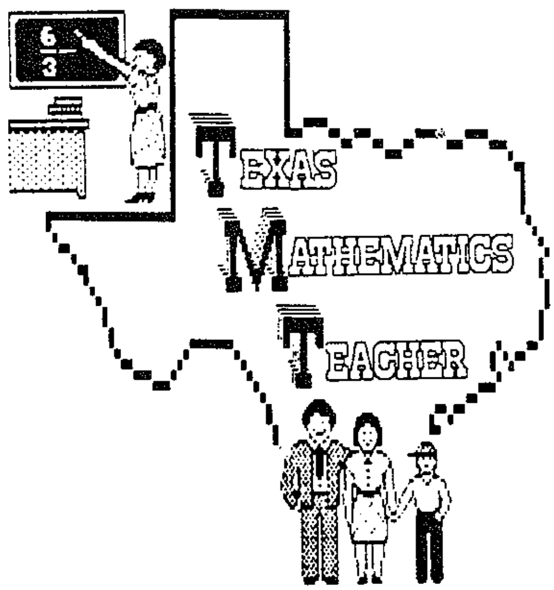

Applying Seldom Used Logarithmic Properties

Exploring Math in the Real World–The Math Workshop

The Distance from a Point to a Line

The Woodrow Wilson Summer Institute Program
for Secondary and Middle School Mathematics Teachers

Imposing a Mathematical Structure on the Set of
Logarithm Functions

New Age Math

MARCH 1994

EDITOR

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

March 1994

TABLE OF CONTENTS

ARTICLES

ANNOUNCEMENTS

President's Message

On October 2nd, I was one of more than 1500 educational professionals at 140 sites nationwide who participated in an interactive video conference "Creating a Climate of Change: Math Leads the Way." The conference was designed to promote a positive environment for change in mathematics education. The video conference was administered by the Math Connection, a consortium of six education associations coordinated by the Mathematical Sciences Education Board. At that conference, the following information was distributed.

WHAT IS THE ROLE OF TEACHERS OF MATHEMATICS IN CREATING A CLIMATE FOR CHANGE IN MATHEMATICS EDUCATION?

What is my stake in this change? Why should I care?

- I am the direct link to children in the classroom. If changes do not happen in the classroom, the community loses faith in the ability of educational system to reflect the real world. My success as a professional educator impacts the future options of my students and reflects my own capacity to learn and grow along with my students.

- Parents expect me to develop the skills and focus the creativity of their children, making them productive learners. They want me to enhance the natural curiosity of their children in fields such as mathematics and science. Parents will support a school system that delivers on its promises.

- I have a professional responsibility to work with my colleagues in mathematics, including those who are elementary generalists, to make mathematics an exciting and productive experience for the children we teach. I need to find ways to cross disciplinary lines–mathematics is in every subject.

What should I do?

- Actively participate in professional development programs related to mathematics teaching and learning. These may be special sessions at the school or district level or opportunities for in-service courses at a local institution for higher education.

- Attend local and regional meetings of teacher professional associations, particularly those with special interest in mathematics education.

- Engage in conversation with other teachers at your school or district about strategies for implementing change in the mathematics program.

- Share with other teachers and administrators in your school or district the new approaches you develop for the teaching and learning of mathematics.

There was a time when a good mathematics classroom was a quiet classroom. The teacher, armed with no more than a textbook and a box of chalk, was the "sage on the stage." Those days are gone, and hopefully will not return. Mathematics education **IS** changing, and we as teachers are the key to making this change occur. The Governors Conference has set the goal that our nation will be number one in the world in mathematics and science by the year 2000. We had better get busy!!!!!

Susan Thomas

APPLYING SOME SELDOM USED LOGARITHMIC PROPERTIES

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In high school and in advanced junior high school, students have opportunities to work with logarithms. Some of the rules or properties of logarithms are:

(1) $\log_b(MN) = \log_b(M) + \log_b(N)$

(2) $\log_b\left(\frac{M}{N}\right) = \log_b(M) - \log_b(N)$

(3) $\log_b(M^k) = (k)\log_b(M)$

(4) $\log_b(b) = 1$

(5) $\log_b(1) = 0$

(6) $\log_b\left(\frac{1}{M}\right) = -\log_b(M)$

We now go beyond these basic properties and state two more logarithmic rules that are often unused.

If $\log_{a^k}(M) = p$, *then* $\log_a(M) = (k)(p)$.

A proof of this rule follows:

Given: $\log_{a^k}(M) = p$. We rewrite this into its corresponding exponential form: $(a^k)^p = M$ or, $a^{kp} = M$. Rewriting this statement logarithmically using a base a yields:

$\log_a(M) = kp$, the desired conclusion.

Another seldom used property or rule states:

If $\log_b(M) = p$, then $\log_M(b) = \frac{1}{p}$.

This may be proved by rewriting $\log_b(M) = p$ into exponential form, $b^p = M$, and raising both sides to the $\left(\frac{1}{p}\right)^{th}$ power: $(b^p)^{\frac{1}{p}} = (M)^{\frac{1}{p}}$, or

$b = M^{\frac{1}{p}}$. Rewriting into logarithmic form using M as the base yields

$\log_M(b) = \frac{1}{p}$, which was desired.

