TEXAS MATHEMATICS TEACHER

SPRING 1992

TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published three times each year, in the Fall, Winter, and Spring. Address all manuscripts and editorial correspondence to Joe Dan Austin at the address listed above.

Manuscripts should be neatly typed and double spaced, with wide margins, on 8 ½" by 11" paper. Authors should submit three copies. Only one copy should identify the authors, and this copy should include a complete return address for one author. Illustrations should be carefully prepared in black ink on separate sheets, the original without lettering and one copy with lettering added. As soon as possible after refereeing, authors will be notified of a publication decision. Accepted manuscripts may be edited for length. One copy of articles not accepted for publication will be returned to the authors. Two copies of the journal in which a manuscript appears will be sent to the first author.

SUBSCRIPTION and MEMBERSHIP information are found on the last page of the journal.

TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

Spring 1992

TABLE OF CONTENTS

ARTICLES

ANNOUNCEMENTS

President's Message

Karen Hall
Kinkaid School; Houston, Texas

Do you have a computer with a modem? If you answered yes, then you are ready to be tele-connected to other teachers in Texas. The Texas Education Network (TENET) is ready for you to call and become part of a state-wide computer conference for teachers, students, and administrators.

Connie Stout, who is the Project Coordinator for TENET, has lead the effort to provide this service to all educators in Texas. TENET is sponsored by the Texas Education Agency's Technology Division. Using TENET you can access major state university libraries, such as the University of Texas, Texas A&M, and out of state university libraries such as the University of California. For a $5 annual fee ($25 for private schools), teachers, students, and administrators will have unlimited access to TENET.

There are many options you will discover when you use TENET. Some of the options are electronic mail, electronic bulletin boards, electronic conferences, and electronic data base access.

To participate in TENET, you must have an IBM compatible computer with at least 512K of internal memory or a Macintosh computer with at least one megabyte of internal memory. Your computer must also have a modem, and you must be able to connect your modem to a telephone line On your modem, dial the toll free number 1-800-25TENET to obtain information on becoming a subscriber. Be prepared to give your name, the name of your school, and your school address.

TENET is a new idea that you will learn to appreciate only if you use it. It can become a resource for you and your students as you can easily communicate state-wide with other students and teachers. You can use it to continue to communicate with people you met at CAMT. Join today!

Mathematics Communication in the Elementary Classroom

Elizabeth Wadlington
Southeastern Louisiana University
Hammond, LA 70402

The Curriculum and Evaluation Standards for School Mathematics (Standards) (1989) published by the National Council of Teachers of Mathematics (NCTM) advocates that the study of mathematics in grades K-8 should include diverse opportunities for children to communicate about mathematics. Effective communication, similar to whole language, emphasizes meaningful exchange of ideas in real-life contexts. Listening, talking, reading, and writing help children construct and clarify their own understanding as well as give teachers valuable information about pupils' thought processes. Historically, mathematics communication activities have dealt almost exclusively with technical terms and symbols. Communications such as explaining one's thinking, justifying answers, and describing solutions to problems have been used infrequently.

While most teachers see the need to use motivating communication activities, they may not know how to start with elementary students. This paper gives some ideas to assist teachers in providing children opportunities to become expressive mathematicians. Modify and extend these suggestions to the needs and interests of your students.

1. Mathematics Show and Tell

At a regularly appointed time, allow children to demonstrate or tell about something mathematical. Initially, children my exclaim, "I do not have any mathematics to talk about." However, after participating in this activity once or twice, they will become very creative in selecting their topics. Possible topics include a toy or game which uses mathematics, a map showing the route and mileage of a family trip, a story involving mathematics, a chart showing the child's daily schedule, and nutritional information on a favorite cereal.

2. Mathematics Journals

Have children keep mathematics journals in which they write and reflect on mathematics activities each day. Initially, you may need to pose questions which help children reflect on what they learned. For example, "What was the most important thing you learned today?" or "How can you use what you learned today in mathematics?" However, such questions will soon become unnecessary. Encourage children to use pictures, diagrams, symbols, and words to communicate their reflections. Journals may be exchanged among children on a voluntary basis. Teachers should also keep a mathematics journal to share with their children.

3. Mathematics Pen Pals

Have children correspond with mathematics pen pals regarding what they are learning in mathematics and how they are using mathematics. Arrange for pupils to write to children in other schools or in other classes at their school. This communication can often be done electronically. Initially, students may write only short notes, nominally fulfilling the assignment. However, as time passes, they will become enthusiastic about communicating and receiving responses. They will often spend their free time in and out of school working on their correspondence.

4. Jig-sawing

Provide problems for small cooperative groups of children to solve. However, given each group member only one piece of the information needed to solve the problem. Consequently, cooperation and communication among children are needed to develop possible strategies and solutions as well as to obtain all information necessary to solve the problem. Encourage talking, listening, reading, and writing within the group. Occasionally, use a variation of this activity in which children are allowed to communicate only through reading, writing, or drawing.

