Texas Mathematics Teacher

Hats Off to Mathematicians

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MAY 1993

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

May 1993

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ANNOUNCEMENTS

President's Message

Snow capped Mt. Rainier...espresso...Pike Place Market...over 13,000 mathematics teachers...I write this from Seattle where I am representing you as a delegate to the 71st annual meeting of NCTM. It has been 5 hectic days of meetings, exchanges of ideas, and renewed enthusiasm for the profession I love. I want to share some observations.

TCTM is one of 250 affiliated groups of NCTM, and at the delegates' assembly we passed resolutions to have NCTM "appoint a task force to provide models that assist teachers with additional accommodations and support of special needs students placed through the inclusion process in regular education classes." Another resolution asked NCTM to "act as an advocate for the establishment and maintenance of an office within every state, province, or District of Columbia Department of Education whose primary focus and expertise is leadership in mathematics education." These resolutions will now go to the NCTM Board of Directors.

Rapidly changing technology was the focus of many sessions, including a presentation by Microsoft chairman Bill Gates who told us of a not so distant future that will include wallet sized PC's and digital money. The capabilities of CD ROM, the newly released Geometric Sketchpad for Windows, and the soon to be released TI-82 graphing calculator all have great potential for the classroom.

A major topic at the conference was alternative assessment. Volume III of the NCTM Standards will address assessment, and should be available in two years. I came away with some easy-to-implement ideas including: a final test item asking students to explain a question they wish had been asked on the test but wasn't, then pose the question, and show the solution (suggested by Miriam Leiva, NCTM Board member); having students write problems based upon newspaper advertisements or articles with the promise that the best one(s) will appear on the next test and the author will automatically get it right (from Steve Leinwand, Connecticut Department of Education); and final test questions that ask "What did you learn as a result of taking this test that you did not know before?" or "What do you still not understand about this topic?" (Texas teacher and author Paul Foerster).

Funding crises were a concern of teachers across the nation. Teachers in Portland, Oregon, reported recent budget cuts that included the athletic program. Students in Portland will now have to pay to play!

An inspiring closing session was given by Texas' own Cathy Seeley. She spoke about the importance of math for everyone. Addressing the changing role of the teacher, she described teachers who generate rich tasks, ask good questions, manage the environment, value all students, and keep on learning.

If you couldn't get to Seattle, you will have the opportunity to attend an exciting conference closer to home, August 11 - 13 in Dallas. The Spring issue of the TCTM newsletter highlights 1993 CAMT. Plan now to attend. Use the form inside the newsletter to sign up for the TCTM breakfast on Friday the 13th. TCTM is in charge of registration at CAMT, so you may use the same form to indicate your willingness to work at the registration desk during the conference. I hope I'll see you there!!

Susan Thomas
TCTM President

R C D P M . . . .
Research Council for Diagnostic and Prescriptive Mathematics

THE 3 E'S OF MATHEMATICS: EMPOWERING THROUGH EQUITY IN EDUCATION

21st Annual Meeting

February 10-12, 1994
Radisson Plaza Hotel
Fort Worth, Texas

CALL FOR PRESENTERS

The Twenty-first Annual Meeting of the Research Council for Diagnostic and Prescriptive Mathematics will focus on student populations that are presently underserved or under-represented in the field of mathematics. These special populations include minority, at-risk, ESL, learning disabled, and female students. Other topics that might be of interest to mathematics educators are also welcomed. All topics, however, must be directly related to the mission of the RCDPM.

RCDPM Mission: To stimulate, generate, coordinate and disseminate research efforts designed to understand and overcome factors that inhibit maximal mathematics learning.

Research or dissemination reports and general interest sessions are to be scheduled for the conference. Workshops, poster discussions, and presentations will be the available formats.

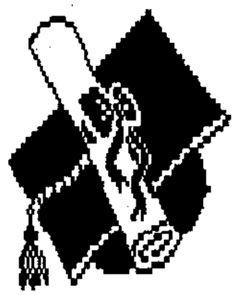
To request more conference information or a proposal submittal form, please contact Frances M. Thompson, Program Chair, at 2946 Housley Drive, Dallas, TX 75228, or call her at work (817/898-2166) or at home (214/681-4184).

PROPOSAL SUBMISSION DEADLINE: JUNE 1, 1993

HATS OFF TO MATHEMATICIANS

Valerie Childress

Tyler, Texas



After polling a group of 25 seventh and eighth grade students regarding the identities of Pythagoras, Rene Descartes, Sir Isaac Newton, Blaise Pascal, Archimedes, Carl Friedrick Gauss, George Boole, Nicole-Reine Hortense LePaute, John Napier, Sonya Vasilievna Kovalevsky, Albert Einstein, Rosalyn Yalow,Charles Babbage, Galileo and others, it was apparent that the students were not acquainted with the world's great mathematicians.

