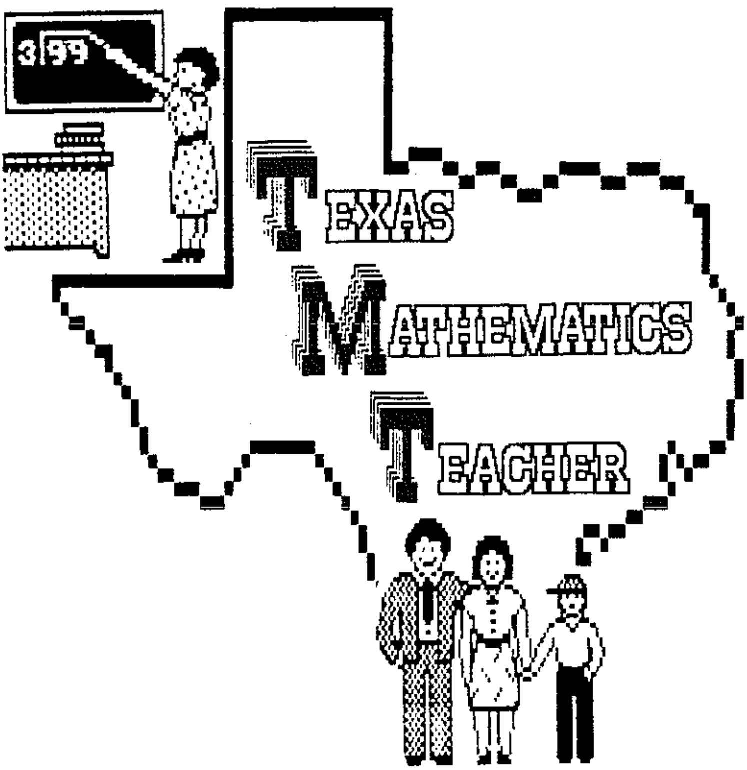


Teaching 'Critical Thinking'--Why Should We Do It?

Making the Counting Connection

The Randomness of Lotto Drawings: A Chi-Square Analysis

Statistics in the Secondary Schools
and the NCTM Standards

OCTOBER 1992

TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published four times each year, in October, January, March, and May. All manuscripts and editorial correspondence should be addressed to George H. Willson, listed above.

Manuscripts should be neatly typewritten and double spaced, with wide margins, on 8 1/2 " by 11" paper. Authors should submit the signed original and four copies. Illustrations should be carefully prepared in black ink on separate sheets, the original without lettering and one copy with lettering added. As soon as possible after refereeing, authors will be notified of a publication decision. Originals of articles not accepted for publication will be returned to the authors. Two copies of the issue in which an author's manuscript appears will automatically be sent to the author.

SUBSCRIPTION and MEMBERSHIP information will be found on the back cover.

TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council
of Teachers of Mathematics

October 1992

TABLE OF CONTENTS

ARTICLES

ANNOUNCEMENTS

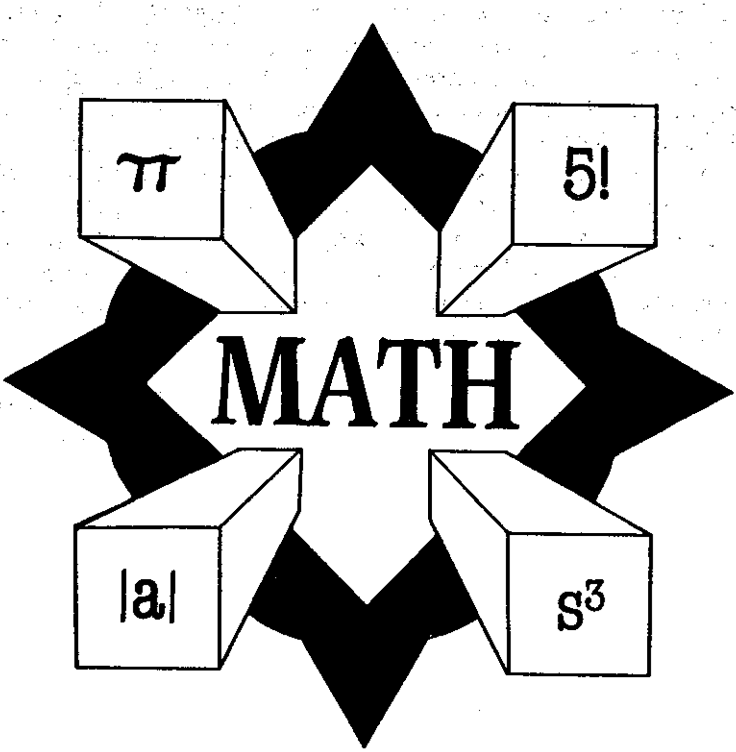
President's Message

A few weeks ago I was enjoying CAMT in San Antonio and thinking about my professional goals for the 1992-93 school year. Dr. Cathy Seeley was speaking about the many roles of a teacher. As I listened to her speak about accepting responsibility for educating all students, being willing to take risks, and aggressively pursuing better ways to teach, I could feel my enthusiasm for returning to the classroom increasing. She spoke of creating ways to dialogue with peers and extending your leadership in mathematics education. That evening I received a phone call from a colleague asking if I would consider running for President of TCTM, since the president elect was not going to be able to take on the president's position. So, Cathy, I said I would, and here I am!