5. Open-ended Questions

Ask children open-ended questions such as, "How should we go about assigning points for the science fair projects?" Children may answer aloud or in writing. Encourage children to consider and respond to each others' responses.

6. Build What I Build

Seat two children next to each other with a barrier between them, e.g., a large book or a tall box. The barrier should prevent each child from seeing what the other child is doing. Ask one child to build a structure using blocks or with other manipulatives. The child then describes his or her construction to the other child who tries to build an identical construction from the verbal instructions.

7. **Acting Out Problems**

Provide groups of children with story problems to solve. Encourage the groups to act out the problems to find solutions. Allow the groups to act out the problems and solutions for other classmates. Interesting problems may be teacher written or adapted from problems in articles in the Arithmetic Teacher. Children may also create their own problems to act out. A variation of this game is to allow only pantomime.

8. **What Is It?**

Ask one child to select a manipulative, e.g., Cuisenaire rod, color tile, centimeter rule, and hide the object behind his or her back. As the child describes the manipulative aloud, the other children try to guess what it is. The child who correctly guesses then chooses an object and describes it for the class. A variation of this game is to have the class ask only questions with a "yes" or "no" answer when they attempt to identify the object.

9. **Mathematics Stories and Poems**

Encourage children to create and illustrate their own stories and poems involving mathematics. Stories and poems may be true, e.g., How I use the Metric System, Construction a kite, or fictitious, e.g, The Martian number system, What's square about a square. Collect children's creative writings and bind them in a simple binder for independent reading in the class library.

10. **Mathematics Bulletin Boards**

Have groups of children be responsible for mathematics bulletin boards. Developing themes, dividing tasks, making materials, and putting up the display require communication among group members. The bulletin board material is then read and discussed by other students.

11. **Real-life Settings**

Allow groups of children to create real-life settings such as restaurants, stores, and other places of business. Child-created props may include menus, price lists, order forms, etc. As children pretend to be to proprietors and customers, they see the need for clear mathematics communication in everyday situations.

12. Surveys

Provide opportunities for children to take surveys and gather other data concerning tastes, interests, and habits of other students and people in the community. They should communicate their findings using charts, graphs, oral reports, and written descriptions.

13. Mathematics Newsletter

Let children write and edit a mathematics newsletter. Possible stories include the children's own mathematical interests, famous mathematicians, people who use mathematics, famous mathematics problems, careers in mathematics, or problems to solve.

14. I Spy

Encourage one child to describe familiar objects in the classroom using spatial terms which are simple, e.g., above, below, over, or under, and which are more mathematical, e.g., parallel, perpendicular, or at a right angle. Other children listen and/or draw a diagram to guess the object. Young children may describe a door as, "Something that's a tall rectangle under the exit sign." Older children progress to descriptions such as, "A parallelogram with four right angles measuring about two meeters by three-quarters of a meeter."

Summary

Teachers who have adapted and used ideas such as these report that children's attitudes toward mathematics improve. Children, who once viewed mathematics as a rigid, unimaginative discipline, begin to see mathematics as a creative subject with opportunities for self-expression. Through communicating mathematics in a variety of ways, they discover that mathematics integrates with other subject areas and with everyday life. Consequently, they voluntarily begin to use mathematics outside school. As they do this, mathematics anxious students often become more confident.

In addition, many children improve in mathematics. As they strive to communicate mathematical ideas to others, they analyze and refine these ideas in their own minds and make mathematical terminology a part of their active vocabulary. They also listen to and read about other students' mathematical ideas. This helps them consider different points of view and to learn from each other.

Teachers should use these ideas as starting points for effective communication in mathematics. After observing children's enthusiasm and enjoyment when engaging in these activities, teachers will discover other ways to foster meaningful communication of mathematical ideas and to help children see mathematics as a vital part of their learning.

References

National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Assessment Book Arrives!

The National Council of Teachers of Mathematics (NCTM) addressed the assessment issue in the Curriculum and Evaluation Standards for School Mathematics published in 1989. Assessment, with respect to classroom education, is a process by which educators evaluate their students' understanding. Using the results, teachers may re-evaluate their own teaching methods.

The long-awaited NCTM publication, Assessment: Myths, Models, Good Questions, and Practical Suggestions, expounds on the issues addressed in the 14 evaluation standards and helps to eliminate much of the uncertainty involved in adopting new assessment techniques.

The publication addresses a wide range of assessment issues. These include current practices, needed changes, alternative assessment, performance assessment, observations, mathematics portfolios, and student self-assessment.

The book costs $8.50 and can be purchased through NCTM. Call toll-free (800) 235-7566 for orders only.

What's So Hard About Graphing a Parabola?