Seventeen knew who Albert Einstein was. The other mathematical geniuses received from zero to two identifications. Galileo had two students recognize his name. However, one called him an astrologer instead of an astronomer. One student remembered Archimedes said, "Eureka," but the student wasn't sure why. The student did know that Archimedes was naked when he shouted "Eureka." Sir Isaac Newton was remembered for "some-thing about an apple."

The same students knew even less about the more contemporary leaders of the mathematical world. Such names as Alonso Church, Grace Hopper, Max Dehn, Stephen Hawking, Ronald Aylmer Fisher and Frank Charles Hoppensteadt evoked little recognition from the students. Hawking, the most recognized name, was not considered to be a mathematician by the students.

Math is a much needed and much used skill as students slowly come to realize. Feeling that students should recognize names of the great mathematicians and the current mathematical leaders and wanting students to understand that math classes can use the library productively, we implemented a library/math project.

Named "Hats Off to Mathematicians," the project was designed by the library staff with the assistance of math teacher Cindy Gaddis. Gaddis had been looking for a project to incorporate library usage into her 7th and 8th grade math classes at J. W. Holloway Middle School in Whitehouse, Texas.

The project objective was to acquaint math students with reference materials located in the library and to identify those men and women who have made and are making significant contributions to the field of mathematics.

Students were given the following instructions:

1. Select a mathematician to research (the math teacher provided a list from which students could select a name);
2. Look up 10 facts about this mathematician and his or her life (the library staff was available to help locate sources);
3. List the facts in sentence form on the back of the mortarboard (graduation hat) given to you;
4. On the front of the mortarboard write the mathematician's name in large letters;
5. On the lower part of the "hat" write the mathematician's greatest contribution to his or her field (formulas were acceptable);
6. Decorate;
7. Prepare a short oral report on the mathematician to be given before the class. Explain his/her contributions to the field of math. Be prepared to explain the facts and symbols used on the "hat."

Each student was given a thick cardboard "hat." We used the packing boards from cartons of X-ray film. A friendly technician saved a supply for us. Students were told they would be given a research grade (daily) for the work they did on the day the class came to the library. The completed "hat" was to be counted as a test grade. Ninety percent (90%) of the grade would come from the 10 facts listed. Another 15 percent (15%) evaluation would be on creativity in decorating the hat. A student could earn up to 105 points.

Wrapping paper scraps, fabric scraps, wall paper ends and yarn were available for students to use. Students were asked NOT to buy materials, but to recycle leftover materials they might have at home. Tassels were made from yarn, materials, string, ribbons, etc.

Grace Hopper and John Napier were two of the mathematicians selected to be researched by the students.

A hat for Grace Hopper, American mathematician, naval officer and computer pioneer, was decorated in navy blue with gold trim. The student highlighted Hopper's teaching and interpreting computers to others as her chief contribution to the discipline of math and its applications. The student selected 10 facts including the following five facts about Hopper to incorporate into the mortarboard:

1. Received her PhD in mathematics from Yale in 1934;
2. Developed operating programs for Mak I computer while in the WAVES;
3. Developed concept of automatic programming which was incorporated into COBOL (Common Business Oriented Language);
4. Was one of five women elected to the National Academy of Engineers; and
5. Was promoted to Captain in 1973 while on retired list, a precedent in the Naval Reserve.

John Napier's mortarboard was decorated in a plaid to denote his Scottish heritage. The student doing this project selected the invention of logarithmic tables as Napier's greatest contribution to math. Facts about Napier included the following:

1. Invented first primitive calculator;
2. First used and popularized the decimal point to separate the whole number part from the fraction part of a number;
3. Napier's "bones" illustrated how square roots could be extracted by the manipulation of counters on a chessboard;
4. Was known as the "Marvelous Merchiston" for his various accomplishments; and
5. Never occupied a professional post in his life although his favorite intellectual pursuit was astronomy.

Each student gave an oral report on his or her chosen mathematician explaining the facts and symbols they elected to use on the mortarboard. Classmates could ask questions at the end of each report. Students then had a classroom discussion on how the contributions, etc. of the researched mathematicians are used in day to day activities. They also discussed how math was important in different professionals and occupations.

The completed colorful and creative hats were hung with red yard from the ceiling of the school's front office. Students learned about the mathematicians by their individual research and by reading the displayed hats created by fellow students.

Copernicus, Carl Friedrick Gauss, Christian Felix Klein, Pascal, Galileo and others became known by the math students. They also learned that the library was a source for math formulas, definitions and theorems.