Some problems sampled from mathematics contests will now be introduced to allow us to put these seldom used properties to work.

Example 1. Determine the value of $\log_9(\frac{1}{27})$.

Solution: Set the expression equal to p and rewrite as

$$\log_{3^2}\left(\frac{1}{27}\right) = p \text{ ; then, } \log_3(27^{-1}) = 2p \text{ , or}$$

$$(-1)\log_3(27) = 2p. \text{ Since } \log_3(27) = 3, \text{ we have } (-1)(3) = 2p,$$

and, finally, $p = -\frac{3}{2}$.

Example 2. The equation $\log_{x^3}(4) + \log_8(x) = 1$

has two roots. Determine the product of the roots.

Solution: Let $\log_{x^3}(4) = p$; then $\log_x(4) = 3p \Rightarrow$

$$2[\log_x(2)] = 3p \Rightarrow \log_x(2) = \frac{3p}{2} \Rightarrow \log_2(x) = \frac{2}{3p} \Rightarrow p = \frac{2}{3\log_2(x)}.$$

Further since $\log_{2^3}(x) = \log_8(x)$,

let $\log_{2^3}(x) = q$, or $\log_2(x) = 3q$.

Therefore, $q = \left(\frac{1}{3}\right)\log_2(x)$.

So, the original equality is equivalent to:

$$\frac{2}{3[\log_2(x)]} + \left(\frac{1}{3}\right)\log_2(x) = 1, \quad \text{or}$$

$$2 + [\log_2(x)]^2 = (3)\log_2(x), \quad \text{or}$$

$$[\log_2(x)]^2 - (3)\log_2(x) + 2 = 0, \quad \text{or}$$

$$[\log_2(x) - 2][\log_2(x) - 1] = 0.$$

By the zero-product theorem, we have either:

$\log_2(x) = 2$ or $\log_2(x) = 1$.

Therefore, it is implied that either $x = 2^2 = 4$ or that $x = 2^1 = 2$. So, the product of the roots then is: $(4)(2) = 8$.

Example 3. If $\log_{400}(5) = x$ and $\log_{400}(2) = \underline{y}$, express $\underline{y}$ in the form $Ax + B$ with $\underline{A}$, $\underline{B}$ rational.

Solution: Notice that $\underline{x}$ can be rewritten as $\log_{20^2}(5)$;

therefore, $\log_{20}(5) = 2x$.

Further, $\underline{y}$ can be rewritten as $\log_{20^2}(2)$, so

$\log_{20}(2) = 2y$. Therefore, $2\log_{20}(2) = (2)(2y) = 4y$, and

$\log_{20}(4) = 4y$.

It follows that the sum:

$\log_{20}(5) + \log_{20}(4) = 2x + 4y$, and that

$\log_{20}[(5)(4)] = 2x + 4y$, or

$\log_{20}(20) = 2x + 4y$ which implies $1 = 2x + 2y$.

From this relationship, we isolate y as:

$$y = -\frac{1}{2}x + \frac{1}{4}.$$

It is to be noted that standard logarithmic properties have been applied right along with the often unused logarithmic rules.

Students drilling as members of junior high and high school contest teams may find that using these seldom used logarithmic rules adds some zest by allowing alternative solution procedures.

EXPLORING MATH IN THE REAL WORLD – THE MATH WORKSHOP

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This article is an overview of a workshop approach to math developed by the authors, primary teachers. What follows is a personal account of its implementation in three primary classrooms.

Why a Math Workshop?

When we began the school year in the Fall of 1992, we were searching for a workshop approach for teaching math. For several years we had immersed ourselves in reading and writing research. We had developed a philosophy that enabled us to create a reading/writing program that allowed the students to experiment, apply, and spiral at a developmental pace. We wanted to transfer the same concepts and procedures to our math program. After discussing how we would implement a math workshop, we realized our basic beliefs about math instruction had changed from delivering a set sequence of skills to a program that honors approximations. Responses that are not exact, but almost so, are used to guide future instruction. In reading/writing we go from the whole and let students apply the conventions as they are ready, allowing for approximations along the way. Why not for math? It is concrete, it is logical, and with the appropriate manipulatives and guidance, students can develop concepts using their own scope and sequence.

Workshop Overview

A reading/writing workshop introduces and revisits skills many times during the year. To accomplish this in our math workshop (based on the National Council of Teachers of Mathematics Standards), we chose problem solving as an umbrella and 5 basic strands to apply problem solving on a daily basis: geometry, operations, number concepts, measurement, and patterns. We set up workstations for each of the strands with a wide variety of manipulatives to meet a wide range of abilities and needs. The materials, mini lessons, and guided math instruction provide children with the spark to generate their own activities. This contrasts the traditional weekly teacher directed centers. We have more time to assess and guide student learning and we involve our students more in the process. (See Figure 1.)

