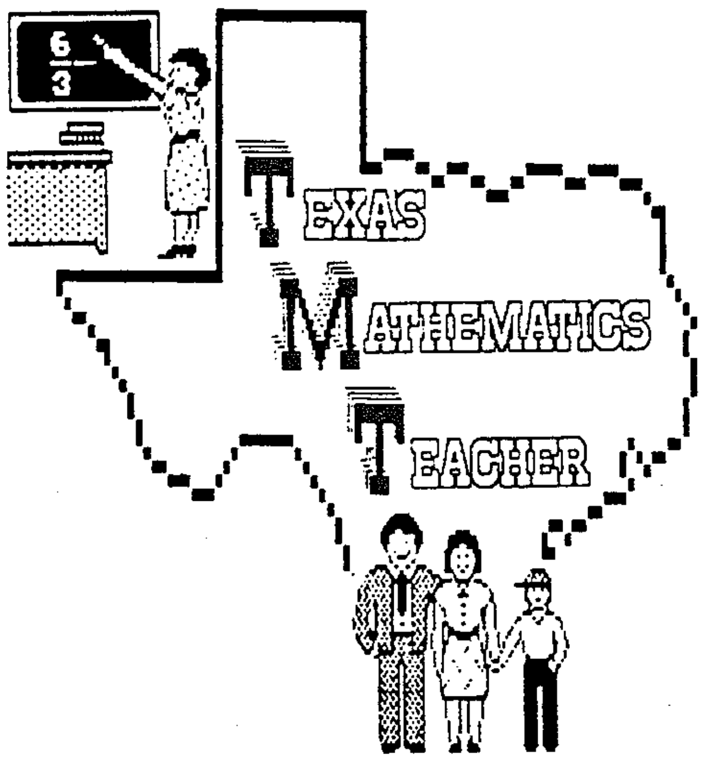

Motivating Second Quadrant Trigonometry

Calculator Activities for Grades K-3

Repeating Decimals

A Construction of Finite Systems From Remainders

JANUARY 1993

TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published four times each year, in October, January, March, and May. All manuscripts and editorial correspondence should be addressed to George H. Willson, listed above.

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council
of Teachers of Mathematics

January 1993

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President's Message

My New Year's resolution: *I resolve to increase the use of multiple methods of assessment in my classes.* There! Now that it is in print, I will feel compelled to do something about it!

I have been reading about, hearing about, and talking about alternative assessment since 1989 when the Mathematical Sciences Education Board publication Everybody Counts stated "we must ensure that tests measure what is of value, not just what is easy to test. If we want students to investigate, explore, and discover, assessment must not measure just mimicry mathematics." NCTM's Curriculum and Evaluation Standards for School Mathematics addresses purposes and methods of assessment. At recent CAMT sessions there have been many presentations about alternative assessment, including outstanding sessions by Steven Leinwand and Marilyn Rindfuss.

Two recent publications have helped convince me that I can and should change my methods of student assessment. The first is the recent NCTM publication Mathematics Assessment, Myths, Models, Good Questions, and Practical Suggestions, edited by Jean Kerr Stenmark. This booklet offers examples of assessment techniques that include comments and suggestions from teachers who implemented them. The second publication is the November issue of the Mathematics Teacher, which is a theme issue dedicated to alternative assessment. (It includes a "Soundoff" column by past TCTM president Ralph Cain and Patricia Kenney.) Articles addressing assessing various student processes, assigning grades, and using reflection as a means of assessment.

Last month the National Middle School Association held their annual conference here in San Antonio. I had the opportunity to talk with teachers from other states who were also struggling with how to do more with alternative assessment. As we talked it became clear to all of us that our best resources are one another. Both in planning assessment and in reviewing results of student work we should gather groups of teachers to discuss ideas. We should share good problems with good questions. We should get involved in mathematics organizations and talk with others who attend their meetings.

Change is never easy, and there never is enough time to do what I want to do, but I am committed to implementing change in assessment in my classroom in 1993. Wish me luck! And if you see me next summer at CAMT, put me on the spot and ask me how it is going! I challenge each of you to pick out one area in your teaching that you wish to change, expand, or improve and make your own resolution for 1993.

Have a wonderful New Year.

Susan Thomas
TCTM President

MOTIVATING SECOND QUADRANT TRIGONOMETRY

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Often in mathematics, we extend ideas by appealing to patterns. For example, once b^n has been defined for positive integers n, we motivate the definition of b^0 by suggesting to our students that we would like to preserve the property that $b^m b^n = b^{m+n}$, and, therefore, we would expect that $b^0 b^n = b^n$. Similarly, we motivate the definition of 0! by appealing to the pattern $n! = \frac{(n+1)!}{n+1}$, thereby expecting $0! = \frac{1!}{1} = 1$. In this article we shall show how the trigonometric functions can be defined in the traditional context of right triangles and then extended to non-acute angles by appealing to well-known patterns that occur in acute triangles.

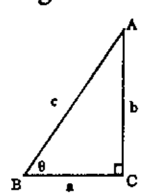

Figure 1

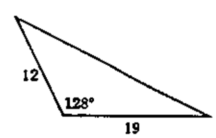

Figure 2

Although the unit circle provides a setting for a comprehensive development of the trigonometric functions, from a motivational point of view it is preferable to make these definitions using right triangle ratios. Specifically, given a right triangle ABC (Figure 1), define

$$\sin\theta = \frac{b}{c}, \quad \cos\theta = \frac{a}{c}, \quad \tan\theta = \frac{b}{a}. \tag{1}$$

Using these definitions, a scientific calculator can be used immediately to solve interesting application problems. (For example, what is the height of a tree if at a point on the ground 30 feet from the base of the tree the angle of elevation of its top is 54°?) The disadvantage of using right triangles is, of course, that they do not provide the groundwork for extending the trigonometric functions outside the interval (0°, 90°). To effect such extension, most textbooks abruptly discard a triangle setting for angles, replacing it with the Cartesian coordinate system. Here the angles are placed with their vertices at the origin and one of their rays

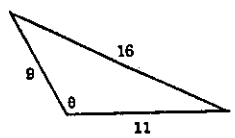

Figure 3

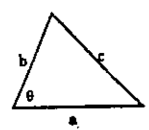

Figure 4

coincidental with the positive horizontal axis. We have seen students become bewildered as to why they are no longer dealing with triangles and why they are suddenly dealing with the Cartesian coordinate system. We would like to suggest an approach with which most of the precalculus applications of the trigonometric functions can be developed prior to the study of their relationship to the Cartesian coordinate system, the unit circle, and the wrapping function, and prior to the study of periodicity.

