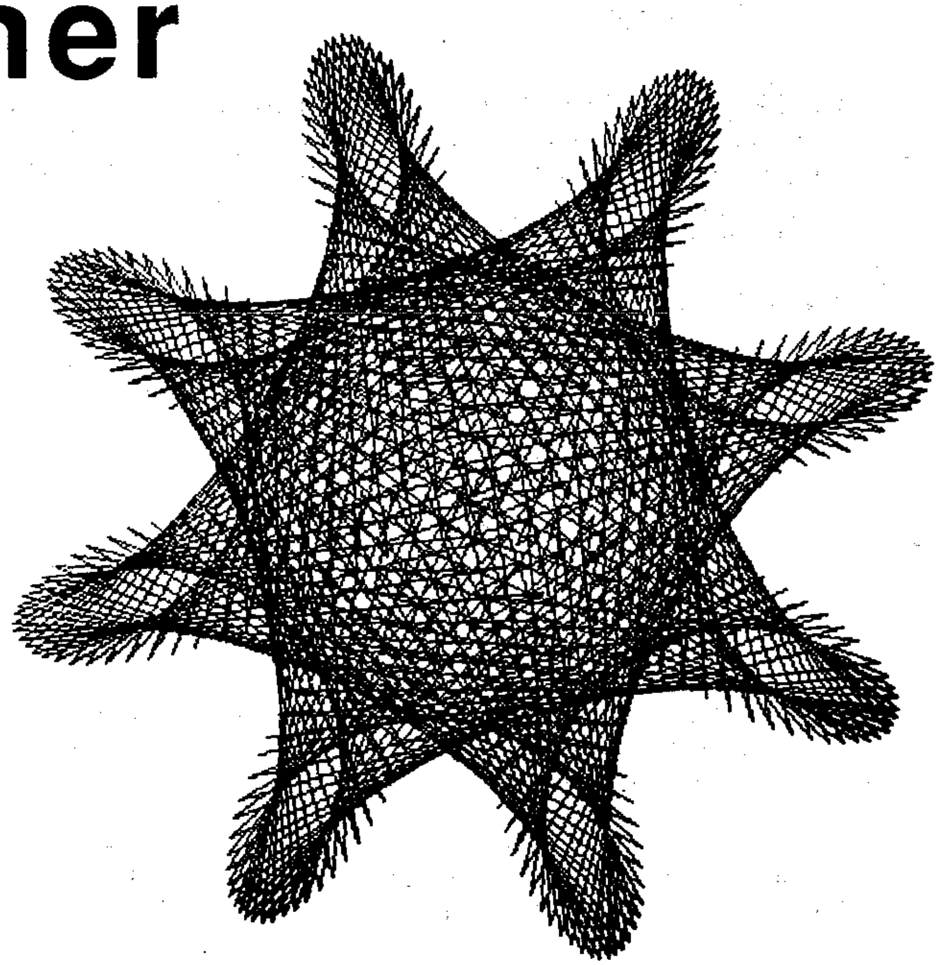
The Texas Mathematics Teacher



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The **TEXAS MATHEMATICS TEACHER**, the official journal of the Texas Council of Teachers of Mathematics, is published four times each year, in October, January, March, and May. Editorial correspondence and regular article manuscripts should be addressed to the editor, as listed above. Manuscripts for the *Voices from the Classroom* section should be addressed to the appropriate Associate Editor(s).

Potential authors should adhere to the following guidelines:

(1) An article for *Voices from the Classroom* should be relatively short, and contain a description of the activity sufficient in detail to allow readers to incorporate it into their teaching, including a discussion of appropriate grade level and prerequisites for the lesson. Whenever possible, these articles should include camera-ready activity sheets that can be directly photocopied by classroom teachers.
(2) Manuscripts should be word-processed or neatly type-written, double-spaced with wide margins on 8 1/2"-by-11" paper. No author identification should appear on the manuscript. Illustrations should be carefully prepared in black ink on separate sheets, the original without lettering and one copy with lettering added.
(3) Submit the original and four copies. If possible, please include a Macintosh or IBM 3 1/2" diskette containing the manuscript.
(4) Include a cover letter containing the following information: author(s) name, address, and phone number; intended level; and, if a disk is submitted, name and version number of computer word-processing or graphics packages used.

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We also need
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(2) Dates, times, and contact people for activities, workshops, and conferences that would be of interest to mathematics teachers.
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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

May 1995

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MESSAGE FROM THE PRESIDENT

I have decided to focus on change for my first message to you. Changes are occurring with the Texas Mathematics Teacher journal. This change was forced by the resignation of long-time journal editor George Wilson. We greatly appreciate his service to the Texas Council of Teachers of Mathematics and to the mathematics educators of Texas. Vince Schielack of Texas A&M University will serve as journal editor. We have decided to add some new material to the journal. The journal will include contributions from teachers for classroom activities and information from local affiliated councils of teachers of mathematics regarding upcoming activities. Please contact the president of your local organization to submit information for the next journal to the TCTM regional director for your area. We want to make the journal a vehicle for communication between affiliated groups and for sharing effective teaching strategies. Articles on specific teaching activities at any level or on effective ways to deal with block scheduling or school clusters would be appreciated by other teachers. Please submit your teaching ideas to Dr. Schielack or the appropriate Associate Editors.

Changes are occurring in mathematics education in the state of Texas. Change is not made just for the sake of change, but because our teaching environment has evolved. The students we have in our classroom are not like those we had twenty, ten, or even five years ago. Technology has changed not only how we teach, but what should be included and what no longer needs to be included in the curriculum. Mathematics teachers are being forced to change not only what they teach, but the way they teach. High school teachers must now teach algebra to everyone. Elementary and middle school teachers are being challenged to prepare all students for algebra. Inclusion has brought more students into the regular classroom. We are being asked to try alternate forms of assessment of student progress. Block scheduling has brought challenges to teachers. The same techniques and material that we have been using for many years are no longer as effective.