David Slavit
Arkansas College
Batesville, AK 72501

Graphing a line is perhaps the easiest concept to learn in a beginning algebra course. All that is required is to plot two points (perhaps the intercepts) and draw the line through them. Why then should there be a quantum leap when the degree of the polynomial is increased by only one? What's so hard about graphing a parabola?

Since the vertex is obviously a key point on the graph of a parabola, finding it is essential to any graphing procedure. The question is how to do this. The traditional approach is to complete the square in order to write a quadratic function in the form

$$y = a(x - h)^2 + k. \tag{1}$$

For quadratic equations such as

$$y = x^2 - 3x + 1, \tag{2}$$

completing the square is fine. However, when the coefficient of the x^2 term is not equal to one, completing the square is very complicated for students. Plotting points until a pattern is found is time consuming, though perhaps a time honored tradition, but seldom leads students to the line symmetry.

A simple, well-behaved curve should be sketched quickly, efficiently, and accurately.

How, then would you graph an equation of a quadratic when the lead coefficient is not equal one? Since the vertex is either a maximum point or a minimum point, setting the derivative equal zero and solving is perhaps the easiest method. For a general parabola

$$y = ax^2 + bx + c, \tag{3}$$

setting the derivative equal to zero gives that

$$y' = 2ax + b = 0. \tag{4}$$

Solving equation (4) gives

$$x = -\frac{b}{2a}, \tag{5}$$

a "magical formula" for the x-coordinate of the vertex. The corresponding y-coordinate can then be found by substituting this value back into equation (3). However, since method without meaning motivates memorization, some justification to the student is in order.

There are several ways to justify this result. The method I prefer is to use the quadratic formula backwards. I find this gives students an intuitive feel for symmetry and maxima and seems the easiest to understand. The explanation is simple. On the graph of a parabola, each y-coordinate has two distinct x-coordinates except the y corresponding to the vertex. Since

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{6}$$

produces two values for x except when $b^2 - 4ac$ equals zero, this must be the vertex, i.e., the x-coordinate of the vertex can be found by simply erasing the radical in the quadratic formula. This approach usually produces smiles and retention. It nearly always answers the question of why $-b/2a$ without any need to refer to the derivative.

To illustrate more carefully why this works, subtract y from both sides of equation (3). Using the quadratic formula gives that

$$x = \frac{-b \pm \sqrt{b^2 - 4a(c - y)}}{2a}. \tag{7}$$

When equation (7) has only one solution, this corresponds to the vertex. This corresponds to the case

$$b^2 - 4a(c - y) = 0, \tag{8}$$

which gives

$$x = -\frac{b}{2a} \text{ and } y = -\frac{b^2 - 4ac}{4a}. \qquad (9)$$

Substituting the x-value from equation (9) into equation (3) gives the y-value in equation 9.

I have yet to go through the rigor of the previous paragraph in class. However, the inquisitive student may find this explanation enlightening if delivered personally.

A second way to justify the method is to complete the square for the general quadratic equation in equation (3), writing it in the form

$$y = a(x + \frac{b}{2a})^2 - \frac{b^2}{4a} + c. \qquad (10)$$

This procedure is very abstract and must be accompanied with a numeric example to have any chance of being meaningful. However, it does tie in well with the quadratic formula and leads to a nice proof. There are two main advantages for going through this process. The first reason is that it reinforces the idea that h and not -h is the x-coordinate of the vertex. For example, if $y = -2(x - 3)^2 + 4$, then "whatever makes the x-term equal 0" is the x-coordinate of the vertex, in this case x is 3. One can quickly follow up with an explanation of why y = 4 gives the y-coordinate. The second reason is that if horizontal and vertical translations have been studied, then this method reinforces these ideas and actually gives a "proof" of the formula for the vertex. If these concepts have not been taught, then this becomes a nice introduction.

Once the vertex is found, there is nothing hard at all about graphing a parabola. Quickly determine whether the parabola opens up or down, locate the intercepts (an maybe a few additional points to check), and voila - a perfect graph!

Completing the square is a tried and true method of graphing a parabola. It does have some advantages. However, for speed and efficiency, the magical formula - which is understood - cannot be beat. In fact, with a good explanation, there is nothing at all magical about it!

Mathematics For the 21st Century

Darla Lacy
Seagraves High School
Seagraves, Texas 79359

The school mathematics curriculum has experienced less change than most other disciplines. Even after the so-called "new-math revolution" in the 1950s and 1960s, the mathematics curriculum remains basically the same as before and is still taught in much the same way (Glatthorn, 1987).