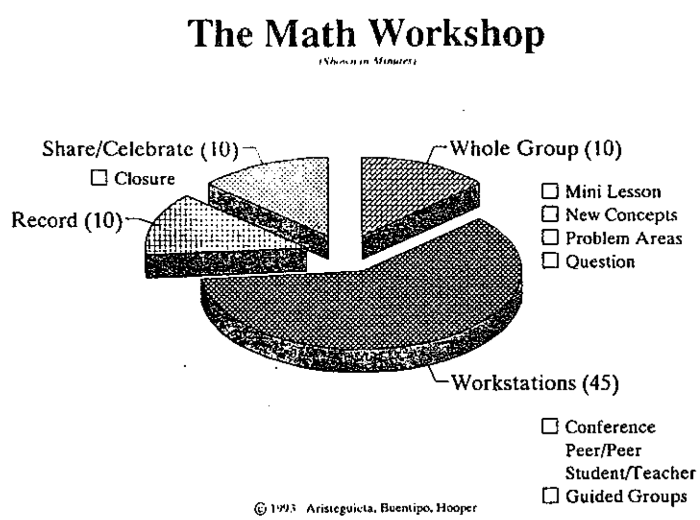


Figure 1.

Whole Group Mini Lessons

Math time starts with a whole group mini lesson. A typical mini lesson lasts no longer than 15 minutes and addresses only one specific procedure, skill, or concept. Similar to our reading/writing workshop mini lessons first address procedures and move into concepts and skills when appropriate. As students become more independent, their exploration is directed through questions that spark their learning in the workstations.

Workstations

The heart of the math workshop lies in the workstations. This is where the action takes place. Traditionally the math strands have been taught as a unit once during the year. The concept is not revisited whether the students master it or not. In comparison, our students are exposed to and given opportunities to apply all strands simultaneously throughout the year. As a result of teacher guidance, students are continuously spiraling upward through the math curriculum at their own rate.

Writing as a Model

In developing our workstations we used our writing workstation as a model. First we created our operations workstation. Both of these workstations are low maintenance in that we resupply as needed and add new materials during the year to spark student activities.

Writing Workstation	Operations Workstation
markers	variety of counters
variety of paper	flannel board
colored pencils	felt pieces
stapler	flash cards
tape	place value models
erasers	paper
pencils	pencils
stamps	books

Problem Solving
(Math Process in Workstations)

Strands	Mini/Question	Materials Students Might Use[1]	Possible Observations	Conference Questions
Operations and Computations	•Combining sets •Regrouping How many fingers in the class?	•Variety of counters •Flannel board/pieces •Place value	•Student using two sided counters to combine sets •Students counting fingers by ones, fives, multiplying, tally marks	•How did you solve the problem? •Why did you regroup?
Number Concepts and Numeration	•One to one correspondence #1-10 •Place value ones/tens/hundreds •How many groups of ten can you find in the room?	•Variety of counters •Place value •Number cards •Number books	•Making sets of #1, 2, 3, etc. •Students counting and recording by tens, students counting isolating groups of ten.	•Do your pieces match? •What number did you make? Show me another way
Geometry	•Edges and sides •Properties of solids ○Graph the solids you can find.	•Geometric solids •Graphing mat •Real life solids	•Tracing •Sorting •Graphing •Building	•Show me an edge, show me a side, how do you know? •How are the solids in this column the same?
Measurement	•Measuring length •Comparing length ○Who's the tallest in the room?	•Tape measure •Unifix cubes •Paper clips •Rulers	•Measuring without lining up ends •Lining kids up, measuring with a tape measure	•What did you use to measure? •Can you measure it another way?
Patterns, Relations and Functions	•ABC patterns •Venn diagram for sorting ○Sharing songs with patterns	•Pattern cards •Pattern blocks •Sorting boxes •Venn diagrams	•Extending pattern cards •Sorting, resorting •Experimenting with new song patterns	•How do you know it is a pattern? •What is another way to read your pattern?

[1]Materials not directly related to mini lessons are also in workstations.

Table 1

The Math Process

In the writing workstation students use the writing process to arrive at a final product: the process is emphasized and the product is celebrated through publication. Likewise, in our math workstations, students use the math process to arrive at a final product: the process is emphasized and the product is celebrated during the share/celebrate time. Traditionally in math, only the correct answer is honored. Students go through a process to reach an answer, although the process is not considered during the final evaluation. In contrast, the process is the essence of our program. Like the writing process, students use the math process to work through math concepts.

The Math Process	The Writing Process
Question/Predict/Plan	Brainstorm
Attempt problem	Draft #1
Self-check with manipulatives	Conference
Peer Conference	Draft #2 or revise
Record final solution	Conference
Teacher conference	Self edit
Work through process again if necessary	Teacher edit
Share - Celebrate	Publish

Guided Math Groups

While students work on problem solving using the math process in the workstations, the teacher circulates and questions the students' processes and pulls guided math groups. Groups are established based on needs. Instruction is given on specific skills to help students transition to conventional math norms. In addition, some concepts are previewed in small groups to spark the students' exploration in workstations.

Recording

An integral part of our Math Workshop is daily student recording. Each of our students keeps a math scrapbook. These scrapbooks allow the students to reflect on concepts practiced and through writing, clarify their understanding of what was learned. They use a recording sheet to record their workstation and what was learned for each day of the week. Students must evaluate their investigations as Easy, Just Right, or Challenge. All math products are kept in the scrapbook throughout the year. Our students use them to note their own progress. We use them as the basis for future math instruction and evaluation (see Figures 2-5).

