Texas Mathematics Teacher

A Statistical Analysis of the Texas Lottery

Modification of the Key Number Method of Factoring

Monte Carlo Cootie

Congressional District Data Analysis: Counting County Intersections

JANUARY 1994

TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published four times each year, in October, January, March, and May. All manuscripts and editorial correspondence should be addressed to George H. Willson, listed above.

Manuscripts should be neatly type-written and double spaced, with wide margins, on 8 1/2 " by 11" paper. Authors should submit the signed original and four copies. Illustrations should be carefully prepared in black ink on separate sheets, the original without lettering and one copy with lettering added. As soon as possible after refereeing, authors will be notified of a publication decision. Originals of articles not accepted for publication will be returned to the authors. Two copies of the issue in which an author's manuscript appears will automatically be sent to the author.

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

January 1994

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President's Message

Last year in this space I told you about my new year's resolution for 1993: To use more alternative assessment in my classroom. I set out to accomplish that task with only a few guidelines--the NCTM Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions was available; articles had appeared in professional journals; and I had attended some helpful talks at CAMT and various other meetings, especially some from Steven Leinwand, that provided some good ideas, but I still felt that I was proceeding without a whole lot of knowledge or direction......WHATA DIFFERENCE A YEAR MAKES!!!

If you want to know more about assessment, several publications this past year will provide you with an **abundance** of knowledge and direction. I highly recommend the following for your 1994 professional reading list:

1. Assessment in the Mathematics Classroom - 1993 Yearbook, NCTM. This collection of articles about assessment contains sections with information specific to K - 4 assessment, 5 - 8 assessment, and 9 - 12 assessment. Included are specific examples and descriptions of assessment in the 90's. Assessment is viewed as a means to achieve educational goals rather than the end of education experiences. Texas educators contributing to the yearbook include Jane Schielack and Dinah Chancellor, with an article entitled "From Multiple Choice to Action and Voice: Teachers Designing a Change in Assessment for Mathematics in Grade 1."

2. Assessing Student Performance: Exploring the Purpose and Limits of Testing, was published in October by Jossey-Bass. Author Grant Wiggins is president and director of the Center on Learning, Assessment, and School Structure. Wiggins examines testing and suggests that "the criterion of a good test is its congruence with reality." An article adapted from the book can be found in the November issue of the Kappan.

3. Measuring What Counts, a Conceptual Guide for Mathematics Assessment, and Measuring Up, Prototypes for Mathematics Assessment, are two recent publications of the Mathematical Sciences

Education Board. Measuring What Counts provides research-based connections between standards and assessment. Measuring Up gives specific classroom activities from fourth grade students demonstrating inquiry, performance, communication, and problem solving in mathematics education. Three Texans were on the 16 member study group on guidelines for mathematics assessment. They are Janice Arceneauz, Houston ISD, Cathy Seeley, Charles A. Dana Center, University of Texas Austin, and Marilyn Rindfuss, Psychological Corporation.

4. Assessment Standards for School Mathematics - Working Draft, October 1993. This recently released draft will be a companion document to the Curriculum and Evaluation Standards for School Mathematics, and Professional Standards for Teaching Mathematics. Tom Romberg directed this project, and NCTM id currently seeing comments on the draft document.

Happy reading, and have a wonderful 1994!

Susan Thomas

A STATISTICAL ANALYSIS OF THE TEXAS LOTTERY

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Introduction

Analyzing statistical data has become an important part of the curriculum in our schools beginning with the elementary level. Learning about means, modes and medians along with standard deviations and other statistical methods can be rather dry even when utilizing a computer for all the calculations. One way to get students interested in statistical analysis is to apply statistical methods to current and exciting events. Texas teachers now have just such an event--the Texas Lottery.

The Rules of the Game

Currently, twice a week a machine is used to select six balls numbered from 1 to 50. Lottery players attempt to pre-select the winning numbers in order to win various amounts of money. Each playboard contains the numbers one through fifty. Six numbers can be selected for any or all of the playboards. Provision is made for these numbers to be entered into more than one drawing by marking a multi-draw number from two to 10. Players can win in the following ways:

- match all six of the numbers drawn - odds 1 in 15,890,700,
- match five of the six numbers drawn - odds 1 in 60,192,
- match four of the six numbers drawn - odds 1 in 1,120, or
- match three of the six numbers drawn - odds 1 in 60.

The over-all odds of winning for each play board played are 1 in 57.

Probability of Winning or Losing

The probabilities of the preceding events occurring are calculated as follows. The probability of selecting all six numbers correctly is 1/C(50,6) which is 1/15,890,700 = .000000063. The probability of selecting five correctly is 1/C(50,6) times C(6,5) times C(44,1) which is approximately 1/60,192 = .000016. The probability of selecting four correctly is 1/C(50,6) times C(6,4) times C(44,2) which is approximately 1/1,120 = .0009. Finally, the probability of selecting three correctly is 1/C(50,6) times C(6,3) times C(44,3) which is 1/60 = .0167. The sum of these probabilities is .0176 which is approximately 1/57--the probability of winning anything. These numbers are printed on the back of each Lotto playslip as odds of winning. Recall the odds of winning are the probability of winning divided by the probability of losing, so the odds of selecting all six numbers correctly would be 1/15,890,699. However, since the denominators of the probabilities above are so large, the *odds* of winning are approximately the same as the *probability* of winning.

The probability of losing is also interesting to calculate. The probability that none of the numbers will be chosen is 1/C(50,6) times C(6,0) times C(44,66) which is .444. The probability that exactly one number will be chosen is 1/C(15,6) times C(6,1) times C(44,5) which is .41. Finally, the probability that exactly two numbers will be chosen is 1/C(50,6) times C(6,2) times C(44,4) which is .128. Thus, the probability of losing is the sum of these numbers which is .9824. Of course this number is 1 - .0176, the probability of winning anything at all. All of the above calculations should be useful in helping students understand probabilities and also in demonstrating the reason the Texas Lottery is a revenue producer for the state treasury.