This "new math" was deemed a failure and was followed by the "back to basics" movement of the 1970s which emphasized "skills" acquired by drill and practice (Adler, 1990). In the 1980s, the National Council of Teachers of Mathematics (NCTM) recommended significant changes in the teaching of mathematics. Some of their recommendations include the following:

- Expand the "basics" to include problem solving; apply mathematics in everyday situations; be alert to the reasonableness of results: estimate and approximate; use appropriate computational skills; use geometry, measurement, reading, interpreting, tables, charts, and graphs; and use mathematics to make predictions.
- Develop curriculum materials which integrate and require the use of calculators and computers in imaginative ways.
- Require computer literacy of all students.
- Include algebra in the program for all capable students but perhaps delaying it until grades 11 or 12 for some students.
- Reevaluate and possibly deemphasize the role of calculus in a differentiated mathematics curriculum (Glatthorn, 1987).

Since the late 1980s, much attention has been given to the need to reform the mathematics curriculum and the teaching of mathematics. A central feature of the proposed new curriculum is a devaluation of "skills" for their own sake and an emphasis on using mathematics. The premium is to be on thinking and communicating. The use of calculators, "hand-on" activities, estimation, and an emphasis on logic, problem-solving, and the practical side of mathematics are critical (Adler, 1990).

Mathematics is the key to opportunity for workers who must absorb new ideas, perceive patterns, and solve unconventional problems. These are the skills which will be needed in the 21st century. "To help today's students prepare for tomorrow's world, the goals of school mathematics must be appropriate for the demands of a global economy in an age of information " (Steen, 1989). "Mathematics must come to be seen as a helping discipline, not as a subject area that sorts and rejects students on their inabilities to perform ... Students must be involved in 'doing mathematics' through the use of manipulatives, discussion of results or investigation, and writing results of experiences" (Dossey, 1989).

Curriculum and Evaluation Standards

In 1989 NCTM responded to the need for changes in the mathematics curriculum by publishing the Curriculum and Evaluation Standards for School Mathematics. These Standards

- reflect the mathematical community's response to the call for reform in teaching and learning mathematics;
- reveal a consensus that all students need to learn more, and often different, mathematics; and
- reflect the belief that instruction in mathematics must be significantly revised (NCTM, 1989).

The NCTM Standards focus on two major goals. The first goal is to create a coherent vision of what it means to be mathematically literate in a world where calculators and computers perform mathematical computations and algorithms in many diverse fields. The second goal is to create a set of standards to guide the revision of the school mathematics curriculum and the revision of the evaluation of student understanding of this curriculum.

The "need" for new standards resulted from societal changes: the shift from an industrial to an information society, the need for mathematically literate workers, and the need for all to learn mathematics (NCTM, 1989).

The Standards identified five broad goals to meet student's mathematical needs for the 21st century. These goals are for students : to value mathematics, to reason mathematically, to communicate mathematics, to solve problems, and to develop mathematical confidence (Steen, 1989). Other recommendations by NCTM include additional teacher preparation, involving teachers as co-investigators in the learning and structuring of mathematical knowledge, spending more time on investigation and less on over-practicing computational procedures, smaller class loads, using "what-if" teaching methods, and using new methods of assessment (Dossey, 1989).

Technology has decreased the need for extensive skill in computation. Using computers and graphing calculators, mathematics can become exciting even to students who previously were unable to see any practical uses for mathematics (Dossey, 1989).

In the past, many lay-persons as well as teachers have resisted the use of technology, especially calculators, in teaching mathematics. Many people were concerned that basic skills would not be learned, that students would not learn to think, or that students would not learn to graph functions if graphing calculators were used. Dion (1990) argues that the successful use of calculators requires a higher level of understanding than required by rote computation. The need to interpret answers encourages critical thinking for all students.

Professional Teaching Standards

In 1991, NCTM published the Professional Standards for Teaching Mathematics. These Teaching Standards are based on two basic assumptions. The first assumption is that teachers are critically important in changing how mathematics is taught and learned. The second assumption is that any successful change requires that teachers have long-term support and adequate resources (NCTM, 1991).

The Teaching Standards recommend five major changes for the mathematics classroom. These changes are as follows:

	TOWARD	AWAY FROM
1.	classroom as mathematical communities	classrooms as a collection of individuals
2.	logic and mathematical evidence as verification	teacher as sole authority for correctness of responses
3.	mathematical reasoning	memorizing procedures
4.	conjecturing, inventing, and problem solving	emphasis on mechanistic answer-finding
5.	connecting mathematics, its ideas, and its applications	treating mathematics as a body of isolated concepts and procedures (NCTM, 1991).

The Teaching Standards provides guidance for developing professionalism in mathematics teaching. The major components focus on teaching mathematics across the grade levels by setting goals and selecting or creating mathematical tasks to help students achieve these goals; by stimulating and managing classroom discourse so both students and teacher are clear on what is being learned; by creating a supportive classroom environment for teaching and learning mathematics; and by analyzing student learning, mathematical tasks, and the environment to make instructional decisions.

