Texas Mathematics Teacher

Inequalities

It's Irrational and It's Not In Our Normal Domain!

The Use of the University of Chicago Fraction Factory Pieces in the Teaching of Arithmetic with Fractions--Part I

Using the Manufacturing of Plastic Gloves to Develop Mathematical Literacy

Math-In-A-Flash?

Teaching Math Through "Sylvester and the Magic Pebble"

MAY 1994

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

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TEXAS MATHEMATICS TEACHER
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President's Message

One of the brochures released with last fall's interactive videoconference "Creating a Climate for Change. . . Math Leads the Way" is intended for parents. It asks "What Should I Look for in a Math Classroom?" and suggests that parents should look closely at the math classroom of today to see if "it is teaching the same old stuff in the same old way, turning out students grossly unprepared for the real adult world." As I read the list of things that the brochure says students and teachers should be doing, I decided it should be on every mathematics teacher's reading list, as we challenge ourselves to continue to strive for change. It would also be helpful for administrators—especially those who are involved in teacher evaluation. And, let's get the material reprinted in school newsletters to parents! Here are some highlights:

WHAT ARE STUDENTS DOING?

- Interacting with each other, as well as working independently, just as adults do at work.

- Using textbooks as only one of many resources. Manipulatives such as blocks and scales and technology such as calculators and computers are useful tools, and students should be learning how and when to use them.

- Becoming aware of how math is applied to real life problems, not just learning a series of isolated skills. And as in real life, complex problems are not solved quickly.

- Realizing that many problems have more than just one "right answer." Students can explain the different ways they reach a variety of solutions and why they make one choice over another.

WHAT ARE TEACHERS DOING?

- Raising questions that encourage students to explore several solutions and challenge deeper thinking about real problems. They are not just lecturing.

- Moving around the room to keep everyone engaged and on track. They are not glued to the chalkboard.

- Allowing students to raise original questions about math for which there is no "answer in the book," and promoting discussion of these questions, recognizing that it may be other students who will find reasonable answers.

- Using manipulatives and technology when it is appropriate, not just as busy work.

- Drawing on student discovery and creativity to keep them interested. The teacher knows that boredom is the enemy of learning.

- Working with other teachers to make connections between disciplines to show how math is a part of every other major subject.

- Using assessments that reflect the way math is being taught, stressing understanding and problem-solving skills, not just memory.

> The brochure is available through The Math Connection, coordinated by the Mathematical Sciences Education Board. Phone 202/334-1289 for information on obtaining brochures.

As the school year concludes for most of us, we again have the luxury of time . . . for shifting gears, relaxing, studying, enjoying family, summer jobs, travelling, etc. An outstanding opportunity to learn how to implement the changes highlighted above will be the 41st annual Conference for the Advancement of Mathematics Teaching, July 27 - 29, in Houston. Details about TCTM's activities at CAMT will be outlined in the spring issue of the newsletter *Math Talk*. Plan on attending the TCTM breakfast Friday morning, and visiting STEAM and the NCTM materials display in the exhibit area. I hope I'll see you there!!

Susan Thomas

INEQUALITIES

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Introduction

Solving inequalities is very important in the learning of Calculus; however, most high school students are afraid of doing inequality problems. The reason may be that they don't understand the need for solving inequalities or they tire of doing a lot of tedious calculations. Through proper introduction, solving inequalities could be very interesting for students. The method described in this article is a version of the method shown in *A Line System to Aid Teaching Inequalities* [1], which should prove to be more straightforward for many students.

Concepts:

I) Let f(x) be a polynomial in x such that

$f(x) = k(x - x_1)^{t_1}(x - x_2)^{t_2} \cdots (x - x_n)^{t_n}$ *where* $x_1, x_2, \cdots, x_n$

are constants with $x_1 < x_2 \cdots < x_n$; $t_1, t_2, \cdots, t_n$ are

positive integers; k is the leading coefficient of f(x). The constants x_1, $x_2, \cdots x_n$ are roots of the equation
$k(x - x_1)^{t_1}(x - x_2)^{t_2} \cdots (x - x_n)^{t_n} = 0$, *thus* $t_1, t_2, \cdots, t_n$

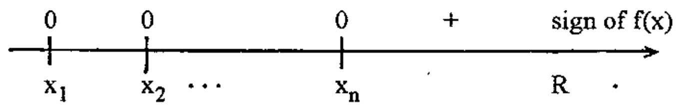
are the multiplicities of $x_1, x_2, \cdots, t_n$, respectively.

I i) For $v > x_n$, we have $v - x_i > 0$ for $i = 1, 2, \cdots, n$.

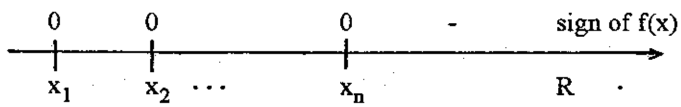
Thus, $(v - x_i)^{t_i} > 0$ *for* $i = 1, 2, \cdots, n$. This implies

$f(v) = k(v - x_1)^{t_1}(v - x_2)^{t_2}\cdots(v - x_n)^{t_n} > 0$ *if* $k > 0$; and $f(v) < 0$ *if* $k < 0$.

In short: a) For $k > 0$, we have



b) For $k < 0$, we have



I-ii) For $x_{m-1} < v < x_m < u < x_{m+1}$ $m = 1, 2, \cdots, n$ (note: x_0 stands for $-\infty$ and x_{n+1} stands for $+\infty$), we know that

$v - x_i > 0$ and $u - x_i > 0$ for $1 \le i \le (m - 1)$;
$v - x_i < 0$ and $u - x_i < 0$ for $(m + 1) \le i \le n$;
$v - x_m < 0$ and $u - x_m > 0$. Therefore,

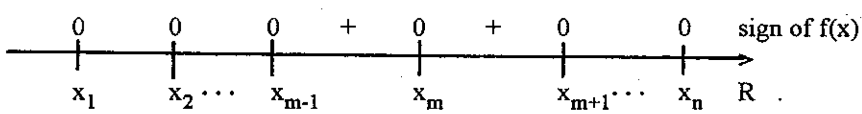
a) *if* t_m *is even, then* $(v - x_m)^{t_m} > 0$, and $(u - x_m)^{t_m} > 0$,
this implies f(v) and f(u) have same sign;

b) *if* t_m *is odd, then* $(v - x_m)^{t_m} < 0$, and $(u - x_m)^{t_m} > 0$,

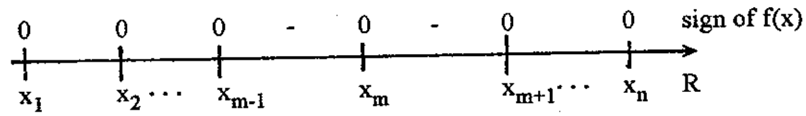
this implies f(v) and f(u) have different sign.