Allowing students to problem solve using the math process has enabled us to meet the individual needs in math like we do in reading and writing. Students are no longer responsible for memorizing facts, but rather for applying concepts in their own unique and meaningful way.

NAME: Giselle

Monday Date: 5-10	Workstation: Geometry Question: Qan I maq a Star weth a geoboard? Today I learned to meiqa Star weth an x and a gras.	(E) JR C (☺) ☹ (finished) unfinished
Tuesday Date: 5-11-93	Workstation: measurement Question: How mach das a book weigh? Today I learned a book Yo Arx a mi Familia weghs 18 Cheins	(E) JR C (☺) ☹ (finished) unfinished
Wednesday Date 5-12-93	Workstation: Number ConCept Question: How qan I meiq the Number 14? Today I learned to meiq 14 with 1 10 and 4 ones.	(E) JR C (☺) ☹ (finished) unfinished
Thursday Date: 5-13-93	Workstation: Operations Question: Qan I subtract 15 Cubs From 10 Cubs? Today I learned I Canot subtract 15 Cubs From 10. I Can subtract 10 Cubs From 15.	E (JR) C (☺) ☹ (finished) unfinished
Friday Date: 5-14-93	Workstation: Patterns Question: wat Can I maq with pattrn blacks? Today I learned to mece a man with 5 difrint Chaps.	E (JR) C (☺) ☹ (finished) unfinished

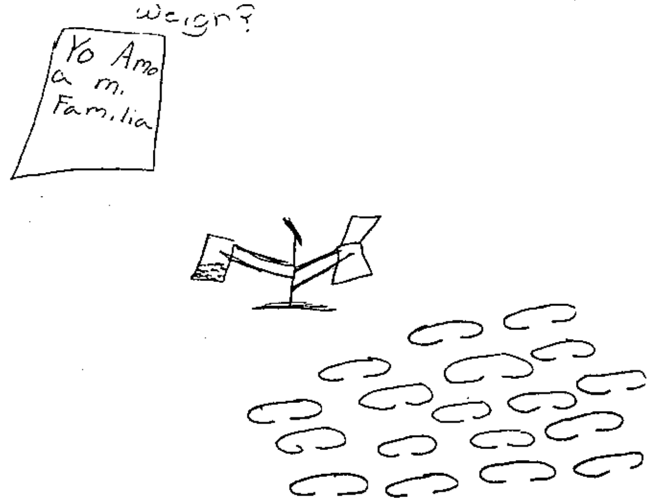
ABH Math Workshop

Figure 2.

Date 5-11-93 Name Giselle

Workstation Measurment

Question: How mach bas a book weign?



Today I lrned a book Yo Amo a mi Familia Wais 18 Cheins

ABH Math Workshop

Figure 3.

THE DISTANCE FROM A POINT TO A LINE

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Presented here is another proof for finding the distance from a point to a line that high school students should be able to comprehend. When the concept is repeated in geometry, Algebra II, and advanced math, the teacher may opt to add a new proof for variety and for the enhancement of student learning.

Interestingly, similar methods with different geometric figures were used in proofs by Schielack, V. (1987), and Eisenman, R. (1969). The authors feel that their method uses similar triangles that students will find easy to visualize.

Assume one is given the line ℓ with the equation $y = mx + b$; the point P(h,k) is off the line as shown in Figure 1 and 2. Dropping a perpendicular line from (h,k) to the x-axis forms two similar triangles having proportional sides.

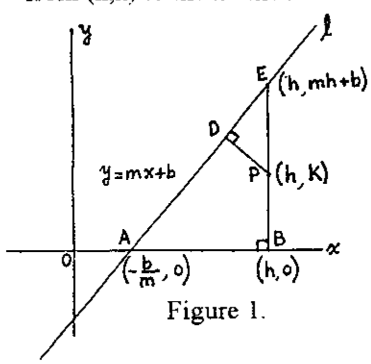

Figure 1.

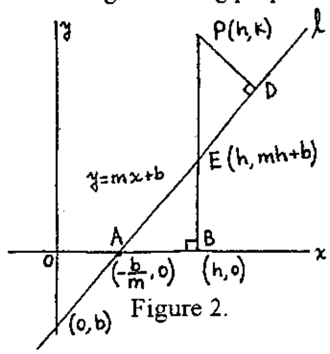

Figure 2.

Two of the points, (h,0) and (h, mx + b), are found by solving the equations y = mx + b and y = 0 simultaneously with the line x = h. The

point $(-\frac{b}{m}, 0)$ is found by solving the equation y = mx + b and y = 0.

Now one has $\triangle ABE \sim \triangle PDE$.

Therefore, $\frac{PD}{AB} = \frac{PE}{AE}$.

$$AB = \left| h + \frac{b}{m} \right| = \left| \frac{mh + b}{m} \right|.$$

$$PE = |k - (mh + b)| \qquad AE = \sqrt{(h + \frac{b}{m})^2 + (mh + b)^2}.$$

Let $d = PD$.