A major evaluation theme is for the improvement of teaching to come from a reexamination of evaluation. Teacher evaluation should focus both on mathematical understanding and on classroom teaching.

Conclusion

As a mathematics teacher, one must realize that change is necessary. A changing society has changed the mathematical understanding which students and citizens need. While mathematical instruction has changed little over the years, change may be particularly difficult because we tend to teach as we were taught.

Students have also changed. Teachers continually compete for students' attention with "MTV," "Nintendo," and action-packed television shows. The challenge is to help students see that mathematics is relevant to their lives. Mathematics must become alive and not just a body of facts to memorize. This will require much thought, planning, and hard work on our part. The Standards - both curriculum and teaching - are a beginning. We must all try to prepare both ourselves and our students to achieve the vision in these Standards of the needs of the 21st century!

References

Adler, J. (1990). "Creating problems." Newsweek Special Issue, 16-22.

Glatthorn, A. A. (1987). Curriculum Leadership. Glenview, IL: Scott Foresman.

Dion, G. (1990). "The graphic calculator: A tool for critical thinking." Mathematics Teacher, 564-571.

Dossey, J. A. 1989). "Transforming mathematics education." Educational Leadership, 47(3), 22-28.

National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1991). Professional Standards for Teaching Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Steen, L. A. (1989). "Teaching mathematics for tomorrow's world." Educational Leadership, 47(1), 18-22.

Mathematics-Language Connection: Using Effective Communication in the Mathematics Classroom

Linda Eastwick Covington
Webb Elementary School
Olton, TX 76203

With the increased emphasis on problem-solving abilities in the mathematics curriculum today, effective communication has become imperative. The National Council of Teachers of Mathematics (NCTM) recommends in their recently issued Curriculum and Evaluation Standards for School Mathematics that "numerous opportunities for communication" be provided in mathematics (NCTM, 1989). In order to comply with these standards, the ways in which we have thought about teaching will need to be reassessed and revised.

It is not enough to know facts. What educators must ensure today is that their students can use facts to solve real-world problems. These problem-solving abilities require a multiple of processes to be taught as well as the traditional basics. Communication is encouraged in the math class so that strategies can be shared, thought processes can become clarified, and abstractions can be related in a language that has meaning.

Communication implies more than the traditional "sage on the stage" approach to teaching. Teachers must be ready to provide experiences that will help students bring meaning to the abstract words and symbols encountered in reading mathematics. They must be willing to step back and listen to their students discuss ideas and help guide them to develop sound approaches to problem-solving. In addition, they must allow students time to reflect upon what they have learned and provide opportunities for the sharing of such information by writing.

Language development takes place only when abstract symbols can be transferred to previously acquired concepts. Smith (1975) explains that children use language to make sense of their world. It is a process that becomes difficult only when they do not have sufficient prior knowledge to construct meaning from words. They must be able to relate words to an existing concept in their minds. Children with limited experiences will have more difficulty in this comprehension process. The math teacher can assist students in making these conceptual connections by providing positive "hands-on" experiences, practicing reading strategies, fostering dialogue, and encouraging the use of writing in the classroom. These types of communication are essential for the successful math class of the 1990s.

Reading

Studies on the relationship between reading ability and mathematical problem-solving ability have been conducted since the 1940s. Treacy (1944) found that vocabulary was a determining factor in ability to solve mathematical problems. Because of the abstract nature of mathematics, however, more than vocabulary needs to be addressed. Bye (1975) concluded that terms traditionally thought to be "easy" are quite abstract in nature and require cognitive processes that are not yet available to some students. For example, the term "all" can present problems to some children in the five to nine age bracket. When red and blue squares and circles were placed in front of children in this age group, and the question posed as to whether all the squares were red, Bye reports that a reply typical of a 5-8 year old may be, "No, because there are some red circles, too." Symbol representation, as well, can be difficult for some children, who will agree that 3+4=7, but not that 7=3+4.

To counter this problem, Bye suggests that teachers provide students with broader experiences, This conclusion is supported by the theories of Bruner (1960) and Wirtz (1976) who suggest that manipulation of real-world objects is necessary for concept development in mathematics. Thus, the problem which reads, "I have three apples and my mother buys four more at the grocery store. How many do I have in all?" could be solved with beans, chips, or real apples to develop the concepts of numeracy and addition.

Schell (1982) claims that mathematics is the most difficult subject to read and suggests that teachers provide opportunities for students to read problems in as many different forms as possible. This conclusion is supported by Dienes (1967) who maintains that a multiple-embodiment approach to mathematics is necessary for proper conceptual development. Thus, using the same example, the teacher could change the problem to, "I have seven apples. My mother bought four at the grocery store this morning. How many did I have before she went to the store?" Students could then manipulate their objects to determine the answer, and in turn, "discover" that 3+4=7 is the same as 7=3+4.

