthenes.

Figure 2. Sieve of Six: Sieve of Eratosthenes, six columns.

<del>1</del>	<b>-2</b>	3–	-4-	<b>5</b> -	<del>6</del>
7—	-8-	9	10-	-11-	12
13-	-14	<del>-15</del> -	<del>-16</del> -	-17-	<del>-18</del>
19–	20	21	22	<b>-23</b> -	24
25	26	27	28	<b>-29</b> -	<del>-30</del>
			34		
-			-40-		
			<del>-46</del> -		
	lso d			•	,

Ask the students to study Figure 2 for patterns. Unlike the Sieve of Eratosthenes with ten columns, there is a discernible pattern concerning the distribution of prime numbers in the 6-sieve. Let your students become mathematicians! Ask the students to give a verbal or written statement that would characterize prime numbers based on the pattern in the 6-sieve. Or, saying it another way, challenge them to develop a "rule" that would enable them to tell if a number might be prime. Once a conjecture is made, it would be up to the students to verify it, and alter the conjecture, if necessary.

When the pattern is examined, it is noticeable that all the primes, except for 2 and 3, fall either in the first column or the fifth column of the 6-sieve. Numbers in those columns are divisible by 6 with a remainder of 1 or 5. Does that mean all numbers that have this characteristic are prime? Certainly not! Both 25 and 35 are not primes (these numbers are multiples of 5) but have remainders of 1 or 5 when divided by 6. Other conjectures could be: Do all primes, except 2 and 3, when divided by 6 have a remainder of 1 or 5? Is it true only for primes less than 100? Can we be sure all primes are of this form? How can we test primes of very large values? There is a lot of mathematics to do to respond to these questions, and others the students will generate, especially since all primes are not known as there are an infinite number of them.

The apparent pattern of primes in the 6-sieve will also require the students to realize that while pattern-finding (i.e., inductive reasoning) is a very powerful tool for generating mathematical conjectures, it does have its flaws. An obvious conjecture students usually make about prime numbers is that all of them (except for 2) are odd numbers. Students are usually interested in knowing if you can tell if a number is prime simply by looking at the number. The observation that all primes after 2 are odd helps only a little. But the "proof" of why all primes except 2 are odd is an interesting exercise in problem solving.

Another interesting pattern for primes builds on the 6-sieve. It does involve mathematical content not ordinarily found in today's elementary school curriculum, but we think it bears mentioning here. The idea is to use base 6 numerals in the 6-sieve, rather than base 10. Again, using the same process of the Sieve of Eratosthenes, we obtain the results presented in Figure 3.

Figure 3. Sieve of Six, base 6 numerals.

4	<b>_2</b>	3–	4	<b>5</b> -	<del>-10</del>
11	12	13	-14	-15-	<del>-20</del>
21	22	23	24	<b>–25</b> –	<del>-30</del>
31-	32	-33	34	35-	<del>- 40</del>
41	42	43	44	<del>-45</del> -	<del>- 50</del>
51-	-52-	-53	-54-	55-	<del>-100</del>
101	<del>-102</del>	103	-104	<b>1</b> -105	<del>- 110</del>
and	d so d	on.			

In this attempt, another pattern emerges. It appears that, after 2 and 3, a prime number's representation in base 6 must end in either 1 or 5 only. Of course, this is a logical extension of the 6-sieve written with base 10 numbers, but this table makes the connection a little more obvious. Perhaps students could try this with other bases as well. Another advantage to looking at primes through other bases is that it allows us to "realize" or "discover" that the primes are the same regardless of their base of representation.

An extension on this investigation is to explore the primes modulo 6. Every number can be written in the form 6n, 6n+1, 6n+2, 6n+3, 6n+4, and 6n+5. Thus, all numbers are congruent to 0, 1, 2, 3, 4, or 5 modulo 6. The primes, except for 2 and 3, will be congruent to 1 or to 5 modulo 6. Furthermore, using this notation, students can "prove" that only those numbers that are congruent to 1 or 5 modulo 6 can be primes since if the numbers are congruent to 0, 2, or 4, the numbers must be multiples of 2. If the numbers are congruent to 0 or 3 they are multiples of 3. Thus, only those numbers which are congruent to 1 or 5 module 6 could be primes.

#### **CONCLUSION**

If we truly want our students to believe that mathematics is a "science of patterns," then we need to make an effort to help them see where patterns can exist. Once found, patterns lend themselves naturally to conjecture making and testing. Altering conjectures with "what if" statements allows students to take many roads in their investigations, roads that they are choosing to travel. Validating conjectures can give students a sense of accomplishment, self-confidence in doing mathematics, and an appreciation for the subject. Students will then begin to see mathematics as a cohesive, dynamic body of knowledge instead of a group of isolated facts. Students will then be performing as mathematicians.

## REFERENCES FOR FURTHER INVESTIGATIONS WITH PRIMES

Middle Grades Mathematics Project. (1986). Factors and multiples. Menlo Park, CA: Addison-Wesley Publishing Company

National Research Council. (1989). *Everybody counts*. Washington, D. C.: National Academy Press.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards. Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). Patterns and functions, addenda series, grades 5 - 8. Reston, VA.

#### It's Time to Register Your School for the

#### AMERICAN HIGH SCHOOL MATHEMATICS EXAMINATION February 13, 1997

This national contest covers the material of the high-school curriculum before calculus. Over 300 Texas schools competed in 1996. The test takes 90 minutes and is given at your school, and students compete for intramural as well as state and national awards. Ten students can be registered for a total of \$30.00, and additional groups of ten for a total of only \$10.00 each. For registration brochure or more information, contact Vince Schielack, Department of Mathematics, Texas A&M University, College Station, TX 77843-3368, phone (409)845-2531, fax (409)845-6028, e-mail vinces@math.tamu.edu.

## A Comparison of Secondary School Mathematics in the United States and Russia

Victor A. Zinger, Mathematics Teacher/Visiting Scholar Science Education Center The University of Texas at Austin Austin, Texas

Charles E. Lamb \*\*, Associate Professor of Mathematics Education Department of Educational Curriculum and Instruction Texas A&M University

College Station, Texas

Russians often quote that "experience is the best teacher". Victor Zinger, a mathematics teacher from St. Petersburg, Russia recently saw how true this quotation is when he had the opportunity to observe a number of schools in central Texas. As part of his visiting scholar program, Victor sought to compare the U.S. educational system with the system in Russia. In that effort he conducted many meetings with faculty members, students, and school administrators in formal and informal situations. He also taught a geometry class at the secondary school level. He learned that teachers from all over the world encounter similar problems teaching mathematics at the secondary school level. He learned that solving problems associated with the teaching of mathematics depends on many factors such as traditions, culture, and the socioeconomic level of the country. The purpose of this paper is to share impressions about American schools, specifically in mathematics education. There will be a discussion of both similarities and differences between the Russian and American educational systems.

#### AN OVERVIEW OF THE SYSTEMS

It is essential for each society to recognize the place of education within its system of values. What is the place of the American educational system in American society? What opportunities are available for the people in this system? What is the purpose of education? Should education

prepare people for real life beyond formal schooling? Or should schooling provide students with basic minimum skills? What are the expectations of society? Who controls or should control and direct school education?

Generally, the American educational system is based on the premise that education is for all people. Gary Althen (1988) describes the attributes of the American educational system in terms of literacy for all, equal opportunity for all, local control of schools, the importance of parental involvement, and the aspiration to strive to produce well-rounded citizens. The basic attributes of the Russian educational system are similar to those of the American system, with the exception of "literacy for all" and the fact that parental involvement and local control of schools are rather limited in Russia. Currently in Russia, compulsory school education is not required of all students, making "literacy for all" an impossible task. Yet, many possibilities exist to continue education beyond formal schooling in Russia. For example, people have the opportunity to attend evening schools, technical schools, and external classes that are taught on an independent basis. Students after the ninth grade may take tests that are necessary to attend two additional years, grades 10 and 11; these school grades are equivalent to those grades in U.S. high schools.

For many years, the two educational systems were closed to each other, both politically and economically. It has only been in recent years that opportunities for the free exchange of ideas have been possible. As the result of perestroika, these opportunities became available to Victor Zinger. In August 1991, Victor was invited to participate in the first Soviet-American Conference of Science Teachers in Moscow organized by the National Science Teachers Association and Moscow State University. During the conference more than 500 American and 500 Russian teachers came to share how they taught students science and mathematics. The Americans shared ideas and their experiences and problems concerning teaching and learning with the Russian teachers. Victor observed many exciting and innovative methods, especially in integrated mathematics and science. For example, he learned that some schools in the U.S. used blocked scheduling and integration to teach physics, mathematics, and technology. Since Victor had taught mathematics in secondary schools for about 10 years and had spent time working in school administration, he was very interested in finding ways to improve the teaching of mathematics in his school in St. Petersburg. Since Victor supported the notion of integration of science and mathematics curriculum, he sought ways to change and modify the curriculum at his school to make it more integrated and meaningful for his students.

<sup>\*\*</sup> Charles E. Lamb was a faculty member at The University of Texas at Austin from 1975-1994, when he joined the faculty at Texas A&M University.

At the Conference, Victor met a science educator (Dr. James P. Barufaldi) from The University of Texas at Austin, who encouraged him to continue his work dealing with integrated science and mathematics. He invited Victor to The University to pursue his interests in integrated mathematics education. The interesting presentations, information, and ideas at the Moscow Conference and Victor's professional association with the science educator from The University provided him the impetus to begin his work of focusing on an integrated curriculum. Victor believed that integration extends beyond the context of mathematics or science in the school program. Ideas concerning an integrated mathematics curriculum presented by teachers from different countries can also help educators find solutions and new approaches in mathematics education.

## COLLABORATION IN MATHEMATICS AND EDUCATION

In the same way that many important discoveries in science have been made simultaneously by scientists in different places in the world, at times almost independently from each other, similar advances in mathematics education often occur independently. A classic example of simultaneous discovery is that of non-Euclidean geometry, specifically, refuting Euclid's Fifth postulate. This geometry was discovered by Lobachevsky (Russia, 1793-1856), Gauss (Switzerland, 1777-1855), and Bolyai (Hungary, 1802-1860) in the first half of the nineteenth century. Lobachevsky's discovery was presented at a scientific symposium in Russia. Gauss' discovery was not realized until well after it occurred; Bolyai was quite disappointed and disturbed to find out that another scientist made the same discovery about the same time as he did. The above example shows that many general ideas and ways of thinking develop similarly for people throughout the world. Leonard Euler (1707-1783), educated in Europe, was invited to Russia and worked at the Russian Academy of Science for about 30 years. His ideas were shared internationally and promoted a high level of fundamental mathematics in Russia. In today's world community, the continuation of international cooperation is extremely important in all fields of human activity, especially in the area of education. The continuation of international cooperation will provide numerous opportunities to reach new horizons in modern mathematics.