"Eureka!" as Archimedes said when he figured out a way of mathematically checking whether or not the King of Syracuse's crown was pure gold.

Eureka! The math students are using the library.

GRAPHING AND THE YOUNG CHILD

Barba Patton

Victoria, Texas

In many curricula today, the student is faced with countless topics in every major discipline. Educators agree that all students need a good strong foundation but this is where the agreement ceases to exist. The definition alone is an area of confusion. What constitutes a good strong foundation to one educator may or may not constitute a good foundation to another educator. Educators have well meaning intentions to help students develop "good foundations" but are these intentions enough when there exists so many differences concerning what constitutes a good foundation?

The topic of graphs and graphing is one of these topics that is in need of much additional research. Are the young students in grades K-5 being expected to perform graphing skills which are developmentally inappropriate? Are some of the approaches that are used with these young children actually setting them up for failure in upper grades?

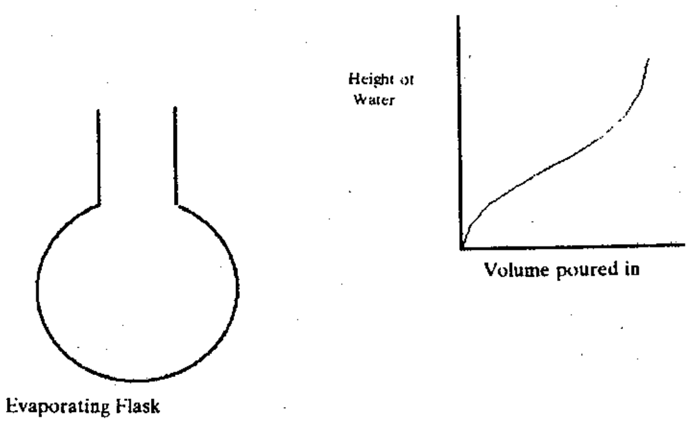
While graphs and graphing appear on the surface to be simple and non-threatening, research has indicated that this is only a surface appearance. Researchers including Piaget agree that construction and interpreting graphs require abstract thinking (Preece, 1983). Piaget further advocates that the structures utilized by formal mathematics may differ greatly from the structures utilized by the natural or informal mathematics of the child. Therefore a new structure reflecting a satisfactory coordination between the two must be developed (Groen & Kieran, 1983).

It is virtually impossible to predict all pitfalls, however, some of the well prepared materials may actually be seeds which tend to cause students to make misinterpretations. Some students will believe the misconceptions so strongly that they will have great difficulty altering their way of believing (Swan, 1982).

One graphing misconception is to refer to the graph as a picture or pictoral representation. Graphs have conventions and ambiguities all their own. Perceptual experience is not sufficient to interpret graphs correctly, students also need mathematical knowledge and expectations. When students are taught that a graph is a complete picture, seeds of misconceptions are being sown. Strategies used to interpret real-world scenes may not be appropriate when interpreting many graphs of infinite and relatively featureless objects such as in functions. Another example of this misconception will result in a graph which will resemble the container's shape if the student is making a graphic representation of the liquid being added to the container. Figure 1 illustrates this misconception.

Figure 1

Interpretation of Graph as Picture or Pictoral Representation

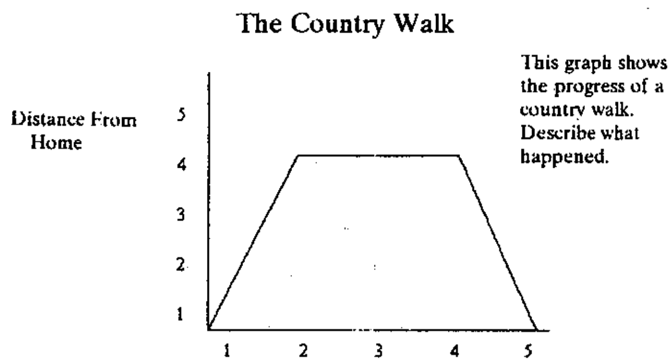


(Swan, 1982, p. 212.)

Still another example of this type misconception is that the student will appear to literally treat the graph as a picture of the problem situation. This occurs when the student is asked to draw a graph of a situation such as the time vs the distance a student walks (Swan, 1982). The student's drawing will resemble a hill. Figure 2 illustrates this type of general error.

Figure 2

Interpretation of Graph as Picture or Pictoral Representation



The people on the country walk were walking up a very steep hill. When they finally got to the top they were quite tired. They carried on walking for a bit and then they went back down the hill on the other side. As they were going down they went at quite a speed* (Swan, 1982, p. 214).