What these findings translate to the classroom teacher is the necessity for early and diverse use of manipulatives in the mathematics classroom. Manipulatives help students understand concepts before they deal with them symbolically. They provide students the opportunity to acquire meaning through common experiences and reduce the chance for misconstruction of meaning. As in any other area, if the student cannot transfer a symbol to an already existing concept in his/her learning, the actual amount of material that is learned is questionable.

Dialogue

Dialogue in the mathematics classroom should be encouraged because it helps students share strategies and clarify concepts; in addition, it provides an assessment tool for the teacher. Easley, Taylor, and Taylor (1990) assert that without "problem-solving activities and dialogue" to encourage the formulation of concepts, teachers stand a good chance of missing out on what students are actually thinking.

One way to foster this type of communication in a problem-solving setting is by cooperative grouping. Slavin (1983) reports many studies have shown that cooperative learning is effective in increasing student achievement. Social skills are increased, as well, since verbal communication and cooperation among group members become essential in order for the group to succeed.

Another way to encourage dialogue is by making problem-solving relevant to the students. Too often, students are bored by text book problems which have little to do with their daily lives. By presenting problems such as "Who is the best broad jumper in the fifth grade?" students have to become involved in the problem-solving process by determining the problem to be solved, selecting a strategy, determining appropriate measurements, and using a suitable method of representing the answer.

My fifth grade class had to decide if the word "who" meant one person, or more than one person (they decided it meant one person due to the singular verb, but the results of more than one person could be represented) where a suitable testing spot might be (outside on the grassy field, the track, or inside), how the jumps would be measured (yardstick, meter stick, tape measure, ruler) and what form of measurement would be used (inches, feet, Yards, meters). During the actual jumping, they decided to have more than one "heat." We could have averaged our three attempts, but they decided to record the best jump. They also predicted who would be the winner and why (longest legs, previous jumping experiences). They then compared their predictions with the results and wrote about the event and the conclusions they drew (the tallest students could jump the longest, some tall students who had no previous jumping experience could not jump far, some short students who were wiry and who had jumping experience could jump far). They also had to figure out how to present the results. After discussing different methods, they decided to report the results of the top six jumpers using a bar graph.

This activity allowed me to assess my students' ability to read and interpret problems, to use appropriate measuring devices, to relate their predictions to outcomes in a logical manner, and to choose suitable representations for their results. Any difficulties were corrected immediately, rather than waiting for tests to be corrected. In addition, the assessment was non-threatening. There were no red marks, no grades, only guided assistance provided by both peers and teacher. It allowed the students the chance to talk, listen, share strategies and ideas, measure, collect data, interpret data graphically, draw conclusions and write about them. All of this, as well, took place in a real-world setting, and with everyone actively involved within a cooperative environment.

Other problems which provide similar language and assessment opportunities include: What is your favorite time of day (connect with literature by collecting data and graphing results after reading Eric Carle's "The Grouch Ladybug"); What is the volume of trash collected in the cafeteria in one week (to be connected with an ecology unit); How much would it cost to feed a family of four for one week (to be connected with a nutrition unit); Would you rather be paid $1,000 per month, or earn 1¢ on the first day and double that amount each of the following days to the end of the month (to be connected with a social studies unit).

Encouraging each group member to share ideas on what the problem asks and to listen to other students fosters a positive learning climate by showing that every contribution is valued. In addition, a common mathematical language is developed while meanings are constructed and concepts clarified. Writing about results encourages reflective thought processes. With teachers encouraging these types of communication, students become masters of their own learning.

Writing

To further reinforce the math-language connection, students should be encouraged to write in math classes. Writing can take place in several areas. Students can be asked to explain a chosen strategy, they can be asked to compare the effectiveness of several strategies in solving a particular problem, they can be asked to record how they arrived at a solution to a particular problem, or they can be asked to write their own problems. The latter writing exercise is an excellent method of obtaining relevant problems for the class, as well as assessing the students' conceptual knowledge.

For instance, in reinforcing a division concept, the students could be required to write their own problems involving division as the operational strategy. By watching who needs peer assistance and who attends to tasks without assistance, the teacher can see immediately who might need reinforcement. Then by having students share their problems, teachers can easily assess who has mastered a concept and provide extra assistance where needed.

Ford (1990) suggests that writing does not just happen. It, too, is a process which must be taught in steps. The first is the "prewriting" stage where the idea is presented and some brainstorming takes place. In the "writing" stage, the problem is written without regard to syntax and grammar. Next is the "conference" stage in which the student receives feedback on the effectiveness of his writing. The student then moves to the "revision" stage where the problem is more clearly defined and errors are corrected. Last, the student "publishes" the problem. Publishing can be in the form of a file box of student-created problems to be located in a mathematics center, a book for the school library, or a challenge to another class in the building. Ford found that by using the writing process in problem solving, interest increases, attitudes improve and problem-solving becomes meaningful due to the ownership involved.