Returning to my thoughts for the new school year, I realize what a wonderful opportunity that we have as classroom teachers to bring closure to one school year and have the time to reflect on it before beginning a new year. I am especially appreciative of that after a family vacation this summer. I watched my husband return to an office with a full in basket, numerous phone messages, and the instant feeling of being two weeks behind where he had been when he left for vacation. Reflecting on last year and dreaming about his new one, I think about the many challenges that face us as mathematics teachers. Advances in technology provide us with newer and better calculators and computer hardware and software for use in our classrooms - yet we deal with funding difficulties and face standardized tests that do not allow calculator use. We strive to provide rich applications and mathematical connections for our students while some parents wonder why students aren't bringing home "real" math homework of drill pages. High school teachers prepare lessons to implement new Pre Algebra textbooks, while learning that Pre Algebra will soon be eliminated at the high school level. We work hard to develop alternative means of assessment while, at the same time, working to prepare students for TAAS and NAPT. We attempt to design new support structures for students who come to us without prerequisites so that ALL high school students will take Algebra I, yet we strive not to increase failures or water down curriculum. Yes, it is a challenging time, but also, an exciting one, as we realize how quickly mathematics education in Texas is changing and how important it is for each of us to be a part of that change.

As I accept the responsibilities that go with being TCTM president, one of my goals for this year is to continue the excellent work that has been done by past president Karen Hall. TCTM is an affiliate of NCTM and serves a crucial role for the classroom teacher of mathematics in the state of Texas. As your voice on the CAMT steering committee, we can bring your ideas and suggestions for CAMT to that committee. This journal and the STEAM newsletter are our ways of communicating with you. We are always looking for manuscripts, articles, or comments. Let us hear from you!! I challenge you to become involved in your professional organizations and be a positive voice for mathematics education in your school and your community.

Susan Thomas
TCTM President



IS FUNCTIONAL

TEACHING 'CRITICAL THINKING'-- WHY SHOULD WE DO IT?

Dale R. Bedgood

Department of Mathematics
East Texas State University

For several decades educators and mathematicians have expressed the need to teach students "how" to think. Many students simply memorize a procedure and manage quite well in the early grades, but they have great difficulty in higher levels of math in high school and college.

Teaching mathematics is a complex process that is further hindered by the many students who have a definite aversion to the subject. That these students see mathematics as a pile of useless facts to be memorized is the fault of today's math programs. The "back to basics" movement focuses on fundamental skills and facts at the expense of true thinking just so students will excel on standardized tests.[7] This emphasis on mere computation has produced a generation of students who rely on being told exactly how to do something, with prompts; students think that doing the problem is sufficient for learning how to solve problems.[2] Unfortunately, today's system values the answers at the expense of the processes used to obtain them.[7] The problem with this approach is that today's technology has progressed to the point where machines can do computation; humans are still needed, however, to do the actual problem solving.[9] Thus, problem solving is thinking about what to do, not simply doing it; the problem must be solved before any computation can be completed.[6] Assuming that teachers themselves are competent, their task is to create efficient problem-solvers out of their math students. "Elementary grade children spend an estimated 90 percent of their school mathematics time on paper-and-pencil computation practice, most often learning computation skills by rote. Many students learn the rules and are able to do the computations, but computation success often masks lack of understanding and reasoning skill."[3] Teaching mathematics is not just teaching rules and expecting completed computations as proof of mastery. Although many students follow the rules and do computations, they fail miserably when asked to reason.[3] To survive in mathematics courses, many students attempt to compensate for their lack of understanding by memorizing procedures and formulas. Analytical reasoning, however, is not learned through memorization of specific facts, but is gained by immersing oneself in the problem-solving process and learning to apply past experience and knowledge to the problem at hand.[4] Despite the problems of just memorizing mathematical facts or using rote learning to make it through the math courses, teachers can develop students' reasoning, critical-thinking, and problem-solving skills. The main deterrent may be the system within which they are forced to conform. Too often students are told to "copy and complete." Teachers tell students exactly what to do instead of directing students to discover for themselves. Teachers often feel too rushed to allow students sufficient time for discovery.

During the 1970's there occurred a trend referred to as a "back-to-basics movement." This affected math instruction since the public demanded that emphasis should be on teaching only the basic computational skills. Therefore, serious deficiencies have arisen concerning problem-solving skills and transfer of learning.[7]

Many students see math as simply memorizing rules and strategies that will enable them to answer assigned problems. Even though students memorize rules, it does not mean they understand them. A memorizing strategy is inadequate because most children are unable to memorize <u>all</u> the different steps needed to solve problems. Students may remember parts of rules and try to use these in situations which results in incorrect answers. They tend to attack math problems mechanically. Critically thinking about the problem and understanding the rules is not often considered to be necessary. Children need to be taught that meaning and understanding are important in math as well as every other course they take. Research shows that rules which are not understood are easily forgotten.

Ever since the turn of the century, American schools have regarded the mastery of thinking skills as a major goal of instruction in most subject areas.[1] However, a survey conducted by the National Assessment of Educational Progress revealed that only a small percentage of students could draw correct inferences from a set of facts, and only one out of seven students could write a persuasive essay.[7] Educators across the curriculum are experiencing growing dissatisfaction due to the students' apparent ability to reason.[4] Our schools are producing "lesson-learners," that is, children who learn lessons factually, but are unable to process and apply facts intelligently.[10] If indeed the mastery of thinking skills is a major goal of instruction, why then is there such a discrepancy between the objective and the product?

What the teacher uses as instructional materials and the very ways those materials are used are key variables in teaching for thinking. If students are repeatedly rewarded for performing tasks in which there is one right answer or a single method of completion, then cognitive flexibility functioning will be extinguished over a period of time. There is little argument among parents, educators, media, and the public at large that children are graduating without minimal competency skills in mathematics. More argument lies in who is responsible. Is it administrators, school boards, inappropriate student-teacher ratios, student anxiety, parents, teachers, and/or textbooks? Sadly, many students enter college without having ever solved a problem; they simply do rote computation that leads to an answer.