The student appears to be making a figurative correspondence between the shape of the graph and some visual characteristics of the problem scene. Students making this type of error appear to have difficulties differentiating between the problem and graph. Attention must be paid to the correspondence between the symbols and mathemati-

*Student's response.

cal ideas if the learning and the teaching of mathematics is to be successful. This is not an easy task. Teachers must remember the existance of vast differences in the mathematical repertoire which they as teachers bring to the classroom and the mathematical repertoire that their students bring, if their teaching is to be effective. As a result of this difference, students do not relate symbols and mathematical ideas in the same manner in which the teacher does. Teachers and students may agree on the relationship between symbols only to not be able to agree on the mathematical ideas represented or the relationship between the ideas (Sesay, 1982).

The symbolic nature of graphs requires that a teacher constantly examine his/her teaching strategies in order to be effective in the classroom. Attention must be paid to the correspondence between symbols and mathematical ideas if there is to be effective learning and teaching of mathematics. Teachers must always be aware of the differences that exist between the way they personally relate symbols to mathematical ideas and the way that their students relate symbols to mathematical ideas. Teachers must remember at all times that their own repertoire of mathematical background is very different from the repertoire which students bring to the mathematics classroom (Sesay, 1982). In conclusion, these misconceptions along with the others cited in the research conducted by Patton (1992) are in no way intended to be a gestalt but rather a genesis.

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Groen, G., & Kieran, C. (1983). The many faces of Piaget. In H. P. Ginsburg (Ed.), The development of mathematical thinking (pp. 352-375). New York: Academic Press.

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BALANCING BASEBALL SALARIES & PRODUCTIVITY: A STATISTICAL ACTIVITY

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Teachers are constantly looking for ways in which the analysis of real world data can be incorporated into the classroom, as encouraged by the NCTM Standards. A rich source of data occurs in baseball, a topic of interest to many students.

In this article two separate sets of major league data for 1992 are integrated--salaries and performance. Both sets of data are readily available. Our sources were both published by Gannett Company; they were USA Today (April 2, 1992) for salaries and USA Today Baseball Weekly (October 7-13, 1992) for individual and team performance data.

These statistics can provide a rich source of data for student exploration. Such exploration should be encouraged because the process of forming and verifying conjectures involving real world data is an important task in any mathematics class.

We will describe two questions that you and your class may wish to investigate. The first question is an elementary example of exploratory data analysis which requires no special background in statistics. The second involves the use of the coefficient of correlation, a somewhat more sophisticated technique in descriptive statistics.

Activity 1

An examination of salary/performance data indicates that a number of players at the lower end of the "salary scale" appear to have had a productive 1992 season. Let us first compose a team of the highest paid player in each non-pitching position. An interesting activity is to select a second team composed of "low paid" players who each had a more productive 1992 season than their "rich" counterpart.

To make such comparisons we must specify a performance measure which will enable us to compare players' seasons. This performance measure might be composed in many ways. We will illustrate one composite measure which will involve several offensive categories.

For each non-pitcher we defined a performance measure (PM), which encompassed all the bases that a given player accounted for offensively. Specifically, we credited a given player with one base for each single or walk, two bases for each double, three bases for each triple, four bases for each home run, and one base for each "net stolen base." A net stolen base is the difference between successful steals and unsuccessful attempts. Thus. $PM = W + H + D + 2T + 3HR + (SB - CS)$ where

W = Number of walks
D = Number of doubles
T = Number of triples
HR = Number of home runs
SB = Number of stolen bases
CS = Number of times caught stealing

We did not include pitchers because they did not have a meaningful offensive measure. You and your students may wish to design a pitching performance measure.

To form Table 1, we first identified the highest paid player in each of the eight non-pitcher positions, and calculated each player's PM for the 1992 season. We then selected examples of players of the same positions with significantly lower salaries whose PM's exceeded those of their rich counterparts.

TABLE I

MOST EXPENSIVE TEAM				
POSITION	PLAYER	TEAM	SALARY	PM
Catcher	Benito Santiago	San Diego	$ 3,300,000	166
First Base	Cecil Fielder	Detroit	$ 4,500,000	345
Second Base	Steve Sax	Chicago (White Sox)	$ 3,575,000	241
Third Base	Kelly Gruber	Toronto	$ 3,633,333	183
Shortstop	Barry Larkin	Cincinnati	$ 4,300,000	316
Left Field	Danny Tartabull	New York (Yankees)	$ 5,300,000	309
Center Field	Andy Van Slyke	Pittsburgh	$ 4,250,000	377
Right Field	Bobby Bonilla	New York (Mets)	$ 6,100,000	256
Total			$34,958,333	2,193