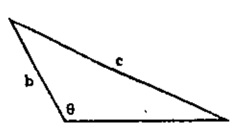

Figure 5

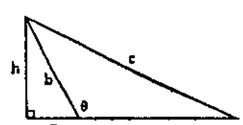

Figure 6

As a first step in exploring alternate ways of developing the trigonometric functions, we should ask why they need to be defined beyond the interval (0°, 90°). A look at the curriculum of trigonometry courses reveals that applications at the precalculus level rarely require evaluation of the trigonometric functions at angles outside the interval (0°, 180°). Specifically, the values of cos θ in the second quadrant arise most often when computing triangle areas or when computing triangle parts using the Law of Cosines or the Law of Sines. For example, the triangle in Figure 2 has area $\frac{1}{2}(12)(19)\sin 128°$ and, in the triangle in Figure 3, the unknown angle θ has approximate measure 106°. Since obtuse triangles provide a more relevant setting than the Cartesian coordinate system for obtuse angles, why not use obtuse triangles to motivate extending the domain of trigonometric functions to include obtuse angles?

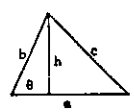

Figure 7

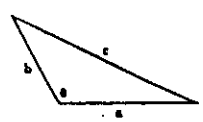

Figure 8

Let us consider how this can be accomplished. Suppose we have already defined the cosine function as in (1) and have established the Law of Cosines for acute triangles. The latter states that if θ is an acute angle (see Figure 4), then

$$\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}. \qquad (2)$$

Consider now a triangle with obtuse angle θ (see Figure 5). Dropping an altitude of length **h** as shown in Figure 6, we obtain

$$c^2 = h^2 + x^2 + 2ax + a^2.$$

But, $b^2 = h^2 + x^2$ and $x = b\cos(180° - \theta)$. Therefore,

$$c^2 = a^2 + b^2 + 2ab\cos(180° - \theta),$$

yielding $$\cos(180° - \theta) = -\frac{a^2 + b^2 - c^2}{2ab}. \tag{3}$$

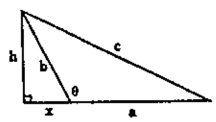

Figure 9

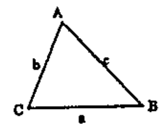

Figure 10

However, we can reason from (2) that if cos θ had a meaning (recall, θ is now an obtuse angle), we would "expect"

$$\cos\theta = \frac{a^2 + b^2 - c^2}{2ab}.$$

That is, we would "expect" $\cos\theta = -\cos(180° - \theta)$. Thus, based on our prior knowledge (2) for an acute angle θ and our finding (3) for an obtuse angle θ, we define

$$\cos\theta = -\cos(180° - \theta). \quad (90° < \theta < 180°)$$

We now have extended the domain of the cosine function to include obtuse angles based on a pattern found in the Law of Cosines and in such a way that the Law of Cosines still "works." Further, with $\theta = 90°$, we would "expect" that $\cos 90° = -\cos 90°$, implying that $\cos 90° = 0$.

Suppose we have already defined the sine function as in (1). If K denotes the area of the acute triangle show in Figure 7, then

$$K = \frac{1}{2}ah = \frac{1}{2}ab\sin\theta.$$

Therefore,

$$\sin\theta = \frac{2K}{ab}. \tag{4}$$

Consider now a triangle with obtuse angle θ and area K (see Figure 8). Dropping an altitude of length h as shown in Figure 9, we obtain

$$K = \frac{1}{2}ah$$

and $$h = b \sin(180° - \theta).$$

so that $$K = \frac{1}{2}ab \sin(180° - \theta).$$

Thus, $$\sin(180° - \theta) = \frac{2K}{ab}. \quad (5)$$

However, we can reason from (4) that if $\sin\theta$ had a meaning (recall, θ is now an obtuse angle), we would "expect"

$$\sin\theta = \frac{2K}{ab}.$$

That is, we would "expect" $\sin\theta = \sin(180° - \theta)$. Thus, based on our prior knowledge (4) for an acute angle θ and our finding (5) for an obtuse angle θ, we define

$$\sin\theta = \sin(180° - \theta). \quad (90° < \theta < 180°)$$

With this definition, we can state that, given any triangle ABC (see Figure 10), acute or not, $Area = \frac{1}{2}ab \sin C.$ By symmetric arguments,

$$Area = \frac{1}{2}bc \sin A = \frac{1}{2}ac \sin B = \frac{1}{2}ab \sin C.$$

If we divide each of the above terms by $\frac{1}{2}abc$, we obtain the Law of Sines:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

We now have extended the domain of the sine function to include obtuse angles based on a pattern found in the law of Sines and in such a way that the Law of Sines still "works."

We conclude by observing that the above definitions together with the right triangle identity $\cos\theta = \sin(90° - \theta)$ can be used to motivate further extension of the domain of the trigonometric functions. For example,

$$\cos(-25°) = \sin(90° - (-25°)) = \sin 115° = \sin 65° = \cos 25°,$$

$$\cos 205° = -\cos(180° - 205°) = -\cos(-25°) = -\cos 25°,$$

$$\sin(-25°) = \cos(90° - (-25°)) = \cos 115° = -\cos 65° = -\sin 25°,$$

and so on.