Textbooks are under constant debate since the textbooks are the source of most of the systematic information children receive. According to Paul Mussen, a professor of psychology at the University of California, Berkeley, "There has been a real 'dumbing down' of the texts." William Trombley of the Los Angeles Times, wrote "in chilling detail" the "dumbing" of American textbooks." The over-simplified textbooks have stopped challenging the student and turned them soft intellectually," stated Trombley.

Probably one of the biggest reasons for the lack of critical thinking in math involves the students' attitude and anxiety towards the whole science of mathematics. Math is a cumulative science; one has to build one idea or concept on top of another. When a gap occurs the student often tends to avoid math whenever possible. Often these students feel helplessly dumb, and inadequate. This feeling of helplessness compounds anxiety, causing a vicious circle which leaves math as a "critical filter," stopping people from succeeding in math-related fields, even when there is no question of their intellectual ability.

Ability to solve problems is a process by which one used previously acquired knowledge, skills, and understanding applied to different situations that occur in various classes or to problems in everyday life. A person is a good problem solver when able to combine what has been learned and apply it in different situations. By contrast, the memorizer prepares for a test, and very shortly thereafter forgets what was "learned" for that test. Minimal growth takes place.

Researchers find it hard to explain what it is that makes some people good problem-solvers while other equally intelligent persons are not. Curtis Miles, author of the article "Good Versus Poor Reasoners," believes that a good problem solver uses diverse thinking strategies.[8] He feels that these people are able to sort through their repertoire of skills and select the ones most likely to work.

The failure of students to learn "thinking skills" has been cited as the basic explanation for the poor achievement of students in cognitive task.

Many teachers may fail to teach thinking skills because they do not like mathematics themselves. They are uninspired by their own mathematics education and studied only what was minimally required. Thus, they rely primarily on the textbook for their teaching, i.e., the textbook actually dictates what happens in the classroom.[3]

By promoting thinking and reasoning, one increases independent pupil functioning. This results in students asking more questions, accepting fewer answers at face value, becoming more critical in their appraisals, and questioning assumptions. In the classroom those students who sit quietly, who do not ask too many questions, who give the pat answers, who never question the teachers, and who do as they are instructed, are generally regarded more favorably by the teacher.[10]

Many students are able to work within the system and achieve a distinguished record. Indeed, some may even be "honor" students, yet must begin in a program of remediation upon entering college.

It is interesting to note that just in recent years when college freshmen are questioned concerning apparent lack of effort in solving "word problems" a glib, flippant response is "Oh, I don't do word problems." This is very much like the domestic worker who "doesn't do windows."

References

1. Beyer, Gary. "Improving Thinking Skills - Defining the Problem." Phi Delta Kappa 65 (March, 1984): 486.

2. Birken, Marcia. "Teaching Students How to Study Math: A Classroom Approach." Mathematics Teacher 81.3 (March, 1988):410-413.

3. Burns, Marilyn. "Helping Your Students Make Sense Out of Math." Learning 88 (1988):31-36.

4. Chisko, Ann, and Lynn Davis. "The Analytical Connection: Problem-Solving Across the Curriculum." Mathematics Teacher 79.8 (November, 1986):592-6.

5. Holmes, Emma E. Children Learning Mathematics: A Cognitive Approach. New Jersey: Prentice-Hall, Inc., 1985:2-36.

6. Lichtenberg, Donovan. "The Difference Between Problems and Answers." Arithmetic Teacher 31.7 (March 1987):44-45.

7. Marback, William D. and Connie Leslie. "Why Johnny Can't Reason." Newsweek 88 (January 1988):59.

8. Miles, Curtis. "The Fourth R: Good vs Poor Reasoners." J of Educational Development 8.2:24.

9. Shesnick, Twila. "Problem Solving: Some Thoughts and Activities." Arithmetic Teacher 31.7 (March 1984):41-3.

10. Wasserman, Selma. "Promoting Thinking in the Classroom." The Education Digest (December 1984):49.

MAKING THE COUNTING CONNECTION

Frances M. Thompson

Texas Woman's University
Mathematics and Computer Science Department
Denton, Texas

1, 2, 3, 4, 5, . . . Hearing a child practicing his or her counting is a familiar and delightful sound to most parents and grandparents. Brothers and sisters encourage younger siblings to count any and everything they see. To count all the way to 100 is a special thrill and something worthy of boasting. So once a child learns to count, what comes next? Do we teachers move on to other seemingly unrelated ideas? Often we do.

The natural way to learn is to explore and extend from those things we already know. Our existing knowledge structures act as mental tools for acquiring new knowledge (Skemp 1985, 449). The Professional Standards for Teaching (NCTM 1991, 25) under Standard 1 states that as teachers, we should present tasks to children that will cause them to "make connections and develop a coherent framework for mathematical ideas." We should encourage them to talk about the relationships they see between old and new ideas.

One way we can promote such connecting of ideas is to look within our mathematics curriculum for "interlocking" or "global" concepts or skills. Such ideas naturally connect various mathematical strands together that otherwise might seem unrelated. They help students see mathematics as an "integrated whole" rather than as a collection of disjointed concepts and skills. A good example of this "global" approach is the role counting plays in mathematics.

Counting interlocks a variety of concepts found in the elementary and secondary curriculums. This skill must be involved any time quantities are combined (addition) and a simple number name sought for the total. It is applied to units considered alike in some way. This use occurs in many different settings. When primary students are first asked to find the total blocks after 5 blocks and 8 blocks are joined together, most will "recount" all the blocks to find the single number name 13. (Later other counting strategies will be presented.) This counting process continues through the upper grades, but is seldom emphasized as a connection to past skills. Several mathematical examples of counting have been outlined in the remainder of this article for the reader to consider. Some concepts are simple and require only a brief discussion, while others are more complex and less familiar, thereby requiring a more lengthy description of the counting action involved.