Randomness of the Lottery

The remainder of this article examines the statistical behavior of the numbers chosen in 100 drawings in order to determine if the lottery is "fair." That is, are the numbers, as chosen so far, truly random? Obviously, 100 drawings will not establish randomness. There are ways to demonstrate to students that as the number of drawings increase, certain measures of randomness also change. The methods considered in this paper are: frequency of the numbers chosen, the mean, standard deviation and Chi squared, distribution of the numbers chosen and distribution of the sums of the numbers chosen. A GWBASIC computer program was used to tabulate the frequency of each number's occurrence and to calculate the probability of its recurrence. Averages and standard deviations were also computed to help determine whether or not the numbers were chosen at random.

Frequency of Numbers Chosen

The frequencies of each number chosen so far are shown in the following table and chart:

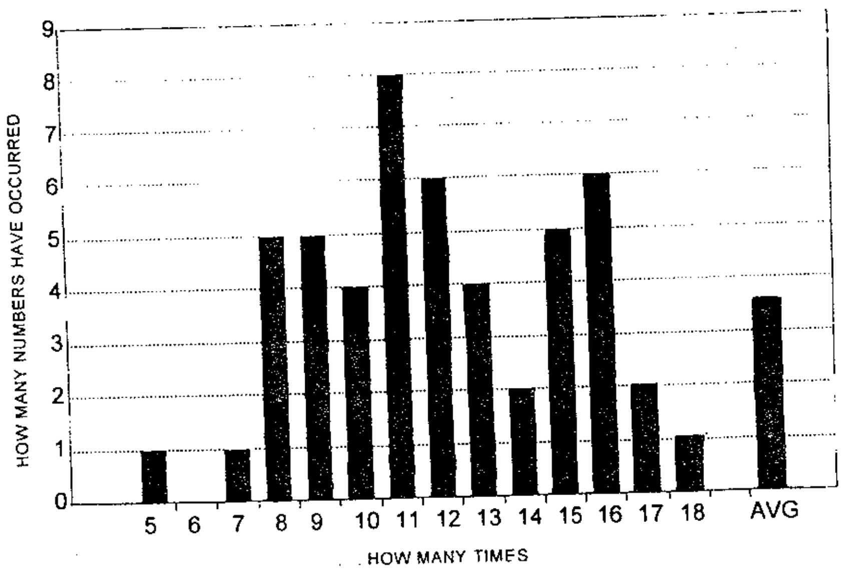
Table 1
Frequency of Lotto Number Occurrences after 100 Drawings

Numbers occurring 18 times: 10
Numbers occurring 17 times: 38 40
Numbers occurring 16 times: 1 4 5 29 32 45
Numbers occurring 15 times: 14 21 22 23 31
Numbers occurring 14 times: 3 33
Numbers occurring 13 times: 13 20 28 39
Numbers occurring 12 times: 24 27 34 35 37 46
Numbers occurring 11 times: 11 15 16 25 30 36 41 47
Numbers occurring 10 times: 7 8 42 50
Numbers occurring 9 times: 2 6 9 17 48

Table 1 (continued)

Numbers occurring 8 times: 18 26 43 44 49
Numbers occurring 7 times: 12
Numbers occurring 6 times:
Numbers occurring 5 times: 19

Chart 1
Frequency of the Fifty Numbers Chosen in 100 Drawings



Theoretically, the probability P(x) that any given number x will be one of the six drawn is:

$$P(x) = \frac{C(1,1)^{*}C(49,5)}{C(50,6)} = 6/50 = .12$$

If y is the number of times x occurs in n drawings then

$$f(y) = C(n,y)\,(.12)^{y}\,(.88)^{n-y},$$

so the number of times we expect x to occur in n drawings is n times .12. The above function is known as the binomial probability function. Since there have been 100 drawings at the time of this writing, x should have occurred 100 times .12 or 12 times. In order to assess the randomness of the first 100 drawings, compare these theoretical frequencies and probabilities with the actual frequencies and probabilities given in the following tables:

Table 2
Frequency of Occurrence of the Fifty Lotto Numbers

1 occurred 16 times	18 occurred 8 times	35 occurred 12 times
2 occurred 9 times	19 occurred 5 times	36 occurred 11 times
3 occurred 14 times	20 occurred 13 times	37 occurred 12 times
4 occurred 16 times	21 occurred 15 times	38 occurred 17 times
5 occurred 16 times	22 occurred 15 times	39 occurred 13 times
6 occurred 9 times	23 occurred 15 times	40 occurred 17 times
7 occurred 10 times	24 occurred 12 times	41 occurred 11 times
8 occurred 10 times	25 occurred 11 times	42 occurred 10 times
9 occurred 9 times	26 occurred 8 times	43 occurred 8 times
10 occurred 18 times	27 occurred 12 times	44 occurred 8 times
11 occurred 11 times	28 occurred 13 times	45 occurred 16 times
12 occurred 7 times	29 occurred 16 times	46 occurred 12 times
13 occurred 13 times	30 occurred 11 times	47 occurred 11 times
14 occurred 15 times	31 occurred 15 times	48 occurred 9 times
15 occurred 11 times	32 occurred 16 times	49 occurred 8 times
16 occurred 11 times	33 occurred 14 times	50 occurred 10 times
17 occurred 9 times	34 occurred 12 times	

Table 3
Empirical Probability of the Fifty Lotto Numbers Being One of the Six Chosen after 100 Drawings
(the expected probability is .12)