References

Graham, John A. and Robert H. Sorgenfrey. *Trigonometry with Applications.* Boston: Houghton Mifflin, 1987.

Senk, Sharon L., et al. *The University of Chicago School Mathematics Project - Advanced Algebra.* Glenview, IL: Scott, Foresman and Company, 1990.

Will the person willing to review papers (whose correspondence is postmarked Corpus Christi) please contact the editor.

Calculator Activities for Grades K-3

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This article presents calculator activities which have been tried in classrooms in grades K-3. These activities have been used in a "lab" setting with children having a range of ability levels. Too often classroom teachers have limited calculator use to students in grades 4 and above. However, early experiences with calculators in grades K-3 enable children to explore many mathematical ideas and the concepts behind the arithmetic operations. According to the Mathematical Sciences Education Board, as ideas of what constitutes "basic skills" changes, the calculator will play an ever increasing role.

> A growing volume of research supports appropriate use of calculators in any grade. It is now clear that an understanding of arithmetic can be developed with a curriculum that uses estimation, mental arithmetic, and calculators, with reduced instruction in manual calculation. Indeed, mental arithmetic may replace written methods as the basic skill of our computer [calculator] age. (Reshaping School Mathematics, p. 19)

The activities in this article reflect the NCTM Curriculum and Evaluation Standards (1989) statement regarding technology and its use.

> *The K-4 curriculum should make appropriate and ongoing use of calculators and computers.* Calculators must be accepted at the

> K-4 level as a valuable tool for learning mathematics. Calculators enable children to explore number ideas and patterns, to have valuable concept-development experiences, to focus on problem-solving processes, and to investigate realistic applications. The thoughtful use of calculators can increase the quality of the curriculum as well as the quality of children's learning. (p. 19)

The Mathematical Sciences Education Board noted, "Goals for student performance are shifting from a narrow focus on routine skills to development of broad based mathematical power," (p. 5). As goals for student learning change, the calculator can be used in ways to enable students to gain mathematical power.

Only a few calculator activities are presented here. However, teachers should be encouraged to use their own creativity to expand or develop new activities. Notes are made regarding mathematical insights gained in working through these activities with students. The first activity, "Calculator Counting" involves children in exploring addition and the use of the repeating function on the calculator. In Activity 2, "Calculator Dollars and Cents," children use the decimal point on the calculator to determine problem situations involving money. Place value concepts are explored in Activity 3, "Wipe-Out" as children play a game with their partner.

Activity 1

Calculator Counting

1. Read all directions from beginning to end before you begin.

2. You will need: A Calculator (with repeating function)
   Centimeter Cubes or Counting Chips
   (Any small counters could be used such as, buttons, beans, Teddy Bear counters, color tiles, etc.)
   Worksheet 1: Hundreds Chart
   Worksheet 2: Information Sheet

3. Directions

- Put the items in a pile.
- Move one from the pile to start a second pile
- When you move one item you must enter a 1 [+]

  on your calculator.

  The window will show 1
- Move another item to the second pile.
- Enter [=] to count the item.

  The window will show 2
- Move another item to the second pile.
- Enter [=] to count the item.

  The window will show 3
- Keep on doing this:

  Move one to the second pile and enter [=]

  Move one and enter [=]

  Move one and enter [=]
- Continue until all the items are in the second pile.

  The calculator window shows the total number of items.

NEXT

- Use the same number of items as when you counted by

  1 [+]

  Do not add or take away any items.

  Look at your calculator information sheet

  for the number of items.

- Put the items in a pile.

  Move 2 counters to start a new pile.

  Enter 2 [+] in your calculator.

  Move 2 more items and enter [=]

  The window will show 4.

  Keep on moving 2 from the first pile to the

  second pile and enter a [=] after each move.

  Continue until all items have been counted.

  If you have only 1 counter left in the first pile,

  enter [+] 1 [=] to count it.

- The final number displayed in the calculator window

  should be the same as when counting by 1's.

  If it is, put a ✓ by the circled number on

  your Hunderds Chart (Worksheet 1).

NEXT

- Use the same number of items you had before.
- Do the same calculator counting by 5's

    Move 5 items from the first pile to a new second pile

    and enter 5 [+]

    Move 5 items from the first pile to the second pile

    and enter [=]

    Move 5 items from the first pile to the second pile

    and enter [=]

    If you have 1, 2, 3, or 4 items left over at the end,

    be sure to enter [+] 1 [=]

    or [+] 2 [=]

    or [+] 3 [=]

    or [+] 4 [=]

    The final number in the window should be the same as when counting by 1's or 2's. Put a ✓ by the circled number on the Hundreds Chart (Worksheet 1) if it is the same.

- Use the same number of items you counted by 1's.
- Work in groups of 4 students. Each member is to choose a different number to count by from the list below.

  Count by 3's, 4's, 6's, 7's, 8's, 9's, 10's

  Each time the number is the same as when counting by 1's put a put a ✓ by the circled number on the Hundreds Chart (Worksheet 1)
- <u>Remember</u>: If you have left over items at the end of your count, enter [+] (the number left over) [=]
- Record your answer on Question #8 of the Calculator Information sheet.

Worksheet 1: Calculator Counting

HUNDREDS CHART

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Worksheet 2: Calculator Counting

Name:____________________________

Date:____________

INFORMATION SHEET

1. What items did you count? ________________________
2. What was your estimate of the number of items in your pile? ____________
3. What was the actual number in the calculator window when you counted by 1's? ____________
4. Was your estimate higher or lower or the same as the actual calculator count? ____________
5. What is the difference between your estimate and the actual number of items? ____________
6. Write a number sentence to show the difference between the estimated number of items and the actual number shown on your calculator.