1. Multiplication of whole numbers. Students count the number of equal rows of chips that they have placed on their desktops and record their construction with specific symbols, e.g., 3 rows of 4 chips is written as 3 X (4 chips). They then recount all the chips to get 12 chips total. The final sentence is 3 X (4 chips) = 12 chips.

2. Perimeter. As readiness for a general perimeter-sides relationship, students count a given unit, e.g., a paperclip, to find how many paperclips are needed to fit along each side of a polygonal cardboard cut-out. All the paperclips are then recounted together to find the total used. This action is then recorded with a number sentence (e.g. 3 clips + 2 clips + 3 clips + 2 clips = 10 clips). With certain standard shapes, other counting strategies, like combining two adjacent side lengths of a rectangle and doubling, are eventually applied and lead to a final formula for perimeter.

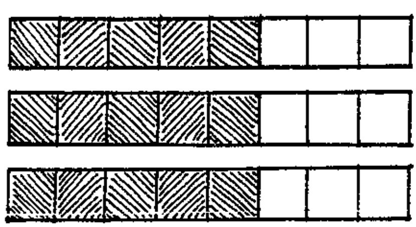
3. Area. Students build a rectangular region with paper squares as units, then count the squares in each row and the number of rows. This is recorded as, e.g., 4 X (5 squares) = 20 squares. ("Square unit" might be used here instead of "square".) Mira (1970, 25-26) observes that the multiplier or "counter of rows or sets" is always an abstract number; that is, it does not have a label of measurement. To teach initially that area is, e.g., 4 inches X 5 inches instead of 4 X (5 sq.in.) is misleading. Using linear measures in place of the area factors should only come after students discover that the numbers for the amount of rows and the row size match the numbers that tell the

side lengths. The numbers may look the same, but their roles are different.

4. Addition of Fractions. When fraction bars are used, students can show 2 sixths with 3 sixths. To find the sum, they simply recount the total sixths. They then write 2 sixths + 3 sixths = 5 sixths. Later they write $\frac{2}{6} + \frac{3}{6} = \frac{5}{6}$ to show their results. Unlike part sizes (different denominators) must be traded to the same size before they can be recounted for a total, then named.

5. Multiplication of Fractions. When 3 sets of 5 eighths are needed, students can show 5 eighths with fraction bars, followed by two more such sets (Ott 1990). They count all the eighth bars they see to find a total of 15 eighths. (See figure 1.) The product can be recorded as 3 X (5 eighths) = 15 eighths at first and later as $3 \times \frac{5}{8} = \frac{15}{8}$. The answer should not be rewritten as a mixed number in simplest form until the pattern for multiplication of a whole number by a fraction has been discovered by the students. Reduction destroys the number patterns that are needed to generate the algorithm. Also, the recording of the number sentence initially with words rather than fraction symbols seems to help students better understand the process involved.

Figure 1: Fraction Multiplication



3 sets of 5 eighths

6. Simple Probability. To find the probability for randomly drawing a green tile out of a box of plastic square tiles, students show all the possible outcomes by pouring all the tiles out on a desk. They then count to find the total number of tiles and the number that are green. The following is written on paper: Probability of drawing a green tile = 5 ways out of 9 total ways to draw a tile. More abbreviated recording forms are developed later.

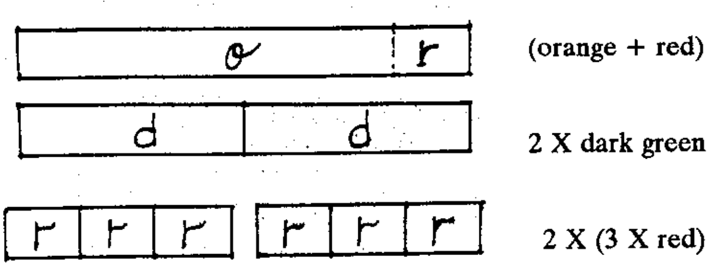
7. Integer Addition. If three yellow tiles (+3) and seven black tiles (-7) are joined together, each black-and-yellow pair of tiles formed will represent a zero-pair that has neutralized. If any tiles are left after the zero-pairs are made, they will be the same color, so can be counted and labeled. In the example, four black tiles will remain unpaired, so the students will record the number sentence:

$$(+3) + (-7) = -4,$$

showing a negative for their answer because the left-over tiles they counted were black or negative tiles.

8. Prime Factorization. Cuisenaire rods can be used to develop prime factoring. Students might begin with an "(orange + red) train," which represents the value 12. (See figure 2.) They must then try to build

Figure 2: A Prime Factorization for 12
with Colored Rods



In Symbols: 12 = 2 X (3 X 2)

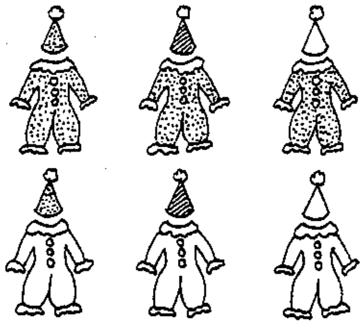
another train of the same length that uses a prime amount of one rod color (excluding the white or unit rod). Two dark green rods are selected; this is recorded as (orange + red) = 2 X dark green, where 2 is the "counter" for the dark green rods used. A new train is now built by replacing each dark green rod with a prime amount of a second color, in this case, with three red rods. The number sentence is extended: (orange + red) = 2 X dark green = 2 X (3 X red), where 3 is the counter for the red rods in each group. Finally, since the rod being used to make the train (here, the red rod) can only be replaced with white rods, the building or factoring process stops. So in this example, using prime groupings and 2 as the value of the last (red) rod, the final factored form for 12 will be 2 X (3 X 2). Each prime factor then represents a counting of specific rod or group sizes that occurred at different stages of the construction activity or the size of the final rod used.