1 has probability .0266	11 has probability .0183
2 has probability .015	12 has probability .0116
3 has probability .0233	13 has probability .0216
4 has probability .0266	14 has probability .025
5 has probability .0266	15 has probability .0183
6 has probability .015	16 has probability .0183
7 has probability .0166	17 has probability .015
8 has probability .0166	18 has probability .0133
9 has probability .015	19 has probability .0083
10 has probability .03	20 has probability .0216
21 has probability .025	36 has probability .0183
22 has probability .025	37 has probability .02
23 has probability .025	38 has probability .0283
24 has probability .02	39 has probability .0216
25 has probability .0183	40 has probability .0283
26 has probability .0133	41 has probability .0183
27 has probability .02	42 has probability .0166
28 has probability .0216	43 has probability .0133
29 has probability .0266	44 has probability .0133
30 has probability .0183	45 has probability .0266
31 has probability .025	46 has probability .02
32 has probability .0266	47 has probability .0183
33 has probability .0233	48 has probability .015
34 has probability .02	49 has probability .0133
35 has probability .02	50 has probability .0166

– The average number occurring is: 25.20834 (we expect 25.5)
– The standard deviation is: 14.27579 (we expect 14.4)

Mean, Standard Deviation, and Distribution of Numbers Chosen

If the machine is choosing the numbers randomly, the average number chosen from the numbers 1 to 50 should be 25.5 and the standard deviation should be 14.577. The actual average number chosen by the Texas Lottery machine in 100 drawings is 25.2 and the standard deviation is 14.3.

Since the actual sample of numbers drawn as of this writing is relatively small--only 100 of over 15 million possibilities--the authors chose to analyze intervals of number occurrences instead of individual occurrences. Table 4 shows how many numbers have occurred in the intervals 1 to 5, 6 to 10, etc., up to the interval 46 to 50.

Table 4
Lotto Number Distribution Over Intervals after 100 Drawings

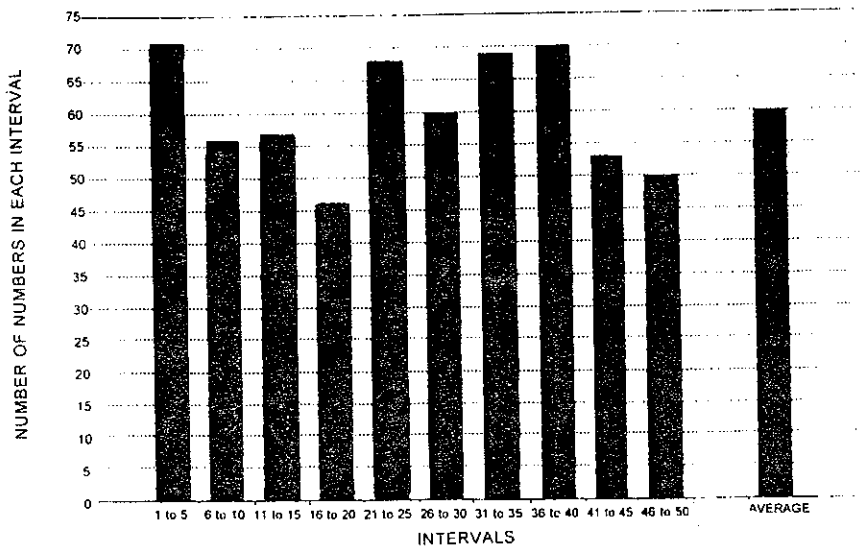
In the interval 1 to 5, 71 Lotto numbers occurred
In the interval 6 to 10, 56 Lotto numbers occurred
In the interval 11 to 15, 57 Lotto numbers occurred
In the interval 16 to 20, 46 Lotto numbers occurred
In the interval 21 to 25, 68 Lotto numbers occurred
In the interval 26 to 30, 60 Lotto numbers occurred
In the interval 31 to 35, 69 Lotto numbers occurred
In the interval 36 to 40, 70 Lotto numbers occurred
In the interval 41 to 45, 53 Lotto numbers occurred
In the interval 46 to 50, 50 Lotto numbers occurred

A total of 600 numbers have occurred;
therefore 60 are expected in each of the ten intervals.
The standard deviation is 9.043107
Chi-squared = 12.26667

The distribution of the numbers in the intervals can best be demonstrated by Chart 2.

Chart 2

Interval Distribution of Numbers
Chosen from 11-14-92 to 10-27-93



Since 600 numbers have been chosen so far and there are ten intervals, the average at the right of the chart is 60. The standard deviation is 9. Since the intervals are five units in length, the expected number of numbers in each interval after 100 drawings is 12 times 5 or 60. The Chi-squared test can now be run on the data in the intervals for 100 drawings as follows:

$$x^2 = (71 - 60)^2/60 + (56 - 60)^2/60 +$$

$$(57 - 60)^2/60 + (46 - 60)^2/60 +$$

$$(68 - 60)^2/60 + (60 - 60)^2/60 +$$

$$(69 - 60)^2/60 + (70 - 60)^2/60 +$$

$$(53 - 60)^2/60 + (50 - 60)^2/60$$

$$= 736/60 = 12.3 < 14.7.$$

Since, according to a table of critical values of Chi square[1], the Chi square value needs to be at least 14.7 to indicate non-randomness with a probability of at least 0.9, we can not say at this point that the number selections are non-random with an error of 10% or less.

Sums of Numbers Chosen and Distribution of the Sums

Another property associated with the lottery numbers involves the sums of the six numbers chosen. Table 5 shows the sums of the six numbers chosen in the first 100 drawings:

Table 5
Sums of the Six Numbers Chosen in 100 Drawings

Drawing Number	Sums of Numbers Drawn	Drawing Number	Sums of Numbers Drawn
1	148	51	209
2	233	52	157
3	161	53	190
4	192	54	107
5	149	55	160
6	130	56	131
7	170	57	151
8	139	58	178
9	172	59	138
10	172	60	159
11	138	61	158
12	149	62	202
13	130	63	171
14	204	64	120
15	145	65	141
16	123	66	151
17	165	67	94

Table 5 (continued)