In short:

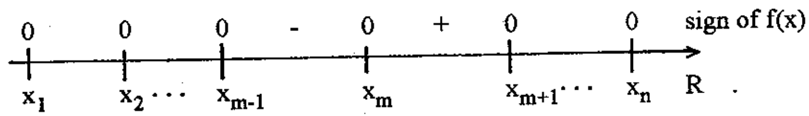
1) If t_m is even and $f(u) > 0$ when $x_m < u < x_{m+1}$, we have



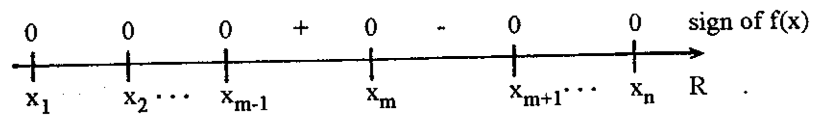
2) If t_m is even and $f(u) < 0$ when $x_m < u < x_{m+1}$, we have



3) If t_m is odd and $f(u) > 0$, when $x_m < u < x_{m+1}$, we have



4) If t_m is odd and $f(u) < 0$, when $x_m < u < x_{m+1}$, we have



Conclusion: It is clear now that if t_m is even, then the value of f(x) never changes its sign when x is changed from $x \in (x_m, x_{m+1})$ to $x \in (x_{m-1}, x_m)$; if t_m is odd, then the value of f(x) changes its sign when x is changed from $x \in (x_m, x_{m+1})$ to $x \in (x_{m-1}, x_m)$.

II) For $a \neq 0$, $ax^2 + bx + c = a\left(x^2 + \frac{b}{a}x + \frac{c}{a}\right)$

$$= a\left[\left(x + \frac{b}{2a}\right)^2 + \frac{-b^2 + 4ac}{4a^2}\right],$$

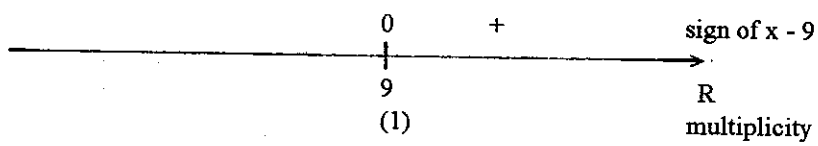
if $-b^2 + 4ac > 0$ (i.e. $ax^2 + bx + c = 0$ has complex roots), then the value of $ax^2 + bx + c$ is always positive if a is positive, and negative if a is negative. This means the sign of the value of $ax^2 + bx + c$ is always the same as that of when $ax^2 + bx + c = 0$ has complex roots.

III) Extension: Concepts I) and II) can be easily and interestingly combined to solve polynomial inequalities and rational inequalities (see the examples below).

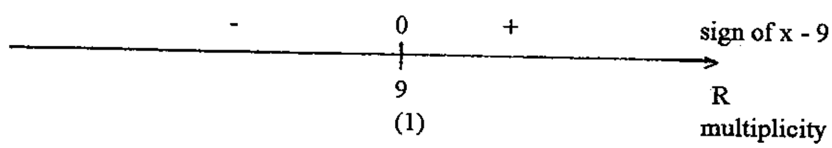
Examples:

1) Solve $x - 9 > 0$.

[Solution]: We are looking for all the values of x that make $x - 9$ greater than zero. Let's first set $x - 9 = 0$. Then $x = 9$ with multiplicity 1. Now use a real number line R to locate the point corresponding to 9, and $x - 9 = (1)(x - 9)$ implies the leading coefficient is 1, which is positive. By concept I-i) we have



next, since the multiplicity of 9 is 1, which is odd, by concept I-ii) we know the sign of the values of $x - 9$ is changed from positive to negative when x is changed from greater than 9 to less than 9, i.e.,



We are looking for all the values of x that make $x - 9$ greater than zero. From the above diagram it is clear that the solution set is $(9, \infty)$.

2) Solve $-x^2 + x + 12 \leq 0$.

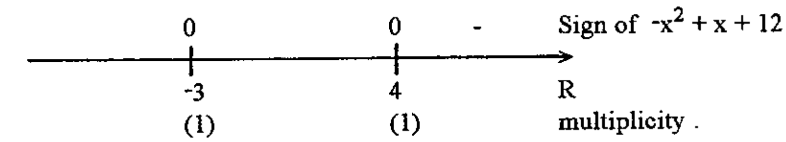
[Solution]: Set $-x^2 + x + 12 = 0$,

$$x^2 - x - 12 = 0,$$

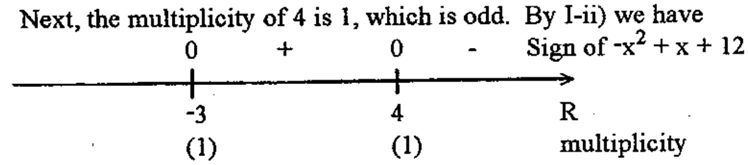
$(x - 4)(x + 3) = 0,$

then	root	multiplicity
	4	1
	-3	1 .

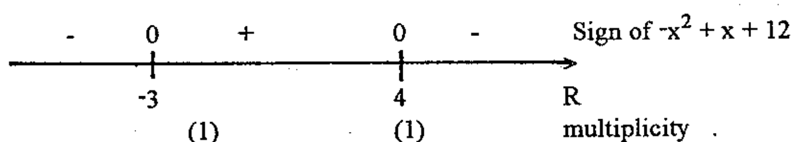
In $-x^2 + x + 12 = (-1)(x - 4)^1(x + 3)^1$ the leading coefficient is -1, which is negative. By concept I-i) we have



Next, the multiplicity of 4 is 1, which is odd. By I-ii) we have



finally, the multiplicity of -3 is 1, which is odd. By concept I-ii) we have



Since we want all the values of x that make $-x^2 + x + 12$ non-positive, the solution set is $(-\infty, -3] \cup [4, \infty)$.

3) Solve $x^2 - 2x + 2 < 0$.

[Solution]: Set $x^2 - 2x + 2 = 0$, then $x = 1 \pm \sqrt{-1}$, i.e., the roots are not real numbers; we can't locate these roots on the real number line. By concept II), we know the value of $x^2 - 2x + 2$ is always greater than zero no matter what x is; therefore, the value of $x^2 - 2x + 2$ can never be less than zero. The solution set is $\emptyset$.

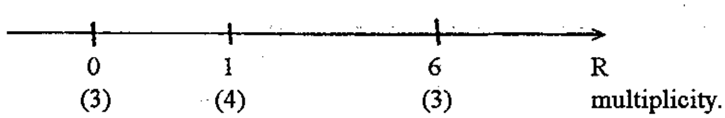
4) Solve $-x^3(x - 6)^2(-2x + 12)\,(6x - 6)^4(-x^2 + x - 1)^3 > 0$.

[Solution]: Set

$$x^3 = 0,$$
$$(x - 6)^2 = 0,$$
$$-2x + 12 = 0,$$
$$(6x - 6)^4 = 0,$$
$$(-x^2 + x - 1)^3 = 0, \text{ then}$$

<u>root</u>	<u>multiplicity</u>
0	3
6	3
1	4
$(-1 + \sqrt{-3}) / (-2)$	3
$(-1 - \sqrt{-3}) / (-2)$	3

By concept II, we can just locate the real roots (since the inequality is now equivalent to $-x^3(x - 6)^2(-2x + 12)\,(6x - 6)^4(-1)^3 > 0$), and their multiplicities on a real number line.



Now that the leading coefficient of $-x^3(x - 6)^2(-2x + 12)\,(6x - 6)^4(-1)^3$ is $(-1)(1)^2(-2)(6)^4(-1)3$, which is negative. We therefore enter a negative sign into the far right portion (the portion of $x > 6$); the portion of $1 < x < 6$ with a positive sign (for the multiplicity of 6 is odd); the portion of $0 < x < 1$ with a positive sign for the multiplicity of 1 is even); the portion of $x < 0$ with a negative sign (for the multiplicity of 0 is odd), i.e.,

-	0	+	0	+	0	-	sign
	0		1		6		R
	(3)		(4)		(3)		multiplicity.