Students should continue to factor the (orange + red) train in other ways, always trading each previous rod for prime amounts of the new rod. For example, if three purple rods are used instead of two dark green rods at the beginning, the final factored form for 12 will be 3 X (2 X 2). Students will soon discover that, although the factorings were done in different orders, the prime factors that result are the same. Therefore, the prime factorization for 12 is not unique in the order of its factors, but the prime factors will always be 3, 2, and 2.

9. Combinations. Paper cut-outs of different clown suits and caps can be used to count all the possible ways a suit can be matched with a cap. For example, if there is a dotted suit and a solid suit with a dotted, a striped, and a solid cap (students will have several cut-outs of each), different outfits can easily be made by taping each cap and suit together. The outfits are placed on the desktop and arranged so that the dotted suits are side by side and the solid suits are side by side. (See figure 3.) Students will count and find that there are three different dotted outfits because they used three different caps with the dotted suit. Similarly, they will count and find that three different solid outfits were made. But there are now two sets of three outfits each, so there are six outfits in all. This result can be recorded as 2

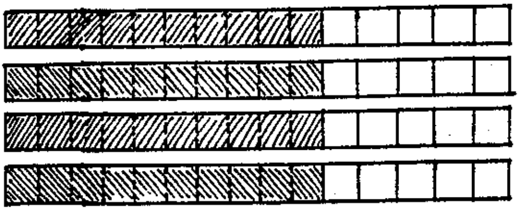
suit designs X (3 cap types for each suit design) = 6 suit-cap outfits. It helps students to use words within number sentences.

Figure 3: Clown suits and Caps: Combinations



10. Distributive Property. The Dienes or base 10 blocks can be used effectively to show the distributive property for whole numbers, particularly its role in mental arithmetic. For example, students might build four sets of 15 by making four rows of blocks with one ten-stick and five ones joined together in each row. (See Figure 4a.)

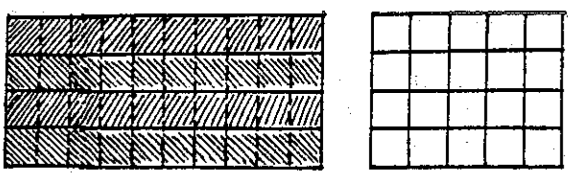
Figure 4a: Reorganizing with the Distributive Property



4 X (10 + 5)

This arrangement is recorded as 4X (10 + 5). The multiplier 4 counts the number of sets made. The students then vertically separate the tens blocks from the ones blocks. The left group can be recounted to find 4 rows of 1 ten each; the right group recounted has 4 rows of 5 ones. (See Figure 4b.) This new arrangement is recorded as (4X 10) + (4X 5). Since the total amount of blocks has not changed, but has only been rearranged, students now equate the two expressions: 4X (10 + 5) = (4X 10) + (4X 5). So four sets of 15 can be mentally computed, using 40 + 20. The flexibility of thinking that such a distribution allows is an indicator of "intelligent thinking" rather than "habit thinking," the latter of which comes from rote memorization (Skemp 1985, 449).

Figure 4b: The Vertical Separation

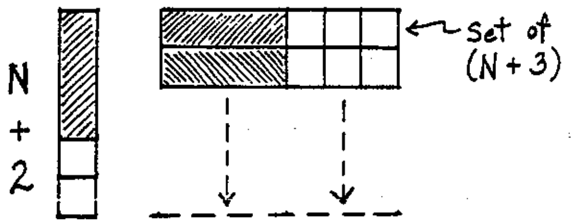


(4 X 10) + (4 X 5)

11. Binomial Multiplication. The above construction method for the distributive property can be extended to variables in algebra. Manipulatives like the commercially-available Algebra Tiles can be used to show the meaning of an expression like (N + 2)(N + 3). The multiplier (N + 2) again tells how many sets of (N + 3) to build. Students can lay out the variable tile with two +1 tiles as a measuring rod for "counting out" the sets needed. Since N is really unknown, the emphasis is not on an exact quantity of sets but rather on enough sets to match the physically observed length of the "counter" tiles. A "train" formed with a variable tile and three +1 tiles is laid down.

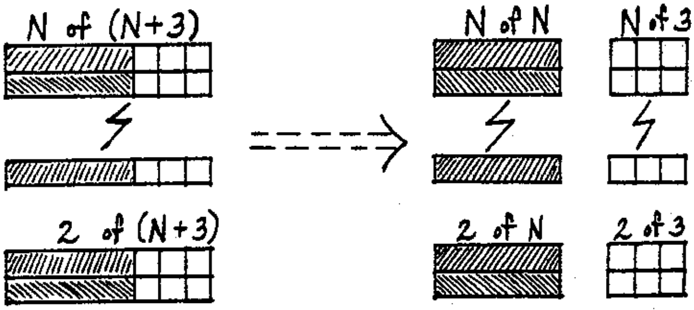
(See figure 5a.) Once the "counting" is completed, the "counter" tiles can be removed.

Figure 5a: Binomial Multiplication with Tiles



The product of tiles can be separated horizontally according to the multiplier's variable tile and its +1 tiles. Students can now "recount" the upper group to find an N amount of (N +3) and the lower group to find an N amount of (N + 3) and the lower group to find 2 rows of (N + 3). This is the first distribution. Continuing with the upper group, an N amount of the variable tile can be vertically separated from an N amount of +3. Likewise, the lower group can be separated to show 2 rows of the variable tile and 2 rows of +3. (See figure 5b.) This is the second distribution. The original product and

Figure 5b: Forming Four Regions



its different partial products can now be recorded as follows:

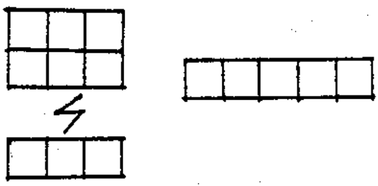
$$(N + 2)(N + 3) = N(N) + N(3) + 2(N) + 2(3).$$

Since an N amount of the set N can also be named N^2, the large square Algebra Tile can replace the N rows of N. This application of equivalent tile areas is based on the definition of area as a countable amount given earlier in this article. Similarly, N rows of +3 have the same area as 3 rows of the N tile, so students can recount the N tiles in the partial products to get 5 N tiles total. Two rows of +3 make +6, and the original product through distribution is finally renamed as N2 + 5N + 6. Binomial multiplication is now complete without the premature use of memory devices such as the "foil" method. This method can easily be extended to trinomials if two different variable tiles are available for students to use.