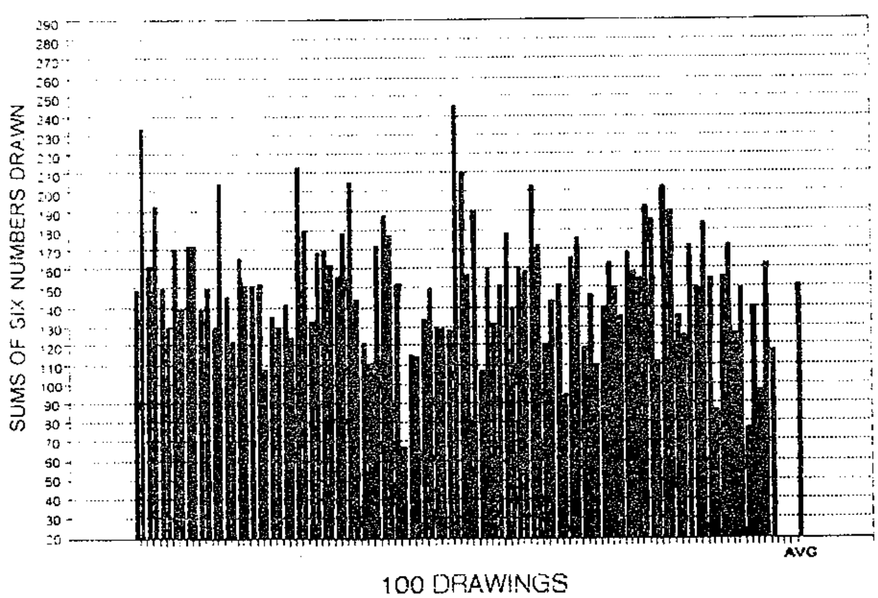
Drawing Number	Sums of Numbers Drawn	Drawing Number	Sums of Numbers Drawn
18	151	68	165
19	151	69	175
20	152	70	119
21	108	71	146
22	135	72	110
23	130	73	138
24	141	74	163
25	124	75	150
26	213	76	135
27	180	77	168
28	132	78	158
29	168	79	155
30	169	80	192
31	162	81	185
32	156	82	112
33	178	83	202
34	205	84	189
35	144	85	135
36	121	86	125
37	111	87	172
38	171	88	149
39	187	89	183
40	177	90	154
41	151	91	86
42	67	92	156
43	115	93	171
44	114	94	126
45	133	95	149
46	149	96	77
47	130	97	140
48	130	98	96
49	128	99	162
50	245	100	117

Average Sum is: 151.25 Minimum Sum is: 67

Maximum sum is: 245 Standard Deviation is: 12.84938

The average sum for these numbers is 151.25 and the standard deviation is 12.83. The following chart demonstrates how these sums are distributed.

Chart 3
100 Sums of the Six Lottery Numbers Chosen from
11-14-92 to 10-27-93



In order to better understand the distribution of these sums, one needs to know how many ways each sum can be obtained. The sum of 21 is the lowest possible sum and can only be obtained one way as 1 + 2 + 3 + 4 + 5 + 6. At the other extreme, 285 is the highest sum possible and can be obtained in only one way as 50 + 49 + 48 + 47 + 46 + 45. Sums between 21 and 285 can be obtained in one or more ways. Since there are 15,890,700 ways to choose six numbers for the Texas Lottery, and the total number of all possible sums is 2,431,277,100, the mean mean of these sums is 153. The standard deviation is 33.5. The sum of 153 can be obtained in the most number of ways, 184,430, and the number of ways for obtaining the other sums are symmetric about this number.

If the possible sums are grouped into 53 intervals, each containing five sums, the theoretical probabilities that a sum will be in one of the intervals are shown in Table 6.

Table 6
Theoretical Probabilities that a Sum will Occur in One of the 53 Intervals

Interval	Probability	Interval	Probability	Interval	Probability
1	.0000007	19	.0300862	37	.0205225
2	.0000049	20	.0351794	38	.016342
3	.0000187	21	.0402313	39	.0126819
4	.000053	22	.0450187	40	9.571399 E-03
5	.0001252	23	.0493294	41	.0070134
6	.0002592	24	.0529445	42	.0049749
7	.0004895	25	.055686	43	.0034092
8	.0008587	26	.0573943	44	.0022486
9	.0014233	27	.0579785	45	.0014233
10	.0022486	28	.0573943	46	.0008587
11	.0034092	29	.055686	47	.0004895
12	.0049749	30	.0529445	48	.0002592
13	.0070134	31	.0493294	49	.0001252
14	9.571399 E-03	32	.0450187	50	.000053
15	.0126819	33	.0402313	51	.0000187
16	.016342	34	.0351794	52	.0000049
17	.0205225	35	.0300862	53	.0000007
18	.0251427	36	.0251427		

The expected number of sums in each interval and the actual number of sums in each interval after 100 drawings are shown in Table 7.

Table 7
Expected Number of Sums and Actual Number of Sums
in the Intervals after 100 Drawings

Interval #	Expected #	Actual #	Interval #	Expected #	Actual #
1	.00007	0	28	5.73943	7
2	.00049	0	29	5.5686	6
3	.00187	0	30	5.29445	4
4	.0053	0	31	4.93294	7
5	.01252	0	32	4.50187	4
6	.02592	0	33	4.02313	2
7	.04895	0	34	3.51794	3
8	.08587	0	35	3.00862	2
9	.14233	0	36	2.51427	0
10	.22486	1	37	2.05225	4
11	.34092	0	38	1.6342	1
12	.49749	1	39	1.26819	1
13	.70134	0	40	.95714	0
14	.95714	1	41	.70134	0
15	1.26819	1	42	.49749	0
16	1.6342	1	43	.34092	1
17	2.05225	0	44	.22486	0
18	2.51427	3	45	.14233	1
19	3.00862	4	46	.08587	0
20	3.51794	3	47	.04895	0
21	4.02313	4	48	.02592	0
22	4.50187	7	49	.01252	0
23	4.93294	6	50	.0053	0
24	5.29445	5	51	.00187	0
25	5.5686	4	52	.00049	0
26	5.73943	8	53	.00007	0
27	5.79785	8			

Chi square = 27.85177

The Chi-square value for this data is 27.85 which is less than 63.17. Therefore, according to a table of critical values of Chi square[1], we can not say the sums are occurring non-randomly with an error of 10% or less.