The same processes — active cooperation, collaboration between educators, teachers, and curriculum specialists from different countries, should be implemented in mathematics education. Working together with little political pressure, and infusing new technologies such as computers, fax machines, and e-mail systems, help improve educational systems for the world society for the twenty-

first century. To encourage this level of cooperation, one should move step-by-step toward studying and researching the quality of mathematics taught at different educational levels, determining the effectiveness of school mathematics, identifying methods and teaching techniques, determining the type of course offerings in mathematics, identifying standards for the teaching and the learning of mathematics, motivating, challenging, and assessing students, encouraging cooperation among countries, and recognizing necessary changes to improve the quality of mathematics education.

#### THE RUSSIAN SYSTEM

To begin the comparison, consider the organization of mathematics education in Russia. The unique place of mathematics and mathematical preparation in all kinds of education and development of the individual are the main reasons for studying mathematics in secondary schools in Russia. Studying mathematics enables students to master the system of mathematical knowledge and skills for successful life in modern Russian society, to learn another discipline, to continue learning beyond formal school education, to form representations about ideas and methods of mathematics, to determine its role in the knowledge of reality, and to enhance personal development through the learning of mathematics.

The general course in school mathematics has three stages of study in accordance with age requirements, historical experience, and traditions of the Russian school: grades 5-6; 7-9 and 10-11. During the first stage, basic mathematics is studied. The second and third stages include algebra and geometry in grades 7-9, and algebra and the elements of analysis and geometry in grades 10-11. After the 9th grade, students take exams in both subjects, written tests in algebra and a verbal assessment in geometry. At this point, students decide to continue or not to continue their school education. After the completion of a full school program, students are required to take a final mathematics examination, including a written portion (the test is as long as five hours). It is noted that mathematics is taken by all students at each grade level. Similar concepts in mathematics are spiraled throughout the grade levels. Currently, the teacher is the key to this learning process. For many years in Russia, the teacher did not play an important role in the development of curriculum since the government controlled all phases of the curriculum and determined what was taught. Certain topics such as probability and mathematical statistics were not included in the curriculum because these topics violated various philosophical underpinnings of socialism.

Sometimes the same teacher teaches the same students from the fifth grade through the eleventh grade. Other times, teachers focus on middle or high school mathemat-

ics. If students wish to enter a university and major in disciplines such as psychology, mathematics, physics, biology, engineering, chemistry, or economics, they must take a special entrance examination. Each university has its own entrance examination and requirements.

The majority of the Russian schools are either general (regular), government, or special schools for advanced mathematics. Special classes in mathematics are also taught in the general school. The most wide-spread style of lesson presentation is a combination between academic lecture and drill work. Most Russian teachers prefer to work with students using the chalkboard to show homework, to have students write exercises or to prove theorems or problems. Teachers have limited use of technology such as video recorders and computers. They do have greater access to simple function calculators. Multiple function calculators such as graphing calculators are rarely found in the schools.

Russian teachers also pay attention to homework because they consider it to be a very essential and important component in the learning of mathematics. The students complete their homework assignments in special notebooks and keep the notebooks during the entire academic year. During their mathematics lessons students write notes in this special book and use the book to prepare for their tests and final examinations. In geometry, Russian teachers teach their students many problems based on different types of proofs. Proving theorems and problems is one of the essential ways to learn mathematics. For example, in geometry courses in Russia, almost all theorems require a proof.

Competition in mathematics is very much part of the Russian school tradition. These competitions include activities such as mathematics Olympiads, class contests in mathematics, an entire school day focusing on mathematics, contests between classes from different schools at the school district, city, national, and international levels. Many reports have indicated that during the past 10 years Russian students have performed extremely well at the international level in mathematics. Students also meet with university professors, scientists, and engineers from various scientific and applied fields such as computer science, microbiology, genetics, and aeronautics. The universities are interested in attracting quality students to their institutions. Since the universities are also interested in increasing the level of understanding and interest in mathematics among students, some professors of mathematics teach in public schools, present lectures, and organize contests for students in mathematics, thereby increasing the level of mathematics teaching and learning in the public school system.

The teachers are supported by having access to professional journals in mathematics education such as Mathematics in School and the excellent scientific magazine, Quantum, which was established by many famous scientists in the former Soviet Union. While Quantum is designed for students, it also addresses the teaching audience. The first issue of the magazine appeared in 1970. Victor remembers this issue very well because he met the first editor of the journal, the world famous academician Kikoin, during a lecture at Victor's school in 1969. At that time, Kikoin discussed his idea to design a journal for students and teachers interested in mathematics and physics. A few years ago, an American version of the magazine was published by the National Science Teachers Association. The publication represents a good example of a cooperative, professional activity between teachers from Russia and the U.S. The magazine has received favorable responses from science and mathematics teachers in the United States Teachers use information from the magazine to improve the teaching and learning of science and mathematics.

#### GENERAL IMPRESSIONS OF THE U.S. SYSTEM

Initially, Victor's impressions were formed by his interactions with teachers and students in a middle school where he was asked to teach one period of an algebra class. He found that the students were quite interested in learning about algebra, but also about Russia! In particular, they were interested to learn about his students in Russia and also various words in the Russian language. They also enjoyed hearing Victor speak in Russian. This was the first experience these students had with a Russian and with the language. This surprised Victor because most Russians know much about American culture and the English language. During his presentation he was also quite perplexed that not all students took notes in their notebooks because in a Russian classroom all students diligently write in their notebooks. He also learned that the students had a choice in those courses in mathematics in which they enrolled. The use of technology, calculators, overhead projectors, and videos, were widely used in this mathematics classroom. A variety of software and supplemental programs were readily available. These resources provided a unique opportunity for teachers and students to move away from simple "drill and practice" toward the creation of new exploratory and discovery activities in mathematics. Complex geometric shapes from the program, The Geometer's Sketchpad, were easily presented via the computer. Victor was also amazed to observe the many ways in which teachers use computers. He observed that computers were used for tracking attendance, recording grades, designing and writing worksheets, and constructing tests. Discovering that it only took a few minutes to duplicate 100 tests for the classroom was truly a revelation for Victor. Unlike schools in Russia, the American classroom environment was very informal; students freely moved throughout the room and talked quietly to each other, and they had a very positive attitude toward their teacher and learning. The students appeared to be well-organized and worked cooperatively on various tasks. Teachers attempted to present information in a very informative interesting manner and demonstrated a very warm, nurturing attitude toward their students and encouraged them to do their very best.

Since then, Victor has had many opportunities to observe and work in classrooms in central Texas teaching mathematics and has reflected on his first American classroom experience. In general, he is concerned about a number of problems in the classrooms. For example, he believes that requirements should be strengthened in mathematics; some students are not prepared to enroll in advanced courses and he found that many students do not have the necessary prerequisites to be successful in these courses. This situation may be due to the fact that little articulation exist between courses in mathematics and in the sciences. He also believes that mathematics is presented at a very low level for most students and recommends a more challenging course of study; he noted that many teachers do not require their students to present oral solutions to problems, theorems, and proofs. He also found that calculators are overly used for the very basic calculations, many of which should be computed mentally. Sometimes students do not recognize the appropriate use of calculators; Victor observed that some students attempted to use calculators to simplify trigonometric expressions! He also found that computers were not always used in an educationally sound way. Certain software programs only required brief student responses and appeared to be of little assistance to the student in learning mathematics. The computer format may not be appropriate to teach all topics in mathematics. A piece of chalk and the board are still very important tools in the mathematics classroom. He also observed that homework plays a minimal role in the mathematics classroom and strongly supports the assignment of homework as an important learning strategy.

#### **A COMPARISON**

Table 1 shows a comparison of a secondary school geometry course in central Texas with one in St. Petersburg, Russia. Russian students take geometry for a total of five years, approximately 357 hours of course work; in the Texas school geometry is taught usually in the ninth or tenth grade, for two semesters, for a total of 180 hours of class time. (see table 1) In Texas, a variety of print materials are used in the classroom in addition to

computer software, worksheets, puzzles, and games; in Russia, two types of textbooks are used, one for the "general" school, the other for advanced students. Russian teachers may also write their own books for classroom use. Such books, must be approved by specialists from the education committee. All students are required to take geometry in Russia, unlike the Texas students. The grading system varies significantly. The Texas school uses letter grades and the 100-point system; in Russia a five-point system is used. Five represents "excellence"; four is equivalent to "good"; three is satisfactory; two is "poor" and one is not acceptable. The geometry textbook in Texas suggests that students prove approximately 50% of the theorems; almost all theorems are proven by students using the Russian textbook. Plane and three-dimensional geometry are included in the study of geometry in Texas; in Russia these are taught at different grade levels. In Texas, computer laboratories and calculators are widely used in teaching geometry; in Russia only simple-function calculators are used. The geometry course in Russia is well-articulated with other mathematics and science courses. Victor did not observe this articulation among courses in the Texas school. Cooperative learning, special projects, application of geometry in everyday life formed an integral part of the geometry course in Texas. In Russia, more traditional approaches are widely employed such as lectures, board work, and oral presentations by students.

#### SUMMARY AND CONCLUSION

The authors believe that the same processes and strategies should be operationalized in the field of mathematics education - active cooperation, meaningful implementation of computers, spiraling of concepts and topics in the curriculum, appropriate use of hand-held calculators, wellaligned curriculum in both science and mathematics, and most important, the collaboration between educators, teachers, and curriculum specialists from different countries such as Russia and the U.S. - to provide the education community a timely and unique opportunity to improve the teaching and learning of mathematics. Improving the status of mathematics education could be initiated with the design and implementation of a collaborative project focusing on a common problem the two countries share. The design may include both the infusion of the new standards in mathematics supported by NCTM and a prospectus or a scope and sequence in mathematics that would reflect the concerns and needs of people in the global community. Each system has much to offer. Integrating the "best" from both systems is a fruitful avenue to pursue.