Thus the solution set is $(0, 1) \cup (1, 6)$.

Note: For $-x^3(x - 6)^2(-2x + 12)\,(6x - 6)^4(-x^2 + x - 1)^3 \geq 0$,
the solution set is $[0, 6] \cup \{(-1 + \sqrt{-3}) / (-2), (-1 + \sqrt{-3}) / (-2)\}$.

For rational inequalities, the method is similar to that of the previous examples. Let's try the following example:

Example: Solve $\dfrac{(x + 1)\,(x - 2)^2\,(x^2 + x - 1)}{(-x + 3)^3\,(x + 4)\,(x^2 + x + 1)} \leq 0.$

[Solution]: Note: The inequality is equivalent to

$(x + 1)\,(x - 2)^2\,(x^2 + x - 1)\,(-x + 3)^3\,(x + 4)\,(x^2 + x + 1) \leq 0$ provided $(-x + 3)^3\,(x + 4)\,(x^2 + x + 1) \neq 0$.

Set	Then	root	multiplicity
$(x + 1) = 0$,		-1	1
$(x - 2)^2 = 0$,		2	2
$x^2 + x - 1 = 0$,		$(-1 + \sqrt{5}) / 2$	1
		$(-1 + \sqrt{5}) / 2$	1
$(-x + 3)^3 = 0$,		3	3
$x + 4 = 0$,		-4	1
$x^2 + x + 1 = 0$.		$(-1 + \sqrt{-3}) / 2$	1
		$(-1 + \sqrt{-3}) / 2$	1

The leading coefficient of
$(x + 1)\,(x - 2)^2\,(x^2 + x - 1)\,(-x + 3)^3\,(x + 4)\,(x^2 + x + 1)$ is
$(1)\,(1)^2\,(1)\,(-1)^3(1)(1)$, which is negative. Thus we have

+	x	-	0	+	0	-	0	+	0	+	x	-	sign
	-4		$(-1-\sqrt{5})/2$		-1		$(-1+\sqrt{5})/2$		2		3		R
	(1)		(1)		(1)		(1)		(2)		(3)		multiplicity.

Note: "x" stands for undefined.

The solution set is

$$\left([\,-4, \frac{-1-\sqrt{5}}{2}] \cup [\,-1, \frac{-1+\sqrt{5}}{2}\,] \cup \{2\} \cup (\,3, \infty) \right).$$

Applications:

1) Find the domain of

$$f(x) = \sqrt{\frac{(x-1)(x-2)}{(x-3)(-x+4)}}.$$

[Solution]:

we have $\frac{(x-1)(x-2)}{(x-3)(-x+4)} \geq 0$,

-	0	+	0	-	x	+	x	-	sign
	1		2		3		4		R
	(1)		(1)		(1)		(1)		multiplicity.

The domain is $[1, 2] \cup (3, 4)$.

2) If $f'(x) = -2(x+2)(x+1)^2(x-2)^4(x-3)^3$, what values of x make f(x) a local maximum? A local minimum?

[Solution]: Set $x + 2 = 0$,

$(x+1)^2 = 0$,

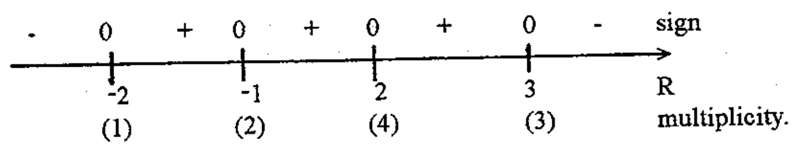
$(x-2)^4 = 0$,

$(x-3)^3 = 0$.

Then,	root	multiplicity
	-2	1
	-1	2
	2	4
	3	3 ,

and the leading coefficient of $f'(x)$ is

$(-2)\ (1)\ (1)^2\ (1)^4\ (1)^3$, which is negative. Therefore



Thus, x = 3 makes f(x) a local maximum.

x = -2 makes f(x) a local minimum.

Reference

A Line System to A id Teaching Inequalities, Cheng-Chi Huang, Alabama Journal of Mathematics, Vol. 10 No. 1 (1986), 27-30.

IT'S IRRATIONAL AND IT'S NOT IN OUR NORMAL DOMAIN!

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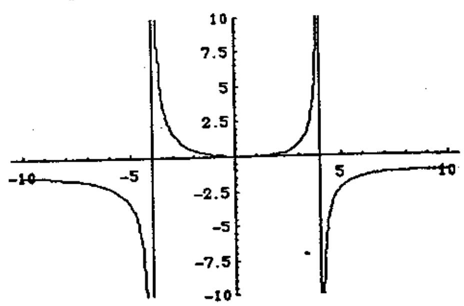
I was surprised last summer while revising an interactive computer laboratory[1] using Maple, a computer mathematics system. I wonder if you, the reader, would work through the following sequence and see if you have the same reaction I did.

Define a function named y:	y:= (x^2 - x) / (16 - x^2); $y := \frac{(x^2 - x)}{(16 - x^2)}$
Plot the function:	plot (y, x=-10..10, y=-10..10);



1 This example is from a Calculus laboratory written by Dr. Phoebe Judson. See Child, J.D., D'Arcy, C., Havsknecht, A., Judson, P., Kowalezyk, P., Kraines, V., & Myers, R. (1992). *Calculus Laboratories for Brooks/Cole Software Tools.* Pacific Grove, CA: Brooks/Cole.

Find the first derivative of y and name it yp (short for y-prime):	yp:= diff(y,x); $yp := \frac{(2x-1)}{(16-x^2)} + 2\frac{(x^2-x)x}{(16-x^2)^2}$

Find a numeric approximation of at least one of the first derivative:	`fsolve`(yp=0,x); 31.49193338

What was your reaction to the last line? When I first noted that a critical point of y was near x = 31.5, I was very surprised. Even though I am a very experienced user of technology, the first question I asked myself was, "Had Maple made an error?" Next I wondered if I had entered an incorrect command. When a class of 18 students worked through the same problem last fall, almost every student sought me out with the same questions.

What to do next? As shown in Figure 1, most of the students graphed the function on a larger domain. Again, there was no graphical indication of a critical point near x = 31.5.

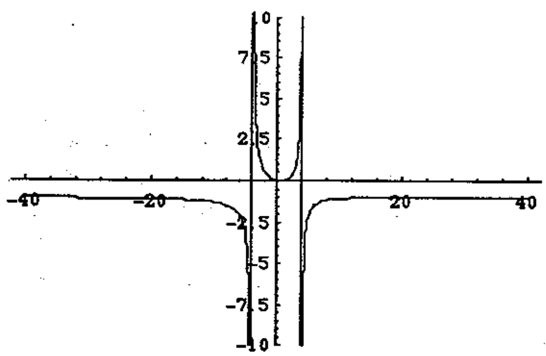


Figure 1.

Finally, as shown in Figure 2, zooming in on x between 30 and 32 revealed that there was a critical point near x - 31.5 and that it produced a local maximum.

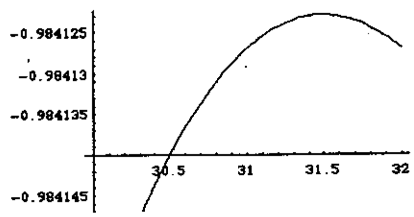


Figure 2.

What are the Lessons from this Example?

This example provides helpful insights as we increase the use of graphing calculators and computer mathematics systems into our curricula.