We must all be willing to try new methods and approaches, but we should not feel alone in our efforts. We can work together with other teachers to effectively alter our teaching strategies and methods of assessment. These changes should be initiated with prospective teachers in their university education. Thus, professors of mathematics education can help make changes by aligning their programs with the NCTM Standards.

Fortunately there are many resources available to help us make changes. The Statewide Systemic Initiative will help guide teachers and school districts to make effective changes. The Texas Mathematics Staff Development project has provided teachers with professional development activities through the modules. The twenty Texas Educational Regional Service Centers provide guidance to teachers and schools. The Conference for the Advancement of Mathematics Teaching provides a wealth of ideas from teachers who have used effective approaches in their classrooms. Plan to attend CAMT next summer in San Antonio on August 2, 3, and 4. The Regional NCTM conference will be held in Houston November 9, 10, and 11. We will all need to work together to face the challenge of changing mathematics education in Texas.

Diane McGowan
TCTM President

Ideas for Motivating Children to Learn Mathematics and Science

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Motivation is lacking in many classrooms because the complexity of many activities is too high. In other instances, children are almost programmed for failure because activities are not interesting enough. Without active participation, learning is unlikely to occur. Some motivational techniques are listed below that may provide children with more positive learning experiences.

1. Create a young scientist and mathematician club. Select a future scientist and a mathematician each week. These students will lead the class in an enjoyable science activity or game. Books such as *175 Science Experiences to Amaze Your Friends* by Brenda Walpole can be very helpful.
2. Select one student to be the teaching assistant for the week. In a large class, the teacher might need two assistants each week. The thoughtful, caring teacher will make sure that every child gets a turn to be the teaching assistant.
3. Integrate art and the mathematics and science activities as much as possible. Curve stitching is an excellent way to incorporate this idea.
4. Let the students compose songs or raps about the mathematics and science concepts being studied. Many times, this helps to reinforce concepts that would otherwise be very difficult. Let students as a class create their own mnemonic devices to aid in the reinforcement of concepts.
5. Allow students to be creative by composing poems and other short descriptive writings about the current mathematics and science classroom topics.
6. Present career role models from the mathematics and science fields. Community leaders are often eager to visit classrooms. Invite local engineers, physicians, research technicians, etc.
7. Present several ideas for experiments and let students select the one they wish to do. Be sure to actively involve students with the planning and implementation of the experiments. Also, be sure to provide students with an opportunity to evaluate the activity as well as their performance. It is important to let students know that their ideas are important and valued. Use their suggestions as soon and as often as possible.

8. Let the class compose a monthly science and mathematics newsletter. In this newsletter, each student should have an opportunity to express thoughts about a science/mathematics activity that occurred during the month.
9. The teacher should have a camera available to take pictures of students participating in their science/mathematics activities. Display the pictures on the school bulletin board. Then let the students place them in a class science scrapbook. (An alternative plan would be to ask each student to provide two rolls of instant develop film at the beginning of the year. Then at the end of the year, divide the pictures among the students.)
10. Video-tape as many of the mathematics and science activities as possible. Place the videos in a library-type checkout system. Thus, children can share with parents the excitement of mathematics and science activities. At the end of the year, the videos should be edited and placed in the school library for future use.

Even with ideas such as those listed above, many children may not be able to experience success in the areas of mathematics and science because the learning experiences are improperly planned. Therefore, some suggestions to aid in the planning and implementation of the ideas follow.

When planning and implementing effective learning experiences for children, several factors must be considered. The psychomotor, cognitive, language development, and learning styles of the child must be considered, as well as the inclusion of appropriate motivation (K. Dunn and R. Dunn, 1978; R. Dunn, 1983; Morrisom, 1988). If these factors are observed, a teacher will be able to provide activities and motivational techniques that are more appropriate and effective. Activities and materials that progress from the very simple to the more complex will be most effective when implementing ideas (Lefrancois, 1992; Smith, 1988). Activities that keep the student actively engaged in stimulating learning experiences will be most effective (Harlan, 1988; Schultz, Colarusso, and Strawderman, 1989).

Effects of improperly presented activities may be long-lasting, if the result is a negative attitude. This negative attitude could lead to low self-esteem and even hamper the student's ability to perform well (Kamii and DeClark, 1985). To avoid problems with self-esteem, teachers must be willing to have learning centers and activities that the students can experience without being concerned about right and wrong answers (Smith, 1988). Better classroom management is likely if the students do not have a high level of concern because the activities are only product-oriented. An indirect benefit of the activities is that children are using higher-level thinking skills and not just memorizing facts.

While implementing ideas such as the ones listed, efforts to present mathematics and science may become more effective and enjoyable. The teacher's attitude most likely will become more positive and enthusiastic, and this positive attitude may influence the level of motivation. Remember, enthusiasm is contagious and it shows and shows and shows ... (in the eyes and actions of successful children).

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Meaningful Contexts, Constructivism, and the Teaching of Mixed-number Fractions

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Introduction and Purpose

The mathematics curriculum "should emphasize the development of children's mathematical thinking and reasoning abilities" and decrease the emphasis placed upon "memorizing rules and algorithms" (National Council of Teachers of Mathematics (NCTM), 1989, pp. 18 and 70). In a constructivist approach to teaching, children would be active participants in both the creation and acquisition of new knowledge, as opposed to passive recipients of cognitive structures invented and communicated by others (Clements and Battista, 1990). Perhaps nowhere in the mathematics curriculum is instruction so heavily algorithmatized as it is in the teaching of operations with common fractions and mixed-numbers. It is the purpose of this article to report the results of an investigation which placed 133 sixth-, seventh-, and eighth-grade children in open problem-solving situations involving mixed-number fractions.