   ____
7. Write the number you got in your calculator window when you counted:
   - by 2's _________
   - by 5's _________
   - group counting project:
     the number you counted by _________
     the number in the calculator window_________
8. When counting large numbers of items is it easier to count them using a calculator or do you think it is easier to count them by hand? Why/why not. ___________________

   ____
   ____
   ____
   ____
   ____

Teacher Notes on
Activity 1: Calculator Counting

1. This activity helps students become familiar with key punching on calculators and using the repeating function.

2. Students become familiar with the concept of multiples of numbers when they county by 2's, 5's, etc.

3. Students confirm their estimated answers in several ways.

4. The concept of remainder is introduced through concrete experiences.

5. Children are presented with the idea that an estimate is not right or wrong, but only the approximation of the actual answer.

6. Children are asked to decide whether or not it is useful to use calculators and then to write their explanation.

Activity 2 (a, b, c)
Grade Level: 2-3

Calculator Dollars and Cents

OBJECTIVES:

- To develop skill in using the decimal point key on the calculator.
- To relate the decimal point to money problems.
- To practice addition of money on the calculator.
- To understand the placement of the decimal point in money problems.
- To apply the addition of money to a practical situation.
- To add money in a column.

Activity 2-a **THE MONEY CHART**

Use the money chart to discuss how money is displayed on the calculator. Discuss the absence of the $ and ¢ signs on the calculator. Relate the decimal point to money. Numbers on the left of the decimal point tell the dollars and the numbers on the right of the decimal point tell the cents. Discuss the constant 0 in the display when the calculator is turned on. This 0 will move to the left as other number keys and decimal point are pressed. **Stress:** It is important to first press the decimal point when we have no dollars.

Tell students to press exactly what is shown on the money chart. For **1¢** they are to press **.01** and the display will be **0.01**. Explain the meaning of each number. This is how **1¢** is shown in the display. It is read as one cent. Write what is displayed on your calculator below each amount on the money chart.

The Money Chart

Name ____________

Cents →	1¢	2¢	3¢	7¢	14¢	17¢
Enter in Calculator →						
Display on Calculator →						

Cents →						
Enter in Calculator →	.04	.05	.09	.16	.20	.21
Display on Calculator →						

Cents →						
Enter in Calculator →						
Display on Calculator →	0.06	0.08	0.11	0.18	0.24	0.25

Cents →	13¢			50¢		
Enter in Calculator →		.26			.45	
Display on Calculator →			0.30			0.35

Activity 2-b

THE ALPHABET MONEY CHART

In a classroom discussion, develop the alphabet money chart.

One version is to have each letter of the alphabet starting with **A** worth **1¢** and ending at **Z** worth **26¢**.

A second version is to have each letter of the alphabet starting at **Z** worth **1¢** and ending at **A** worth **26¢**.

A third version is to assign an arbitrary amount of money to each letter of the alphabet.

Give the students a copy of the Alphabet Money Chart and a piece of 2 cm. grid paper.

Demonstrate the procedure.

1. Put 1 letter of your name in each 2 cm. square. Be sure to find the value of each letter. Be sure to write the decimal point correctly and put a + sign between each value and an = sign after the last letter of your name.
2. Enter into the calculator exactly what was written. (Demonstrate this with "your name.") Write the total down for the first name and correctly place the decimal point. Clear the calculator. Repeat the addition for the first name. If the same amount appears again then circle the number.
3. Go on to the middle name and follow the same procedure as the first name. Go on to the last name and follow the same procedure as the first name. Add all three amounts for a grand total.

Y	O	U	R		
.25 [+]	.15 [+]	.21 [+]	.18 [=]	$0	.79
N	A	M	E		
.14 [+]	.01 [+]	.13 [+]	.05 [=]	$0	.33

$0.79	[+]	$0.33	[=]	$1.12

4. Follow the same procedure for other words and names. Ask children to read their totals. Check for placement of the decimal point.

Activity 2-c

THE COST OF A MEAL

Supplies:

1. Menus from a quick food restaurant.
   • You will need 3-4 menus per group of students
   • Suggested Restaurants: McDonald's, Sonic, Wendy's, Burger King, Denny's, TaCasita, Arby's, Dairy Queen, Jack In The Box, Whataburger

2. Activity Worksheets 2-c and calculators.

WHOLE GROUP (Guided Practice):

Use one menu and have children read food selections and decide on one order for a lunch. List the choices on the chalkboard or overhead projector. Have the students copy the order on one of their own order sheets (Worksheet 2-c). Have them record their money amounts on the grid portion. This will assist in correct placement of the decimal point. Have children use their calculator to add the amounts. Ask students to read the correct total cost of the meal. You could discuss "tips" and how to add them to the total amount of the bill.

SMALL COOPERATIVE GROUPS:

In groups of 2-3 have students use their menus to plan 3 different meals: a breakfast, lunch and dinner. Have them write down their orders and calculate the amount of each bill.

Extension (early finisher activity): Have the group decide on a common menu for lunch and then calculate the cost for the same items in three different restaurants. For example: "Find the cost of a hamburger, small soft drink, and large fries at McDonald's, Burger King, and Dairy Queen." "Compare the costs and write about what you found out."

SUMMARY (Whole Class):

Discuss group results for each meal. Ask the following questions and include some of your own as well:

1. Which breakfast was the most expensive? Least expensive? What is the difference between the most and least expensive breakfast?

2. Which lunch was the most expensive? Least expensive? What is the difference between the most and least expensive lunch?

3. Which dinner was the most expensive? Least expensive? What is the difference between the most and least expensive dinner?