1) Traditionally, most school problems have "nice solutions" on a "nice domain." Our students do not expect answers which are either irrational or very far from the origin.

2) We need to emphasize that graphing technology often may give an overview rather than an exact representation of a function. We may need to use numerical and/or symbolic techniques to discover all the characteristics of a function.

3) On the other hand, the example shows that symbolic techniques may beg for direction from graphical representations. The `fsolve` command found only one critical point. Yet the graph of y makes it clear that there must be another critical point near the origin. Maple did not "make a mistake." Rather, the `fsolve` command finds a "single real root" when applied to rational functions. The example also points out the need to know the limitations of the particular software being used.

What Did the Students Do Next?

Students found the other critical point of y by using an optional parameter in the `fsolve` command to force Maple to look for another zero of the derivative of y (yp) on the interval -1 to 3. The command:

```
fsolve(yp=0, x, -1..3)
```

found the other critical point near x = 0.508. To confirm their understanding, they then did first and second derivative tests with the help of Maple's symbolic routines.

THE USE OF THE UNIVERSITY OF CHICAGO FRACTION FACTORY PIECES IN THE TEACHING OF ARITHMETIC WITH FRACTIONS

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Editor's Note: This is a two part article; Part I is presented here. Part II will appear in a future journal publication.

PART I. FRACTION PIECES AND FRACTIONS

This reader-active article is based on a part of the mathematics curriculum that was designed to be used in a four-week LaSIP project for 37 inservice teachers of mathematics of grades 5-8 held at Southern University-Baton Rouge during the Summer of 1993.

TASK 1. Getting to Know the University of Chicago Fraction Factory Pieces.

There are nine different fraction pieces of different colors. (See Figure 1.) For each of the nine different fraction pieces, determine how many fraction pieces of that color are needed to **exactly** cover the black fraction piece. In each case, put your answer in the blank.

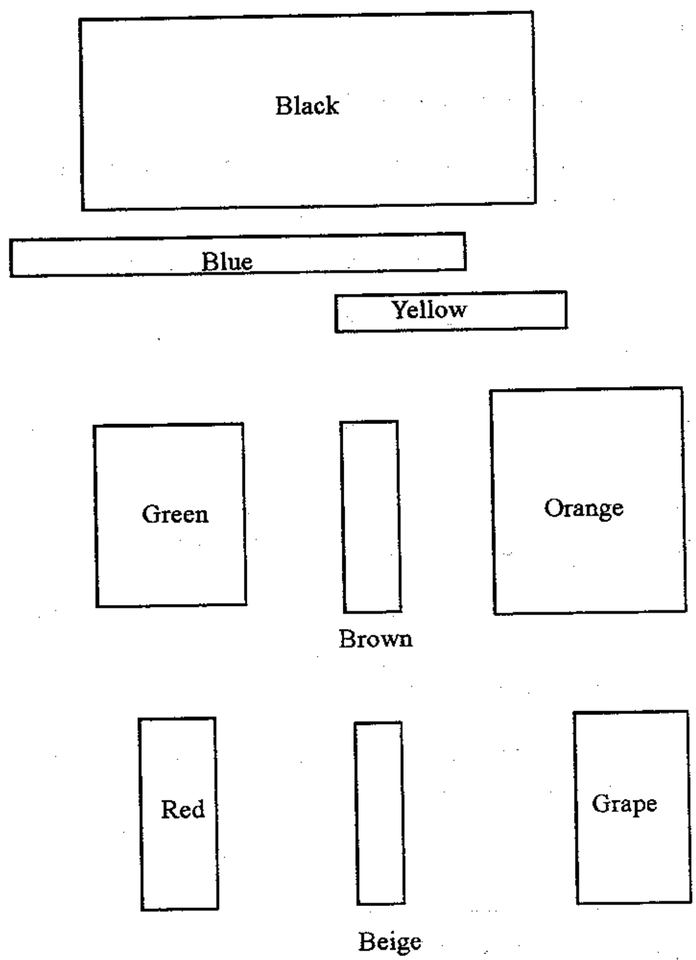

Figure 1.

1. yellow ___ 2. blue ___ 3. black ___ 4. orange ___

5. green ___ 6. beige___ 7. brown ___ 8. grape ___

9. red ___

10. Let us agree, at least in theory, that

   i) for any natural number n, the black fraction piece can be divided into n equal pieces,

   ii) each of the n pieces is called a fraction piece, and

   iii) for different values of n, the fraction pieces are of different colors.

Complete the following table for the value of n for the nine fraction pieces.

Color	Yellow	Blue	Black	Orange	Red	Green	Beige	Brown	Grape
n =									

Table 1.

11. Any **pile** of fraction pieces consisting of one or more fraction pieces is called an **extended fraction piece**.

12. Based on the statements found in 10 and 11, which of the following statements are true? Make a comment.

   i) Any fraction piece is an extended fraction piece.

   ___

   ii) Any extended fraction piece is a fraction piece.

   ___

13. An extended fraction piece is called **proper** if wherever the extended fraction piece is placed correctly on a black fraction

piece, the black fraction piece is **not completely covered**. Otherwise, the extended fraction piece is called **improper**.

14. For each of the following extended fraction pieces, determine if it is "proper" or "improper."

    i) 1 red, 1 green
    ii) 2 blue, 3 yellow
    iii) 1 green, 1 beige, 1 grape, 1 red
    iv) 7 green
    v) 4 red, 3 orange
    vi) 1 black, 5 red

15. Anytime an extended fraction piece **exactly** covers a black fraction piece, the extended fraction piece can be **exchanged** for one black fraction piece. Anytime copies of a smaller fraction piece exactly cover a larger fraction piece, then the extended fraction piece made from the smaller fraction pieces can be exchanged for the larger fraction piece and conversely.

    Determine several exchanges that can be made for each fraction piece.

16. For **each** of the **proper** extended fraction pieces in 14 do the following:

    i) Place the extended fraction piece correctly on a **black** fraction piece. The black fraction piece **should not** be completely covered.

    ii) Pick a fraction piece such that the uncovered part of the black fraction piece can be completely covered with pieces of that color. Now,

    iii) Replace, by use of exchanges, the given extended fraction piece by an extended fraction piece made with fraction pieces of the **same color** as the fraction pieces in (ii).

17. *****BULLETIN.** The **power** of item 16 is that it suggests a way to **replace**, by making exchanges, any **proper** extended fraction piece by an extended fraction piece consisting of fraction pieces of the **same color!!** An extended fraction piece is in a **standard form** if it consists of fraction pieces of the **same color**.

18. Replace **each** improper extended fraction piece in 14 by one of the following:

    i) An extended fraction piece consisting of black fraction pieces only

    ii) Two extended fraction pieces where one extended fraction piece consists of black fraction pieces only and the other extended fraction piece is proper and is in a standard form. (Having this form, it is called **mixed**.)

19. Suppose you are given an improper extended fraction piece that consists of black fraction pieces and fraction pieces of a second color. Tell how that improper extended fraction piece can be replaced by an improper extended fraction piece having all pieces of the **second color**. Write your statement on the back of this page.

20. *****BULLETIN.** Two extended fraction pieces are equal if any one of the following is true:

    i) They cover the same amount of space when placed correctly on a blank fraction piece

    ii) Any one of the extended fraction pieces can be exchanged for the other

    iii) When one is placed on top of the other the fit is **exact**.

    iv) If the extended fraction piece made from the two extended fraction pieces is put in a standard form and each of the extended fraction pieces is put in the standard form involving the fraction piece found in the standard form of the extended fraction piece made from the two extended fraction pieces, then they have the same number of fraction pieces.