TABLE 1: COMPARISON OF GEOMETRY COURSES BETWEEN AMERICAN AND RUSSIAN SCHOOLS

	American Schools	Russian Schools
Grades where geometry is taught	9th or 10th grade/ two semesters.	7th through 11th grades.
Total hours of instruction	180 hours.	357 hours.
Length of class period	55 minutes.	40 - 45 minutes.
Textbooks	A variety of print material.	Two sets per regular school/ one per advanced level.
Elective or required course	Elective course.	Required of all students.
Grading scale	100 points/letter grades: A, B,C, D, F.	5 point system.
Type of assessment	Written tests.	Written and oral tests.
Proportion of proofs	About 50%.	100%.
Mix of Plane and Solid Geometry	Combination of both in each semester.	Separate courses: 7-9th grades, Plane Geometry; 10-11th grade, Solid Geometry.
Use of Technology	Computers in geometry labs, graphic and simple function calculators, overhead projectors.	Limited computers and graphic calculators Some use of simple function calculators, overhead projectors.
Alignment of geometry content with science content	Very limited alignment.	High level of alignment.
Innovative teaching methods	Cooperative learning, special projects. Applications of geometry to everyday problems.	Traditional methods, lecture-based.

#### REFERENCES

Althen, G. (1988). American ways: a guide for foreigners in the United States. Yarmouth, ME: Intercultural Press.

NCTM (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

NCTM (1991). Professional standards for teaching mathematics. Reston, VA: NCTM.

NCTM (1993). Assessment standards for school mathematics-working draft. Reston, VA: NCTM.

# Expanded Opportunities to Demonstrate Learning Equals Student Success

by
Karen Lindig, *Del Valle I. S. D.*Wayne Gable, *Austin I. S. D.*Mary Alice Hatchett, *Round Rock I. S. D.* 

Who is more surprised by a performance task in mathematics? Is it the students, who find they know much more than they realized about estimation and measurement? Or is it the teacher, who discovers information about her students and their abilities? The performance task presented here enables both teachers and students to find out what they know and what they need to learn in order to explore the effect of a force on the distance objects will travel.

#### WHAT IS A PERFORMANCE TASK?

Performance assessment in mathematics can be defined as "presenting students with a mathematical task, project, or investigation, then observing, interviewing, and looking at their products to assess what they actually know and can do (NCTM, 1991, p. 13). One benefit of performance assessment is that it occurs within the context of regular learning and does not distract from the instructional program. The mathematical task presented here, for example, enhances a unit on measurement, in that students are engaged in a realistic investigation which is motivating and gives them opportunities to practice important mathematical skills. Performance assessment is useful as it provides information about students' understanding of concepts such as estimation and linear measurement. This type of assessment allows teachers to know more about students' strengths and weaknesses in understanding and helps teachers make decisions about further instruction.

The Curriculum and Evaluation Standards (NCTM, 1989) calls for increased attention in assessment procedures on determining what students know and their abilities to think mathematically. A performance task such as this one allows teachers to gain such information through multiple sources, such as observation of individuals and groups as they work through the activity and make their oral presentations to the class and through written explanations of their thinking and the processes they used. This information can be used for evaluative purposes such as teacher assessment of students' learning as well as allowing children to reflect on their own performance.

Information gathered through performance assessment is also useful when discussing children's learning with parents and others.

When choosing a performance task for classroom use, a teacher should examine the task to determine if it will be beneficial to mathematical learning. A list of criteria for evaluating performance tasks, adapted from one developed by Leinwand and Wiggins, is presented in Mathematics Assessment (NCTM, 1991). The performance task presented here is beneficial in that it fits into the core curriculum and represents the "big ideas" of estimation and linear measurement. From our own experiences with using this task in elementary classrooms, we can state that the activity engages students and gives them many chances to interact and construct their own meanings of the concepts. One of the greatest advantages of an activity such is this one is that it allows all students to be successful since there are multiple ways to approach the problem.

We used this performance task with one class at the first grade level and one class at the fourth grade level at different schools in the Austin area. Although there were some differences in how the students worked through the activity, the procedures we followed for both of the classes were basically those described below.

#### **NECESSARY MATERIALS**

The objective for this task is for students to use a literature selection as an investigation opportunity for measuring linear distance with a tape measure. The book *The Three Little Javelinas* (Lowell, 1992) will be used as a focus for this activity. Students will be working through this task in groups of four and the following materials will need to be gathered for each group: two metric tape measures, soda straws (one per student), a cotton ball, a toothpick, and a wooden cube. We found that a booklet for each student with a recording sheet (see Figure 1) and a rubric (see Figures 2 and 3) and several sheets of writing paper was the easiest way to organize all of the students writing. They seemed to enjoy working in their own booklets.

## PROCEDURES FOR ENGAGING IN THE PERFORMANCE TASK

The activity begins with a reading of the book *The Three Little Javelinas*. A discussion of the characters in the story may include some information about coyotes and javelinas. The students generally recognize the book as a variation of the three little pigs and you may wish to allow children to identify some of the similarities and differences in the two stories. You may also wish to have students identify some of the references to the Southwest included in the book, such as the landscape of the scenes, the dress of the characters, or the howling of the coyote.

Ask the students to talk about what kind of structures they would blow on if they were the coyote and why they would blow this. When students think back about the book, what things do they recall that the coyote tried to blow? You may wish to write "tumbleweed," "saguaro ribs / sticks" and "adobe bricks" in one column on the blackboard or on a chart. Ask them to describe each of these items and write these describing words next to the objects. Finally, show them the objects they will be using in the activity and have them tell you which of the things from the book the objects might represent. These could be written in a column next to the descriptions. The chart you develop with the class might look something like this.

house made from	description	object
tumbleweed	light / blows easily	cotton ball
saguaro ribs / stick	slight / can roll	toothpick
adobe brick	heavy / doesn't move	wooden block

This chart can be used as a reference by students as they engage in writing throughout the task.

It is beneficial for students to have some points of reference about the measurements they will be using in the activity. Ask students to examine the tape measure and find a part of their hand which is about one centimeter. The width of a pinky, for example is usually about one centimeter. Now ask students to use their hands to estimate the length of 10 centimeters. Young students may find they can open their fingers just a bit to get a hand spread which will match 10 centimeters on the tape measure.

Then ask students to estimate and check on 100 centimeters on the tape measure. Children may relate this to the span of both arms stretched wide or to the length of a table or desk. Now that the students have some general ideas about the length of 1, 10, and 100 centimeters, they are ready to estimate. Have each group use tape to attach one of the tape measures to a desk or table with the 0 mark on the edge. This will be the starting point for each object. The second tape measure is available to measure the distance of objects which travel farther than the distance of one tape measure.

Before students can begin to work on the activity, you need to demonstrate how they will blow each object. Explain and model how to place the object at the end of the tape measure and use one breath through the soda straw to move the object. Although you may wish to discuss how students will measure the distance traveled when the object strays from the line of the tape measure, we found that each group of students was able to determine their own rules. We felt that allowing students to make decisions about methods for measuring and judging the accuracy of the measurements was an important part of the

activity itself. Now that the students are aware of the materials and procedures for this task, it is time to think and make some predictions about what they believe may happen. You may wish to display questions such as these to get students started in their thinking.

#### Think About These Questions As You Work

- A. As you work, what helps you to make better estimates?
- B. What will affect the distance each object travels?
- C. What mathematical knowledge will you use to complete this activity?

Students should label one of their writing pages with the word "before" and should write their thoughts about what they predict will happen. Following an appropriate writing period, ask several students to read their predictions to the class.

One of the important aspects of this performance task is the students' self-evaluations of their performance. At this point, it is necessary to discuss how students will identify their levels of success in the activity. For younger students, a rubric may be created by the teacher (see Figure 2) which uses faces to show how a child feels about his or her performance on each of the criteria. Older students may use a rubric with numbers to complete the self-evaluation (see Figure 3). This same rubric will be used for teacher evaluation of each student's level of success. Some students may be able to determine their own indicators of success to create a rubric which is unique for their class.

Each student should use a recording sheet (see Figure 1) to write an estimate for the distance each of the three objects (cotton ball, toothpick, and wooden block) will travel with the force of one breath. We emphasized that the estimates were only guesses at this point and that we were not concerned about correctness. Having the students write their estimates with crayons discouraged them from wanting to change the estimates after they did the actual blowing and measuring. After all of the members of a group of four students has made estimates for the distance traveled by each of the three objects, the group is ready to begin. Each group is given a bag of materials. Each student blows the cotton ball and uses the tape measure to determine the distance traveled. Students in each group can work together to determine distances. When everyone has the chance to blow the cotton ball, the group continues with the toothpick and finally the wooden block. Students in our classes were actively engaged in the task at this point and there was much discussion about their results. In many cases, the children were surprised by the actual distances and they talked about the effect of different types of breaths on each of the objects.

When each of the objects has been moved and the actual distances have been written on the recording sheet, the differences between the estimates and the actual measurements need to be determined. Calculators may be used for this. Students should now be asked to reflect upon their experiences in doing the activity the first time. Allow for about two minutes to reflect quietly. Ask students to use the information they found to make a second round of estimates on the bottom half of the recording sheet. After each of the members of the group has written estimates, the group is ready to complete the activity a second time with a new round of blowing, measuring, and finding the differences between estimates and actual distances.

We learned a great deal about our students while observing the procedures they used for estimating. Most of the children, although not all, recognized that a heavier object like the wooden block would not move as far as a lighter object such as the cotton ball and wrote decreasing numbers for their estimates of the distance the three objects would travel. Some students predicted that the toothpick would roll well and estimated a distance for that object similar to the cotton ball. The second round of estimates informed us about how the students were using the information they had gathered. The students who seemed to best understand how the information could help, used the numbers from the actual measurements from their first try to help them make better estimates the second time. A number of students, especially in the fourth grade class, discussed how a different type of blow through the straw could effect the distance and estimated accordingly.

Now that the students have engaged in the activity, we want them to reflect upon their experiences. Questions such as these may be displayed to get students started in their thinking about what occurred as they worked through the activity.

Think About What Happened As You Worked

- A. Why did you make the estimates that you did?
- B. What were the mathematical processes you used to do this activity?
- C. Why did the objects move the distances they did?
- D. How does this activity relate to things you've done before?

Students should label one of their writing pages with the word "after" and should write their thoughts about what happened. Following a appropriate period for writing, ask several of the students to read their reflections to the class. There was a variety of responses by students as they reflected on the activity. A number of the first grade children wrote about how their estimates related to the actual distances: "I thought that the cotton ball was going to go 90 cm but I wasn't right. It was 149 cm." Several of the children wrote about the activity itself, talking about how it was easier than they thought it would be or mentioning how they liked doing it. With the fourth grade students, it was possible for them to respond to each of the four questions.