Fostering creativity and providing children with open-ended problem-solving opportunities to invent new knowledge can pose many problems for the classroom teacher. One difficulty is finding a source for these problems. Kamii and Lewis (1990) advocate using situations from daily living that are thought to be of interest or relevant to the children. An important dimension of the model for understanding mathematics proposed by Miller (Miller and Kandl, 1991; Miller, Malone, and Kandl, 1992) involves acquainting students with the knowledge of knowing when to apply the mathematics they are learning.

Background of the Students

The current investigation involved 133 sixth-, seventh-, and eighth-grade students who, with the possible exception of a few transfer students, had been introduced to traditional algorithms for multiplying and dividing mixed-number fractions in fifth grade. The algorithms taught to the students involved changing the representation of the mixed-number fraction to an improper fraction, and then following the traditional algorithms for multiplying and dividing common and improper fractions. At the time of the administration of the inventories all students had reviewed this material earlier in the current school year, but not in explicit preparation for the inventories. The regular teachers were not advised of the content addressed by the inventories. Thus, no special reviews of this material had been conducted as they often are for standardized testing at the end of the school year.

Methodology

The students were provided with three-page inventories. The first page contained four symbolic items involving mixed-number fractions, such as "$5\ 1/4 \times 18 =$ ______". Two of the items involved multiplication and two involved division. These items were administered to assess students' knowledge of the traditional algorithms. The second page of the inventory contained three verbal items involving mixed-number fractions that could be solved using the traditional multiplication algorithm, and the third page contained three verbal problems that could be solved using the traditional division algorithm. The students were not permitted to return to the symbolic items on the inventory after beginning the verbal items. Table 1 contains some sample verbal items.

Table 1. Some Sample Inventory Items

1. Sally and her dad collect aluminum cans to recycle. They have 6 1/4 boxes of cans with 28 cans per box. How many cans do they have altogether?
2. Billy works at a camera store. In the back room he has 8 1/3 cases of cameras. Each case contains 24 cameras. How many cameras does he have?
3. Maria works in a restaurant. She has 8 1/2 pies to cut for selling. Each pie will be cut into pieces which are 1/6 of the whole pie. How many pieces will she get from the pies?
4. Joe has 4 3/4 pizzas for his party. Each pizza will be cut into pieces which are 1/8 of the whole pizza. How many pieces of pizza will he have for his party?

Results

The students were coded as demonstrating some level of mastery of the traditional multiplication algorithm if it was employed successfully on at least one of the two symbolic multiplication items. Only 15 of the 133 students, mostly eighth-graders, were coded as having some mastery of the traditional multiplication algorithm. Students were coded as having mastered the traditional division algorithm if they successfully used it to solve at least one of the two symbolic division items. Only nine students, all eighth-graders, were so coded in regard to the traditional division algorithm. It is important to note that no other methods were used successfully to solve the symbolic items.

The most interesting results were obtained when students' performances on the verbal items were evaluated. First, as might be expected, none of the children who forgot to use the traditional algorithms on the symbolic items switched to using these algorithms on the verbal items. Additionally, one disappointing result was that only seven of the 15 students who did employ the traditional multiplication algorithm when solving the symbolic items applied it to the verbal multiplication items. On the verbal division items only two of the nine students who employed their traditional algorithm on the symbolic items used it in the solution process. Thus, of the 133 students, most of them did not exhibit mastery of the traditional algorithms for solving multiplication and division items involving mixed- number fractions, and the few who did employ it generally did not extend its use to address appropriate application items when given an opportunity to do so.

What many of the students *did* do was to invent their own algorithm. Consider item 1 from Table 1. Instead of changing the mixed-number "6 1/4" to "25/4", the students first worked with the six cases that were full. They multiplied "6×28" to produce the partial result of "168". They then computed "1/4 of 28", producing "7", and added the two partial results. They used the same approach with the division items, first working with the whole pies to see how many pieces would result, and then combining this result with the number of pieces obtained from the partial pie. Each of the teachers whose students participated in the investigation were questioned regarding this alternate algorithm and all maintained that it had never been a part of regular instruction. In fact, the textbook series used by the school contained very few verbal items, and the teachers confirmed that the students had little in-class experience with verbal items. There were variations of this non-traditional approach, such as picturing a pizza and cutting it. However, all variations involved the general approach of first working with the whole units, and then working with the fractional component.

Of the 133 students, 24 successfully employed the non-traditional algorithm on the verbal multiplication items, and another 54 students set the items up correctly but did not produce a correct result. There were 31 students who successfully employed the non-traditional algorithm on the division items, and another 16 students who set the items up correctly without producing a correct result. While it cannot be claimed that these students "invented" the non-traditional algorithm during this investigation, it is apparently the case that the students' insight was not gained through formal instruction in the use of this method.