TABLE I Continued

LESS EXPENSIVE TEAM				
POSITION	PLAYER	TEAM	SALARY	PM
Catcher	Chris Holles	Baltimore	$ 175,000	210
First Base	Frank Thomas	Chicago (White Sox)	$ 600,000	432
Second Base	Carlos Baerga	Cleveland	$ 500,000	342
Third Base	Daniel Hollins	Philadelphia	$ 180,000	354
Shortstop	Travis Fryman	Detroit	$ 300,000	323
Left Field	Larry Anderson	Baltimore	$ 345,000	415
Center Field	Marquis Grissom	Montreal	$ 300,000	380
Right Field	Felix Jose	St. Louis	$ 300,000	276
Total			$ 2,700,000	2,732

Note that:

- Each player on the expensive team earned considerably more than the total of the eight salaries on the less expensive team.
- Each person on the less expensive team had a higher PM than the player at the same position of the highest price team.

Most baseball fans would probably contend that most of the players on the expensive team had good 1992 seasons. But a team owner who could have predicted in advance which lower paid players would have a productive 1992 season could have save a great deal of money!

Activity 2

The "bottom line" for a baseball team is to win games. Salaries are only a means to accomplish this goal. What is the relationship between teams' regular season wins and their total team payrolls? Table 2 displays these two statistics for each of the 26 major league teams for the 1992 season.

TABLE 2

TEAM	NUMBER OF REGULAR SEASON WINS	TOTAL SALARIES FOR ALL PLAYERS, IN-CLUDING PITCHERS
1. Atlanta	98	$32,975,333
2. Toronto	96	$42,663,666
2. Pittsburgh	96	$32,589,167
2. Oakland	96	$39,657,834
5. Milwaukee	92	$30,253,668
6. Minnesota	90	$27,432,834
6. Cincinnati	90	$35,203,999
8. Baltimore	89	$20,997,667
9. Montreal	87	$15,869,667
10. Chicago (White Sox)	86	$27,813,500
11. St. Louis	83	$26,634,836
12. San Diego	82	$27,454,167
13. Houston	81	$13,352,000
14. Chicago (Cubs)	78	$29,435,833
15. Texas	77	$28,245,667

TABLE 2 Continued

TEAM	NUMBER OF REGULAR SEASON WINS	TOTAL SALARIES FOR ALL PLAYERS IN-CLUDING PITCHERS
16. Cleveland	76	$ 8,111,166
16. New York (Yankees)	76	$34,462,834
18. Detroit	75	$25,557,834
19. Boston	73	$42,203,584
20. California	72	$33,529,834
20. Kansas City	72	$31,783,834
20 New York (Mets)	72	$44,464,002
20. San Francisco	72	$32,488,168
24. Philadelphia	70	$23,804,834
25. Seattle	64	$22,204,834
26. Los Angeles	63	$43,788,166

A method frequently used to determine the strength of the relationship between two sets of paired data involves the coefficient of correlation. The formula for this descriptive statistic for X - Y pairs is:

$$r = \frac{n(\Sigma XY) - (\Sigma X)(\Sigma Y)}{\sqrt{[n(\Sigma X^2) - (\Sigma X)^2]\,[n(\Sigma Y^2) - (\Sigma Y)^2]}}.$$

While this formula appears daunting, it is not overly hard to use with small sets of data. Many scientific/statistical calculators automatically compute r internally when data is entered in an appropriate mode; this eliminates the necessity for any paper and pencil calculations.

It can be shown that $-1 \leq r \leq 1$. The type of correlation is often interpreted as follows:

1. If r is close to +1 or -1, there is a strong relationship between the variables. If $r > 0$, this relationship is direct (small x's tend to be paired with small y's while large x's tend to be paired with large y's). If $r < 0$, the relationship is inverse (small x's tend to be paired with large y's and vice versa).

2. If r is close to 0, there is a weak relationship between the variables. For all 26 teams, the correlation coefficient is only 0.022. There is essentially no relationship between total payroll and team success as measured by winning games. This is quite surprising!

Challenges to the reader and his/her students:

1. Calculate the salary per base ratio for each of the players of Table 1. Compare these ratios. Recall that the total number of bases that a player produces is his PM.

2. Design measures of productivity for pitchers and make comparisons similar to those of this article. This is difficult to do since pitchers have many distinct roles (e.g. starters, long relievers, set-up relievers, and closers).

3. Determine other productivity measures for non-pitchers and investigate them in the style of this article.

4. Apply this process to school sports. Examples might include softball, basketball, or volleyball. What might be used as a substitute for salary?

Reference for correlation coefficient:

Miller, C., Heeren, V., and Hornsby, E. Mathematical Ideas (6th edition), Harper Collins, 1990.