By substitution $$\frac{PD}{\left| \frac{mh + b}{m} \right|} = \frac{|k - (mh + b)|}{|mh + b| \sqrt{1 + \frac{1}{m^2}}}.$$

$$d = \frac{|mh + b|}{|m|} \cdot \frac{|mh + b - k|}{|mh + b| \sqrt{\frac{m^2 + 1}{m^2}}}.$$

$$d = \frac{|mh + b - k|}{|m|\sqrt{\dfrac{m^2 + 1}{m}}} = \frac{|mh + b - k|}{\sqrt{m^2 + 1}}.$$

Notice that if $m = \frac{-A}{B}$ and $b = \frac{C}{B}$, *then the familiar form*

$$d = \frac{|Ah + Bk - C|}{\sqrt{A^2 + B^2}}$$ *is obtained.*

This approach uses only the elementary algebra and analytical geometry which most students of high school geometry should be able to understand.

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Schielack, V. "Mathematical Applications of Geometry." *Learning and Teaching Geometry*. Reston, VA.: The National Council of Teachers of Mathematics, 1987.

THE WOODROW WILSON SUMMER INSTITUTE PROGRAM FOR SECONDARY AND MIDDLE SCHOOL MATHEMATICS TEACHERS

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Introduction

The *Professional Standards for Teaching Mathematics* recommends that teachers of mathematics take an active role in their own professional development. More specifically, activities advocated include participation in mathematical education opportunities such as workshops, discussions with colleagues regarding teaching techniques and issues in the teaching and learning of mathematics, and participation in the design and implementation of professional development programs for mathematics teachers.

The Woodrow Wilson National Fellowship Foundation (WWNFF) has devised, implemented, and propagated a unique program of national scope that promotes these recommended activities for the practicing secondary or middle school mathematics teacher -- the National Science and Mathematics

Leadership Program. This program is based on the powerful paradigm of "teachers teaching teaching," the foundation of which is a pair of firm convictions: that professionals entertain and incorporate new techniques and ideas most readily when these are conveyed by their peers in an atmosphere of professional collegiality, and that teachers are the key to revitalizing mathematics education, since they have final authority about what actually happens in their classrooms and are most fit to decide what works. The WWNFF program has three major goals:

(1) To strengthen the content knowledge and pedagogical skill of mathematics teachers by facilitating the dissemination of up-to-date mathematical information and teaching techniques that have been proven effective through classroom use.

(2) To increase the development and retention of mathematics teachers by improving their professional status, sense of community, and scholarly growth.

(3) To raise the level of mathematical motivation and achievement of students by promoting the use of teaching strategies, materials, and technologies that involve all students as active learners.

The Program

The National Science and Mathematics Leadership Program, begun in 1984, has two major interconnected facets. The first is a four-week institute held at Princeton University each summer. This institute brings together 50 of the country's most outstanding secondary school mathematics teachers for an intensive examination of recent developments in research and methodology, aided by distinguished college and university personnel from across the nation. Using their classroom expertise, the teachers extract from this material the mathematical and pedagogical ideas that they feel will be most useful and develop instructional units that can be incorporated into the existing curriculum. They then test these self-generated materials in their own classrooms during the next year. Teachers can disseminate materials that are proven successful to other teachers in their schools and localities; these "outreach" activities are supported through small grants available from WWNFF. More that 80 percent of the Princeton institute participants utilize these and other funds for this purpose, often in collaboration with local teacher and professional organizations, business associations, and college and universities.

The second major Woodrow Wilson activity consists of a series of one-week institutes held during the summer of each year since 1985 at sites across the nation, usually at colleges and universities. The most dynamic and effective of the Princeton institute teachers are selected to form four-person teams of "teacher leaders" to conduct four or five of these institutes each summer. (Many of these individuals are recipients of prestigious recognition as outstanding teachers, such as Presidential Awards.) Each institute has a site coordinator, a faculty member from the hosting institution who, with the help of WWNFF, recruits up to 32 teachers from the site's vicinity to attend the institute, and also handles local arrangements for the institutes, such as meals and lodging for participants and teacher leaders and appropriate workrooms, facilities, and computer and demonstration equipment. The teacher leaders present the materials they developed during their Princeton institute experience and refined in their own classrooms and share their expertise with the participant teachers in an interactive environment. Each participant receives a notebook containing the institute materials, additional relevant papers on content or instructional methods, and computer software. In the presentation of institute materials, the teacher leaders promote active investigation, introduce timely and relevant "real-world" applications, and demonstrate effective means for utilizing technology to aid learning, as participants work in a collegial, collaborative atmosphere under the tutelage of their peers. Participants themselves present techniques or demonstrations that have worked well in their own classrooms. At all times, the teacher leaders model teaching behaviors that emphasize the acquisition of higher-order thinking skills and problem-solving behaviors within the current curriculum.

After the institute, participants return to their schools and incorporate their institute experiences into their own teaching. This can be done by using institute demonstrations in whole or in part, or by developing new materials based on concepts learned during the institute. These teachers often transfer these materials to colleagues following the example modelled by the Woodrow Wilson teacher leaders, producing a multiplier effect for the impact of the one-week institutes. A one-day follow-up session is held at the original site during the spring of the school year following the institute. This follow-up session is directed by one of the teacher leaders, but the majority of the day is used for participants to recount personal stories of successes and problems encountered during the year. One of the most beneficial aspects of the institutes is the creation of informal local networks of teacher colleagues who can support each other in their endeavors to improve mathematics instruction in their schools.