4. If I had only $5.00 to spend what could I get at three different restaurants?

Restaurant: ________________ Item: ____________
____
____

Restaurant: ________________ Item: ____________
____
____

Restaurant: ________________ Item: ____________
____
____

Name____________________
Name____________________
Name____________________

Activity 2c
Worksheet 1

Breakfast at

____

ITEMS	COST
______________________________	
______________________________	
______________________________	
______________________________	
______________________________	
______________________________	
______________________________	

TOTAL COST [ ]

Activity 2c
Worksheet 2

Name____________________
Name____________________
Name____________________

Lunch at ______________________________

ITEMS	COST

TOTAL COST

Name____________________
Name____________________
Name____________________

Activity 2c
Worksheet 3

Dinner at

____

ITEMS	COST

TOTAL COST

Activity 2c
Early Finisher Activity

Name________________________
Name________________________
Name________________________

Cost of a Meal at 3 Restaurants

Restaurant #1:______________________________

Restaurant #2:______________________________

Restaurant #3:______________________________

ITEMS	COST #1	COST #2	COST #3
____________________			
____________________			
____________________			
____________________			
____________________			
____________________			
TOTAL COST			

* * * On another paper write about your findings.

Teacher Notes
on
Activity 2 (a, b, c): Calculator Dollars and Cents

1. This activity develops skill in using the decimal point key on the calculator.

2. The activity helps students relate money to its decimal representation.

3. Students practice adding and subtracting money.

4. Students have real world application of mathematics.

5. Children work in cooperative groups to collect and organize information, make decisions and solve problems.

Activity 3

Wipe-Out

Wipe-Out Game: This game requires one person as the Number Giver and player (from 1 to the total class). Students can play as partners with each partner taking turns as the Number Giver.

Object of the Game: To remove (Wipe-Out) one digit of a given number entered into the calculator without changing other numbers and to end up with 0 in the display. The game can use numbers as large as the calculator will display.

Prerequisite Skills: Students should have an understanding of place value from the tens to the millions place.

DIRECTIONS: The Number Giver selects a number and tells the student(s) to enter that number into the calculator. Then the Number Giver tells the student(s) which digit in the number to Wipe-Out. There are three examples below the children can use for Guided Practice before they begin to play.

Number Giver Says	Player(s) Action	Calculator Display	Write on Overhead or Chalkboard
"Enter 9"	Press 9 Key	9	9
"Wipe-Out 9"	Press ⊟ 9	0	-9 / 0
"Enter 29"	Press 29 Key	29	29
"Wipe-Out 9"	Press ⊟ 9	20	-9 / 20
"Wipe-Out 2"	Press ⊟ 20	0	-20 / 0
"Enter 123"	Press 123 Key	123	123
"Wipe-Out 1"	Press ⊟ 100	23	-100 / 23
"Wipe-Out 3"	Press ⊟ 3	20	-3 / 20
"Wipe-Out 2"	Press ⊟ 20	0	-20 / 0

Teacher Notes on
Activity 3: Wipe-Out

1. Place value concepts are reinforced via the calculator.
2. Mental arithmetic skills with small numbers are practiced.
3. The children are given activities which prepare them for expanded forms of large numbers.

Summary

These are three examples of calculator activities for children in the early elementary grades. Teachers should extend these as well as develop new ones of their own. The calculator is a powerful tool for the development of mathematical ideas. Most importantly, students will feel a source of control over their own learning. In particular, "If I push buttons (in the correct way), I will get the correct answers!" "Calculators only do what I tell them to do, but they do it correctly!"

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REPEATING DECIMALS

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Introduction

In this article an easier way to convert repeating decimals into fractions is discussed and the definite initial position of the repeating block of a nonterminating repeating decimal is determined. All of these short-cuts follow from the process commonly given in current algebra textbooks.

Notations and Definitions

Let b be an integer. We say the integer a is a factor or a divisor of b if there exists an integer c such that $b = ac$. We denote this by $a|b$. If a is not a divisor of b, we write $a \nmid b$.

Let a and b be integers. The greatest common divisor of a and b is denoted by (a,b).

Definition: A repeating decimal is a decimal with a block of digits, called the repetend, repeated infinitely many times.

Notes: 1) A terminating decimal can be regarded as a repeating decimal ending in all zeros or nines.

Example:

$$\frac{3}{8} = 0.375 = 0.375000 \cdots = 0.375\overline{0}$$

$$= 0.374999 \cdots = 0.374\overline{9} \, .$$

2) A repeating decimal which does not end in all zeros or nines is called a nonterminating repeating decimal.

Example:

$$\frac{1}{3} = 0.333 \cdots = 0.\overline{3} \, .$$

Results

The following lemma is known [1]. It states that if p and q are prime to each other, the p/q is a nonterminating repeating decimal if and only if q has a prime factor other than 2 or 5.

Lemma 1: Let p, q be positive integers such that $(p,q) = 1$, then p/q is a nonterminating repeating decimal if there exists a prime number t ($t \neq 2, 5$) such that $t|q$.

[Proof]: "$\Rightarrow$" If the only factors of q are 1, 2 or 5, then $p/q = u/10^v$, for some positive integer u and nonnegative interger v. Thus, p/q is not a nonterminating repeating decimal, which is a contradition.

"$\Leftarrow$" If t is a prime number with $t \neq 2, 5$ such that $t|q$, then, $p/q = x/10^y$ for any positive integers x and y, thus p/q is a nonterminating repeating decimal.

The following lemma enables us to convert from any repeating decimal to a fraction.