#### COMPLETING THE RUBRIC

After students have participated in the activity, they are ready to gauge their levels of success. Review with the students the indicators of success they examined at the beginning of this lesson. A transparency could be used to demonstrate to students how to mark the rubric to identify their levels of success on each of the criteria. The rubric with the faces (Figure 2) can be used by having each child color one face in each row to tell about feelings of success for that criteria. The teacher may describe reasons for marking each of the faces, such as "I did great! I really knew what I was doing." with the happiest face down to "I'm not sure if I knew what I was supposed to do here, I didn't feel that good about it" at the unhappy side. If using the rubric with numbers (Figure 3), be sure to have the children mark under (S) for student. The teacher will use the (T) column.

Discuss how each of the ratings for each criteria would be received. For example, tell students how to look at the differences between estimates and actual distances. "Were the differences smaller on the second round than on the first round? Was this because you used the information from the first round to make better estimates the second time? If this is true, then you may wish to give yourself a rating of "quality' or 'exceptional quality' next to number 1 'Improvement of estimation skills' on the rubric. If you don't feel that your estimates were better, you may feel you deserve a 'satisfactory' rating for this criteria. If you see that you did not use the information to help you make a second round of estimates, you may wish to mark 'not satisfactory' for number 1." Specific examples like these for each of the criteria helps students conduct the selfevaluation more effectively.

At this point students have participated in the activity and have rated their performance. It is time for them to reflect and write about what they have learned. The students revisit their recording sheets and the writing they did before and after the activity to write their reflections. Tell students that you will use what they have written as part of your information for rating their performance, so they should include mathematical information and write as

completely as possible with details to let the reader understand the activity. You may wish to display suggestions such as these to help students in their reflections.

Things to Remember as You Reflect and Write

- Show what you know
- Do your best work
- Explain your thinking fully
- Make your ideas clear

In many cases, the students were able to write about what they learned in the activity. The first graders talked about learning to measure and about their successes in the activity. Although these children had some experiences with nonstandard measurement and using rulers to measure, the measuring tape was a new tool and many of them wrote about it. One student related the measuring tape to a more familiar object, the number line. A big revelation for the fourth grade students was how altering the force of their blowing would effect the distance the objects travel. Their writing about the activity was generally complete and competent. Many of the fourth graders remarked about using the rubric and how they judged themselves critically.

#### TEACHER ASSESSMENT

The teacher collects the students' work and uses the written predictions and reflections, recording sheet, and rubric to evaluate each student's success on each of the criteria for the task. The teacher then conferences individually with each student to discuss performance. The rubric with numbers (Figure 3) has a place for the teacher (T) to mark her rating. At this conference, the teacher and the student together may determine an overall score (1, 2, 3, or 4 on the bottom of the rubric) for each student. It is possible for teachers to use this information for regular grading purposes by assigning a grade to the different levels.

## THE PERFORMANCE TASK IN THE CLASSROOM

One thing we learned by doing this with our students is that a performance task such as this will last more than one math period. At the first grade level, we were able to read and discuss the book, prepare for doing the activity, write our predictions, and complete the two sets of estimates and actual measures in about 2 hours on the first day. It took about an hour on the second day to review the activity, use calculators to find the differences, and write reflections on the activity. The teacher later met with each group of four students to help them mark their rubrics and to get them to write about what they had learned.

In the fourth grade class, the students were scheduled for hour-long math periods. On the first day, the students read and discussed the book and wrote their predictions. The second day was used for students to estimate, conduct the actual measures and find the differences between the two. On the third day, the students wrote about what happened as they worked and completed the rubric. The students wrote their reflections on what they had learned on the fourth day. The teacher collected the materials and met with students individually at a later time.

Although the performance task required more than one day to complete, we felt that the time was well spent. The students demonstrated their abilities to solve a problem, to estimate and measure, and to communicate mathematically in written and oral language. The task was fun for students and kept them involved at all points. By taking the task through to a self-evaluation, the concepts were strengthened as we revisited the things we had done and the students reflected on what they had learned.

#### **BIBLIOGRAPHY**

Lowell, Susan (1992). *Three Little Javelinas*. Flagstaff, AZ: Northland.

National Council of Teachers of Mathematics. (1989)

Curriculum and Evaluation Standards for School

Mathematics. Reston, VA: The Council.

National Council of Teachers of Mathematics. (1991)

Mathematics Assessment: Myths, Models, Good

Questions, and Practical Suggestions. Reston, VA:
The Council.

#### **Criteria for Completing the Rubric**

#### Concepts to be Examined

E: Estimation

M: Mathematical skills

(linear measurement and computation in this activity)

S: Problem solving Strategies

C: Communication

#### Level 1: Unsatisfactory Response

- E: Estimates are based on guessing without thoughtful reasoning.
- M: The student has difficulty with linear measurement and/or computation.
- S: The student demonstrates limited understanding of the strategies needed to complete the activity.
- C: Communication is limited.

#### Level 2: Satisfactory Response

- E: Some thoughtful reasoning is revealed in the estimations, but the reasoning is flawed or incomplete.
- M: The student has some understanding of linear measurement and/or computation.
- S: The student demonstrates adequate understanding of the strategies needed to complete the activity.
- C: The responses are good, but may be somewhat unclear or incomplete.

#### Level 3: Quality Response

- E: Estimates are based on logical and thoughtful reasoning, although there may be some flaws in the reasoning.
- M: Measurements and computation are reasonably correct.
- S: The student understands the strategies needed to complete the activity and includes an explanation of the strategies.
- C: The explanations are clear and may include a drawing.

#### Level 4: Exceptional Response

- E: The estimations and responses reveal exceptional logical reasoning.
- M: All measurements and computations are correct.
- S: The students written responses show excellent communication of strategies.
- C: There is evidence that the student has made connections that go beyond basic understanding of the activity.

## **The Coyote Sneezed**

Recording sheet for	 
	•

object	estimation	distance	difference
80° -			
• •		**	1:00
object	estimation	distance	difference
			· .
		·	
			· ·

Figure 2: Example Rubric (used with first grade students)

Rubric for	
I know how to estimate.	(E) (E) (E)
I know how to measure.	\(\text{\texi}\text{\text{\text{\texi}\text{\text{\texi}\text{\text{\texi}\text{\text{\texi}\text{\text{\texi}\text{\texi}\text{\texi}\text{\texit{\texi}\text{\texi}\text{\texi}\ti
I got better at estimating.	\(\text{\ti}\}\text{\texi}\text{\text{\texi}\text{\text{\text{\texi}\text{\text{\texi}\text{\text{\texi}\text{\text{\texi}\text{\texi}\text{\texi}\text{\texit{\texi}\text{\texi}\text{\texi}\ti
I can show someone how I worked on this problem.	(E) (E) (E)

Figure 3: Example Rubric (used with fourth grade students)

## **RUBRIC**

List of Criteria for Success on the Task

	No Satisfa		Sati	sfactory	Qı	ality	Exce Quali	ptional ity
	S	T	S	1	S	T	S	1
1. Improvement of estimation skills.								
Better understanding of linear measurement.								
3. Show strategies used to accomplish task.								
4. Understand the meaning of estimation.								
S = student $T = Teacher$	(	Overall S	Score					

## "Ah-ha", the Answer

by Virginia Kester Fisher Elementary School 2220 Gruenwald Pasadena, Texas 77502 (713) 948-4340

"What can I do differently to help them understand?" I had tried everything I knew to try, but several of my second graders still struggled with concepts and skills in math. One of the reasons for this struggle was that they had not experienced that "ah-ha" or felt meaning as described by Caine and Caine in their book *Making Connections*.\* This lead me to research new approaches to teaching math.

The procedure of pairing upper and lower grades attracted my attention and best suited my situation. A fifth grade teacher in my school seemed as eager to try this as I did. After talking with administration, we began our planning. She would bring her class to my room for about forty-five minutes once a month. We formatted our lessons with a presentation of a mini-lesson, opportunity for students to divide into small groups while the teacher monitored the small group activities, and a time for the group to share their results.

Concepts and skills we used in these pairing sessions included: estimation, graphing, place value, telling time, and fractions. The most memorable was the lesson on measuring. My second graders were having trouble learning to read a ruler as well as recording measurement. This session paired one second grade student with one from fifth grade. They used a tape to measure their partner's height and some body parts, then record and compare results.

As the fifth grader measured a part of the second grader, for example the forearm, the procedure was explained and modeled while the second grader observed. The younger student then followed the example and measured the forearm of the partner with help as needed. Together they recorded the information and compared the results. They took turns in the same way and measured their hands, feet, waist, and height. Comparisons were recorded on a chart and posted in the classroom.

This presented an enjoyable approach to student learning. Both classes were delighted and eager for the pairing sessions. My class frequently asked, "When are the fifth graders coming back?" The atmosphere of this was anything but quiet as with any small group activity. Although the noise level of the room increased because of active learning within the groups, the problem of keeping stu-

dents on task decreased. As with any new procedure, improvements can be made. One addition would be to present a writing connections in order for the students to present their ideas and concerns about what they had done.

This project proved beneficial in helping students learn. As the students worked together, the older students could present a fresh, new approach while assisting the younger in their understanding. The challenge of explaining a skill or concept helped the older student's understanding as they formulated their approach to the problem. Eagerness to please was apparent in the younger students. At the same time, there seemed to be an increase in self-esteem for the older student. Our pairing project appeared to have worked as well as we had hoped. Both teachers realized that understanding and mathematical skills had improved for all the students.

"Ah-ha", this procedure will definitely be continued.

\*Caine, R. N., and Geoffrey Caine, (1991) *Making Connections: Teaching and the Human Brain*. Alexandria, VA: Association for Supervision and Curriculum Development).

## Conference on the Teaching of Secondary Mathematics

January 24 and 25, 1997 Sam Houston State University Huntsville, Texas

Conference Theme: Applications Across the Curriculum

Feature Speakers on Friday, January 24:

3:30 p.m. Bill Hopkins, Director of Mathematics, Texas Education Agency

4:45 p.m. Chuck Vonder Embse, Professor of Mathematics, Central Michigan University

#### Saturday, January 25

#### Presentations

- illustrating the use of applications in the middle and high school mathematics classroom
- · on the TEKS and activities to model the TEKS
- workshops and demonstrations using the latest technology
- exhibits of textbooks, calculators, software and other materials

For additional information contact Dr. Max Coleman, Sam Houston State University, Huntsville, Texas 77341-2206,

Phone: (409)294-1574

FAX: (409)294-1882

## VOICES FROM THE CLASSROOM



#### **TEXTEAM INSTITUTES**

On the following pages you will find exerts from the five new TEXTEAM institutes.