Content

In 1993, fourteen Woodrow Wilson teams presented 53 mathematics institutes at sites in 31 states. The fourteen included five algebra teams, two teams working with each of geometry, functions, and middle-school mathematics, and one team presenting material dealing with each of applications, statistics, and modelling. All institutes encourage introducing appropriate technology throughout the curriculum, using manipulatives to facilitate student learning, and experimenting with new directions in the teaching and content of the mathematics curriculum that reflect the emphases of the *Curriculum and Evaluation Standards for School Mathematics*, in order to make mathematics more appealing and accessible to all students.

The flavor of the Woodrow Wilson institutes can be sensed through a partial listing of topics included in the 1993 Summer Mathematics Institutes: mathematics applied to human decision-making; the discrete and applied nature of mathematics; graphing calculators; data analysis; geometric probability; matrices; technology and symbol manipulation; teaching strategies to encourage students at all levels to move freely among geometric, algebraic, and numerical representations of a problem; linear programming; cooperative learning; helping students learn mathematics through writing; non-traditional geometric topics; a critique of the existing geometry curriculum; implementation of the statistics and probability strands in the NCTM *Standards*; simulations; probability; sampling and inference; a deeper appreciation and understanding of the role of applied mathematics; the use of mathematical modelling to enhance the mainstream secondary-school curriculum; a focus on shape and dimension as integral facets of the middle-school mathematics curriculum; and origami constructions.

Evaluation

The Woodrow Wilson one-week institute program has been the subject of an ongoing evaluation by Norman Webb of the University of Wisconsin based on surveys of institute participants. Six months after the 1989 institutes, a questionnaire was mailed to each participant; over 77% were returned, and it was determined through analysis of demographic data that the sample was representative of the total population, and that this response rate was sufficiently large to estimate the response proportion on items for the total population within one percent at a 95% confidence level. (The extremely high response rate is indicative of the regard teachers have for the program.) A summary of major survey results follows.

(1) Eighty-three percent of participants reported the overall impact of attending the institute as being moderate or higher, and 19% rated the impact as being very high or extremely high.

(2) The "teachers teaching teachers" approach was highly valued by participants when compared to other inservice experiences. Over 98% found this approach to be of the same or greater value with regard to topics covered, materials demonstrated, teaching techniques demonstrated, participants' ideas shared, and the teacher image modelled by the teacher leaders. On the different dimensions, from 79% to 89% found the "teachers teaching teachers" approach to be of greater value than others.

(3) At least 94% of participants reported that the institute increased their knowledge of each of general content, applications, and teaching techniques. Of those reporting a great deal of increase, 36% did so for general content, 50% for applications, and 48% for teaching techniques.

(4) Over 88% of participants reported using information received at the institute from each of material on teaching techniques, background information in the institute notebook, and ideas shared by other participants. Of those reporting a great deal of use, 32% did so for

teaching techniques, 26% for notebook information. and 24% for participant ideas.

(5) Ninety-two percent of participants reported increasing their use of having students utilize critical thinking skills (34% a great deal). Ninety percent increased use of student responses to open-ended questions (32% a great deal).

(6) At least 96% of participants experienced an increase in each of enthusiasm for teaching, interest in curriculum reform, and interest in continued professional growth as a result of the institute. Of those reporting a great deal of increase, 59% did so for enthusiasm for teaching, 61% for interest in curriculum reform, and 71% for interest in further professional growth. Seventy-nine percent reported increased participation in curriculum planning for their school or district.

(7) A large number of participants reported a positive impact on their students as a result of the institute. Ninety-four percent reported an increased interest in content, 91% reported an increased involvement in learning, and 85% reported an increase in achievement.

Webb repeated his analysis for the fourth time after the 1991 institutes. The results for 1991 were nearly identical to those for the previous three years. He noted that "the consistently high percentage of teachers who year after year testify to the impact of the one-week institutes affirms the quality of the institutes and the model for delivering professional development experiences to teachers" (Webb, 1992, p. 1).

The Woodrow Wilson Mathematics Program in Texas

Since the first Texas Woodrow Wilson mathematics institute in 1988, Texas has had a large number of Woodrow Wilson mathematics institutes at sites across the state, including eight in 1993 at Amarillo College, Texas A&I University, Texas A&M University, Texas Southern University, the University of Houston, The University of Texas at Arlington, and the University of Texas at Dallas. The Texas participants echo the sentiments of their national counterparts in their end-of-institute evaluations. When asked what they enjoyed most about the institutes, many cited the associations and fellowship with other teachers and the friendliness, enthusiasm, and professionalism of their group. When asked for a free-response overall rating of the institute, at least one of the following occurred on the large majority of responses: "great," "exceededexpectations," "A+," "excellent," "fantastic," "outstanding," "without equal," "wonderful," "super," "very good," "superior," and "11 on a scale of 1 to 10!"