12. Word Problems. Students do not seem to have much trouble with word problems in which the set size is unknown, but the number of sets is known. An example might be as follows: "Henri bought 5 pairs of blue socks and 3 pairs of red socks. A red pair cost the same as a blue pair. If Henri paid $7.84 in all for the socks, how much did each pair cost?" Students can easily model the problem by using a block to represent each pair of socks, whether red or blue, and counting out blocks for the "pairs" needed. Then 5 blocks combined with 3 blocks can be recounted to find a total of 8 blocks or 8 pairs of socks purchased. The number sentence for this is 5 pairs + 3 pairs = 8 pairs. Since the same cost can be associated with each pair of socks, the second number sentence for this situation might be 8c = $7.84, from which c, the cost per pair, is found by division.

When students encounter word problems that involve a variable as the "counter" of known set sizes and attempt to solve them only by means of abstract notation, it is difficult for the students to write equations for the problems (Küchemann 1981). An example of such a word problem might be as follows: "Lupe bought several cassette tapes on sale at $3 each. She also bought a new tape holder for $5. the total cost was $32. How many tapes did Lupe buy?" In this example, the "counter" of the tapes is the unknown, so the total cost of the tapes should be written as N($3), not $3(N), with the final number sentence being N($3) + $5 = $32. The manipulative model would use an N amount of +3, not 3 rows of the N tile. (See figure 6.) This distinction may seem trivial and totally unnecessary to successful mathematicians, but to untrained students, making the correct connection to the "counter" as a variable and using a consistent format to record the product symbolically is very important.

Figure 6: Modeling a Word Problem



(N of $3) + $5

Conclusion

Many other concepts besides the twelve discussed here depend upon counting for their development. For example, the absolute value of an integer can be considered the countable amount of units (not their positive or negative value) that must be joined to the integer to change its value to zero. Teachers need to become more aware of the role of counting in mathematics and stress its importance to their students. The significance of counting becomes even more obvious when manipulatives are used in instruction. Building new skills or concepts upon a counting action, when appropriate, can provide security for the learner through frequent connections with more familiar or more meaningful knowledge gained in earlier school experiences. The process of counting provides a natural background against which to organize new ideas (NCTM 1989). Through continual application of such a "global" idea, students may begin to view mathematics as a unified whole rather than a collection of unrelated segments of ideas.

References

Küchemann, Dietmar. "Algebra." In Children's Understanding of Mathematics: 11-16, edited by K.M. Hart, 102-119. London: John Murray, 1981.

Mira, Julio A. Mathematical Teasers. New York: Harper & Row, Publishers, 1970.

National Council of Teachers of Mathematics, Commission on Standards for School Mathematics. Curriculum and Evaluation Standards for School Mathematics. Reston, VA: The Council, 1989.

National Council of Teachers of Mathematics, Commission on Teaching Standards for School Mathematics. Professional Standards for Teaching Mathematics. Reston, VA: The Council, 1991.

Ott, Jack M. "A Unified Approach to Multiplying Fractions." Arithmetic Teacher 37 (March 1990): 47-49.

Skemp, Richard R. "A Progress Report." In Proceedings of the Ninth International Conference for the Psychology of Mathematics Education, Vol. 1, Individual Contributions, edited by Leen Streefland, 447-452. The Netherlands: State University of Utrecht, 1985.

THE RANDOMNESS OF LOTTO DRAWINGS: A CHI-SQUARE ANALYSIS

Bonnie H. Litwiller
David R. Duncan

Department of Mathematics and Computer Science
The University of Northern Iowa
Cedar Falls, Iowa

Many states have instituted lotteries in an attempt to raise revenue. Our state (Iowa) recently instituted a Lotto game. To play this game, a person selects six numbers from the set 1, 2, 3,..., 30. The player's choices are recorded electronically on the Lotto computer. At the end of a game period (originally one week but recently twice a week), six winning numbers are randomly selected by the Lotto office and are announced on television. If a player matches all six numbers, he/she wins the jackpot. Matching three, four, or five numbers yields lesser prizes.

There is a great deal of interest on the part of some Lotto players in determining a "winning strategy" for selecting the six numbers. These players keep careful records of the number of times that each of the numbers from 1, 2, 3,..., 30 has been chosen by the Lotto office. Some of these players feel that the numbers which have appeared the most often will continue to do so and should, therefore, be selected. Other players feel that the numbers which have been selected the least often should be played since, by the "law of averages," their turn will come next.

Table I reports the number of times each of the numbers from 1 through 30 has been selected by the Lotto office.

A casual observation of Table I would seem to suggest a disparity among the frequencies with which these were selected. For instance, the number 3 was selected 49 times while the numbers 19 and 26 were selected

Table I

1	36
2	36
3	49
4	26
5	36
6	31
7	35
8	39
9	31
10	30
11	26
12	36
13	37
14	37
15	38
16	31
17	43
18	36
19	25
20	35
21	32
22	36
23	28
24	33
25	46
26	25
27	39
28	37
29	33
30	36

only 25 times each. Do these and other disparities in Table I cast significant doubt on the assumption of randomness in the selection process?