Although the theoretical mean of 153 is close to the calculated mean of 151.25 for the 100 sums of numbers, the theoretical standard deviation of 33.5 is not close to the calculated standard deviation of 12.85 which means that the actual sums chosen so far are abnormally close to the mean, which would seem to indicate that Texas Lotto players might do well to choose their numbers so that the sum of the numbers is close to 153.

Conclusion

These are but a few of the applications one might choose to have students explore if a class project were established to track and analyze the Texas Lottery. The process can be started at any drawing as if that drawing were the first.

Utilizing the Texas Lottery as a teaching tool should also provide significant material for student debate regarding random processes. Many other useful teaching techniques could be established that would give students a better understanding of the practical applications of probability and statistics and the unlikely event of their winning the Texas Lottery.

REFERENCE

Mendenhall, William and Robert J. Beaver. Introduction to Probability and Statistics. PWS-Kent Publishing Co., Boston, 1991, pp. 670 - 671.

A MODIFICATION OF THE KEY NUMBER METHOD OF FACTORING

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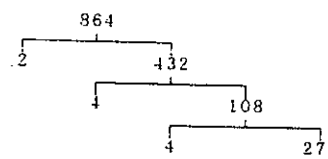
One of the most basic concepts needed to advance in algebra is factoring. For beginning algebra students, the methods of factoring can become formidable especially when learning how to factor a quadratic trinomial with a leading coefficient greater than one. Some of their confusion is based upon the many different ways in which factoring this trinomial can be taught. Each method seems to pose its own particular impediment to the student.

A modification of the key number method, however, may obliterate the confusion. The key number method has a close relationship to the FOIL process of multiplying binomials (Nanney, 107). This technique provides the student a good basis for understanding the placement of terms in the binomial factors. The key number method alone, however, does not guide the student into choosing the correct factors for the first terms of each binomial factor. It relies upon trial and error for that selection. If the leading coefficient has many factors, the trial and error approach is a hinderance to most beginning algebra students. Finding the greatest common factor of the leading coefficient term and the product of the outer terms will overcome this obstacle.

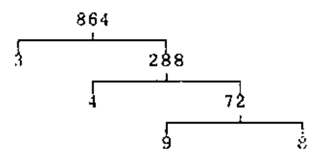
Example: Factor $36x^2 + 59x + 24$.

Multiplying the coefficient of x^2 and the constant together, one obtains the *key number* of 864. The student then must find the set of factor pairs of 864 which add together to give the coefficient of x, 59. Several possibilities exist.

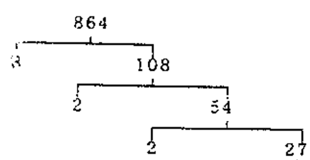
a.



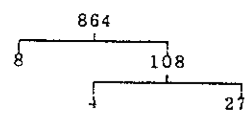
b.



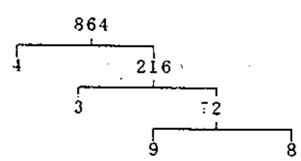
c.



d.



e.



Using **e.** here one obtains 864 = 12 * 72. At this step, 864 = 4 * 3 * 9 * 8, which when grouped form:
a. 864 = 32 * 27 and
b. 864 = 36 * 24.

Since the sum must be 59, group a. exhibits the correct factors. So the middle term 59x = 32x + 27x. Either the 32x or the 27x can be chosen to be the product of the outer terms. If 32x is chosen to be the product of the outer terms, then 27x is the inner product. For the placement of the terms in the binomial factors one can set up the following diagram and ask the following questions:

```
 1       4  2       3
 (          )(          )         product of inner terms
 |        |_27x_|       |
 |                      |
 |________32x___________|         product of outer terms
```

Position 1. What is the greatest common factor of $36x^2$ and $32x$? 4x

So 4x is placed in position 1.

```
 1       4  2       3
 (4x        )(          )
 |        |_27x_|       |
 |                      |
 |________32x___________|
```

Position 2. For the product of the first terms to be $36x^2$, what is the other first term? 9x

```
 1       4  2       3
 (4x        )(9x        )
 |        |_27x_|       |
 |                      |
 |________32x___________|
```

Position 3. For the product of the outer terms to be 32x, what is the term in position 3? +8

```
 1       4  2       3
 (4x        )(9x   +  8)
 |        |_27x_|       |
 |                      |
 |________32x___________|
```

Position 4. For the product of the inner terms to be 27x, what is the term in position 4? +3

```
 1       4  2       3
 (4x   +  3)(9x   +  8)
 |        |_27x_|       |
 |                      |
 |________32x___________|
```

Finally, check the product of the last terms. 3 * 8 = 24

This modification of the key number method can provide algebra students a precise and less time-consuming way of factoring quadratic trinomials with a leading coefficient greater than one. Finding the greatest common factor of the second degree term and the product of the outer terms eliminates the need of the student to find the correct first terms by trial and error.

If the strict key number method had been used, a student could possibly try all five sets of factors of the number 36 before electing the correct one. Incidentally, if a student factors by trial and error alone, then the possibility exists that all twenty combinations of the factors of 36 and 24 could have been tried before selecting the correct one (Sullivan, 84-86). Usually a student will indicate a second-degree trinomial to be prime if there are many sets of factors to consider. In using this method a student can discover the correct factor pairs of the key number and avoid the tendency to indicate the trinomial to be prime.

In addition, the modification is less time-consuming than other methods of factoring since all questions can be answered mentally after finding the correct factor pairs of the key number. Other methods such as factoring by grouping require pencil and paper work which makes factoring a lengthy process (Sullivan, 87).

Since the notion of the greatest common factor is introduced early when teaching factoring techniques, the concept, along with the key number method's association to FOIL, makes this modification a good approach for beginning algebra students to follow when factoring second-degree trinomials.