Conclusion

According to the NCTM Standards, language should be an integral part of mathematics. Wilde (1991) suggests that one of the best ways to encourage language growth is to have students keep a daily journal of their learning experiences. By reading what they have written, students provide their peers a summary of the lesson, as well as different ways of looking at a concept. She cites one first grader who had just learned about fractions by exploring some manipulatives: "Fractions are very important. (You) use them a lot. I learned about fractions and how they work. I know all about them. I know which is bigger and/or... smaller. One-half is bigger than one-fourth." Along with the text, the student drew pictures of fractions ranging from 1/1 to 1/10.

Wilde concludes that the writing experience for this student helped him reflect on his newly acquired knowledge, helped him develop divergent purposes for writing, and provided an assessment tool for the teacher. Students generally are not reluctant to talk about what they know; neither are they reluctant to write about what they know. By encouraging the use of language in the mathematics class, the teacher can assess whether students have mastered a concept, or merely mastered manipulating symbols. The emphasis is on the positive, that is, on what students know and not what they do not know.

Reading, dialogue and writing all make mathematics more meaningful for each child by helping create increased understanding of abstract ideas, fostering a positive learning climate, and creating an environment of discovery. By encouraging more use of language in the mathematics classroom, we provide our students with the skills to become thinkers and problem-solvers, rather than work-sheet processors.

References

Bruner, J. (1960). "On learning mathematics." Mathematics Teacher, 53, 610-619.

Bye, M. (1975). "Reading in mathematics and cognitive development." Educational Resources Information Center (ED 124 926).

Dienes, Z. (1967). Building Up Mathematics (3rd ed.). London: Hutchinson Publishing Group.

Easley, J., Taylor, H., & Taylor, J. (1990). "Dialogue and conceptual splatter in mathematics classes." Arithmetic Teacher, 37, 34-37.

Ford, M. (1990). "The writing process: A strategy for problem solvers." Arithmetic Teacher, 37, 18-22.

National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: NCTM.

Schell, V. (1982). "Learning partners: Reading and mathematics." Reading Teacher, 35, 544-548.

Slavin, R. (1990). "Research on cooperative learning: Consensus and controversy." Educational Leadership, 48, 52-54.

Smith, F. (1975). Comprehension and learning - A conceptual framework for teachers. New York: Owens.

Treacy, J. (1944). "The relationship of reading skills to the ability to solve arithmetic problems." Journal of Educational Research, 38, 86-95.

Wilde, S. (1991). "Learning to write about mathematics." Arithmetic Teacher, 38, 38-43.

Wirtz, T. (1976). Banking on problem solving. Carmel, CA: Curriculum Development Associates.

Place Value: A Problem in Abstraction

Patricia M. Lamphere
University of Montana
Missoula, Montana 59812

and

Charles E. Lamb
University of Texas at Austin
Austin, Texas 78712 - 1294

Much of the mathematics education literature supports the belief that concepts can be presented at three levels of abstraction. These three levels are the concrete, the pictorial, and the symbolic (Troutman and Lichtenburg, 1987). During the early years of school, the concept of place value should be presented using the concrete-pictorial-symbolic sequence. Such an approach provides children with opportunities to manipulate objects, to visualize pictorial images, and finally to deal with abstract mathematical symbolism.

As part of the instructional process, children should be guided through developmental stages of understanding by using several different concrete representations of the same idea (Post, 1988). This variety of experiences allows students to see relations in many different contexts and builds a broad, in-depth understanding of the concept of place value. The emphasis on multiple representations is exemplified by recent widespread calls for the use of manipulative materials in the teaching of mathematics. Students then make pictorial records of their investigations while using concrete materials as a beginning to forming a link between their understanding of the concrete and symbolic. Useful manipulatives for teaching place value include rods, bead frames, counting cubes, and multibase blocks. Experiences in using bundling sticks or unifix cubes help students make the transition to other consciously made proportional models. Non-proportional models such as bead frames and counting chips become meaningful when students begin to transfer their understanding of the place value system developed through use of more concrete proportional materials.

As an illustration, consider the following possible sequence for the teaching of place value ideas.

- Popsicle sticks or similar objects which can be easily grouped provide a useful physical representation of the number 13.

  |||||||||| ||| 1 ten and 3 ones

- Pictures or diagrams showing groupings of popsicle sticks, rocks, or other concrete objects can give a pictorial representation of the number 13.

|||||
||||| ||| 1 ten and 3 ones

- Mathematical symbols and words can give a symbolic representation of the number 13.

Thirteen equals one ten and three ones.
13 = 1 ten and 3 ones.
13 = 1 ten + 3 ones.