21. Given the five fraction pieces: orange, grape, beige, green, red. Based on **sight**, arrange them in the order largest to smallest.

22. Do (21) by use 20 (iv). Put your results on the back of this page. Include a discussion.

23. Suppose you are given an improper extended fraction piece in a standard form. Tell how that improper extended fraction piece can be replaced by an equal extended fraction piece of one of the following forms:

    i) An extended fraction piece having all black fraction pieces. This is a standard form.

    ii) A mixed extended fraction piece.

24. Replace each of the following mixed extended fraction pieces by an equal extended fraction piece in a standard form.

    i) 3 black, 7 yellow
    ii) 4 black, 4 blue
    iii) 5 black, 1 orange
    iv) 7 black, 1 red

25. Replace each of the following improper extended fraction pieces by an equal improper fraction piece consisting of all black fraction pieces or a mixed extended fraction piece.

    i) 33 yellow
    ii) 30 blue
    iii) 13 red
    iv) 19 brown

v) 23 grape
vi) 18 orange

26. Given an extended fraction piece in a standard form. If it is possible to replace the given extended fraction piece by an equal extended fraction piece in a standard form having **fewer** fraction pieces, then we say that the given extended fraction piece can be **reduced**. Otherwise, the given extended fraction piece is said to be **reduced to lowest terms**.

Reduce each of the following extended fraction pieces to lowest terms.

i) 4 brown
ii) 2 grape
iii) 3 beige
iv) 4 beige
v) 6 beige
vi) 5 yellow
vii) 3 grape

TASK 2. An Association Between Extended Fraction Pieces and Fractions.

1. Given a **fraction piece** x. Let us agree that if it takes n copies of x to **exactly** cover the black fraction piece, then we will associate with x the symbol 1/n and **call 1/n a fraction**!!

Complete the following "association table," Table 2:

X	Yellow	Blue	Black	Orange	Red	Green	Beige	Brown	Grape
Fraction									

Table 2.

2. Given an extended fraction piece y *where y is in a standard form*. If it takes n of the fraction pieces of y to completely cover the black fraction piece and y consists of m fraction pieces, then we

associate with the extended fraction piece y the extended fraction m/n.

3. Associate with each extended fraction piece in FUNSECTION (1), a fraction.

4. Given a fraction s/k. To associate with the fraction s/k an extended fraction piece, one might do the following:

   i) Pick a fraction piece that has the property that it takes t copies of that fraction piece to completely cover the black fraction piece.

   ii) Associate with the fraction s/t the extended fraction piece consisting of s copies of the fraction piece in (a).

5. In the Table 3, associate with **each** fraction, an extended fraction piece:

   i) where the extended fraction piece is in a standard form.

   ii) where the fraction pieces of the extended fraction piece are **two** different colors.

Fraction	(i)	(ii)
5/6		
6/5		
8/3		
7/8		
3/4		
7/12		

Table 3.

6. In Table 3 of problem 5,

determine many correct answers to (ii) for 8/3; for 7/12. Put your answer on the back of this page. Answer the question if we insist that the extended fraction piece is reduced to lowest terms.

7. ***BULLETIN. Suppose the extended fraction piece x is associated with the fraction m/n and the extended fraction piece y is associated with the fraction s/t. Then

   i) the fractions m/n and s/t are equal if and only if x and y are equal extended fraction pieces.

   ii) m/n is a proper (improper fraction) if and only if x is a proper (improper) extended fraction piece.

   iii) m/n is reduced to lowest terms if and only if x is reduced to lowest terms.

8. **BULLETIN! BULLETIN! BULLETIN!**

At this stage in the lesson, one way to use information about extended fraction pieces to gain information about fractions is to do the following when asked a question about fractions:

   i) Associate with the fractions extended fraction pieces.

   ii) Change the question about the fractions to the correct question about extended fraction pieces.

   iii) Answer the question concerning the extended fraction pieces.

   iv) Associate with the answer in (iii) an answer about fractions. This is a desired answer to the given question about fractions.

9. FUNSECTION 1.

By use of the BULLETINS in 7 and 8 and facts about extended fraction pieces, do the following for fractions:

1. Reduce the following fractions to lowest terms:

   i) 8/10
   ii) 40/12
   iii) 30/8
   iv) 10/12

2. Arrange the fractions 2/3, 1/2, 3/4 in the order largest fraction to smallest fraction.

3. Show that the following pairs of fractions are equal:

   i) 3/4, 6/8
   ii) 1/2, 5/10
   iii) 2/3, 8/12

4. In (iv) of the summary, show that an extended fraction piece can be replaced by an extended fraction piece in a standard form in **more than one way**. Give an example to do so.

5. Based on the nine fraction pieces, prove that the fraction 3/4 is reduced to lowest terms.

10. SUMMARY OF TASKS 1 AND 2

By now you should *understand* and *know* the following facts about extended fraction pieces and fractions:

i) A pile of fraction pieces consisting of one or more fraction pieces is an **extended** fraction piece.

ii) An extended fraction piece is **proper** if whenever the fraction pieces in the extended fraction piece are placed correctly on a black fraction piece, the black fraction piece **is not** completely covered. Otherwise, it is called **improper**.

iii) An extended fraction piece is **mixed** if it consists of two extended fraction pieces, one consisting of black fraction pieces and the other a proper extended fraction piece in a standard form.

iv) An extended fraction piece, by use of exchanges, can be replaced by an extended fraction piece that is in a standard form.

v) Two extended fraction pieces are equal if and only if, by use of exchanges, any one extended fraction pieces can be replaced by the other extended fraction piece. In fact if given two or more extended fraction pieces and asked to determine how are they related (equal to, greater than, less than), one might do the following:

a) Make a *new* extended fraction piece from the given extended fraction pieces.

b) Replace the new extended fraction piece by an extended fraction piece that is in a standard form.

c) Replace each of the given extended fraction pieces by an extended fraction piece consisting of copies of the fraction piece in (b).

d) The decision is easily made by use of the information in (c).

vi) Given an extended fraction piece that is in a standard form. The extended fraction piece can be **reduced** if and only if it is equal to a new extended fraction piece in a standard form having fewer fraction pieces than the given fraction piece.

vii) An extended fraction piece in a standard form is **reduced to lowest term** if and only if it is impossible to replace it by an equal extended fraction piece in a standard form having fewer fraction pieces than the given extended fraction piece.

viii) Given an extended fraction piece. To associate with the extended fraction piece a fraction, one might do the following:

a) Replace the extended fraction piece by a new extended fraction piece in a standard form.

b) Count the number of fraction pieces in the **new** extended fraction piece. Suppose it has m fraction pieces.

c) Determine the number of copies of the fraction piece in the *new* extended fraction piece needed to completely cover the black fraction piece. Suppose n copies are needed. Then associate with the given extended fraction piece the fraction m/n.

ix) Given a fraction s/t. To associate with the fraction s/t an extended fraction piece, one might do the following:

a) Pick a fraction piece that has the property that it takes t copies of that fraction piece to completely cover the black fraction piece.

b) Associate with the fraction s/t the extended fraction piece that consists of s copies of the fraction piece in (a).