Table I represents 173 selections of six numbers each; thus, 1038 separate numbers were chosen. The assumption of perfect randomness would suggest that each number would be selected $\frac{1038}{30}$ or 34.6 times.

Do the actual frequencies of Table I differ enough from the ideal of 34.6 to cast significant doubt on the assumption of randomness?

The Chi-Square method[1] is used to test this type of "goodness of fit." First the differences between the observed frequencies (o) and the expected frequencies (e) are calculated and then squared. Each of those squares is then divided by the expected frequency and these quotients are summed. The resulting sum is then compared with the number listed in a Chi-Square table for 29 degrees of freedom. The 29 degrees of freedom result from the fact that if 29 frequencies are independently chosen, the 30th is known because the sum of the frequencies must be 1038. Table II reports the results of the Chi-Square computations.

Informally, we note that if the observed frequencies matched the expected frequencies very closely, the differences between the two would be very small and the sum of the entries of the last column would also be small. If, on the other hand, the expected and observed frequencies were frequently widely different, the sum of the entries of the last column would be very large. We must judge whether the sum of the entries of the last column is large enough to cast doubt on the assumption of randomness.

The Chi-Square table entry for the 10% significance level is 39.09. That is, <u>if</u> the numbers were picked at random, the probability that the computed Chi-Square statistic would exceed 39.09 is 10%, a rare event. Such a computed Chi-Square statistic of 39.09 or larger would be evidence that the process is <u>not</u> random. Until this threshold level is reached, the assumption of randomness is reasonable and should not be rejected.

Iowa Lotto players who are trying to beat the system may still have fun composing strategies based on the results of Table I; however, the results of Table II indicate that these strategies will not be effective.

Table II

Number	o	e	o-e	$(o-e)^2$	$\frac{(o-e)^2}{e}$
1	36	34.6	1.4	1.96	0.06
2	36	34.6	1.4	1.96	0.06
3	49	34.6	14.4	207.36	5.99
4	26	34.6	-8.6	73.96	2.14
5	36	34.6	1.4	1.96	0.06
6	31	34.6	-3.6	12.96	0.37
7	35	34.6	0.4	0.16	0.00
8	39	34.6	4.4	19.36	0.56
9	31	34.6	-3.6	12.96	0.37
10	30	34.6	-4.6	21.16	0.61
11	26	34.6	-8.6	73.96	2.14
12	36	34.6	1.4	1.96	0.06
13	37	34.6	2.4	5.76	0.17
14	37	34.6	2.4	5.76	0.17
15	38	34.6	3.4	11.56	0.33
16	31	34.6	-3.6	12.96	0.37
17	43	34.6	8.4	70.56	2.04
18	36	34.6	1.4	1.96	0.06
19	25	34.6	-9.6	92.16	2.66
20	35	34.6	0.4	0.16	0.00
21	32	34.6	-2.6	6.76	0.20
22	36	34.6	1.4	1.96	0.06
23	28	34.6	-6.6	43.56	1.26
24	33	34.6	-1.6	2.56	0.07
25	46	34.6	11.4	129.96	3.76
26	25	34.6	-9.6	92.16	2.66
27	39	34.6	4.4	19.36	0.56
28	37	34.6	2.4	5.76	0.17
29	33	34.6	-1.6	2.56	0.07
30	36	34.6	1.4	1.96	0.06
				TOTAL:	27.09

Challenges:

1) If your state or neighboring state has a Lotto game, compose a frequency distribution like that of Table I. Investigate the assumption of randomness.

2) Use the Chi-Square method to investigate "goodness of fit" for other situations, such as the distribution of births during months of the year.

* * *

<u>References</u>

[1]Freund, John E. <u>Modern Elementary Statistics</u> (7th edition), Prentice Hall, 1988. pp. 367-371.

[2]__________, <u>Modern Elementary Statistics</u> (7th edition), Prentice Hall, 1988, p. 513.

STATISTICS IN TEXAS SECONDARY SCHOOLS AND THE NCTM STANDARDS

Ralph W. Cain

Department of Mathematics
The University of Texas
Austin, Texas

The purpose of this paper is to raise some questions and discuss some issues related to the role of statistics in the secondary school curriculum in Texas in light of positions that are stated or implied in the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989), hereinafter referred to simply as the Standards. Many of the questions and issues raised have been generated from the results of a study by Vaquiax Alvarado (1991). The study was concerned with the perceptions of Texas secondary mathematics teachers with regard to the status of statistics in secondary school curricula, the ability of schools to offer courses or units in statistics, the feelings of teachers toward statistics and their ability to teach the content, and what the Standards have to say about statistics in the curriculum. Based upon that study and discussions with teachers, graduate students, persons who were directly involved in the development of the Standards, and others, many questions about the feasibility of some of the suggestions presented in the Standards, strategies for their implementation, scheduling of such implementation, the economic implications for possible implementation (costs of technological equipment, retraining teachers, etc.) in a time of economic recession, and even space utilization have been raised. Some of those questions and the issues they embody are the focus of this paper.

Among the more important of the questions raised are those concerning the preparation of secondary teachers in the mathematical content of statistics. Certification programs have generally not required courses in this area, which would lead one to believe that there are many otherwise competent teachers who have not had any formal courses in statistics. A valid question that could be raised a this point is whether formal courses in statistics are necessary to prepare secondary mathematics teachers to do their jobs, especially since courses in statistics are quite rare at the secondary level. So two closely related questions emerge, one regarding the extent, both in breadth and depth, to which statistics are likely to be inserted into, or expanded within, the curriculum, and the other related to the amount and the nature of statistics content to be required of prospective teachers and what steps need to be taken to upgrade, if necessary, the competence of inservice teachers in that field.