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MONTE CARLO COOTIE

AN ACTIVITY FOR

MIDDLE-SCHOOL

STUDENTS

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Probability is an important component of the National Council of Teachers of Mathematics' (NCTM) Curriculum and Evaluation Standards for School Mathematics for Grades 5-8. Standard 11 states, in part, that the mathematics curriculum in grades 5-8 should include explorations of probability in real-world situations so that students can "model situations by devising and carrying out experiments or simulations to determine probabilities" and "make predictions that are based on experimental or theoretical probabilities." The game of Cootie works well for carrying out these objectives.

Directions for Cootie

Most students have played Cootie as children. The Milton Bradley game consists of six types of plastic Cootie pieces (body, head, eye, nose, antenna, leg) and a six-sided die. The object of the game is to be the first player to complete a cootie. A completed Cootie has one body, one head, two eyes, one nose, two antennae, and six legs. A player must roll a one on the die to qualify for a body piece. Then the player must roll a two for the head. The body must be first, and the head must be second. After obtaining both a body and a head, a player may get the remaining eleven pieces in any order.

Simulation of Cootie

This probability activity does not use the plastic pieces but simulates the game of Cootie by using a die and the chart in Table 1. The first question to ask middle-school students is: How many tosses of the die do you think are necessary to complete a Cootie? Each cooperative group of students can then write down their estimate and proceed to make a Cootie. One person in the group tosses the die, another group member tallies the tosses, and checks when a body piece is obtained, and the third member of the group makes sure that all the rules are followed correctly. Table 2 shows one groups' result. After all groups have completed their Cooties, have them compare their experimental results with their estimates. Finally, find the average number of tosses required by the groups in the class.

TABLE 1

Tosses of die	**COOTIE**
	Body (1) ☐
	Head (2) ☐
	Eye (3) ☐ ☐
	Nose (4) ☐
	Antenna (5) ☐ ☐
	Leg (6) ☐ ☐ ☐ ☐ ☐ ☐

TABLE 2

Tosses of die	**COOTIE**
	Body (1) ☑
𝍸 𝍸 𝍸	Head (2) ☑
𝍸 𝍸 𝍸	Eye (3) ☑ ☑
𝍸 𝍸 𝍸	Nose (4) ☑
𝍸 III (53)	Antenna (5) ☑ ☑
	Leg (6) ☑ ☑ ☑ ☑ ☑ ☑

Next, ask the students: If you repeat the experiment would you expect to get the same results? and Would you like to change your estimate of the number of tosses required to make a Cootie? The students may repeat the experiment and average the results with the first set of results. After repeating the experiment as many times as desired, discuss estimates and what the expected number of tosses is. Then ask: How could one be more certain of the number of tosses required? Students may suggest continuing the experiment and averaging results. This is a good solution but time-consuming. Using a computer program to simulate the experiment is a logical next step. The computer program (a Turbo Pascal program is included in Table 3) can make 500 Cooties in a few seconds or a few minutes (depending on the computer capabilities available) and report the average results. The computer program can also be modified to make more or fewer Cooties by changing the line, play:=500. This computer follow-up will help to broaden students' understanding and give students the opportunity to see how useful the computer can be to increase the number of trials in a short period of time. The students should also observe the law of large numbers, increasing the number of trials helps to refine the experimental expected value. The experimental expected value of the number of tosses required to complete a Cootie is somewhere between 48 and 52.

Table 3

Cootie Turbo Pascal Program

```
Program Cootie (input, output);
{How many tosses are required on the average to make a Cootie?}

VA
  part: array[1..6] of integer;
  number, i: integer;
  tosses, games, play: longint;

procedure toss;
begin
  number:=random(6) + 1;
  inc(tosses);
end; {toss}
```

```
begin                          {Main program}
  play:=500;
  if paramcount = 1
      then val(paramstr(1),play,i);
  randomize;
  writeln(output);
  writeln(output);
  games:=1;
  tosses:=0;
  while games <=play
      do begin
          for i:=3 to 6
              do part[i]:=0;            {initialize array}
          while number <> 1             {get body first}
              do toss;
          while number <> 2             {get head second}
              do toss;
          while (part[3] < 2) or (part [4] < 2) or (part[5] < 1) or (part [6] < 6)
          do begin
          toss;
          part [number]:= part[number] + 1;
          end; {while}
      inc(games);
  end; {while}
  write (output, 'Out of', play, 'simulations of the game of Cootie');
  writeln (output, 'the average number of');
  writeln (output, 'tosses required to complete a Cootie is: ', tosses/play:6:2);

end.
```

Discussion

Now discuss what this experimental expected number of tosses means. Can one expect to make a Cootie with just 50 tosses of the die? What is the least number of tosses necessary to make a Cootie? (13) Would one expect to do this often? What would one think is the highest number of tosses it might take to make a Cootie? Is 100 tosses unreasonable? (Probably at least one group required close to 100 tosses in the student experiments.) What is the estimated number of tosses to get the body only? (The theoretical expected value is six. Since the probability of getting a one on the die is 1/6, on the average it will take $^{1}/_{1/6}$ tosses of the die for a one to appear). What is the estimated number of tosses required to get the body and the head? (The theoretical expected value of getting both the body and the head is twelve.) It is much more difficult to compute the theoretical expected value for the entire Cootie since the parts, after the body and head, can be obtained in any order.

The game of Cootie provides an excellent opportunity for a fun activity which also teaches some basic probability and statistic concepts. Following this activity students should be able to give the probability of getting a "six" when a die is tossed and also the expected number of tosses required to get a six. Students can also be introduced to mean, median, and mode, using the Cootie data collected by the class. These probability experiences should help students grow in their ability to use mathematics in modeling real-world situations. Discussion may lead to the question: What is the probability of your wining the state lottery? For more activities of this type, there are excellent examples of Monte Carlo simulations, as well as many other probability activities, in the Grades 5-8 NCTM Addenda book, Dealing with Data and Chance. One Addenda activity, very closely related to the Cootie game, is the collection of rock star photos. The problem states that a gum company packs a rock star picture in each of its packages of gum and there are six different pictures. The question for students to consider is: How many packages of gum will you need to buy to have one of each of the photos? (Zawojewski, p. 15.)