USING THE MANUFACTURING OF PLASTIC GLOVES TO DEVELOP MATHEMATICAL LITERACY

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The curriculum of the middle grades is often viewed by students as being "irrelevant, dull, and routine," emphasizing "computational facility at the expense of a broad, integrated view of mathematics" (National Council of Teachers of Mathematics [NCTM], 1989, p. 65). The areas of probability and statistics, combined with the natural settings of society, can often provide students with the opportunity to view mathematics as a subject vital to their future needs. In addition, these topics can also provide the students with opportunities to employ scientific methodology, and see how new knowledge can be gained. One such opportunity can be created in a classroom by focusing upon a quality control process used in the manufacturing of plastic gloves. The purpose of this manuscript is to provide an outline for a week to three weeks of class work which has been found to be appropriate for middle and secondary school students. As recommended by the NCTM Standards, the students will be involved in "using statistical methods to describe, analyze, evaluate data, and make decisions," as well as "creating experimental and theoretical models of situations involving probabilities" (NCTM, 1989, p. 70).

To best meet the needs of a class of students the scope and direction of some classroom activities should be based upon the interests of the students and the nature of the school's local community. The activities described within this manuscript will provide many opportunities for a wide variety of individual and group assignments which will arise during class discussions. Teachers are urged to move away from the idea of the teacher as a source of knowledge to one in which the teacher provides minimal assistance in helping the class solve its own problems. Thus the teacher becomes a catalyst for student investigations which will ultimately provide answers to questions which arise. The curriculum will not be textbook-driven, but will employ a constructivist-based approach in which the data collection activities will provide students with the opportunity to construction their own meanings. This project will also provide ample opportunities to evaluate students in non-traditional activities such as collaborative efforts, independent projects, oral presentations, and written reports.

Problem Context

The factory the class will investigate produces plastic gloves, such as those often used in housework. Although each stage of the manufacturing process is carefully monitored, at times things go wrong. Some of the problems can be detected if the plastic gloves tear apart while attempting to put them on. Based upon past experience the students should be told that about 5% of the gloves manufactured are defective in this way.

Plastic gloves are manufactured in batches of 500 at a time, although any number can be used. It should be obvious to the students that trying on 500 pairs of gloves is a time consuming process. The question to be investigated by the class is whether a smaller, random sample can be used to effectively predict the quality of a batch that has been manufactured. Such a procedure can be discovered and developed to whatever degree of precision that the class decides upon, and the activities now presented focus upon just how small that random sample can be.

The investigation can be facilitated if the class is divided into four to six small groups. Each small group should be presented with a shopping bag of 500 congruent objects such as straws, tongue depressors, or metal washers.

Additionally, 50 similar objects should be presented to the group leader, each one marked by a magic marker or a pen to distinguish them from the original set of 500. These marked objects will be used to indicate defective plastic gloves. The group leader should replace 25 unmarked objects with 25 "defective" objects. Thus, 5% of the objects in the bag are defective.

In the first activity data will be collected to determine the smallest sample size which can be used to accurately predict the number of defective objects in the bag. Each group will investigate different sample sizes. Thus, Group One will collect data for sample sizes of five, and Group Two will investigate sample sizes of ten, and so on, in increments of five each time. Each group will randomly draw ten samples of the specified size it is investigating, replacing the sample and mixing the contents of the bag prior to the next drawing. For each of the ten drawings for each sample size the group will record the number of defective objects. After completing the ten drawings each group will then average the number of defective objects for each drawing and use the average as a basis for reporting the effectiveness of the sample size under consideration. When Group One finishes collecting data it will be assigned a new sample size to investigate. This work will continue until sample sizes of at least 75 have been investigated.

After sample sizes of 75 have been investigated the groups will meet as a class and report their findings. The data should then be charted on a graph. Up to a point, generally speaking, the data will suggest that the larger the sample size the better the sample size is at suggesting that 5% of the objects in the bag are defective. However, the students should see that there is a point of diminishing returns and that the graph will level out with only small fluctuations after the sample sizes increase to a certain point.

The point at which the sample sizes start leveling out is the starting point for the second activity. Suppose the initial data suggest that this occurs when the sample size is 30. The first activity was designed to provide a rough approximation of the optimal sample size. The second activity will provide a more refined estimate. The groups will now focus attention upon sample sizes ranging from 20 to 40 in increments of one, repeating the procedures from the first activity. After completing their investigations the groups will again assemble as a class and a decision will be made concerning the optimal sample size.

In the third activity, the students will examine the reliability of their choice of sample size. Assume that the class has decided upon a sample size of 26 objects. As a parenthetical note students may inquire as to whether a different sample size might have been selected if they had started with bags which contained a different percentage of defective objects. Bags in which 50% of the objects are defective can be quickly used to suggest that the initial percentage of defective objects does matter. A factory must know its history to achieve the best results.

The first step in the third activity is for the class to discuss the question of what constitutes an acceptable percentage of defective objects. The theoretical considerations are the cost of melting down a defective batch in comparison to the damage done to the factory's reputation and the cost of replacing defective gloves that are returned by customers. Assume that the class decides that whenever the percentage of defective objects is 10% or higher that the entire batch should be rejected and melted down.

The students should return to their groups. The instructor should arrange for Group One to work with a bag in which 10% of the objects are defective. Group Two should work with a bag in which 15% of the objects are defective. The remaining groups should work with bags in which the percentage of defective objects is increasingly higher in increments of 5%. Each group will randomly select 26 objects from its bag, and repeat this procedure three or four times. Prior to each selection, the preceding set of 26 objects should be placed in the bag and the contents of the bag should be thoroughly mixed. Each group should record the number of times its random sample suggests that the entire batch should be rejected. After completing the experiments the class should see that the higher the percentage of defective objects in the bag, generally speaking, the more accurately the small sample size correctly suggests that the entire batch should be rejected.

The activity described in the previous paragraph provides the justification for testing the reliability of sample sizes of 26 objects **working only** with bags in which 10% of the objects are defective. Bags in which 10% of the objects are defective are more difficult to identify than bags in which 11% of the objects are defective.

The students should realize that the procedure used is set up to identify unacceptable batches. Occasionally acceptable batches in which 8% or 9% of the objects are defective will be rejected. Rejecting an acceptable batch is an error in the decision-making process as is marketing a batch with too many defective objects. The teacher should raise the question of whether one of these errors is more serious than the other. The quality control procedures should be designed to avoid making the more serious of the two errors.

To estimate the reliability of using sample sizes of 26 objects each group should begin working with a bag in which 10% of the objects are defective. Group leaders should insure that 50 of the 500 objects are defective in each bag. In examining the sample size of 26 whenever three or more of the objects are defective the entire batch is rejected. Each group should test ten samples of 26 objects and record the data. The data should then be compiled. The percentage of times in which the groups failed to reject a batch should then be reported. This figure is an **estimate** of the reliability of the sample size of 26 objects.

For purposes of discussion assume that 15% of the time the decision rule failed to reject unacceptable batches. The estimate of reliability is therefore 85%. Experience has shown that most students will believe that a figure of 85% is an acceptable number. The teacher should help the students see that the degree of security provided by a reliability figure of 85% may depend upon the risk involved. A birth control method that is 90% effective means one person in ten will have an unexpected pregnancy. The class may then decide to investigate larger sample sizes. The class might first determine what constitutes an acceptable reliability figure. Assume that the class decides that they desire a sample size that is accurate 95% of the time. The class should then investigate sample sizes until an acceptable one is identified.