Implications for Instruction

The students' inability to recall the traditional algorithms is not surprising even though this material is continuously reviewed from sixth-grade through general mathematics courses in high school. Given the dearth of application items in many textbooks it is also not surprising that most students who did employ the traditional algorithms did not apply them to the verbal items. What is interesting, and most encouraging, is the students' success on the verbal items with methods of their own choosing. Instead of being placed at the end of a unit, perhaps these items should be used to introduce the unit. In doing so, two important objectives can be accomplished. First, the items suggest to the students why it is important to be able to work with mixed-number fractions. Brown (1986) has suggested that both minority students and females perform better when the relevance of the material is made clearer. Second, it is reasonable to assume that students will bring to the classroom many previously acquired skills and experiences that will relate to the content being introduced. Educators can gain insight into the students' knowledge by placing them in open problem-solving situations and determining what insights the students already possess. Formal classroom instruction should then build upon this foundation. These open problem-solving opportunities foster the development of alternative methods of working problems. Too often mathematics students acquire the misconception that not only is there only one correct answer, but only one appropriate way to solve a problem. It is also the case that what students invent or create for themselves will probably be better understood and retained longer than knowledge that is conveyed in other ways.

The results of this investigation do not necessarily mean that traditional algorithms should be abandoned. However, the role and importance of the traditional, symbolic algorithms should change to some extent in a constructivist approach to the curriculum. For most students, these algorithms will remain the ultimate goal of instruction, and in a sense will provide closure to the study of mathematical content. These algorithms represent fast, efficient methods of working certain classes of problems. However, some students invariably will not have sufficient exposure to these algorithms to remember them when the need arises. Other students may not possess the academic talents necessary to acquire them at all. If a student does encounter a need for a formal algorithm, but is unsure of it, having had the opportunity to build a meaningful foundation to fall back on may be sufficient to overcome the problems faced. For those students not as talented as others, the opportunity to build at least some skills in a constructivist approach may be all the students gain from instruction.

Conclusion

The investigation reported here represents an important "action study" that teachers can conduct in their own classrooms. Students do bring to their classrooms powerful knowledge and experiences that can be used to introduce and develop new material more effectively. A constructivist approach to instruction that places students in meaningful and interesting real life situations and provides opportunities for students to invent their own methods should be at the heart of classroom instruction. Previous research efforts have suggested that this approach will better address the needs of all students, especially female and minority students, and provide each student with a better opportunity to develop the talent that she or he possesses.

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A New Look at Summing Arithmetic Sequences

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The sum of the first n terms of an arithmetic sequence can be obtained by the following well-known formula:

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right].$$

This formula can also be used to find the sum of the first n terms of the natural numbers

$$S_n = \sum_{k=1}^{n} k$$

since

$$S_n = \frac{n}{2}\left[2(1) + (n-1)\right] = \frac{n(n+1)}{2}$$

where $a_1 = 1$ and $d = 1, \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

Rudd and Shell (1990) have proven formulas for arithmetic sums of the first n positive integers by mathematical induction. The authors present the following method for the sum of the arithmetic progression.

Let us consider the sum of n ones:

$$\sum_{k=1}^{n} 1 = n. \tag{1}$$

Squaring both sides of number (1) we obtain:

$$\left[\sum_{k=1}^{n} 1\right]^2 = n^2.$$

Then the left side will be

$$
\begin{array}{rcl}
 & & 1 + 1 + \cdots\cdots\cdots\cdots + 1\\
 & & 1 + 1 + \cdots\cdots\cdots\cdots + 1\\
\hline
1 & = & 1 + 1 + 1 + 1 + \cdots\cdots + 1\\
3 & = & 1 + 1 + 1 + 1 + \cdots\cdots + 1\\
5 & = & 1 + 1 + 1 + 1 + \cdots\cdots + 1\\
\vdots & & \vdots\\
(2n-1) & = & 1 + 1 + 1 + 1 + \cdots\cdots + 1
\end{array}
$$

The sum of the ones in the spaces of backward L's gives

$$\sum_{k=1}^{n}(2k-1) = n^2. \tag{2}$$

Observe that the last row and column have $(2n-1)$ ones since the corner one is counted only once.

Now, adding equations (1) and (2), and using methods similar to Allison (1993), one has

$$\sum_{k=1}^{n}(2k-1) + \sum_{k=1}^{n} 1 = n^2 + n,$$

where the left side will be

$$\sum_{k=1}^{n} 2k = n^2 + n. \tag{3}$$

Dividing both sides of number (3) by (2) we obtain

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}. \tag{4}$$

Squaring both sides of equation (4) we get

$$\left[\sum_{k=1}^{n} k\right]^2 = \left[\frac{n(n+1)}{2}\right]^2. \tag{5}$$

The expansion of the left side of equation (5) will result in

$$\begin{array}{cccccccccccccc}
 & & 1 & + & 2 & + & 3 & + & 4 & + & \cdots & + & n \\
 & & 1 & + & 2 & + & 3 & + & 4 & + & \cdots & + & n \\
\hline
1^3 & = & 1 & + & 2 & + & 3 & + & 4 & + & \cdots & + & n \\
2^3 & = & 2 & + & 4 & + & 6 & + & 8 & + & \cdots & + & 2n \\
3^3 & = & 3 & + & 6 & + & 9 & + & 12 & + & \cdots & + & 3n \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\
n^3 & = & n & + & 2n & + & 3n & + & \cdot & \cdot & \cdots & + & n^2
\end{array}$$

Looking at the sum in the "n^3" backward "L", we have

$$\begin{aligned}
&2n + 2(2n) + 2(3n) + \ldots + 2[(n-1)n] + n^2 \\
&= n\left[\sum_{k=1}^{n-1} k\right] + n\left[\sum_{k=1}^{n} k\right] \\
&= n\left[\frac{n(n-1)}{2}\right] + n\left[\frac{n(n+1)}{2}\right] = n^3.
\end{aligned}$$

Again, summing the numbers in the spaces of backward L's produces the familiar sum

$$\sum_{k=1}^{n} k^3 = \left[\frac{n(n+1)}{2}\right]^2.$$

The authors leave it to the reader to prove the details of the following formula for the summation using the methods given here:

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3+3n^2+n}{6}.$$

It is relatively simple to find a summation that sums to n^3 using the product of the summations for n^2 and n. Then, find the summations for $2n^3, 3n^2$, and n; divide the sum of the summations by 6 to get the desired result.