Funding Sources

Nationally, the Woodrow Wilson institutes have been supported by a number of doners. These include a variety of private foundations, corporations, and individuals. Grants have also been obtained from governmental sources such as the National Science Foundation, the United States Department of Education, and the Dwight D. Eisenhower Mathematics and Science Education Grants Program through collaboration between the WWNFF and state coordinators. The Dwight D. Eisenhower Mathematics and Science Education Grants Program administered by the Texas Higher Education Coordinating Board has supported six Woodrow Wilson mathematics institutes at Texas sites for each of the four years 1990-93.

A participant registration fee of $150 is used to offset material and meal costs associated with each one-week institute. Since they are the major beneficiaries of the program, schools and districts are encouraged to compensate participants for expenses involved with the institutes.

Additional Information

Additional information about the Woodrow Wilson National Fellowship Foundation and its programs can be obtained by writing the Woodrow Wilson National Fellowship Foundation, CN 5281, Princeton, New Jersey, 08543-5281. The WWNFF also has programs in chemistry, physics, physical science, biology, and history that are similar to the mathematics program.

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IMPOSING A MATHEMATICAL STRUCTURE ON THE SET OF LOGARITHM FUNCTIONS

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At one time logarithms were important because they could be used as a tool to do computation. Once better tools were in place to take care of computation, emphasis shifted to logarithms as functions. The importance of logarithm functions is well known. What would calculus or differential equations be without them? The emphasis in these courses is on facts about logarithm functions as elements of a set. Jean Piagét has suggested that in the study of mathematics, attention should also be given to mathematical structures (a non-empty set together with a binary operation).

The purpose of this note is to illustrate a way to impose a mathematical structure on the set of all logarithm functions. Throughout this note x is a positive real number, $b \neq 1$ is a fixed positive real number, and Q is the zero function ($Q(x) = 0$ for all positive real numbers x).

1. The Set of All Logarithm Functions As a Group

Remark 1. If $n \neq 0$ is a real number, then

$$n \log_{b^n}{}^{(x)} = \frac{n \log_b^{(x)}}{\log_b^{(b^n)}} = \frac{n\log_b^{(x)}}{n} = \log_b^{(x)}.$$

So, the two functions $\log_b n$ and $\frac{1}{n}\log_b$ are equal. That is to say,

$\log_b n = \frac{1}{n}\log_b$.

Remark 2. Let $G = \{\log_b{}^n \mid n \neq 0 \text{ real}\}$. Let $a \neq 1$ be a positive real number. Since $b^k = a$ when $k = \log_b{}^{(a)}$, it follows that $\log_a = \log_b{}^k$, $\log_a$ is an element of G, and G is the set of all logarithm functions.

Remark 3. Let + be the binary operation "function addition." Then

$(\log_b{}^n + Q)(x) = \log_b{}^{n\,(x)} + Q(x)$ where $n \neq 0$ is a real number. But $\log_b{}^{n\,(x)} + Q(x) = \log_b{}^{n\,(x)}$. Hence, $\log_b{}^n + Q = \log_b{}^n$ and Q is an identity element on G relative to +.

Remark 4. Let $n \neq 0$ and $m \neq 0$ be real numbers with $n + m \neq 0$. Then $(\log_b{}^n + \log_b{}^m)(x) = \log_b{}^{n\,(x)} + \log_b{}^{m\,(x)}$. But from Remark 1, $\log_b{}^{n\,(x)} = \frac{1}{n}\log_b{}^{(x)}$ and

$\log_b{}^{m\,(x)} = \frac{1}{m}\log_b{}^{(x)}$.

So we have $\log_b{}^{n\,(x)} + \log_b{}^{m\,(x)} = (\frac{1}{n} + \frac{1}{m})\log_b{}^{(x)} = \frac{m+n}{(nm)}\log_b{}^{(x)}$.

From Remark 1, if $k = \frac{nm}{m+n}$, then

$\frac{(m+n)}{nm}\log_b{}^{(x)} = \log_b{}^{k\,(x)}$. It follows that

$\log_b{}^n + \log_b{}^m = \log_b{}^k$, where $k = \frac{nm}{n+m}$.

Remark 5. Let $n \neq 0$ be a real number. Then both $\log_b{}^n$ and $\log_b{}^{-n}$ are elements of G and $\log_b{}^n + \log_b{}^{-n} = Q$ and each element of G has an inverse in G relative to +.

Remark 6. Let $G_1 = G \cup \{Q\}$. From Remarks 3 and 4 and the fact that $Q + Q = Q$, G_1 is closed under +. Remark 3 assures that G_1 has an identity element relative to + and Remark 5 assures that each element in G_1 has an inverse in G_1 and hence G_1 is a GROUP under +.

Remark 7. The set R of all real numbers is an Abelian group under addition. What can be said about the mapping T from G_1 to R where $T(Q) = 0$ and $T(\log_b^n) = \frac{1}{n}$?

For $\log_b^n$ and $\log_b^m$ in G1 with $n + m \neq 0$,
$T(\log_b^n + \log_b^m) = T(\log_b^k) = \frac{1}{k}$ where $k = \frac{nm}{n+m}$.

$T(\log_b^n) + T(\log_b^m) = \frac{1}{n} + \frac{1}{m} = \frac{n+m}{nm} = \frac{1}{k}$.