The final activity involves a "real life" simulation of the factory. Divide the class into groups again, and provide each group with bags representing a batch of 500 gloves. The number of defective objects should be varied from group to group, with percentages varying just above and just below ten percent. The actual percentage for each bag should not be revealed prior to testing. Occasionally bags containing unusually high percentages of defective objects should be distributed. The students may then see that the sample size selected does an excellent job of identifying the greatest threats to the factory's reputation. Batches with percentages of defective objects just above ten percent do not pose the same threat to the factory as do these other batches.

Some Closing Comments

Working within the framework of quality control operations will involve fascinating applications of probability and statistics. Traditional, contrived textbook items, such as the number of ways of ordering six books on a shelf, or the number of ways of selecting a president and a vice-president for a club, do not provide students with relevant, interesting applications of these concepts. In addition, working with real life situations may provide opportunities for developing relationships with representatives of the business community. Field trips within the community, or visits from specialists can provide additional exposure to possible classroom activities. Connecting science and mathematics to real life activities should have a profound impact upon both the curriculum and the way it is taught.

References

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MATH-IN-A-FLASH

Robert M. Nielsen

Freelance Writer

After more than 10 years of "restructuring," General Motors lost nearly $8 billion dollars last year. Nearly 10 years after the publication of *A Nation At Risk* and in spite of major efforts to change them, school achievement levels are not measurably different. This is not surprising, for in schools as in industry, the same old things done the same old ways produce the same old results. Until changes are made in the basic product design and the processes by which these products are manufactured, all the "restructuring" and applications of "information technologies" become little more than prescriptions for piling up scrap faster. No, I'm not trying to equate children with Chevys or teachers with automobile workers. It's more a matter of the school program than kids, less product than process.

The old mass production assembly of a car is based on the principle of Taylorism, after Frederick Winston Taylor, father of turn-of-this-century Scientific Management. It means low-skill jobs for untrained workers and uninformed consumers. By Taylor's principle, every worker is given a different pile of parts and each attaches that particular part in the proper place to form an emerging automobile. Like, for example, placing bolt #35 on the left rear wheel. Roughly put, Taylorism assumes a car is the sum of its parts, a notion that consumers have rejected, and the United States automobile industry doesn't understand yet. Taylorism is obsolete.

It is possible that Taylor discovered Scientific Management while in school, for the education industry had been practicing it for centuries. It is still widely assumed that students come to school empty-headed (un-assembled), and that teachers can fill those empty heads with buckets of basic facts. No where is this more true than in early instruction in math.

Elementary school math instruction starts with a ridiculous ordered ranking which begins with arithmetic. Arithmetic is addition, subtraction, multiplication, and division--and each part of this pecking-order must be mastered before the next can be undertaken. Moreover, each of these subdivisions can be represented by a table. In turn, these tables can be cut into pieces (basic number facts) and pasted onto cards. We now have our bucket of bolts--they are called flash cards. A child learns arithmetic by having these parts bolted into their heads by drill and rote. Does it work? Nope. Not any better than it does with cars. The U. S. produces too many automobiles that no one wants to buy; and U.S. school kids get nine years of arithmetic which more than half of them cannot use to solve simple problems.

With the same mechanical, cut-and-paste efficiency and a heavy dose of intellectual sleight-of-hand, these piles of flash cards also are used to assemble text books and standardized tests. They are a marvel of simplicity, math coursework, text, and test, all rolled into one. The flash card system has several other advantages. First, it is teacher proof; any one can hold up a flash card. Second, it is positively biased against thinking; no complex figuring out here. And third, it serves as a simple Intelligence Test; either you've got it, or you don't. flash cards are pervasive in nearly every elementary school in the country. They are there today, as they were for you and me and our mothers and fathers before us, to drill children in some 399 "number facts."

The National Council of Teachers of Mathematics' (NCTM) new *Curriculum and Evaluation Standards for School Mathematics*, and *Professional Standards for Teaching Mathematics* emphasize three fundamental ideas:

1. All kids can learn math.

2. Math is more than arithmetic.

3. Knowing math means being able to use it.

Using flash cards to drill kids in the "basic facts of arithmetic" contradicts every one of these ideas. By reducing *learning* to memorization and recitation, *knowing* to the ability to give instant answers, and *teaching* to close-order drill, flash cards make math meaningless.

Why then, does the use of flash cards to "teach" arithmetic persist? Like General Motors, the schools are struggling to modernize outdated facilities, and to restructure their management systems in order to prepare students for their lives in the 21st century. If a company doesn't get it right, it either goes bankrupt or ventures into another business. Our schools have no such option. the NCTM *Standards* for school math are a hopeful sign. In those schools and classrooms where they are being implemented, flash cards as the principle means of instruction are disappearing.

Each of us has an obligation to help our schools make the transition from the old to the new, from the obsolete to the competitive. We can start by paying attention to what the math reform community has to say. Ban Flashing! Help Stamp Out Math Damage Now!

TEACHING MATHEMATICS THROUGH "SYLVESTER AND THE MAGIC PEBBLE"

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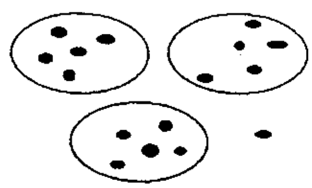
Many papers have been published demonstrating that children's books, in which mathematical concepts are readily apparent, can be used to teach math to young children. A few examples are: *I Can Count the Petals of a Flower* (number recognition), Wohl & Wohl, 1975; *A Kiss is Round* (geometric shapes), Budney, 1966; *Five Chinese Brothers* (ordinal numbers), Bishop, 1938; and *Caps for Sale* (one to one correspondence), Slobodkina, 1984. Mathematics can also be taught with books where the learning activities are neither predetermined, nor prescribed. This article highlights how one such book, *Sylvester and the Magic Pebble* (Steig, 1969), was used to teach math to 31 kindergarten children (Jennings & Scott, 1989) while incorporating five of the National Council of Teachers of Mathematics Standards (1989). (See appendix for a listing of other books used in the study.)

Developing Mathematical Concepts

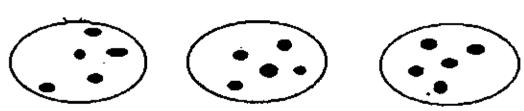
Sylvester and the Magic Pebble is a story about a donkey who lived in the country with his parents, Mr. and Mrs. Duncan. Sylvester's hobby was collecting pebbles on Strawberry Hill. One cold, rainy day, Sylvester found an unusual red pebble. As he studied the pebble, he began to shiver from the cold rain. He wished it would stop raining, and to his surprise, the rain stopped. He wished the sun would shine, and that wish also came true. Suddenly, Sylvester saw a mean and hungry lion approaching and wished that he could become a rock; and he did. Sylvester remained inside the rock for several seasons. There was a great search to find Sylvester, but to no avail. Late one summer, Sylvester's parents decided to go on a picnic on Strawberry Hill. After setting their lunch on a beautiful rock, the Duncans spotted the magic pebble. Picking up the pebble, Mr. Duncan said, "Sylvester would have loved this pebble for his collection." Sylvester who was inside the rock, heard his parents' voices and wished that he could be himself. At that very instant, he was! At last the Duncans were a happy family again. Mr. Duncan put the magic pebble in an iron safe. What more could they wish for than to be together as a family?