It is now easy to see using these methods how one can develop a wide variety of new formulas for arithmetic and non-arithmetic summations.

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Half-Time: Using Children's Books to Explore The Concept of One-half

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Have you ever given half a thought to the number of ways we use the work "half" in the English language? I'll bet you a half-dollar you can't get halfway down the block before you've thought of a half-dozen phrases that contain the word "half." Half a glance at an English language dictionary demonstrates the various ways the concept of a "half" is used. In sports, for example, we find a halfback, a half-nelson, a half-volley, and a half-gainer. Monetary units have often referred to half-portions, with such terms as a halfpenny, a halfcrown, a half-eagle, and a half-dime. And these ideas are just the half of it! It's no half-baked notion to think that, as a whole, we should give the idea of "half" more than just a half-hearted review. Several children's trade books have been selected and activities are described which allow students to explore the concept of one-half. The activities have been organized into three levels: Prekindergarten, kindergarten, and first grade; second and third grade; and fourth and fifth grade. With some modifications, these activities could be adjusted for use at other levels. A resource list of additional trade books which may be used to teach fractional concepts is included.

"Half and Half" (for Levels PreK, K, and 1)

Literature:

McMillan, Bruce. *Eating Fractions*. New York: Scholastic, 1991. (Note: A Big Book version of this book is available.)

Wood, Don & Audrey. *The Little Mouse, The Red Ripe Strawberry, and the Big Hungry Bear*. New York: Child's Play, 1984.

Background:

Bruce McMillan's book of colorful photographs of two children involved in dividing food items into equal portions is the beginning of an activity which allows children to explore the possibilities of separating into halves. The humorous story and illustrations by Don and Audrey Wood provide a context for children to create their own visions of sharing.

Objectives:

The children will work in small groups to divide a basket of fruit equally in half. Each group will represent on paper and share how they divided each of the fruits in their basket.

Directions:

1. Read aloud the book *Eating Fractions* and discuss the methods by which the two children divided each of the food items. Do you think they shared equally? Why do you think it was important for the children to divide the things equally? Are there other ways in which the foods could have been shared? Why would people need to know how to divide things in half? Why would people need to divide things in thirds or fourths?
2. Show the children a few sample food items such as a slice of bread, several crackers, and a bun. These could be real or made from paper. Ask the children to discuss some ways each of these food items might be divided in half to share between two people. After obtaining several suggestions, you should actually divide the items as directed by the students. Have two students actually share the food items. How do you feel about this way of dividing the food to share? Do we have equal parts? Did this way help us to give half to each of the two people?
3. Read aloud and discuss *The Little Mouse ...* with the children. With whom was the mouse sharing the strawberry? Into how many pieces did the mouse cut the strawberry? When we have two equal parts, we call each a *half*. Discuss whether you think this was a fair way to share. How could the strawberry have been divided in half a different way?
4. The children will work in pairs or small groups to divide a selection of fruits in a basket in half. While dividing real fruits in half would be more fun and motivating (the students would actually get to eat what they had shared with someone else!), paper models could be substituted. Whole fruits would be used with younger children. A small variety of fruits, such as a banana, half an apple, two strawberries, and four grapes should be included in each basket for more experienced learners. A small plastic knife (for real fruit) or scissors (for paper fruits) should be provided for each group. As the groups are working, the teacher can walk around to observe and ask some questions of each group.
   - How did you decide on a way to divide that kind of fruit?
   - Do you think you've shared it equally? How do you know?

- What would you do if you had more (or bigger) fruit?
- Are there other ways you might have divided that fruit?
- What would you do if you cut it and then found it wasn't half?

5. Each group will be given one large 11" × 17" piece of paper. The groups should find a way to present their processes for how they divided each of the fruits in their baskets. While many young children will prefer to draw pictures, some learners may use numbers or words to show how they shared. If paper models were used to share, they may be pasted and labeled on the paper.
6. Call all of the students together for a sharing time. Each of the groups should be given an opportunity to share the methods they used for dividing each of the fruits in half. Did everyone share the fruits in the same ways? Which way to divide in half was new to you? Can you think of other fruits which could be divided in half this way?

Extensions:

Encourage the children to be on the lookout for other times when we talk about a half. They may notice, for example, how a paper is folded in half. Milk often comes in half-pint cartons. Children might hear about a half-hour or a half-moon. Where else do we use the word "half"?

"Halfway" (for Levels 2 and 3

Literature:

Pomerantz, Charlotte. *The Half-Birthday Party*. New York: Houghton-Mifflin, 1984.

Background:

Daniel plans a half-birthday party for his sister Katie when she is six months old. Each of the guests bring half a present, such as half of a birthday cake from each grandparent, half of a set of earrings, and half of a pair of slippers. Daniel realizes that he was so busy planning the party that he forgot to bring a present. He surprises everyone (including himself) with his last-minute gift of a half-moon. This book initiates a collection of data about class birthdays and half-birthdays.

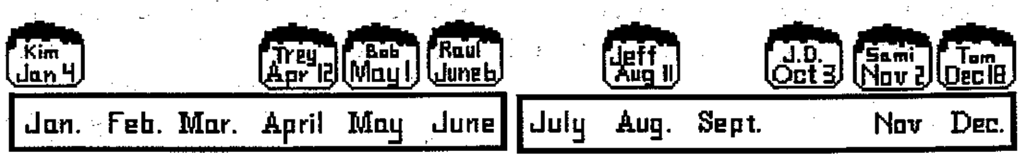
Objectives:

The children will contribute to a class graph of birthdays. Each child will determine his or her half-birthday. The children will read the data from the class graph of half-birthdays to make a table.