AN ANALYSIS OF THE ACCURACY OF THE RULE OF 72

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Since the use of calculators and computers has become more wide spread, students are becoming less and less able to do mental and paper-pencil calculations and are losing their skills with the calculation process. One way to keep students involved in the calculation process is to use rules for estimation. In order to use an estimation method effectively, its accuracy must be known so that actual answers obtained from computers and calculators can be realistically compared to the estimates. Teachers need to know a variety of estimation methods to provide their students with ways to predict answers to problems before they calculate them.

One estimation method in the area of business mathematics is called the "Rule of 72." It estimates the time in years it takes for the principal to double when it is invested at a given compounded interest rate. One simply divides the given rate (in percent) into 72 to get an estimate of the doubling time. For example, the rule predicts that money invested at 9% will double in value in about eight years. Note that we divide 72 by 9, not .09, to get the doubling time.

The question that comes to mind about this and any approximation to an actual amount is: How accurate is it over a given set of data? To check the "Rule of 72," we will compare the approximation to the actual value. The formula for finding the doubling time for a given interest rate i is given by:

$$2 - 1(1 + i)^n$$

where i is the interest rate per compounding period and n is the number of compounding periods. Thus

$$n = (\ln 2)/(\ln(1 + i)).$$

The following Table 1 shows how the "Rule of 72" compares with the actual doubling times in years for interest rates from 1 to 99 percent compounded biannually, annually, semi-annually, quarterly and monthly.

Table I

RULE OF 72 ERRORS FOR FIVE COMPOUNDING METHODS

%	Rule	BiAn	ER	Ann	ER	Semi	ER	Quar	ER	Mo	ER
1	72.0	70.0	2.0	69.7	2.3	69.5	2.5	69.4	2.6	69.3	2.7
2	36.0	35.3	0.7	35.0	1.0	34.8	1.2	34.7	1.3	34.7	1.3
3	24.0	23.8	0.2	23.4	0.6	23.3	0.7	23.2	0.8	23.1	0.9
4	18.0	18.0	-0.0	17.7	0.3	17.5	0.5	17.4	0.6	17.4	0.6
5	14.4	14.5	-0.1	14.2	0.2	14.0	0.4	13.9	0.5	13.9	0.5
6	12.0	12.2	-0.2	11.9	0.1	11.7	0.3	11.6	0.4	11.6	0.4
7	10.3	10.6	-0.3	10.2	0.0	10.1	0.2	10.0	0.3	9.9	0.4
8	9.0	9.3	-0.3	9.0	-0.0	8.8	0.2	8.8	0.2	8.7	0.3
9	8.0	8.4	-0.4	8.0	-0.0	7.9	0.1	7.8	0.2	7.7	0.3
10	7.2	7.6	-0.4	7.3	-0.1	7.1	0.1	7.0	0.2	7.0	0.2
11	6.5	7.0	-0.4	6.6	-0.1	6.5	0.1	6.4	0.2	6.3	0.2
12	6.0	6.4	-0.4	6.1	-0.1	5.9	0.1	5.9	0.1	5.8	0.2
13	5.5	6.0	-0.5	5.7	-0.1	5.5	0.0	5.4	0.1	5.4	0.2
14	5.1	5.6	-0.5	5.3	-0.1	5.1	0.0	5.0	0.1	5.0	0.2
15	4.8	5.3	-0.5	5.0	-0.2	4.8	0.0	4.7	0.1	4.6	0.2
16	4.5	5.0	-0.5	4.7	-0.2	4.5	-0.0	4.4	0.1	4.4	0.1
17	4.2	4.7	-0.5	4.4	-0.2	4.2	-0.0	4.2	0.1	4.1	0.1
18	4.0	4.5	-0.5	4.2	-0.2	4.0	-0.0	3.9	0.1	3.9	0.1
19	3.8	4.3	-0.5	4.0	-0.2	3.8	-0.0	3.7	0.1	3.7	0.1
20	3.6	4.1	-0.5	3.8	-0.2	3.6	-0.0	3.6	0.0	3.5	0.1
30	2.4	2.9	-0.5	2.6	-0.2	2.5	-0.1	2.4	0.0	2.3	0.1
31	2.3	2.9	-0.6	2.6	-0.2	2.4	-0.1	2.3	0.0	2.3	0.1
32	2.3	2.8	-0.6	2.5	-0.2	2.3	-0.1	2.3	-0.0	2.2	0.1