So

$T(\log_b^n + \log_b^m) = T(\log_b^n) + t(\log_b^m)$ when $n + m \neq 0$.

It is easy to verify that $T(P+S) = T(P) + T(S)$ for all P, S in G_1 and T is a group homomorphism. It is also straight forward to see that T is both one-to-one and onto. Hence, T is a group isomorphism.

2. The Set of All Logarithm Functions As A Vector Space

For r any element of R, the field of all real numbers, define * between G_1 and R as follows:

$r*Q = Q$, $r*\log_{b^n} = Q$ when $r = 0$, but $r*\log_{b^n} = \log \frac{n}{b^r}$ when $r \neq 0$.

Remark 8. The Abelian group G_1 is a vector space over the field of real numbers relative to *.

Proof.. The following must be shown to be true for all reals r and t and all P,S in G_1:

i) r*p is an element of G_1
ii) (r+t)*P = r*P+t*P.
iii) r*(P+S) = r*P+r*S.
iv) (rt)*P = r*(t*P).
v) 1*P = P.

Let $P = \log_b^n$, $S = \log_b^m$, $n+m \neq 0$. Then
$r*(P+S) = r*\log_b^k$, $k = \frac{nm}{n+m}$.
But $r*\log_b^k = \log \frac{k}{b^r}$.
$r*P+r*S = \log \frac{n}{b^r} + \log \frac{m}{b^r} = \log_{b^c}$ where
$c = \frac{nm}{r(n+m)} = \frac{k}{r}$, $r*(P+S) = r*P+r*S$, and a part of iii) has been proven.

The remainder of the proof is left to the reader.

Remark 9. The field R can be viewed as a vector space over itself and the group homomorphism T in Remark 7 is a vector space isomorphism.

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NEW AGE MATH

Robert M. Nielsen

Freelance Writer

Recently, I visited my grandson's second-grade math class. I've been puzzling over some of the goings-on in that room ever since. Does anyone know when they got rid of the rows of desks? Small work tables were scattered about the room. There was a desk only for the teacher, and it didn't look like she used it much. You never saw such scampering and milling around the room. I asked the teacher when class was going to begin, and she said, "About 20 minutes ago." There were calculators on every table, a computer on one, a television set in the corner, colored blocks, games, dice, and gadgets of every variety. Everyone was very busy. Children talked to each other and jumped around from one place to another. A group of three with a clipboard, tape measure, and meterstick were running this way and that measuring everything in sight--even each other.

Asking what the lesson was for today, I got an even bigger surprise. The teacher, Mrs. Johnson, smiled politely and said, "There isn't just *one* lesson. These children are working on about nine different activities, each one of them aimed at a particular concept and skill. That group over in the corner is learning to add fractions, the ones over by the window are doing Monster Math--adding ten 10-digit numbers. Those over there are doing long division." Pointing around the room, she added, "Multiplication at those middle tables, statistics and probability next to them, and graphing equations over in that far corner. Yes, there's a lot going on in here. Keeps me busy. Why don't you join some of them and see how they're doing?"

Recalling that ratios were the real test of whether or not a kid could do math, I sat down at the Fraction Table next to a little girl named Chrissy. "What are these things?" I asked. "Fraction Cakes," she answered. "What are you doing with them?" I probed. "Adding fractions," she said. "You're too little to add fractions, " I teased. "Am not. I'll show you," she shot back. In front of her was a printed sheet with six fraction problems on it. The next in line was 1/2 + 1/3 = __. I remembered having trouble with that one in the sixth grade. Chrissy patiently scrabbled around in a box until she found two pie-shaped pieces, one marked 1/2, the other 1/3. Piecing them together on the table, she studied them carefully. Then she placed the 1/3 on top of the 1/2 and deliberated the new arrangement. Suddenly she smiled, reached in the box, pulled out two 1/6's, and placed them on top the 1/3. Nodding agreement with herself, she returned the 1/3 to the box. Quickly exchanging the 1/2 for three more 1/6's, she now pieced all the 1/6's together. Glancing triumphantly at me, she picked up her pencil and began to count: "One, two, three, four, five. One-half and one-third is five-sixths." She grinned and penciled her answer on the sheet. "Do you want me to show you how to do it with the calculator?" she asked. "Sure," I said. "I'd like to see that." So she proceeded. I was in awe; still am.

My first reaction was skepticism and an itch to discount Chrissy's success. However, Chrissy demonstrated that she possessed a far deeper understanding of what the sum of those two fractions meant than I ever did, even when I was several years older than Chrissy. "Welcome to the wonderful world of manipulatives," Mrs. Johnson said when I expressed my amazement to her. "We can teach concepts as well as skills with these marvelous little inventions."

This classroom is one example of a *new learning* in math. It is a product of the new standards in coursework and teaching from the National Council of Teachers of Mathematics and the promulgation of these ideas in cooperation with the Mathematical Sciences Education Board, a part of the National Research Council in Washington, D.C. This is math for every child, not just a special few.

The children in the second-grade class I visited test out at the third-grade level or better on the California Achievement Test. What was happening in that classroom is not occurring in all schools yet, but it should be--the sooner, the better. Our nation's economic survival and my retirement income depend on it.

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