Directions:

1. Read aloud and discuss the book *The Half-Birthday Party*. What are some of the half-presents Katie received? What are some other half-presents which might have been given? What would you have given?

2. Have each child label a small paper birthday cake with name and birthday. On one paper strip write the months from January to June. On another paper strip, write the months from July to December. Have each child come to paste a cake above the correct month.



3. Ask the children to tell what they observe about the graph. Which months have the most/fewest birthdays? Now ask the children to look at the month labels. How many months are in one year? How many months are in half of one year? Switch the two paper strips to change to a half-year from the original birthday. Help the children determine when their half-birthdays are according to the new graph.

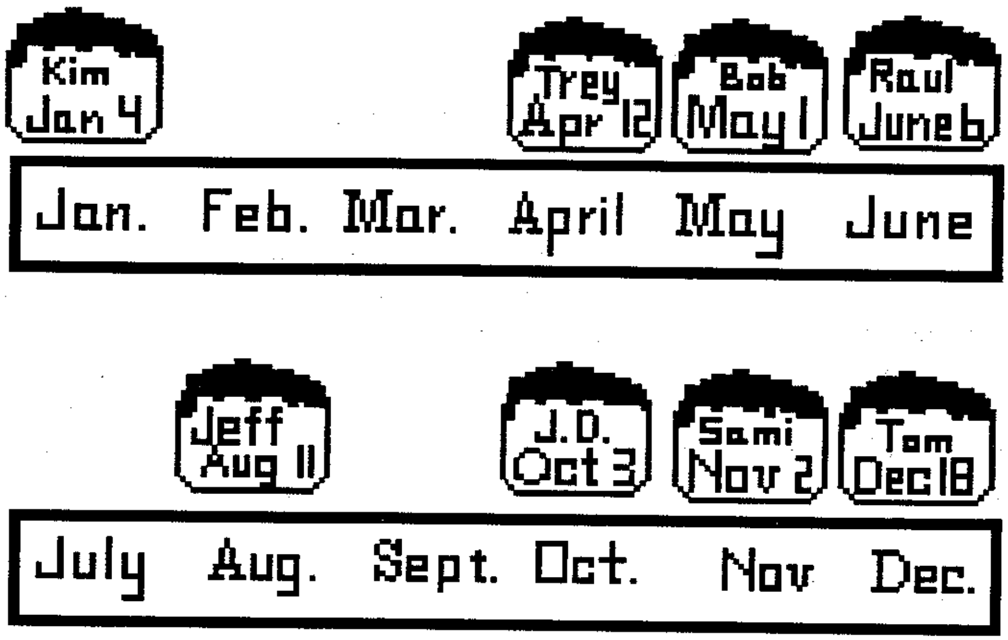
(**Note:** There are several dates which will not work by this method. These are May 31, Aug. 29, 30, and 31, Oct. 31, Dec. 31. Most children will be willing to choose a date before or after these for a half-birthday.)

4. Have the children use the information in the picture graphs to complete the table (see activity sheet) to display the number of birthdays and half-birthdays in each month. When the tables have been completed, ask the children to use the information to answer questions about the class.

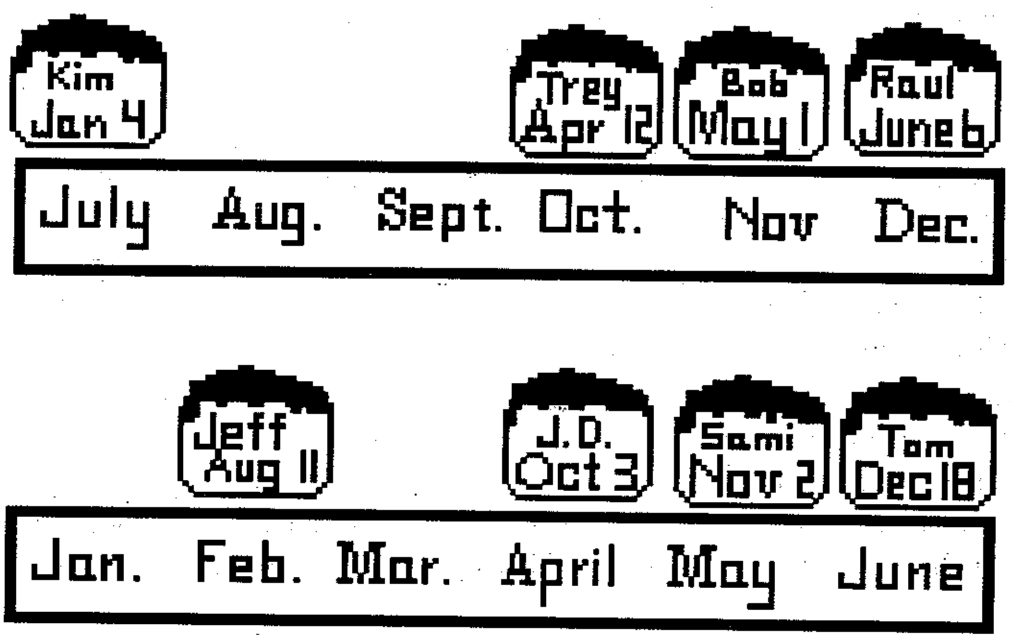
Extensions:

A circle calendar like the one in Figure 1 can also be used to help children determine their half-birthdays. A line of string stretched straight across the circle from the birthday month will show the half-birthday month. Fractional parts of the circle calendar can be used to relate to seasonal concepts. A circle geoboard can also be used for this purpose.