Lemma 2:

$$a_1 \cdots a_m . b_1 \cdots b_n \overline{r_1 \cdots r_t}$$

$$= \frac{a_1 \cdots a_m\, b_1 \cdots b_n\, r_1 \cdots r_t - a_1 \cdots a_m\, b_1 \cdots b_n}{(10^t - 1)(10^n)} .$$

[Proof]: Let

$$c = a_1 \cdots a_m . b_1 \cdots b_n \overline{r_1 \cdots r_t} ,$$

then

$$10^n c = a_1 \cdots a_m b_1 \cdots b_n . \overline{r_1 \cdots r_t} ,$$

$$10^t 10^n c = a_1 \cdots a_m b_1 \cdots b_n r_1 \cdots r_t . \overline{r_1 \cdots r_t} .$$

Thus,

$$10^t 10^n c - 10^n c$$

$$= a_1 \cdots a_m b_1 \cdots b_n r_1 \cdots r_t - a_1 \cdots a_m b_1 \cdots b_n .$$

i.e.,

$$c = \frac{a_1 \cdots a_m \; b_1 \cdots b_n \; r_1 \; \cdots r_t - a_1 \cdots a_m \; b_1 \; \cdots b_n}{(10^t - 1)(10^n)}.$$

Examples:

$$0.2\overline{34} = \frac{234 - 2}{(10^2 - 1)(10^1)} = \frac{232}{990}, \quad 0.\overline{367} = \frac{367}{999},$$

$$1.23\overline{567} = \frac{123567-123}{99900} = \frac{123444}{99900},$$

$$3.6 = 3.5999 \cdots = 3.5\overline{9} = \frac{359-35}{90} = \frac{324}{90},$$

$$3.6 = 3.6000 \cdots = 3.6\overline{0} = \frac{360-36}{90} = \frac{324}{90}.$$

The following theorem can be related to Euler's Theorem [2]. It states that if q is a positive integer which does not have factors of 2 and 5, then there exists t such that $\underbrace{99 \cdots 99}_{[t\ 9\text{'s}]}$ is divisible by q.

Theorem 1: Let q be a positive integer such that $2 \nmid q$ and $5 \nmid q$. Then there exists a positive integer t such that $q \mid (10^t - 1)$.

[Proof]: i) If q - 1, then $q \mid (10^1 - 1)$.

ii) If $q \neq 1$, then $(1,q) = 1$, by Lemma 1, 1/q is a non-terminating repeating decimal, and by Lemma 2 we have

$$1/q = \frac{w}{(10t - 1)(10^n)}$$

for some positive integers t, n and w. This implies $qw = (10^t - 1)(10^n)$. And then

$$q \mid (10^t - 1)(10^n),$$
$$\text{but } 2 \nmid q,$$
$$\text{and } 5 \nmid q$$
$$\text{implies } q \mid (10^t - 1) .$$

Example:

$$7 \mid 999999 \qquad 999999 = 10^6 - 1.$$

Lemma 3:

$$\frac{a_1 \cdots a_t}{10^t - 1} = 0.\overline{a_1 \cdots a_t} .$$

[Proof]:

$$0.\overline{a_1 \cdots a_t}\,(10^t - 1)$$
$$= a_1 \cdots a_t\,.\,\overline{a_1 \cdots a_t} - 0.\overline{a_1 \cdots a_t}$$
$$= a_1 \cdots a_t\,.$$

Thus

$$\frac{a_1 \cdots a_t}{10^t - 1} = 0.\overline{a_1 \cdots a_t}\,.$$

The following theorems 2 and 3 decide the definite initial position of the repeating block of a fraction.

Theorem 2: If p, q are positive integers, and p/q is a nonterminating repeating decimal such that $2 \nmid q$, $5 \nmid q$ and $(p,q) = 1$, then,

$$\frac{p}{q} = a_1 \cdots a_m\,.\overline{r_1 \cdots r_t}\,.$$

Note: The repeating block begins right after the decimal point.

[Proof]: By Theorem 1 there exists a positive integer t such that

$$\frac{p}{q} = \frac{u}{10^t - 1}$$

for some positive integer u

$$= a_1 \cdots a_m + \frac{r_1 \cdots r_t}{10^t - 1}$$

$$= a_1 \cdots a_m.\overline{r_1 \cdots r_t}$$

by Lemma 3.

Notes: 1) $a_1 \cdots a_m$ could be equal to zero.

2) The remainder, $r_1 \cdots r_t$, is not zero but may have leading zeros. (For example, if $t = 5$ and the remainder of u divided by $(10^t - 1)$ is 151 then $r_1 \cdots r_t$ is 00151.)

Example:

$$\frac{31}{7} = 4 + \frac{3}{7} = 4.\overline{428571}.$$

Theorem 3: Suppose p, q are positive integers with $(p,q) = 1$, and $q = 2^m \cdot 5^n \cdot u$, where u is a positive integer such that $u > 1$, $(u,2) = 1$ and $(u,5) = 1$; and m, n are nonnegative integers with $m + n > 0$. Then,

$$\frac{p}{q} = a_1 \cdots a_t . b_1 \cdots b_{\max(m,n)} \overline{r_1 \cdots r_x} .$$

[Proof]: By Lemma 1, it is clear that p/q is a nonterminating repeating decimal. Let M = max(m,n). Then,

$$\frac{p}{q} = \frac{p}{2^m \cdot 5^n \cdot u}$$
$$= \frac{1}{10^M} \frac{v}{u}$$

for some positive integer v such that $2 \nmid v$ or $5 \nmid v$. Now since $u > 1$, $(u,2) = (u,5) = 1$, by Theorem 1 we know that there exists a positive integer x such that $u \mid (10^x - 1)$, thus $ut = 10^x - 1$ for some positive integer t. Since $2 \nmid t$, $5 \nmid t$; and $2 \nmid v$ or $5 \nmid v$, this implies

$$\frac{v}{u} = \frac{vt}{ut} = \frac{c}{10^x - 1}$$

with $c = vt$ and $10 \nmid c$. Therefore

$$\frac{p}{q} = \frac{1}{10^M} \cdot \frac{v}{u} = \frac{1}{10^M} \cdot \frac{c}{10^x - 1} .$$

Without loss of generality we assume that

$$\frac{c}{10^x - 1} = a_1 \cdots a_z + \frac{r_1 \cdots r_x}{10^x - 1},$$

where

1) the remainder, $r_1 \cdots r_x$, is not zero, but may have leading zeros. (For example, if $x = 5$ and the remainder of c divided by $(10^x - 1)$ is 21, then $r_1 \cdots r_x$ is 00021.)