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National Council of Teachers of Mathematics, Curriculum and Evaluation Standards for School Mathematics, Reston, VA, The Council, 1989.

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CONGRESSIONAL DISTRICT DATA ANALYSIS: COUNTING COUNTY INTERSECTIONS

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An important notion in the teaching of mathematics today is that of connections. These connections should link mathematics to real world situations and should also integrate different mathematical concepts. We shall present an idea which involves mathematics and social science.

Following the 1990 census of our state, Iowa had to redistrict its congressional districts both because of population shifts from the previous census and because the size of its House of Representatives delegation was reduced from 6 to 5. The resulting configuration of districts were shaped and located quite differently from the districts which preceded them. However, in both arrangements none of Iowa's counties is split between two districts.

Table I (A and B) reports the assignment of the 99 counties (numbered in alphabetical order) for the six congressional districts of the 1980's and the five congressional districts of the 1990's.

TABLE IA 1980's Congressional Districts

Congressional District	County Numbers
I (16 counties)	4, 26, 29, 44, 51, 54, 56, 58, 59, 62, 68, 70, 82, 89, 90, 92
II (11 counties)	3, 10, 16, 22, 23, 28, 31, 33, 49, 53, 57
III (16 counties)	6, 7, 9, 12, 19, 34, 38, 45, 48, 52, 64, 66, 79, 86, 96, 98
IV (6 counties)	8, 25, 40, 50, 77, 85
V (27 counties)	1, 2, 5, 13, 14, 15, 20, 24, 27, 36, 37, 39, 43, 61, 63, 65, 69, 73, 78, 80, 81, 83, 87, 88, 91, 93, 94
VI (23 counties)	11, 17, 18, 21, 30, 32, 35, 41, 42, 46, 47, 55, 60, 67, 71, 72, 74, 75, 76, 84, 95, 97, 99

TABLE 1B 1990's Congressional Districts

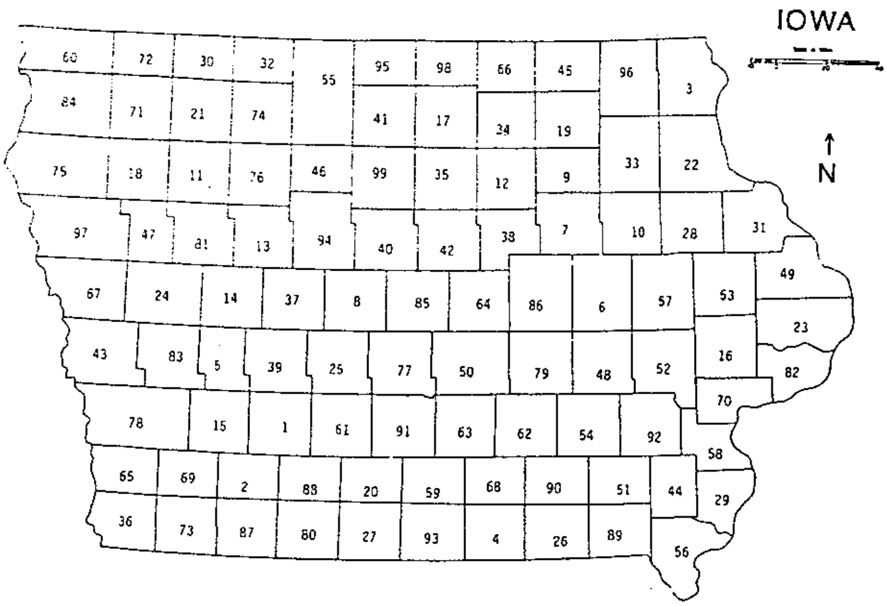
Congressional District	County Numbers
I (8 counties)	16, 23, 52, 53, 57, 58, 70, 82
II (21 counties)	3, 6, 7, 9, 10, 12, 17, 19, 22, 28, 31, 33, 34, 38, 45, 48, 49, 66, 86, 96, 98
III (27 counties)	2, 4, 20, 26, 27, 29, 44, 50, 51, 54, 56, 59, 62, 63, 64, 68, 73, 79, 80, 85, 87, 88, 89, 90, 91, 92, 93
IV (13 counties)	1, 5, 15, 25, 36, 39, 43, 61, 65, 69, 77, 78, 83
V (30 counties)	8, 11, 13, 14, 18, 21, 24, 30, 32, 35, 37, 40, 41, 42, 46, 47, 55, 60, 67, 71, 72, 74, 75, 76, 81, 84, 94, 95, 97, 99

Our first question concerns finding the intersections of the old (1980's) and the new (1990's) congressional districts. This may be accomplished in two ways:

- Using the following Iowa map with the counties numbered as the basic for a Venn diagram.

- Using the sets as tabulated in Table I.

MAP OF IOWA



The resulting intersections are displayed in Table II.

TABLE II

New ∩ Old	County Numbers	Cardinality of Intersection
I ∩ I	58, 70, 82	3
I ∩ II	16, 23, 53, 57	4
I ∩ III	52	1
I ∩ IV	∅	0
I ∩ V	∅	0
I ∩ VI	∅	0
II ∩ I	∅	0
II ∩ II	3, 10, 22, 28, 31, 33, 49	7
II ∩ III	6, 7, 9, 12, 19, 34, 38, 45, 48, 66, 86, 96, 98	13
II ∩ IV	∅	0
II ∩ V	∅	0
II ∩ VI	17	1
III ∩ I	4, 26, 29, 44, 51, 54, 56, 59, 62, 68, 89, 90, 92	13
III ∩ II	∅	0
III ∩ III	64, 79	2
III ∩ IV	50, 85	2
III ∩ V	2, 20, 27, 63, 73, 80, 87, 88, 91, 93	10