Grades 1-2 was developed with project director George Christ of Region 10.

Grades 3-5 was directed by Pam Littleton, Tarleton State University.

Grades 6-8 was developed with project director Elaine Wizdom, Region 10.

The Algebra Institute was first presented in the spring of this year under the direction of Dr. Susan Williams of the University of Houston.

The Geometry Institute under the direction of Dr. John Gilbert of the University of Texas was presented in October and November.

All of these institutes are from 3 to 5 days in length and may be presented by teachers trained through the TEXTEAM. For further information or a list of trainers contact Jackie Jimenez, TEXTEAM director, at jjimenez@tenet.edu or Texas SSI, Dana Center, University of Texas, 2613 Speedway, Austin, TX, 78712, (512) 471-6193

Algebra TEKS, End Of Course Exam, Activities for Algebra,
Algebra Action Team, TEXTEAM ,Block Scheduling, Technology for Algebra

If these Algebra topics interest you, subscribe to the new TENET user group which promotes a discussion of topics of interest to the algebra educators of Texas.

tenet. educator. algebra

Sponsored by TENET and the Statewide Systemic Initiative

If you have a TENET email address, you may subscribe to the user group.

- Press lower case 1 to get a listing of newsgroup folders.
- Arrow down below the News/conf hyphened lines and press a (to add newsgroup).
- At the Subscribe to : prompt, type tenet.educator .algebra
- Press return twice to add the group to your listings. This needs to be done only once.
- Highlight the newsgroup name in the folder listing and press return to enter it.

You may highlight any message in the list which interests you. You may respond by r for reply.

If you have a email address which is not through TENET, please contact the moderator, Diane McGowan, dmcgowan @ tenet.edu. You will be added to a mailing list of non-tenet users and mail will be forwarded through Diane.

The newsgroup is moderated. This means that any message must go through the moderator and be approved for passage on to the news group. Please note that copyright laws are in effect in a user group; reviews of commercial products are acceptable, but posting advertisements is not acceptable, and no lobbying may be posted. All newsgroup postings must adhere to the TENET Acceptable User Policies.

## Section E: Integration Making the Connections

Activity: The Giant Ant March

Concepts: Counting, estimating, multiplication arrays, graphing

Materials: One Hundred Hungry Ants by Elinor J. Pinczes

pinto beans (two hundred per pair of students)

graph paper crayons

#### Introduction:

• Read the book to the children and have them make the ant lines described with their beans. Suggestion: Make a transparency of the lines and put it on the overhead if students are having trouble with the activity.) Pay close attention to the language the children use as they are making their lines.

#### **Exploration:**

- How are the sets of 100 ants alike? How are they different?
- Describe your sets of ant (pinto bean) lines.

#### Assessment:

- Can you show me how many lines there are if you have 20 ants in each line?
- Can you show me how many lines there are if you have 10 ants in each line?

#### **Observations:**

- Were students using appropriate vocabulary?
- Could students justify their sets of lines?

#### Tasks:

- Write a journal entry explaining your sets of lines for 100 ants.
- Write a question about the 100 ants lines for another group to answer
- Have the children work in pairs with beans to predict how long bean ant lines of 5, 10, 20, 25 and 50 would be in inches. Have them make the lines and check their predictions.
- Have the children estimate how many beans it would take to make the initial letter of their first name three inches high using their beans. Have each child write their estimate on a sticky note. Have children make the initial letter of their first name using their beans. Have each child count how many beans it takes, then write that number under their estimate. Ask the children to decide how they could use the beans to find out the difference between their estimate and their actual beans used to make the letter. Record the actual number of beans used to make the initial letters of all students names on a chart.
- Decide which alphabet letters are missing on the class chart. Challenge the students to estimate and then check the number of beans used to make the missing letters (all 3 inches high). (Note: you will probably have "average number" of beans used for each letter. This would be a great place for use of a calculator.) Have the children design a "BEAN ALPHABET" graph.

## **Focus Activity**

Activity:

What Comes Next?

Objective:

Investigate patterns while cutting paper.

Materials:

Tissue Paper

**Scissors** 

Transparency for recording

#### Procedures:

- 1. Fold a sheet of tissue paper in half and then in fourths (in the opposite direction). Cut the corner with two folds off.
- 2. Open the paper and observe the number of holes in the paper (one). Start a table to record the number of cuts and the number of holes in the paper.
- 3. Refold the paper, then fold it once again. Again cut the folded paper across the corner with folds (no edges).
- 4. Unfold and again observe the number of holes in the paper (three).
- 5. Repeat this process until someone sees a pattern. You may have to ask questions referring to the growing table. One approach is to look at differences between consecutive entries in the number of holes column (2, 4, 8, 16, etc.).

#### Extensions:

- 1. Play the Tower of Hanoi and look for patterns with 2, 3, 4, and 5 pegs. The minimum number of moves may not emerge immediately.
- 2. A similar activity can by done by just paper tearing and recording number of tears and number of pieces of paper. The pattern is the powers of 2 (with each tear the number of sheets of paper doubles). This particular activity can be extended(for older students) by measuring the thickness of the stacked paper after about 6 or 7 tears and then estimating how many tears would be required for the paper to stack a particular height. Students are always surprised how few tears (20 or 30) to reach a considerable distance.

22

#### FROM THE GRADES 3-5 TEXTEAM INSTITUTE

#### Notes:

- 1. The number pattern of the number of holes in the paper that develops in this activity is the same with the Tower of Hanoi problem for n pegs and the minimum number of moves to rearrange the disks.
- 2. This can be either a demonstration, or participants can do the activity with you.
- 3. The only reason tissue paper is recommended is because you can make more cuts than with a heavier paper.
- 4. The folding/cutting procedure produces the following table. Many students will give the recursive definition for this pattern. (For the next entry, double the current entry and add one.)

Number of Cuts	Number of Holes
1	1
2	3
3	7
4	15
5	31
•	•
•	•
•	•
n	$2^{n}-1$

## **Relations and Functions Session**

Activity: Big Blank Number Line

Objective: Students will investigate relationships of whole numbers, fractions, and decimals on the number line.

Materials: Big Blank Number Line

Cards with random whole numbers between 0 and 500 printed

one number to a card Cards with random fractions

Cards with random decimals in tenths and hundredths

Double sided tape or tacky stick

#### Procedures:

- 1. Attach Big Blank Number Line to the wall. It should be approximately 12 feet long.
- 2. Prepare cards with whole numbers from 0 to 500. Include in the set 0, 100, 200, 250, 300, 400, and 500 and an assortment of other numbers to make a set of 20 to 30 cards.
- 3. Give the zero card to a participant and ask him/her to place it on the blank number line. (It does not have to be on the far left end, however, if it is placed too far right you might want to verify that it is where the participant wants it.)
- 4. Give the 500 card to a participant and ask that it be placed on the blank number line. (Anywhere to the right of zero will work, but too close will cause problems later. If it is placed too close to zero, do not move it; let the group decide it needs to be moved, along with all the other numbers that have been placed so far.)
- 5. Next give the card with 250 on it to a participant and ask that it be placed in the appropriate place on the number line. (Note that this number must be located relative to 0 and 500.)
- 6. Continue giving cards to participants (or letting them draw from the deck) to be placed on the number line. Ask the group if they agree with the placement and why.
- 7. Repeat the activity with fractions. Make cards with  $0, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{5}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}$ , 1 and an assortment of other fractions to make a set of 20 to 25 cards.
- 8. Repeat the activity with decimals. Make cards with 0, 0.1, 0.10, 0.2, 0.20, 0.25, 0.3, 0.30, etc.

#### Extensions:

- 1. Include in the fraction set mixed numbers less than ten.
- 2. Place whole numbers, fractions and decimal fractions on the same number line.

#### Notes:

- 1. When introducing this activity to younger children, use a set of whole numbers between 0 and 50 or 100. As children become comfortable with the activity, include larger numbers.
- 2. The primary goal of this activity is not only ordering of whole numbers, fractions, and decimals, but proportional thinking. Start with familiar or related numbers then include less familiar fractions such as  $\frac{9}{17}$ . This fraction is a little more than one-half. And that is the thinking we want to encourage.
- 3. The Big Blank Number Line could have a long narrow strip of Velcro down it and the back of each card could have a corresponding piece, therefore, eliminating tape or tacky stick.
- 4. This Number Line can be used for many other classroom activities, such as, graphs, time lines, etc.
- 5. Children's book Every Day Counts by Patsy Kanter is a nice complement to this activity.

# Segment 8: Themtic Learning Proportional Reasoning and Measurement

#### Overview of Thematic Learning

Teaching multiple concepts through a unifying theme can be more motivational to students and helps them find connections and see the 'big picture.' Using themes also allows a natural way for spiraling (revisiting and expanding previously taught concepts) to occur.

In the activities in this segment the following concepts can be taught for the first time or extended through a second visit: measurement, precision, mean, median, mode, ratio and proportion, area of a circle, estimation, and ratio of areas. Once again, as in previous segments, the focus is on active student involvement and creating a mental or visual image.

MAJOR STRANDS: Measurement and Proportional Reasoning

**OBJECTIVES:** Students will be able to measure length using English measure, will determine and find appropriate measure of central tendency, and solve problems using ratio and proportion.

**PREREQUISITES:** Students should have had prior experiences with measurement and solving proportions.

MATERIALS: Yardsticks, rulers, calculators.

TIME RANGE: 3-5 days

**ACTIVITY** "One inch tall"

**Note:** The related supplemental activities in this unit may be taught prior to the investigation for students with less experience with the concepts used.

Launch: Read the poem "One Inch Tall" (Silverstein, 1974) which speculates what the world would be like for someone one inch tall. Also talk about or show the movie "Honey, I Shrunk the Kids."

**Explore:** State "If you were one inch tall and lived in a house that was proportional to your size, how big would your door be?"

Have students discuss how they would approach the problem. Pose questions such as "What would you need to measure in this room?" (Answers: the door and individual heights of students.) "How could you decide on what the size of a typical student is?" (Answer: Measure all students and determine which measure of central tendency best represents the "average" student.) "What unit of measure should we use?"; "What would be an appropriate precision measure?" (Answer: Nearest 1/2 inch or nearest inch depending on grade level of students)

**Share:** Have students complete and share measurement results by completing the measurement chart on the overhead. Then groups should use the information to complete the class data chart and determine which measure of central tendency to use. (The mean is the best representation.)