Birthdays for graph



Half-Birthdays for graph



Let's Reflect on Half

Literature:

Eager, Edward. *Half Magic*. New York: Harcourt Brace, 1982.

Walter, Marion. *Magic Mirror Tricks*. New York: Scholatic, 1971.

Background:

At the turn of the century, Jane, Mark, Katharine, and Martha spend an ordinary summer vacation until they find a coin that is magical. When the children begin to use the magical coin, they discover that the coin will grant them half-wishes. To get a complete wish, they must ask for twice as much as they want. Mark's wish lands them in the Sahara Desert and Katharine's transports them back to the days of King Arthur. During all these incredible happenings, the children acquire a stepfather. The story ends with the children using up all their wishes and secretly giving the coin to another child.

Objective:

The children will be able to use a Mira to complete the line-symmetric figure. The children will develop number sense about whole and half. The children will define and write a story using half words.

Directions:

1. Tell the children the background information about the story and encourage them to read the book. Then read Chapter 3, "What Happened to Mark," and have the children write down all the "half" words they hear. (You may consider reading aloud 10 to 12 pages daily until the book is completed.)

   Ask "What "half" words did you hear?" (halfway, half talk, half-language, half peep, half-see, half-alive, half a wish, half-fire, half-carry, half a mile, half-life, and talk half the time.)

   Tell "When Jane was explaining about the magic charm she said, 'So far we've each got half of what we wished for—all we have to do from now on is ask for twice as much as we really want!'"

   Ask "What did Jane mean by twice as much as half?" (She meant that, to get one whole, you have to ask for two halves.)

   Tell the children to think about a wish they might enjoy making. **Ask** "Can you draw half of your wish?" When the children tell you they can, tell them that they will be using miras to tell if they have drawn exactly half of a picture.

3. Share the book entitled *Magic Mirror Tricks*, by Marion Walton. In this book they will see what one half of an object looks like and what they will see when they use a mirror. Point out that if the children in *Half Magic* wanted this design, they would have to ask for the design plus the design in the mirror.

4. If your students have not used a mira before, you will need to begin by teaching the names of the mira parts. Identify the front face, the back face, the ends, and the beveled edges.

   Ask "Can you name the parts of your mira?" (If "Yes", name parts. If "No", review parts again.)

   Ask "Can you tell which face is the front?" (Point to the front face.)

   Ask "What is mira reflection?" (Begin to explain mira reflection by bouncing a ball on the floor.)

   Tell "Watch the path of a ball as it bounces on a floor. It reflects from the surface of the floor. When you are using a mira, light from the object reflects from the mira (just like a ball reflects from the floor). It seems to come from a position behind the mira. This position is where you see the image, from the point where the reflection of the object might be drawn.

5. Have the students stand the mira on a sheet of plain paper. Tell them to hold it steady with one hand and draw one half of your wish using a sharp pencil. Start and end half of the wish on the mira-line or reflection-line. Remove the mira and draw the other half of the figure. Place the mira midway between the two halves. Test the combined figure for line symmetry. Test for accuracy by trying to reflect the first half drawn onto the other half.

   Ask "Does my figure have a line of symmetry?" (If "Yes", tell how you know. If "No", figure out what you have done wrong.)

   Ask "Did the first half of the figure you drew reflect on the second half?" (If "Yes", tell how you know. If "No", tell why your figure does not reflect on the second half.)

6. Language Arts connection: After completing the mira activity begin a discussion on other meanings for half. Use the list of half words and divide the students into group. Have each group of children define the meaning for 2 to 3 words.

   Ask "Would a dictionary help you with defining your words?" (They may use a dictionary if they choose.)

halfback	second half	half an hour	half past
better half	by half	in half	not the half of
go halves	half-baked	half-and-half	half-blooded
half brother	half-cocked	half dollar	half-heated
half-length	half-life	half-mast	half moon
half-nelson	half pint	half time	half ton
half-truth	halfway house	halfway	half valley
half-wit			

- Give extra credit for the groups that can come up with other half words.
- Let each group have the opportunity to present their findings.
- Allow time for the class to discuss what they have learned.
- Have the students conclude these activities by writing a story using as many of the half words as they can.

Extensions:

- Make a set of cards and have the students draw the reflecting image.
- Draw a reflection-image of line segments and line symmetric figures. Find reflection-congruent figures. Draw the reflection line of each.
- Discuss the idea of receiving three-fourths of your wish each time you wished. What would the children have to wish for in order to receive a whole wish? (Continue to change the fraction.)

Additional Trade Books For Teaching Fractional Concepts:

Conaway, Betty & Midkiff, Ruby Bostick. "Connecting Literature, Language, and Fractions. *Arithmetic Teacher* v. 41, no. 8 (April, 1994), p. 430–435.

Emberley, Ed. *Ed Emberley's Picture Pie: A Circle Drawing Book.* Boston: Little, Brown, 1992.

Hutchins, Pat. *The Doorbell Rang.* New York: Greenwillow, 1986.

Matthews, Louise. *Gator Pie.* New York, Dodd, Mead, 1979.

Merrill, Jean. *The Toothpaste Millionaire.* Boston: Houghton-Mifflin, 1972.

Munsch, Robert. *Moira's Birthday.* Toronto: Annick Press, 1987.

Whitin, David J. & Wilde, Sandra. *Read Any Good Math Lately?* Portsmouth, N.H., 1992.

Trigonometry Toss

Bettye Hall
Houston, Texas

Prerequisite Skill: Definitions of the six trigonometric functions.

Grade: Trigonometry

Objective: The student will identify the quadrant in which each trigonometric function falls according to its sign.