III ∩ VI	∅	0
IV ∩ I	∅	0
IV ∩ II	∅	0
IV ∩ III	∅	0
IV ∩ IV	25, 77	2
IV ∩ V	1, 5, 15, 36, 39, 43, 61, 65, 69, 78, 83	11
IV ∩ VI	∅	0
V ∩ II	∅	0
V ∩ III	∅	0
V ∩ IV	8, 40	2
V ∩ V	13, 14, 24, 37, 81, 94	6
V ∩ VI	11, 18, 21, 30, 32, 35, 41, 42, 46, 47, 55, 60, 67, 71, 72, 74, 75, 76, 84, 95, 97, 99	22

Our second and major question is to determine the extent to which new redistricting has affected the groupings of counties which were present in the old districts. This question is significant both to politicians and to voters. Congressional candidates must organize their campaigns and, if elected, conduct their office in such a way that a majority of the voters generally approve; if not, their campaigns are likely to be unsuccessful; and if elected, their incumbencies will be short-lived. This very practical consideration requires careful attention both to the issues of special concern to the voters of a given district and also to the local political figures who are influential in that district. If a district remains somewhat stable over time, these special concerns and influential figures are known quantities, or at least change gradually. Political figures and ordinary voters know what to expect and how to make their voices heard.

But when a major realignment of the counties and districts occurs, all of these issues and relationships are "up for grabs." A new mix of local concerns and local leaders must be reorganized into coherent election and governing strategies. For this reason, incumbent political figures often prefer stability, while newcomers and challengers often prefer the fresh start provided by a major county/district realignment.

Table III reports the extent of the county/district re-grouping as follows:

Column 1 contains the county number.

Column 2 contains the number of counties in its new congressional district (not including the given county).

Column 3 contains the number of counties of column 2 which were in the same congressional district as the given county in the old districting.

Column 4 contains the ratio of column 3 to column 2 written in decimal notation. This ratio might be regarded as a measure of "stability" in that county's "cohort counties." A high ratio means that the new cohorts were also generally grouped with that county in the previous districting; a low ratio indicates that the county is essentially in a "new" district.

TABLE III

Col 1	Col 2	Col 3	Col 4	Col 1	Col 2	Col 3	Col 4
1	12	10	0.83	51	26	12	0.46
2	26	9	0.35	52	7	0	0.00
3	20	6	0.30	53	7	3	0.43
4	26	12	0.46	54	26	12	0.46
5	12	10	0.83	55	29	21	0.72
6	20	12	0.60	56	26	12	0.46
7	20	12	0.60	57	7	3	0.43
8	29	1	0.03	58	7	2	0.29
9	20	12	0.60	59	26	12	0.46
10	20	6	0.30	60	29	21	0.72
11	29	21	0.72	61	12	10	0.83
12	20	12	0.60	62	26	12	0.46
13	29	5	0.17	63	26	9	0.35
14	29	5	0.17	64	26	1	0.04
15	12	10	0.83	65	12	10	0.83
16	7	3	0.43	66	20	12	0.60

17	20	0	0.00	67	29	21	0.72
18	29	21	0.72	68	26	12	0.46
19	20	12	0.60	69	12	10	0.83
20	26	9	0.35	70	7	2	0.29
21	29	21	0.72	71	29	21	0.72
22	20	6	0.30	72	29	21	0.72
23	7	3	0.43	73	26	9	0.35
24	29	5	0.17	74	29	21	0.72
25	12	1	0.08	75	29	21	0.72
26	26	12	0.46	76	29	21	0.72
27	26	9	0.35	77	12	1	0.08
28	20	6	0.30	78	12	10	0.83
29	26	12	0.46	79	26	1	0.04
30	29	21	0.72	80	26	9	0.35
31	20	6	0.30	81	29	5	0.17
32	29	21	0.72	82	7	2	0.29
33	20	6	0.30	83	12	10	0.83
34	20	12	0.60	84	29	21	0.72
35	29	21	0.72	85	26	1	0.04
36	12	10	0.83	86	20	12	0.60
37	29	5	0.17	87	26	9	0.35
38	20	12	0.60	88	26	9	0.35
39	12	10	0.83	89	26	12	0.46
40	29	1	0.03	90	26	12	0.46
41	29	21	0.72	91	26	9	0.35
42	29	21	0.72	92	26	12	0.46
43	12	10	0.83	93	26	9	0.35
44	26	12	0.46	94	29	5	0.17
45	20	12	0.60	95	29	21	0.72
46	29	21	0.72	96	20	12	0.60
47	29	21	0.72	97	29	21	0.72
48	20	12	0.60	98	20	12	0.60
49	20	6	0.30	99	29	21	0.72
50	26	1	0.04				

Table IV reports the frequencies of the column 4 "stability" ratios of Table III.

TABLE IV

RATIO	FREQUENCY
00.0	2
0.03	2
0.04	4
0.08	2
0.17	6
0.29	3
0.30	7
0.35	10
0.43	4
0.46	13
0.60	13
0.72	22
0.83	11

Observe that in Table IV, 53 (just over one-half) of the 99 counties had stability ratios of 0.46 or less. For each of these 53 counties this means that fewer than one-half of the other counties in its new district were part of its old district. When representatives of these 53 counties go to district conventions, they do not see many old friends; more seriously, a restructuring of interests and coalitions must be undertaken. There has been a substantial rearrangement of counties among districts.

This article displays connections between social science and mathematics. Within mathematics itself, the notions of sets, intersections, ratios and frequency distributions are linked.

Challenges for the Reader and his/her Students

1. Replicate this analysis for your own state.

2. Using Iowa or your own state, redo this analysis using populations rather than numbers of counties.

3. Approach this problem in a different manner. For each of the old districts determine the number of pairs of counties which can be formed. Do this for all districts and total the numbers of pairs. How many of these "county pairs" are still intact in the new districts? This might be regarded as a more sophisticated approach to the question for which this article employed only simple counting.

4. Find other connections between mathematics and the real world.

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