Table 1 Continued

33	2.2	2.7	-0.6	2.4	-0.2	2.3	-0.1	2.2	-0.0	2.1	0.1
34	2.1	2.7	-0.6	2.4	-0.3	2.2	-0.1	2.1	-0.0	2.1	0.1
35	2.1	2.6	-0.6	2.3	-0.3	2.1	-0.1	2.1	-0.0	2.0	0.0
36	2.0	2.6	-0.6	2.3	-0.3	2.1	-0.1	2.0	-0.0	2.0	0.0
37	1.9	2.5	-0.6	2.2	-0.3	2.0	-0.1	2.0	-0.0	1.9	0.0
38	1.9	2.5	-0.6	2.2	-0.3	2.0	-0.1	1.9	-0.0	1.9	0.0
39	1.8	2.4	-0.6	2.1	-0.3	1.9	-0.1	1.9	-0.0	1.8	0.0
40	1.8	2.4	-0.6	2.1	-0.3	1.9	-0.1	1.8	-0.0	1.8	0.0
90	0.8	1.3	-0.5	1.1	-0.3	0.9	-0.1	0.9	-0.1	0.8	0.0
91	0.8	1.3	-0.5	1.1	-0.3	0.9	-0.1	0.8	-0.1	0.8	0.0
92	0.8	1.3	-0.5	1.1	-0.3	0.9	-0.1	0.8	-0.1	0.8	0.0
93	0.8	1.3	-0.5	1.1	-0.3	0.9	-0.1	0.8	-0.1	0.8	0.0
94	0.8	1.3	-0.5	1.0	-0.3	0.9	-0.1	0.8	-0.1	0.8	0.0
95	0.8	1.3	-0.5	1.0	-0.3	0.9	-0.1	0.8	-0.1	0.8	-0.0
96	0.8	1.3	-0.5	1.0	-0.3	0.9	-0.1	0.8	-0.1	0.8	-0.0
97	0.7	1.3	-0.5	1.0	-0.3	0.9	-0.1	0.8	-0.1	0.7	-0.0
98	0.7	1.3	-0.5	1.0	-0.3	0.9	-0.1	0.8	-0.1	0.7	-0.0
99	0.7	1.3	-0.5	1.0	-0.3	0.9	-0.1	0.8	-0.1	0.7	-0.0

For example for 9%, under the heading, "Rule," the Rule of 72 gives 8.0 years to double. The actual times to double and error in years by the Rule of 72 are reported in the columns to the right. For compounding biannually (every two years) the actual time to double is 9.3 years, and the reported error of -0.3 indicates that the Rule of 72 under estimates by .3 years (rounded to the nearest tenth of a year). For 9% compounded monthly, the actual time to double is 7.7 years and the Rule of 72 over estimates by 0.3 years. The table shows an interesting property of the errors for the various compounding methods. Notice that the error for biannual compounding changes sign between 3% and 4% indicating that there is a value of i for which the "Rule of 72" agrees with the actual doubling time formula between these two interest rates. For annual compounding, the error changes sign between 7% and 8%, and for semi-annual compounding, the sign change occurs between 15% and 16%. The quarterly rate error changes sign between 31% and 32% and the error sign change for monthly compounding occurs between 94 and 95 percent.

We noticed that for annual compounding, the error changes sign between 7% and 8%. If we concentrate on the interval from 7 to 8 percent and compute the doubling times for 7.1%, 7.2% etc. we get the following table of values:

TABLE 2

ERROR SIGN CHANGE FOR RULE OF 72
BETWEEN 7% AND 8%

Percent	Rule of 72	Actual Doubling Time	Error
7.0	10.29	10.24	0.0410
7.1	10.14	10.11	0.0360
7.2	10.00	9.97	0.0300
7.3	9.86	9.84	0.0250
7.4	9.73	9.71	0.0200
7.5	9.60	9.58	0.0160
7.6	9.47	9.46	0.0110
7.7	9.35	9.34	0.0060
7.8	9.23	9.23	0.0020
7.9	9.11	9.12	-0.0020
8.0	9.00	9.01	-0.0060

Notice that there is a change of sign between 7.8% and 7.9% indicating the value where the two methods agree will occur between these percentages. We could continue this process and compute a table for values between 7.81% and 7.89% and so on, until we obtained a value that was accurate enough for our needs. However there is another approach that will yield the value more quickly.

Recall that the actual value of n is given by

$$n = \ln(2)/\ln(1 + i). \qquad (1)$$

If we assume that n is also given by

$$n = 72/100i$$

and equate these two expressions for n, we get

$$72/100i = \ln(2)/\ln(1 + i)$$

so

$$\ln(1 + i) = 100i \ln(2)/72. \qquad (2)$$

If we can solve this equation for i, we will have the value for which the two values of n agree.