**Reflect:** Pose question: "Once we have the height of the average student and the height of the classroom door, how can we find the height of the door in a one inch world?

Share: Have students discuss approaches. Discuss proportional reasoning. How can we use the calculator to solve and convert decimal answers to 16ths of an inch? (Example: 1.14 inches converts to 1 7/50 which is close to 1 7/48 which is between 1 6/48 and 1 and 8/48 or 1 3/16 and 1 and 1/4 or 14:100 as x:16 yields x as approximately 2/16 or 1/8.)

**Explore:** Have students discuss the "One inch world" activity sheet questions. Have available containers in English and metric measure, weights in English and metric, and both English and metric rulers.

Share: Have students share and defend their answers. Determine most reasonable answers through discussion and class vote.

Apply: (Assessment)

Design a house (either two or three dimensional) for your family if they lived in the "one inch world." Draw a floor plan including placement and size of furniture or build the house including furniture. Include an accompanying sheet that shows how you arrived at all measurements. Write a story about the family that lives in your house.

## "The One Inch World"

# If you lived in a world where everyone was one inch tall: (Justify all answers with a written statement.)

1.	How high would your front door be?	
2.	Instead of buying a quart of milk, what would you buy?	
3.	In what units would you weigh yourself?	<del></del>
4.	How would you measure temperature?	
5.	What might a thimble be used for?	
6. how	If you went to visit your grandmother (let's say she lives 50 far would that be in a one inch world?	0 miles away),

7. be?_	If it took you a month to walk to the store, how far away would it
8.	What kind of coins might you carry and what would a \$150 CD player
cost	What would be the circumference of a
CD?_	
9.	How long would an hour be?
10. patter	How high would your door be if you were 2" tall? 3"? 4"? Do you see a
Refe	rences:
	rstein, Shel. <i>Where The Sidewalk Ends.</i> Harper Collins: New York, N.Y. (p.55)

## HAVE YOU LOST YOUR MARBLES?

Focus:

Collection, analysis, and interpretation of data.

Objective:

Alg I TEKS, Foundations for Functions #2

The student understands the properties and attributes of

functions.

Terms:

Rate, slope

Set-Up:

Participants should be seated at tables in groups of 3-4.

Materials:

Transparency #56, Activity #40

Each group needs 5-6 children's blocks (or similar building blocks), 39 cm of PVC pipe, marble or steel ball bearing that will fit inside and roll freely through the PVC pipe, metric

tape measure, stop watch

Prerequisites:

Metric measurement

Procedure:

Transparency #56: Have You Lost Your Marbles?

Activity #40: Have You Lost Your Marbles?

Each group should have one person to roll the marble, one person to hold the blocks and pipe, one person to mark where the marble stops, and one person to measure the distance that it traveled. Each participant should record the measurement on

his/her individual activity sheet.

Participants are to roll the marble from heights of 1, 2, 3, 4, and 5 blocks. The marble must be released each time at the edge of the pipe. Participants measure the distance the marble

rolls once it leaves the end of the pipe, record the

measurement in cm on the data table, and graph the data.

#### Questions for Discussion

• Did the height of the slope increase the distance the marble rolled? Why or why not? (Ans: Yes, the marble has more potential (stored) energy with greater height.)

• If the slope keeps getting steeper, will the marble roll farther each time? Why or why not? (Up to a point, yes. When the slope is completely vertical, the marble will not roll far because its energy is absorbed by the ground.)

#### **Extensions**

• use different sized marbles (as long as they still travel freely through the pipe),

• use spheres with different masses (i.e., golf balls, ping pong balls, steel ball berings),

• use different surfaces (i.e., rug, cement, dirt, table top, sheets).

#### Emphasize Alg I TEKS, Foundations/Functions #2

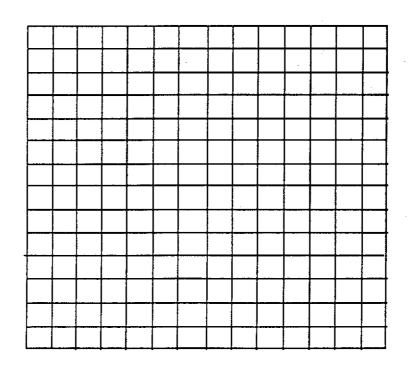
2.d. The student collects data and records results to solve problems, organizes the information, makes scatter plots and interprets the results in order to model, predict, and make decisions and critical judgments.

#### Emphasize Alg I TEKS, Linear Functions #6

6.b. The student relates direct variation to linear functions and solve problems involving proportional change.

## HAVE YOU LOST YOUR MARBLES?

Height (Block)	Distance (in cm)



## HAVE YOU LOST YOUR MARBLES?

- 1. Roll a marble through a PVC pipe from heights of 1, 2, 3, 4, and 5 blocks.
- 2. Release the marble at the edge of the pipe.
- 3. Measure the distance (in cm) that the marble travels once it leaves the end of the pipe.
- 4. Record the height of the pipe and the distance the marble travels.
- 5. Graph the data and describe the relationship between the independent and dependent variables.

#### THE SCALE FACTOR OF MOUNT RUSHMORE

#### Teacher Guide

#### GOAL:

Use the scale factor of the models of Mount Rushmore and the length of George Washington's nose to find the lengths of various features of the presidents on Mount Rushmore.

#### PREREOUISITE:

Scale factor.

#### **MATERIALS:**

Ruler.

#### PROCEDURE:

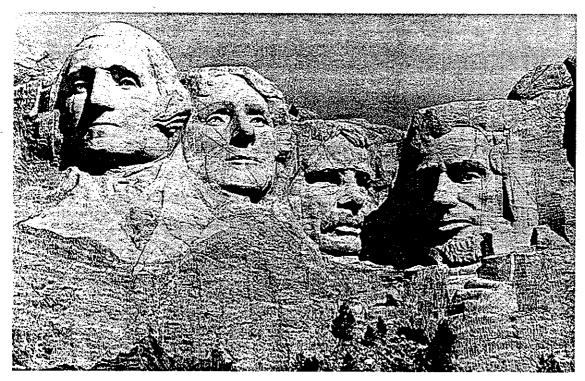
Have students read the paragraphs on Borglum, then discuss how Borglum created his "pointing" machine and used it to transfer the model to the mountain. Possibly make a pointing machine using a protractor, small wire and string with a weight.

Read up on Mt. Rushmore. Students have many questions. Mt. Rushmore was started in 1927 and in 1941 Borglum died. Final touchup work was completed by his son.

#### **SOLUTIONS:**

- 1. 240"
- 2. 11'
- 3. 216", 18'
- 4. 60"
- 5. Between 434' and 496' (7.5 heads = 465')
- 6. Using 5/16" get picture:mountain = 5/16in:20ft or 1/16in:4ft or 1/16in:46in
- 7. 4', 48"
- 8. 3 1/3', 40"
- 9. 4', 48"
- 10.48', 576"
- 11. Not accurate. Measurement of a three dimensional, curved object is made from a two dimensional picture. Estimate made in measurement of Washington's nose affects scale factor and hence all other lengths.

### THE SCALE FACTOR OF MT. RUSHMORE



Mount Rushmore, located in the Black Hills of South Dakota, is the masterpiece of Gutzon Borglum. To create Mt. Rushmore, Borglum made plaster models of each president based on life masks, paintings and photographs. He kept the plaster models on the mountain to guide him and the workers.

To transfer the models to the mountains, Borglum used a ratio of 1:12—one inch on the model equaled 12 inches on the mountain. Borglum created a "pointing" machine. The "pointing" machine was a metal shaft that was placed upright at the center of the top of the model's head (1). At the base of the shaft was a plate (2) marked like a protractor. A metal bar (3) attached to the shaft pivoted on the plate so Borglum could read the angle measurement. A plumb line (4) hung from the bar and slid back and forth to measure the distance from the head to the point on the model.

Each point on the model would receive 3 separate measurements (the degree of the angle, the distance to where the plumb is connected on the bar, and the length of the plumb line). A bigger "pointing" machine was placed on each president on the mountain. Borglum would determine the distances and lengths on the models and multiply them by 12 to transfer them to the mountain. Did he multiply the measure of the angle by 12, also? Why or why not?

Ιn	e scale factor of the models to the mountain is 1:12
10)	ne inch on the model equals twelve inches on the mountain)
1.	On the model, George Washington's nose is 20" long.
	How many inches long is his nose on the mountain?
2.	On the model, the length of George Washington's eye is 11".
	How many feet long is his eye on the mountain?
3.	On the model, the length of Abraham Lincoln's mouth is 1.5 feet.
	How many inches long is his mouth on the mountain?
	How many feet long is his mouth on the mountain?
4.	From Theodore Roosevelt's chin to his forehead is 60 feet on the mountain.
	How many inches is it from Roosevelt's chin to his forehead on the model?
5.	The height of an "average" human is about 7 to 8 times the length of his or her head.
	Confirm this by measuring your head and height
	If George Washington's head is 62 feet long, how tall would George Washington be it
	a figure were made from head to toe?
Us	se the picture from the front page and a ruler to answer the following
	testions.
-	George Washington's nose is 20 feet long on the mountain.
-	Use a ruler to find the scale factor of the picture to the mountain.
	·
	Picture: Mountain $\frac{1}{16}$ ": 20 ft $\frac{1}{16}$ ": ft $\frac{1}{16}$ ": in
	10 10
7.	Using the scale factor from above, find the length of Jefferson's nose on
	Mt. Rushmore.
	in feet: in inches:
8.	Find the length of Jefferson's mouth.
	in feet: in inches:
9.	Find the length of Lincoln's nose from the bridge to the tip.
	in feet: in inches:
10	Find the length from the outer corner of George Washington's left eye to the outer
	corner of his right eye.
	in feet: in inches:
11	. Could you consider these to be accurate measurements of the men on the mountain?
	Explain your answer.

# Mount Rushmore

The Scale Factor of Mt. Rushmore



# Connected Mathematics Project -Texas

#### WHAT IS CMP?

The Connected Mathematics Project (CMP) is a middle school curriculum created to help develop student knowledge and an understanding of mathematics that is rich in connections—connections within mathematics, between mathematics and other disciplines, and with applications of mathematical ideas outside the school. The assumption is that curriculum, instruction, and assessment must all work together for students to learn the powerful mathematics they need for their high school preparation and life skills. CMP was developed at Michigan State University with funding from the National Science Foundation to meet the needs of middle school students.