This sequence allows children to build upon their counting and basic grouping skills while following the suggested developmental order (concrete-pictorial-symbolic). Of course, the symbolic step is often felt to be the desired goal, but an understanding of all three representations and the relation between the three are necessary to understand and apply the mathematics learned.

As instruction progresses, children often acquire place value ideas following a pattern learned only by rote. For example, children may be able to use symbols to write the number 13 but cannot explain what the representation means. At a later stage, children may recognize incorrect responses, such as writing 31 for the number thirteen. Finally, children are able to explain the meaning of the symbols used to represent numbers and what the number means in relation to physical and pictorial representations (Ginsburg, 1977). During this progression of understanding, children are constructing the link between symbolic representation and the physical objects which the numbers may represent. An understanding of how the symbols relate to procedures and to representations is important for solving problems. Understanding is a complicated collection of intuitions and interrelated ideas about mathematics and the real world (Hiebert, 1984). Skemp (1971, 1979) also makes similar observations about understanding concepts.

The ideas presented in this article and the sample teaching sequence suggested provide a good learning environment for children to begin learning. However, the simple logical structure of the base ten positional number system belies the difficulties many children have understanding this structure and its representations (Wheeler, 1981). These difficulties are further amplified when students deal with computation (Ashlock, 1982). The failure of many children to understand computational procedures may be in part due to the absence of experience with manipulatives and to the sole use of abstract representations. A concrete-pictorial-symbolic sequence seems particularly important for such children.

A major point in instruction relates to emphasis. Often instruction focuses only on the "jumps" in levels of abstraction. A fruitful alternative may be to focus on one step in the process. Often, concrete materials are used to present place value. For example, sticks are used to show ones, tens, and hundreds using sticks of different sizes or different groupings. Even though this is a task focused at the concrete level, children must abstract to understand and use the numeration scheme. Similar abstractions are common when beads, rods, and chips are used. The child must make abstractions at each level and be able to relate the abstraction to the physical world in order to understand.

To aide the movement to abstraction and back to the physical world, cubes can be labeled ones, tens, and hundreds. Have children use their imagination to think of one hundred objects in the cubes labeled hundreds, ten objects in the cubes labeled tens, and only one object in the cubes labeled ones. Have the children describe different numbers and how to show that the mathematical representations using multiple die give the number represented by the number system. The authors have observed this procedure successfully used with first graders (Cline, 1987). The procedure, though simple, provides children with a useful mechanism for making the important transition from concrete reasoning to a higher level of abstraction.

Using this and similar physical representations early in a child's learning can provide important building blocks for the child's "jumps" to abstraction later in the curriculum.

References

Ashlock, T. B. (1982). Error patterns in computation: A semi-programmed approach (3rd ed.)., Columbus, OH: Merril.

Cline, A. (1987). Classroom observations by Charles E. Lamb. Austin, TX: Austin Independent School District.

Ginsburg, H. (1977). Children's arithmetic: How they learn it and how you teach it. Austin, TX: Pro-Ed.

Hiebert, J. (1984). "Children mathematics learning: The struggle to link form and understanding." The Elementary School Journal, 84(5), 497-513.

Post, T. R. (1988). Teaching mathematics in grades K-8: Research based methods. Boston, Allyn & Bacon.

Skemp, R. R. (1971). The psychology of learning mathematics. New York: Penguin.

Skemp, R. R. (1979). Intelligence, learning, and action. Chichester, England: John Wiley.

Troutman, A. P. & Lichtenburg, B. K. (1987). Mathematics - a good beginning: Strategies for teaching children (3rd ed.). Monterey, CA: Brooks/Cole.

Wheeler, R. E. (1981). Modern mathematics: An elementary approach (5th ed.). Monterey, CA: Books/Cole.

MEMBERSHIP

Texas Council of Teachers of Mathematics

Affiliated with the National Council of Teachers of Mathematics

Annual Membership Dues $8.00

USE THE FOLLOWING CARD FOR MEMBERSHIP.

Copy or cut on line.

Texas Council of Teachers of Mathematics

______	______	______	______
Last Name	First Name	School	(Leave Blank)
______	______	______	______
Street	Address	City	State Zip Code

To ensure continuous membership for the year, please print your name, zip code, and school. Enclose this card (or copy) with a check payable to TCTM for $8.00. Mail to:

Lynn Walcher, Treasurer
5711 Sugar Hill Apt. #21
Houston, Texas 77057

___ Renewal ___ New ___ Change of Address

Circle area(s) of interest: K-2 (STEAM) 3-5 (STEAM) 6-8 9-12 College

Texas Council of Teachers of Mathematics

Affiliated with the National Council
of Teachers of Mathematics
1991-92

Texas Mathematics Teacher
Joe Dan Austin, Editor
Rice University
P.O. Box 1862
Houston, Texas 77251-1862

NON-PROFIT ORGANIZATION
U.S. Postage
PAID
Houston, Texas
Permit #7549