2) $a \neq r_x$. (If $a_z = r_x$, then, since the last digit of $(10^x - 1)$ is 9, and if we combine

$$a_1 \cdots a_z + \frac{r_1 \cdots r_x}{10^x - 1}$$

into one single fraction, the last digit of the numerator is $a_z(9) + r_x = a_z(9) + a_z = 10\ a_z$, this implies $10|c$, which is a contradiction.)

Thus

$$\begin{aligned}\frac{p}{q} &= \frac{1}{10^M}\left(a_1 \cdots a_z + \frac{r_1 \cdots r_x}{10^x - 1}\right) \\ &= \frac{1}{10^M}\left(a_1 \cdots a_z + 0.\overline{r_1 \cdots r_x}\right) \text{ by Lemma 3} \\ &= \frac{1}{10^M}\left(a_1 \cdots a_z . \overline{r_1 \cdots r_x}\right).\end{aligned}$$

Next, without loss of generality we assume that $z > M$ (we can let $a_1 = a_2 = \cdots = a_w = 0$ for some $w \leq z$ if needed), and since $a_z \neq r_x$ we assure that $r_1 \cdots r_x$ does not match the last portion of $a_1 \cdots a_z$. Therefore,

$$\frac{p}{q} = \frac{1}{10^M}(a_1 \cdots a_z . \overline{r_1 \cdots r_x})$$
$$= a_1 \cdots a_t . a_{t+1} \cdots a_{t+M} \overline{r_1 \cdots r_x} \text{ for some } t$$
$$= a_1 \cdots a_t . b_1 \cdots b_M \overline{r_1 \cdots r_x}$$

where

$$b_i = a_{t+i} \ for \ i = 1, \cdots, M$$
$$= a_1 \cdots a_t . b_1 \cdots b_{\max(m,n)} \overline{r_1 \cdots r_x} .$$

Examples:

$$\frac{7}{12} = \frac{7}{(2^2)(3)} = 0.58\overline{3},$$
$$\frac{7}{15} = \frac{7}{(5)(3)} = 0.4\overline{6},$$
$$\frac{7}{60} = \frac{7}{(2^2)(5)(3)} = 0.11\overline{6}.$$

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A CONSTRUCTION OF FINITE SYSTEMS FROM REMAINDERS

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Finite systems can play useful role in the teaching and learning of mathematics. However, the two usual approaches to construction of finite systems require the learner to have some mathematical maturity and a knowledge of mathematics beyond the usual high school curriculum. For example, the typical notation for elements of finite systems is 0,1,2,3, ... Thus, the systems having 3 and 6 elements, respectively, are denoted by $\{0, 1, 2\}$ and $\{0, 1, 2, 3, 4, 5\}$, but the first set is not a subset of the second!

The purpose of this article is to present a seldom-used approach to constructing finite systems which employs only middle school mathematics through long division. In fact, a discussion is given of the father of long division, the DIVISOR THEOREM, including the two important interactions of the Divisor Theorem with the set $\{1,2,...\}$ of natural numbers and the interaction of the Divisor Theorem with the natural number system, the system consisting of the set of natural numbers and the two binary operations addition and multiplication. The latter interaction is fundamental in the construction of the finite systems.

Given a natural number "a" and a natural number $b \neq 1$, there exist unique whole numbers 1 and r, called the quotient and remainder respectively, such that $a = b \text{ X } q + r$, where $0 \leq r < b$. This result is called the Divisor Theorem. When $b \neq 1$ is a fixed natural number, the Divisor Theorem has a local effect on the set of natural numbers in that it associates with any natural number "a" two unique whole numbers q and r. In fact, q and r are the quotient and remainder gotten when "a" is divided by b by use of long division. For this reason we consider the

Divisor Theorem as the father of long division. Since the association is at the element level, this interaction of the Divisor Theorem with the set of natural numbers is considered a local interaction or result. The remainder, the often rejected step-child in the "quotient" - "remainder" pair, is given the royal treatment in this article.

The Divisor Theorem also has a global effect on the set of natural numbers. In fact, for a fixed natural number $b \neq 1$, the Divisor Theorem induces a unique division of the set of natural numbers into b disjoint subsets, $S_0, S_1, \ldots, S_{b-1}$, where S_i, $i = 0, \ldots, b - 1$, is the set of all natural numbers that leave a remainder of i when divided by b. That is to say, S_i consists of all natural numbers that can be written in the form $bm + i$, where m is a whole number. This interaction of the Divisor Theorem with the set of natural numbers from which b is selected and fixed, is considered global since the action is at the set level. What effect do consequences of the Divisor Theorem have on the operations of addition and multiplication of natural numbers? The answer to this question is the subject considered next.

For a whole number x let r(x) be the remainder gotten when x is divided by 6. Let $m = 10$, $n = 15$, and $k = 20$. It is easily shown that the following are true:

i) $r(mn) = r[r(m)\, r(n)]$.

ii) $r(m + n) = r[r(m) + r(n)]$.

iii) $r[r(k)] = r(k)$.

iv) $r(m + n + k) = r[r(m) + r(n + k)] = r[r(m + n) + r(k)]$.

v) $r(mnk) = r[r(mn)\, r(k)] = r[r(m)\, r(nk)]$.

vi) $r[m(n + k)] = r[r(m)\, r(n + k)] = r[(mn) + r(mk)]$.

If "6" is replaced by any natural number $b \neq 1$, and m, n, and k are replaced by any whole numbers, then each of the six statements just considered will remain true. It is because of this state of affairs that, for any natural number $b \neq 1$, a Finite Arithmetic can be defined for the set of remainders $D_b = \{0, 1, ..., b - 1\}$. How this can be done is the essence of this article.