#### WHAT IS CMP-TEXAS?

In the summer 1996, with support from Michigan State University and the Texas SSI, seven sites from across Texas joined in an effort to implement CMP over the next three years as a complete curriculum for grades six through eight. School districts in the state are hungry for information concerning effective curriculum and instruction for middle school students. An increasing number of schools are responding to criticism by encouraging students to take the traditional algebra course as early as sixth grade. We predict CMP will appeal to many districts as a good alternative.

The Texas SSI sponsored a CMP-Texas conference during the summer of 1996 to introduce the CMP philosophy and sixth-grade materials to site coordinators and teams of three teachers and their principals from each of twenty-one campuses. During 1996-97 teachers will use the materials in their classes and lead other 6th-grade staff in the use of CMP replacement units. Next summer, the SSI will sponsor a conference primarily designed for seventh-grade teachers from the Texas sites to explore the CMP curriculum. Each site has a plan to implement the CMP curriculum fully on their campuses within three years.

CMP Texas sites and site coordinators:

• Austin ISD (3 schools)

Ron Gonzales

(512) 414-4350

· Corpus Christi ISD (1 school)

Julia Hankins

(512) 878-1400

• El Paso USI (8 schools)

Vodene Schultz

(915) 747-5778

Lubbock ISD (3 schools)

Pam Summers

(806) 766-1114

Mary Upton

(806) 766-1026

· North Lamar ISD (1 school)

Joyce Schaffer

(903) 737-2006

• Plano ISD (4 schools)

Jim Wohlgehagen

(972) 519-8160

• Region I ESC (1 school)

Noel Villarreal

(210) 383-5611

#### WHAT ARE INITIAL FACTORS FOR SUCCESS?

Based on other successful systemic change initiatives, this project will actively involve principals from the beginning, encourage community involvement and support, and enlist willing teachers in the pilot projects. It will also involve the entire mathematics departments in the process, and ensure that campuses provide support and time for teachers to learn the curriculum, instructional methods, and assessment in the CMP program. Finally, this project will ensure that participating teachers have time built into their academic schedules to interact with and support each other.

The Texas SSI is responsible for creating and supporting links between the various state sites. We envision a Texas network of CMP teachers and teacher leaders that will provide ongoing mutual support both within original sites and help with training as additional sites are added. The Texas SSI will serve as a common source of information, encourage collaboration among sites, and lead program evaluation efforts. To help accomplish this, the SSI sponsored a conference in September for sites to consider evaluation needs.

#### WHERE CAN I FIND OUT MORE?

For further information contact
Susan Hudson Hull, Mathematics Specialist
Texas Statewide Systemic Initiative
University of Texas at Austin
2613 Speedway
Austin, TX 78712
(512) 475-8153 fax (512) 475-8799
shhull@mail.utexas.edu

For more information about the Texas SSI and related activities, check the SSI web site: www-tenet.cc.utexas.edu/ssi/. For information from Michigan State University about CMP locales around the country: http://www.ns.msu.edu/CMP/cmp.html.

# PRESIDENTIAL AWARDS

The state winners of the National Science Foundation Presidential Awards were honored at the CAMT luncheon in Dallas. Three secondary and three elementary awardees were honored.

The secondary awardees are

- Cindy J. Boyd, mathematics teacher, Abilene High School, Abilene, Texas. Cindy teaches Geometry and Algebra.
- Norma Jost, mathematics teacher, Reagan High School, Austin, Texas. Norma taught AP Calculus, computer math and geometry until she was appointed this year to be a secondary mathematics specialist for the district.
- Diane Reed, Hanks High School, Ysleta, Texas.
   Diane teaches AP Calculus and algebra.

The elementary awardees are

- Judy S. Bishop, Saint MaryÆs Hall, San Antonio.
   Judy is an elementary mathematics specialist for grades 1 through 5.
- Katherine Eileen McCollum, James Starrett Elementary, Grand Prairie, Texas. Katherine teaches fifth grade math and serves as computer technology manager and teacher.
- Merlinda Rodriguez, Highland Park Elementary, Austin, Texas. Merlinda is a full-time third grade teacher in Austin ISD.

The national winners will be announced in November or December. The honored secondary and elementary teachers will be awarded with a week long trip to Washington, D.C. in the spring.

For information on how to nominate someone for this prestigious award contact Barbara Montalto at the Texas Education Agency, 1707 Congress, Austin, TX 78701.

# **Affiliated Group News**

The Texhoma Chapter of the Teachers of Mathematics October 24 meeting featured a panel of experts from kindergarten through college. The subject was "Questions You Have Always wanted to Know About Math and Were Afraid to Ask."

Information on the MAA of Texas may be reached from its web page, www.maa.org.

The Panhandle Area CTM September 28 conference was held at West Texas A&M University, Canyon, Texas.

Fort Bend CTM held its first meeting on October 10 with speaker Dr. Susan Williams, Assistant Professor University of Houston, and lead writer of the TEXTEAM Algebra institute. Their goals this year include providing scholarships for students, teachers returning for professional training, CAMT, and mini-grants for special projects. The new president for FBCTM is Cindy Calander from Fort Bend ISD

The Austin Area CTM spring meeting was hosted by Holt Rinehart and Winston at their Austin headquarters. Dinner was provided by HRW. Members elected new officers and shared ideas learned from attending the NCTM national conference in San Diego. The October meeting was held at Serrano's restaurant. Norma Jost, Austin ISD mathematics specialist, presented on the topic of the concept of area from first grade to calculus. Lake Travis ISD teachers showed how their vertical team presents the concept of rates from middle school to high school. The 1996-97 president is Cynthia Hays, McCallum High School, Austin ISD.

### **Calendar**

#### November 12

Fort Bend Council of Teachers of Mathematics meeting.

#### November 18

Austin Area Council of Teachers of Mathematics meeting hosted by Round Rock ISD , place to be announced. Topic: "Becoming a Math Coach"

#### January 11, 1997

Red River Council of Teachers of Mathematics Meeting, Red Lick School, FM 2148 (northwest of Texarkana. Feature Presentation by Cindy Boyd

#### January 24 and 25, 1997

Conference on the Teaching of Secondary Mathematics, Sam Houston State University, Huntsville, Texas. Contact Dr. Max Coleman for further information.

#### February 1, 1997

Texhoma Chapter Teachers of Mathematics meeting will feature Cindy Boyd and will include various presentations for elementary middle school, secondary and college teachers of mathematics.

#### February 2, 1997

Fort Bend Council of Teachers of Mathematics meeting.

#### February 8, 1997

Convocation of Faculty

Interested in Summer Institutes for High School Students

Southwest Texas State University, San Marcos

Speakers from all over country on successful programs, NSF, USA Math Talent Search, SUMA, MAA, SSI, Ohio State.

For more:

Max Warshauer

Department of Mathematics Southwest State University San Marcos, TX 78666 email: mw07@swt.edu Phone: 512/245-3499

#### March 1, 1997

Austin Area Council of Teachers of Mathematics Annual Mini conference, Mc Callum High School. Teachers will be provided an opportunity to attend sessions on mathematics teaching from kindergarten through calculus.

#### April 3-5, 1997

MAA Texas Section Meeting, Texas Lutheran University, Seguin, Texas

#### April 24, 1997

Fort Bend Council of Teachers of Mathematics banquet.

#### October 17-18, 1997

STEAM II

(Successfully Training Educators As Mathematicians) Texas A&M University-Texarkan/ Texarkana College

#### July 30-August 1, 1997

Conference for the Advancement of Mathematics Teaching (CAMT) is the annual mathematics education conference for the state of Texas, sponsored by the Texas Education agency, The Texas Council of Teachers of Mathematics, The Texas Association of Supervisors of Mathematics and the Texas Section of the MAA. Programs and registration materials will be mailed to persons who attended last years conference in April. TCTM members may request a program using a coupon which will appear in next springs journal.

#### February 12-14, 1998

Mathematics- Deep in the Heart of Teachers, NCTM Regional Conference, Dallas

# TCTM and Prentice Hall Scholarship Winners

The annual scholarship awards were announced at CAMT TCTM breakfast. The winners who were nominated by TCTM members are

- Allison Albrecht, B. F. Terry High School, Rosenberg, Texas, 77471 nominated by Jeanne Koonce
- Karen Lusk, Shallowater High School, Shallowater, Texas, nominated by Gay Bratton
- Tisha Carr, Canadian High School, Canadian, Texas, nominated by Karlyn Benge
- Crystal Emmons, Trinity High School, Trinity, Texas, nominated by Judy Bishop
- Rosendo Gonzalez, Sidneyy Lanier High School, San Antonio, Texas, nominated by Judy Wright

These awards a financed by a donation of \$2000 by Prentice Hall and supplemented by a TCTM donation. The scholarship application is included in this journal. The deadline for application is March 15, 1997.

# E. Glendadine Gibb Award and TCTM Leadership Award.

TCTM has an awards program to honor those who have contributed to the enrichment of mathematics education in Texas and in the nation. The awards were presented at the opening session of CAMT on in Dallas in August. Dr. Cathy Seeley, Texas SSI, was presented the E. Glenadine Gibb Achievement Award. This award is named in honor of Dr. Glenadine Gibb who was the first Texan to be president of the National Council of Teachers of Mathematics and was a professor of mathematics education at the University of Texas at Austin. Dr. Seeley has served on the board of directors of National Council of Teachers of Mathematics. and as Mathematics Supervisor for the Texas Education Agency. She was honored as an inspiration to mathematics teachers for her vision of mathematics education, for her leadership in the development of ideas for systemic change, and for her service as a mathematics education ambassador.

Bettye Forte, Fort Worth ISD, was awarded the Texas Council of Teachers of Mathematics Leadership Award. She was honored for her contributions to the mathematics teachers of Fort Worth and the state, and for her service to raise the level of mathematics instruction in Texas and for her efforts to empower teachers to deliver the best teaching environment.

TCTM will continue this program with awards to be presented at the opening session of CAMT in August of 1997. The E. Glenadine Gibb Award is presented to someone who is nominated by a TCTM member and who is to be honored for his or her contribution to the imporovement of mathematics education at the state and/or national level. The TCTM Leadership Award will be present to someone who is to be honored for contributions at the local and state level for who was designed innovative staff development and lessons and who has promoted the local mathematics council. The person must be nominated by a local NCTM affiliate. Nomination forms are available in this TCTM journal. The deadline for application submission is February 15, 1997.