"*Sylvester and the Magic Pebble*" offers many exciting opportunities to engage children in mathematics as it relates to their experiences. On the first page of the story Sylvester has a collection of pebbles. After hearing the story, children used pebbles, realistic props, to develop concepts of whole number operations, estimation, geometry, measurement, time, and probability and statistics. Realistic manipulative materials are important for young children to engage in problem solving tasks and divergent thinking (Marbach & Yawkey, 1980) in mathematics and for the development of number sense, the genesis of "mathematical maturity." The pebbles were free and accessible to children and they began building their own collections of pebbles from the school grounds and from home. The children's pebble collections contained pebbles of unusual shapes and colors. Estimations of how many constituted a handful, and comparisons of size, color, texture, weight and shape naturally developed as the children began examining and sorting their pebbles. In addition to spontaneous initial activities, additional experiences were designed to develop specific aspects of the mathematics curriculum.

Concepts of whole number operations. Children concretely experienced large numbers. They were asked numerous questions based on their pebble collections. Rebecca's collection contained 25 pebbles. If Rebecca found 6 more pebbles, how many would she have in all? How many would she have if she lost one? If Joey and Rebecca combined their collections, how many pebbles would there be in their composite collection? If Joey decided to share his collection with Chris, and he gave Chris half of his pebbles, how many would Joey have left? If Rebecca divided her collection of 16 pebbles equally among three friends, how many pebbles would each friend receive, and how many would be left? Through questioning and manipulating of the pebbles, children were able to deal concretely with addition, subtraction, fractions, and division with a remainder.



Rebecca's collection divided among three friends:



Friend #1 Friend # 2 Friend # 3

Figure 1.

Rebecca concretely experienced dividing her pebble collection among three friends. Later, when she was developmentally ready, she represented this abstractly as "16 ÷ 3 = 15 with a remainder of 1."

Estimation. Children began developing estimation skills when asked to guess how many pebbles were in a child's collection and then to determine the accuracy of the estimate by actually counting. Through estimating children made predictions about which collections contained the most or the fewest pebbles. Statements such as, "Katie's collection looks a little larger than Rebecca's" and "Robert's collection is about the same size as Danny's," encouraged children to see that approximations are a legitimate part of mathematics. Situations in which an exact count was unnecessary, such as choosing an appropriate sized table on which to display collections, were explored with the children.

Geometry. The story describes Sylvester's encounter with an extraordinary pebble which is red, shiny and perfectly round like a marble. This provided an opportunity for the teacher to explore solid geometry. Children were encouraged to notice that Sylvester's home and the policeman's desk were rectangular prisms; the orange in the basket was identified as a sphere. The concept of a sphere was further explored using balls and marbles. This activity was followed by a discussion of a sphere, for example; spheres roll; they have no edges or corners; and they have no top or bottom. Spheres were compared to the various shapes of wooden blocks (cubes, rectangular prisms, cylinders, and pyramids), and wheels on trucks and cars typically found in the kindergarten construction learning center. At the end of the story children began to look for regularities in the classroom environment and began to express them mathematically.

Measurement/Time. When Sylvester found the magic red pebble and became enclosed in a large rock, there were opportunities for developing the concept of time including the introduction of calendar time, clock time and psychological time (Elkind, 1981). Although Piaget (1971) says that children don't fully understand the concept of time until about age eight, references to calendar time, such as first, last, before, after, day and night, and the passing of months were made. Children were given a calendar and encouraged to turn the page to indicate that each sheet represented a month. Seasons were discussed using the picture in the story showing snow in winter and flowers in summer. Clock time was introduced when Mr. and Mrs. Duncan were preparing for a picnic at about midday. Children were shown a clock indicating 12:00 noon. Psychological time was noted as children began to collect and relate pebbles to objects within their own environment and when they gathered statistical information such as classifying pebbles, tallying information and drawing conclusions.

Elkind (1981) says that kindergarten children need to learn that these terms, calendar, clock, and psychological time have definite meanings. They use these terms in their conversations even though an internalization of the duration of intervals has not yet been developed. Further, exposing children to mathematical vocabulary is crucial for them to grow in mathematical understandings.

When Mr. and Mrs. Duncan took a picnic lunch to Strawberry Hill, Mr. Duncan discovered the magic red pebble. Questions were asked that directed their discussion to incorporate positional and spatial concepts using a large rock and a pebble painted red. For example, "Is the pebble above, on or under the rock?" "Which pebble did Sylvester find first?"

Children's thinking was extended to include measurement of distance when the teacher asked, "How far away from the rock was Mr. Duncan when he spotted the pebble?" Using a large rock found on the roadside, children placed the small pebble near the large rock and measured distances from many directions. They measured the circumference of the large rock and their pebbles.

Statistics and Probability. The concept of probability was explored in two very concrete, understandable and developmentally appropriate ways. Strategies included first giving each child two bags of 10 pebbles each (one with one magic pebble and nine ordinary ones; another with five magic pebbles and five ordinary ones). Children were asked to guess which bag of pebbles might yield the most wishes. This activity was especially motivating as children often showed an increased interest when discussing wishes and magic pebbles. A second strategy involved placing an equal number of pebbles in several small paper bags. Each bag had the same number of ordinary pebbles but contained varying numbers of "magic" pebbles. The bags were distributed to each child or pair of children. They were asked to reach into the bag (without looking) and select a pebble. After several trials children were asked if they noticed anything unusual about the number of times a magic pebble was selected. They were then encouraged to make suggestions to explain the results. Later children were given the opportunity to gather statistical information. The children reached into their individual bags ten times and recorded whether or not a "magic" pebble was selected. The chosen pebble was returned to the bag after each trial. The children were shown how to keep a tally sheet as shown in Figure 2 so that conclusions could be drawn from the information collected.

Trials	Red ●	Not Red ⊙
1	X	
2		X
3		X
4		X
5	X	
6	X	
7		X
8		X
9		X
10		X

● Red 3

⊙ Not Red 7

Figure 2.
Gathering statistical data in the early childhood classroom.

Observation skills were developed by having children collect pebbles and place them in egg cartons for easy sorting and storing. After collecting the pebbles, each child was asked to select one pebble from their collection which was very special to them and carefully examine its characteristics and then give it a name. For example, one child named her pebble "Dot" because there were small dark spots on one side. Another child named his pebble "Sugar II" because it was a golden color like his dog, Sugar. The teacher wrote the name of each child's pebble on a separate index card. With index cards and special pebble in hand, the children formed a circle on the floor and each took a turn explaining their rationale for giving their pebble that particular name. At that point the special pebbles were placed in the center of the circle and "mixed" together. The task was to find one's own special pebble among all of the others. Discussion of how pebbles were identified ensued. Pebbles were passed around the circle with children identifying additional characteristics. Following this activity the children began their classification and graphing activities which went beyond the traditional rough-smooth, large-small categories. For example, categories included pebbles named after favorite people or pets and pebbles named because of specific physical characteristics such as spots and stripes.

Conclusion

Incorporating children's books into math instruction can be an enjoyable activity for both children and teachers. It has been shown that using children's books to teach mathematics concepts increases interests and achievement in mathematics (Jennings & Scott). We know that young children naturally enjoy literature, making it a motivating and useful tool when designing a well-balanced mathematics program. Children's books in which math concepts are not clearly identified provide many opportunities for teachers and children to use their imagination.

Books contain many ideas for using materials in a child's environment. Using the environment enhances the children's understanding of the actual mathematics being taught. It helps them see mathematics as in integral part of their world rather than a series of meaningless abstract tasks. The natural environment develops a sense of ownership within children; Cazden (1976) indicated that it makes them feel in control. They feel they are active participants in mathematics rather than passive bystanders.

Children's literature can serve as the foundation on which advanced mathematical skills are built. These skills are essential for the development of well-rounded, mathematically literate adults.

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Appendix 1

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