Let $D_3 = \{0, 1, 2\}$. For a whole number x, let r(x) be the remainder that is gotten when x is divided by 3. Define $\oplus$ and $\bullet$ on D_3 as follows: For x and y in D_3 define

$$x \oplus y = r(x + y) \text{ and}$$

$$x \bullet y = r(xy).$$

The following tables are the operation tables for D_3 relative to $\oplus$ and $\bullet$:

Table 1. An Operation Table for D_3 Relative to $\oplus$

$\oplus$	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

Table 2. An Operation Table for D_3 Relative to $\bullet$

$\bullet$	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Let $D_6 = \{0, 1, 2, 3, 4, 5\}$. For a whole number x, let r(x) be the remainder that is gotten when x is divided by 6. Define $\oplus$ and $\bullet$ on D_6 as follows: For x and y in D_6, define

$$x \oplus y = r(x + y) \text{ and}$$

$$x \bullet y = r(xy).$$

The following tables are operation tables for D_6 relative to $\oplus$ and $\bullet$:

Table 3. An Operation Table for D_6 Relative to $\oplus$

$\oplus$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Table 4. An Operation Table for D_6 Relative to $\bullet$

$\bullet$	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	1	2	3	4	5
2	0	2	4	0	2	4
3	0	3	0	3	0	3
4	0	4	2	0	4	2
5	0	5	4	3	2	1

Let $b \neq 1$ be a natural number. For a whole number x, let r(x) be the remainder that is gotten when x is divided by b. Let D_b be the set of all possible remainders that can be gotten when whole numbers are divided by b. So, $D_b = \{0, 1, \ldots, b - 1\}$ has "b" elements. For x and y in D_b,

$$x \oplus y = r(x + y) \text{ and}$$

$$x \bullet y = r(xy)$$

define binary operations on D_b because remainders are unique and r(x + y) and r(xy) are elements of D_b.

The system consisting of the set D_b and the two operations $\oplus$ and $\bullet$ is called a Finite System or a Finite Arithmetic. Each natural number $b \neq 1$ determines a unique Finite System. Each system is rich in properties, both similar to and different from properties of the usual number systems. Some interesting properties of the finite systems are the following:

1. The set D_b is closed under the two operations $\oplus$ and $\bullet$.

2. The two operations $\oplus$ and $\bullet$ are commutative on D_b.

3. The two operations $\oplus$ and $\bullet$ are associative on D_b.

4. The operation $\bullet$ is distributive over $\oplus$.

5. The number 0 is the identity element for D_b relative to $\oplus$.

6. If x is in D_b, then $r(b - x)$ is in D_b and $r(b - x)$ is the inverse for x relative to $\oplus$.

7. The number 1 is the identity element for D_b relative to $\bullet$.

8. If b is a prime natural number and $x \neq 0$ is in D_b, then x has an inverse in D_b relative to $\bullet$.

9. If b is not a prime, then there exist x and y in D_b such that $x \neq 0$, $y \neq 0$, but $x \bullet y = 0$.

10. D_b is an Abelian group under $\oplus$.

11. D_b is a commutative ring with unity relative to $\oplus$ and $\bullet$.

12. If b is a prime, then D_b is a field under $\oplus$ and $\bullet$, but D_b is not an ordered field and hence inequalities cannot be defined for D_b. Any subset of D_b closed under $\oplus$ must contain 0.

Some special examples follow:

Example 1. Let H = {1, 3, 5, 7} and let the operation be multiplication Mod(8). By use of an operation table for the system, it is easily seen that H is closed under the operation, the operation is associative, 1 is the identity element of the system, and each element in H has an inverse in H. Put another way, the system H is a group. The system is an Abelian group since the operation is commutative. This is the Klein 4-Group.

Example 2. Let K = {2, 4, 6, 8} and let the operation be multiplication Mod(10). The system K is an Abelian group and 6 is the identity element for the group.

Example 3. Let $L = \{2, 4, 6, 8, 10, 12\}$ and let the operation be multiplication Mod(14). The system L is an Abelian group and 8 is the identity element for the group. Let $S = \{2k | k = 1, \ldots m - 1\}$ where m is an odd prime and let the operation be multiplication Mod(2m). The system S is an Abelian group and $m + 1$ is the identity element for the group.

Example 4. The groups in examples 1, 2, and 3 are Abelian. This example involves a group that is not Abelian (commutative).

Let $G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} \middle| \; a \neq 0, \text{ a, b elements of } D_3 \right\}$

G has 6 elements. They are

$t_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $t_1 = \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$, $t_2 = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix}$, $t_3 = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$,

$t_4 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, and $t_5 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$. Let the operation be matrix multiplication Mod(3).

Two operation tables for the system G are given in Tables 5 and 6. They are interesting in their own right. Or are they?

Table 5. An Operation Table for the System G

	t_0	t_1	t_2	t_5	t_3	t_4
t_0	t_0	t_1	t_2	t_5	t_3	t_4
t_1	t_1	t_0	t_5	t_2	t_4	t_3
t_2	t_2	t_4	t_0	t_3	t_5	t_1
t_4	t_4	t_2	t_3	t_0	t_1	t_5
t_3	t_3	t_5	t_4	t_1	t_0	t_2
t_5	t_5	t_3	t_1	t_4	t_2	t_0

Table 6. An Operation Table for the System G

	t_1	t_4	t_5	t_1	t_2	t_3
t_0	t_0	t_4	t_5	t_1	t_2	t_3
t_4	t_4	t_5	t_0	t_2	t_3	t_1
t_5	t_5	t_0	t_4	t_3	t_1	t_2
t_1	t_1	t_3	t_2	t_0	t_5	t_4
t_2	t_2	t_1	t_3	t_4	t_0	t_5
t_3	t_3	t_2	t_1	t_5	t_4	t_0

The system G is the symmetric group on three symbols.

The Divisor Theorem is fundamental to mathematics. Its interactions with the set of natural numbers and the natural number system make it possible to define meaningful binary operations on a set of remainders to produce rich finite systems.

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