# **Team For the Future**

A Team for the Future, a conference for the preparation of the secondary mathematics teacher, was held on October 3-6 at the Inn on Lake Travis, Lago Vista. The conference was sponsored by the Texas Statewide Systemic Initiative Algebra Action Team. The conference was coordinated by Dr. David Molina, SSI, Pam Alexander, Stephen F. Austin State University, and Diane McGowan, Austin ISD.

Thirty-three teams of three or four members from a university or college, public school administration and

secondary teachers, participated in discussions on issues and concerns, discussed the Texas Essential Knowledge and Skills for Algebra and Geometry, and formulated plans to improve communication between university and secondary educators. They saw presentations on the TEKS in practice by Mary Selcer, College Station ISD, Manipulatives in Algebra; Diane McGowan, Austin ISD, Rate of Change and Linear Functions; Cynthia Hays, Austin ISD, excerpt from the Geometry Institute; and Dr. Susan Williams and Cindy Boyd, an excerpt from the Algebra Institute.

Dr. Uri Treisman, University of Texas professor and director of the Texas SSI, spoke to the participants on mathematics education in Texas. A panel of educators answered questions on current issues and practices. Panel members, Dr. Paul Kennedy, Southwest Texas State University, Dr. Lloy Lizcano, Austin ISD, Dr. Jean Jones, Texas A&M at Texarkana, Dr. Jim Wohgehagen, Plano ISD, and Wendy Reyes, Austin ISD shared their ideas and concerns.

The major areas of concerns which developed in small group discussions were training for secondary and university professors, money for funding activities, development of secondary mathematics methods courses to meet the particular needs of the mathematics teacher, vertical alignment of curriculum, and communication between the secondary teacher and the college educator. Teams set goals for future communication and curriculum development. The conference is the beginning activity to improve the preparation of the secondary mathematics teacher in Texas.

## **Ballot**

Mary Jane Smith who was elected to the position of Vice President for Elementary has moved to California and resigned her position.

Judy S. Bishop has been nominated to serve in this position.

Judy has a BS and M.Ed. from Southwest Texas State University. She has 27 years teaching experience. She has taught at Saint Mary's Hall for 23 years and served as the Campus Level Elementary Mathematics Specialist for the past 3 years. She was the 1992 Exemplary Mathematics teacher for the Alamo District Council of Teachers of Mathematics.

She is a Freelance writer for Psychological Corporation, a TEXTEAM trainer, and a presenter at CAMT for the past ten years.

Please mail this ballot to	Basia Hall,	12306 Piping Rock, Houston,	TX 77077.
----------------------------	-------------	-----------------------------	-----------

Vice President for Elementary Ballot

	Judy	' Bis	hop
--	------	-------	-----

#### TEXAS COUNCIL OF TEACHERS OF MATHEMATICS PRIVATE §

#### MATHEMATICS SPECIALIST SCHOLARSHIP

Amount:	\$1000 or \$500		
Application Deadline:	March 15, 1997		er for getting of the section of the
Eligibility: Any student who will in college in the Fall of 1997 to school or as a secondary school	pursue a career in mathematics	teaching either as a mat	private - and who plans to enroll hematics specialist in elementary
Return all materials in one enve	lope to:		Basia Rinesmith Hall 12306 Piping Rock
			Houston, TX 77077-5916
NIANE.		A Company	
NAME:	First	Middle	
Address:			
Number and Street	Apt. Number		
City	Zip Code		
Phone Number: ()			
Birth date:		10 - 12 - 12 - 12 - 12 - 12 - 12 - 12 -	
Social Security Number:			of the first of the second of
•			
High School(s) Attended:			
What college or university do y check directly to the business o			
oncor directly to the outsited o	2 01 mic voltago. 110 mode me		

To apply, enclose the completed application with each of the following in the same envelope and mail to Basia Hall at the address listed above. YOU MUST INCLUDE 3 COPIES OF ALL REQUIRED MATERIALS.

- 1. List High School Activities including any leadership positions on a separate sheet,
- 2. Official High School Transcript,
- 3. Letter of recommendation from a TCTM member,
- 4. An essay describing your early experiences learning mathematics and any experiences explaining math matics to your classmates or friends. This essay must be no more than two pages, double spaced.
- 5. An essay telling why you want to be a mathematics specialist in elementary school or a mathematics teacher in middle or high school. This essay must be no more than one page, double spaced.

# TCTM LEADERSHIP AWARD APPLICATION

The TCTM Leadership Award is presented to a TCTM member who is nominated by a TCTM -Affiliated Group. This person is to be honored for his/her contribution to the improvement of mathematics education at the local and state level. He/she has designed innovative staff development, and has promoted the local TCTM-Affiliated mathematics council.

Information	about the TCTM-Affiliate	d Group nomina	ating a candidate:	•
	ted Group:			_
President of Co	uncil:		: 	_
Home Address:			·	_
Home phone	Work Phone	E-ma	iil	<del>-</del>
Are you a mem	ber or TCTM? 1	NCTM?		
Information	about the nominee:		-	
Name:				
Home Address:				
	Work Phone			
Is the nominee	a member or TCTM?	NCTM?	Retired?	_
Applications sh	•	ee ivities ces or committees traging involvment/inent s rating why the nomin	mprovement of math of	
Send completed	applications to:			
	Basia Rinesmith	<del></del>		
	12306 Piping R	ock Drive		

Houston, TX 77077-5916

42

Deadline: February 15, 1997

# Glenadine Gibb Achievement Award Application

The E. Glenadine Gibb Achievement Award is presented to someone nominated by a TCTM member to be honored for his/her contribution to the improvement of mathematics education at the state and/or national level.

Information	about the TCTM memb	er nominating	a candidate:	
Name:				<u>-</u>
Home Address:		ş ·-		_
Home phone	Work Phone		E-mail	<u>.                                    </u>
Are you a mem	ber or TCTM?	NCTM?		
Information	about the nominee:			
Name:				<del>-</del>
Home Address:			©\$	
Home phone	Work Phone		E-mail	<u> </u>
Is the nominee	a member or TCTM?	NCTM?	Retired?	<u></u>
Applications sh	State TCTM Local TCTM Staff Develo Honors / Aw	ninee activities ces or committees offices held I-Affiliated Group pment ards dicating why the r	offices held nominee should be hono	red for his/her contribution to the
Send complete	d applications to:	•	•	9 (a) 
.       •	Basia Rinesr 12306 Piping	nith Hall g Rock Drive 1.77077-5916		

Deadline: February 15, 1997

# TCTM MEMBERSHIP APPLICATION

Last Name		First	Name	Sc	hool or Unive	ersity
		W 1011(A # 1011	The second section is a second	使用整理性度,基本的人员。 -	. เดริกับระโดกใช้เมื	
MAILING AD	DRESS	· .		· .	•	್ಕ್ ಕ್ರಿ
	~ .		n in the second of the second of	eran e		
		(a.)	dit	La State House		Service Francis
Street Address		City	La La Marian	State	will per many	Zip
email address _				to professor	n gant og garafig	and the engineering
		4 may =				
CIRCLE AREA	A(S) OF INTEREST:	7 g		4		Contract to the section
K-2	3-5	igati Sigaraga	9-12	an garage en	College	
CHECK ONE:						
<b></b>	NT.	· • • • • • • • • • • • • • • • • • • •	CII.			÷
Renewal	. New	Member	•	of Address 📙		
			·			
	inuous membership, ple				is application	with your check
ffor \$10.00 for	one year payable to To	CTM and mail to	):			
	TCTM		\$ 1.50 m	For the		
·	Barbara Polnick					
	#3 Ridgeway Rd, Wo	odgate Place	1.4			
	Conroe, TX 77303					

Contributions for the spring addition of the journal must be mailed to

Dr. Paul Kennedy
Department of Mathematics
Southwest Texas State University
San Marcos, TX 78666
by January 15, 1997.

#### TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the

National Council of Teachers of Mathematics 1996-1997

President

Basia Hall 12306 Piping Rock Houston, TX 77077 basia @tenet.edu

Past -President

Diane McGowan 4511 Langtry Austin, TX 78749 dmcgowan @tenet.edu

Vice-President, Elementary

Mary Jane Smith 1810 Brushy Creek Dr. Sugarland, TX 77478 mjsmith @tenet.edu

Vice President for Secondary

Pam Wisdom 3508 Gary Drive Plano, TX 75023 pwisdom @tenet.edu

**Northeast Regional Director** 

Olene Brame 3441 Boulder Drive Dallas, TX 75233 obrame @ tenet.edu

**Northwest Regional Director** 

Edith Hays 2120 Crestwood Denton, TX 76201 F\_HAYS @TWU.EDU **Southeast Regional Director** 

Evelyn Dixon 11915 Meadowdale Stafford, TX 7747

**Southwest Regional Director** 

Nora Munguia Rt 1 Box 143M Mission, TX 78572

Secretary

Carol Williams 857 Vista Lane Abilene, TX 79601

Treasurer

Barbara Polnick P.O. Box 125, Chappell Hill, TX 77426-0125 bpolnick @tenet.edu

**NCTM Representative** 

Judy Rice 2400 Old South Drive #707 Richmond, TX 77469

Parliamentarian

Diane Butler 4822 Rollingwood Austin, TX 78746

**Business Manager** 

Dr. Lloy Lizcano 3806 Frodo Cove Austin, TX 78749 lloy @tenet.edu TEA Consultant

Bill Hopkins 11700 Clivden Circle Austin TX 78759 bhopin @ tenet.edu

**NCTM Regional Services** 

Representative Cindy Boyd 2502 Rountree Abilene, TX 79601

**Government Relations** 

Represent ative Dr. Kathleeen Mittag 3400 Magic #159 San Antonio, TX 78229 kmittag@ lonestar.jpl.utsa.edu

**CAMT Board** 

Rachel Pinkston 10618 Caravan Drive Houston TX 77031-1813 pinky@tenet.edu

Journal Editor

Dr. Paul Kennedy 1722 Sunnybrook New Braunfels, TX 78130` pko3@SWT.edu

**Director of Publications** 

Diane McGowan 4511 Langtry Austin, TX 78749 dmcgowan @tenet.edu

#### **Texas Council**

#### of Teachers of Mathematics

Member 1996-1997

NAME \_\_

## TEXAS MATHEMATICS TEACHER

Dr. Paul Kennedy Department of Mathematics Southwest Texas State University San Marcos, TX 78666 Bulk Rate
U.S. Postage
PAID
Austin, TX
Permit No. 1571

11/96

DIANE MC GOWAN 4511 Langtry Austin, TX 78749