The series expansion of $\ln(1 + x)$ is

$$\ln(1 + x) = x - x^2/2 + x^3/3 - x^4/4 + \ldots + (-1)^{n-1}(x)^n/n \ldots \text{, [1]}$$

so replacing x with i and substituting in (2), we have

$$100i \ln(2)/72 = i - i^2/2 + i^3/3 - i^4/4 + \ldots + (-1)^{n-1}(i)^n/n \ldots .$$

Dividing by i, we obtain

$$100 \ln(2)/72 = 1 - i/2 + i^2/3 - i^3/4 + \ldots + (-1)^{n-1}(i)^{n-1}/n \ldots .$$

In the expansion of $\ln(1 + x)$, using MacLaurin's Formula[2], the error beyond the seventh term is less than .000002 when x is between .01 and .02, so if we use a seven-term polynomial to approximate this expansion, we get

$$100 \ln 2 = 72 - 36i + 24i^2 - 18i^3 + 14.4i^4 - 12i^5 + 10.2857i^6$$

Thus $0 = 2.685282 - 36i + 24i^2 - 18i^3 + 14.41^4 - 12i^5 + 10.2857i^6$

One of the solutions of this polynomial, found by the binary Chopping Method[3], is approximately .078469 or 7.8469% which is a value between 7.8 and 7.9 percent as Table 2 predicted.

To find the interest rate for which the "Rule of 72" agrees with the actual doubling for interest compounded quarterly, we alter the annual equation (1) and get the quarterly equation:

$$4n = \ln(2)/\ln(1 + i/4)$$

where n is years and i is the annual rate of interest. If we again assume that n - 72/100i and equate these we obtain

$$4(72/100i) = \ln(2)/\ln(1 + i/4)$$

so

$$\ln(1 + i/4) = 100i \ln(2)/288$$

which is similar to (2) above. Using the same seven-term expansion for $\ln(1 + x)$, we have

$$\ln(1 + i/4) = i/4 + (i/4)^2/2 + (i/4)^3/3 + (i/4)^4/4 - (i/4)^5/5 - (i/4)^6/6 + (i/4)^7/7$$

so

$$\ln(1 + i/4) = i/4 - i^2/32 + i^3/192 - i^4/1024 + i^5/5120 - i^6/24576 + i^7/114688.$$

Thus

$$100i \ln(2)/288 = i/4 - i^2/32 + i^3/192 - i^4/1024 + i^5/5120 - i^6/24576 + i^7/114688,$$

so

$$100 \ln(2)/288 = 1/4 - i/32 + i^2/192 - i^3/1024 + i^4/5120 - i^5/24576 + i^6/114688.$$

Therefore

$$0 = -.240676 + 1/4 - i/32 + i^2/192 - i^3/1024 +$$

$$i^4/5120 - i^5/24576 + i^6/114688.$$

So $$0 = .009323896 - .03125X + .0052083335x^2 -$$

$$.0009765626x^3 + .0001953125x^4 - .0000407x^5 + .00000872x^6.$$

Multiplying by 10,000,000 we get

$$0 = 87.2x^6 - 407x^5 + 1953.125x^4 - 9765.625x^3 +$$

$$52083.335x^2 - 312500x + 93238.96.$$

Solving this polynomial by the Binary Chopping Method yields a root of .313874 or 31.3874% which is between 31 and 32 percent as predicted.

In a similar manner, we could find the values for which the "Rule of 72" agrees with the actual doubling time for interest rates compounded biannually, semi-annually and monthly. Interested readers are encouraged to do so.

After noting that there are specific interest rates and compounding methods for which the "Rule of 72" agrees with the actual doubling time, we wondered if there is a compounding method (for example, compounded monthly, compounded continuously, etc.) that agrees with a "Rule of N" for all interest rates. Consider the formula for continuous compounding:

$$A = Pe^{rt}$$

If A - 2 and P - 1 then the doubling time t in terms of r is

$$t = \ln(2)/r.$$

If we assume t can also be found exactly by a "Rule of N," then

$$t = N/100r,$$

so $$N/100r = \ln(2)/r.$$

Multiplying by r, we get

$$N/100 = \ln(2),$$

so $$N = 100 \ln (2).$$

This means that the "Rule of 100 in (2)" agrees with the actual doubling time for all interest rates that are compounded continuously. Teachers and students may wish to pursue this question further. For example, for all interest rates of r% compounded monthly, is there a single number N such that N divided by 100r gives the exact time to double?

Of further curiosity to some students may be the question of where the "Rule of 72" agrees with the actual doubling time when interest is compounded weekly or daily. We might suspect from the trends shown in Table 1 that these interest values are well beyond 100%. Our suspicions are justified because, the value for weekly compounding turns out to be 408.0404% and the daily compounding value is 2864%.

We hope that this analysis of the "Rule of 72" will provide teachers with a better background to discuss the rule with their students, that their students will experiment with other "rules" to estimate answers to calculations, and students will learn to use rules to check the answers they get from their calculators. We hope we have illustrated material for activities for students of all levels from high school to college.

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