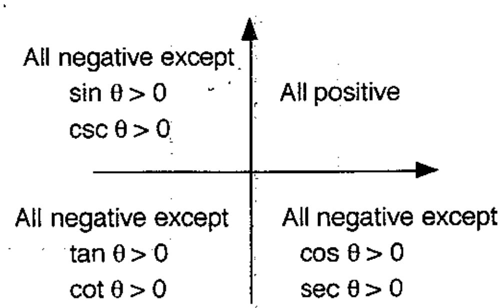
Materials:
Coordinate Grid game sheet
colored pencils or crayons
one die labeled with $\sin\theta, \cos\theta, \tan\theta, \csc\theta, \cot\theta$
one die labeled with three *pos* and three *neg*.

Procedure:

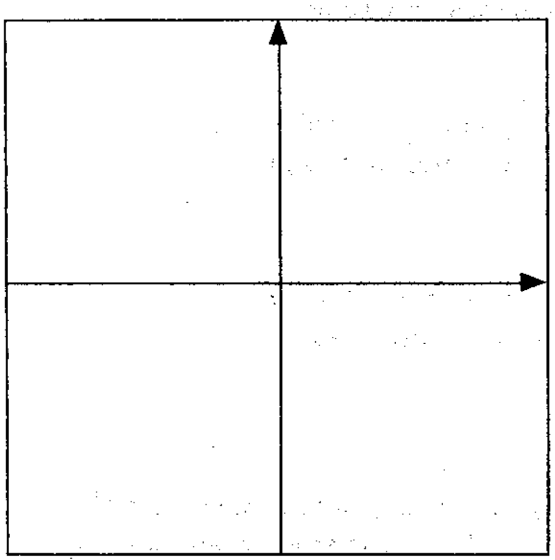
- The activity is done in pairs.
- Player A tosses the dice and states the regions of the graph that contain the function. For example: if the toss is $\sin\theta$ and *neg*, the student announces, "Quadrants III and IV." If Player B agrees with the statement, the student writes $\sin\theta < 0$ in the correct areas of the coordinate system on his/her game sheet. If Player A is incorrect, play passes to Player B.
- After Player A marks his/her game sheet play passes to Player B. The students take turns tossing the cubes. Each player records on his/her own game sheet.
- The winner is the first player that correctly fills in all six functions in all four quadrants of his/her coordinate system.

Note: Let the class decide what the rule will be if a student tosses an area that is already marked on his/her game sheet.

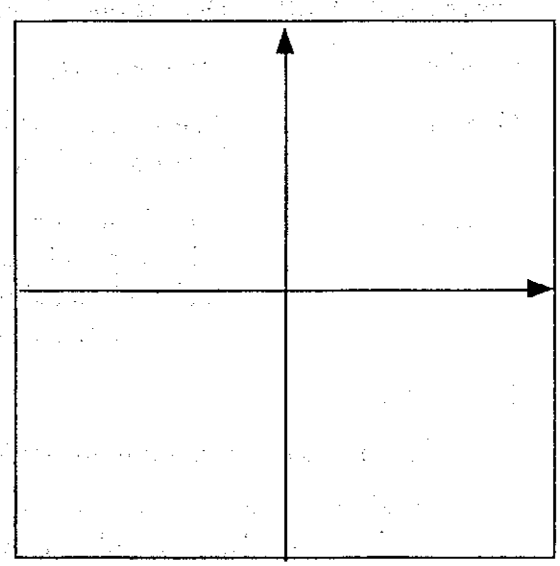
KEY:



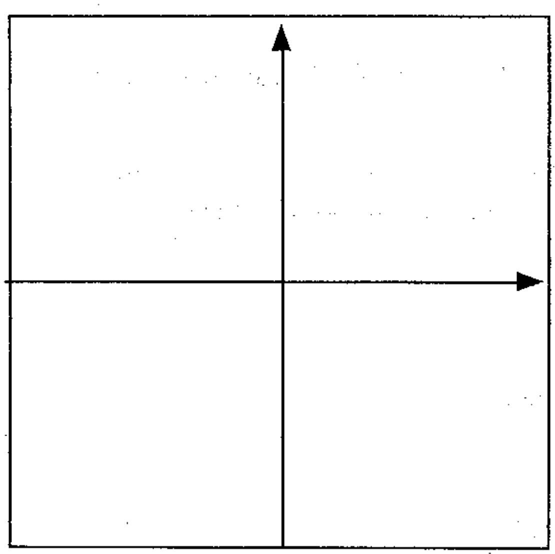
TRIGONOMETRY TOSS



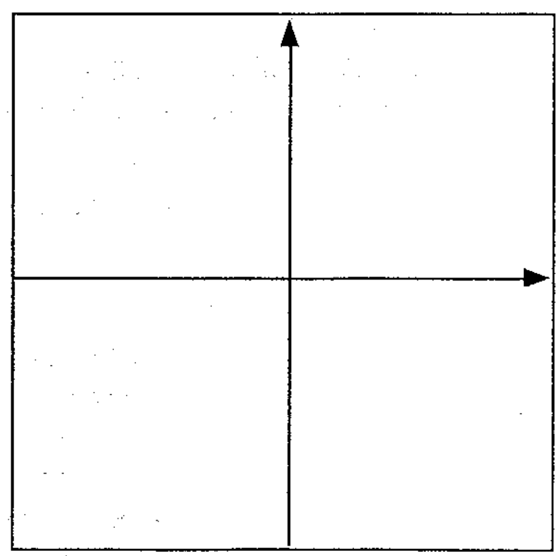
Game 1 - Player A



Game 1 - Player B



Game 2 - Player A



Game 2 - Player B

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