What do the Standards include that might have strong implications for the content preparation of secondary mathematics teachers? There are two categories of standards that need to be considered: (1) the four that reflect general goals and are included at all levels; and (2) those that specifically refer to statistics at the secondary level. The implications of the two categories differ in the types of knowledge and skills required of teachers, but no in their level of importance.

The four general standards (Mathematics as Problem Solving, Mathematics as Communication, Mathematics as Reasoning, and Mathematical Connections) require knowledge and understanding at levels sufficient to allow a teacher to go well beyond the mere "getting answers" by use of formulas or other algorithmic processes. It would appear that a thorough understanding of the <u>principals</u> of statistics is necessary to be able to guide student through problems in a meaningful way, to help them express processes and results clearly, to "think through" problem situations so that formulas and rules are tools and not crutches, and to be able to relate statistics to other areas of mathematics courses. Too often professors (and students) focus on learning the techniques and getting the right answers to problems, ignoring the very aims that are embodied in the general standards. If courses in statistics are to be required of prospective or inservice teachers, it would seem to be mandatory that they include some content that addresses these integrative standards.

The specific standards for statistics at the middle school (grades 5-8) and high school (grades 9-12) are stated in the Standards as general behavior objectives for students. It is obvious that, if students are to be able to demonstrate specified behaviors, teachers must be able to do so, and, further, be able to motivate and guide their students to do so. The mathematical content in statistics at the middle school level (Standard #10) does not appear to be too difficult, and is implied by the following from the Standards (p. 105):

> In grades 5-8, the mathematics curriculum should include exploration of statistics in real-world situations so that students can:
>
> - systematically collect, organize, and describe data;
> - construct, read, and interpret tables, charts and graphs;
> - make inferences and convincing arguments that are based on data analysis;
> - evaluate arguments that are based on data analysis;
> - develop an appreciation for statistical methods as a powerful means for decision making.

While the level of mathematical content may not seem to be high, words such as "interpret," "inferences," and "evaluate" suggest that cognitive levels well above computation are involved. There are also implications for communication, including reading, describing, and constructing arguments.

At the high school level, as one would expect, the level of mathematical content suggested by the standards for statistics (Standard #10) rises significantly, and is implied by the following from the Standards (p. 167):

> In grades 9-12, the mathematics curriculum should include the continued study of data analysis and statistics so that all students can:
>
> - construct and draw inferences from charts, tables, and graphs that summarize data from real-world situations;
> - use curve fitting to predict from data;
> - understand and apply measures of central tendency, variability, and correlation;

- understand sampling and recognize its role in statistical claims;
- design a statistical experiment to study a problem, conduct the experiment, and interpret and communicate the outcomes;
- analyze the effects of data transformations on measures of central tendency and variability;

and so that in addition, college-intending students can:

- transform data to aid in data interpretation and prediction;
- test hypotheses using appropriate statistics.

Thus, for statistics all students are expected to be able to use curve fitting, to compute and understand descriptive statistics, to analyze the effects of data transformations, and to design, carry out, and report and interpret the results of an experiment. Further, college-intending students are expected to perform data transformations and to use inferential statistics to test hypotheses. It is highly likely that there are numerous certified high school mathematics teachers that would be unable to do, much less teach, some of the content suggested by this standard.

There is one other factor related to teacher preparation that is closely related to the Standards and its suggestions for statistics. Throughout the Standards there is a strong emphasis on the use of technology, and the opportunities for the use of calculators and computers in the area of statistics are numerous. Beyond problems that might arise related to the availabilityof the necessary equipment, both hardware and software, there is the question of whether teachers are properly prepared to use the technology when it is provided. Most, if not all, secondary mathematics teachers prepared and certified in recent years have had some experience with computers and, perhaps, with such recent innovations as graphing calculators. On the other hand, many teachers of earlier vintage did not receive any computer training in their teacher preparation programs. It would be hoped that most of those have overcome this deficiency through inservice workshops or further schooling, but there are still many who have not done so. In any case, the use of technology with respect to statistics requires more than basic computer literacy, and means must be found to upgrade teachers' competence to use the power of their machines if all of the suggestions for statistics are to be implemented in secondary schools, especially at the high school level.

Another question raised by the Standards that has implications for teachers of statistics is that of evaluation of student performance, especially in the area of alternative methods of assessment. Because many of the suggestions presented for statistics include varied activities, such as gathering in "real-world" settings, and others that would reasonably be large- or small-group activities, it is appropriate that some sort of group assessment be used to evaluate performance and, eventually, to give students grades. Teachers who are accustomed to using pencil-and-paper tests, or even computer-based but individual tests, may be expected to have difficulty adjusting to new assessment and evaluation techniques, particularly those involving groups. Training in this area would appear to be necessary if teachers are to be effective in the application of some of the new methods suggested in the Standards.

Burrill (1990) has suggested ten principles for implementing the Standards for statistics as follows:

1. Students' activities should focus on asking questions about the students' environment and finding quantitative ways to answer these questions.

2. Problems should be approached in more than one way, with an emphasis on discussion and evaluation of different methods.

3. Real data should be used whenever possible in any statistics lesson, and classroom presentations should give students hands-on experience in working with data.

4. Traditional topics in statistics should not be taught until students have experienced and worked with simple counting and graphing techniques and have established a foundation for those traditional ideas.

5. The emphasis in teaching statistics should be on good examples and building intuition, not on probability paradoxes or using statistics to deceive.

6. Projects by students should be an integral part of any work in statistics.

7. The emphasis in all work with statistics should be on the analysis and communication of this analysis, not on arriving at a single numerical result.

8. Statistics need not be taught as a separate course in the curriculum but should be introduced whenever appropriate to illustrate and expand on traditional mathematics concepts and to form interdisciplinary links for students.

9. The presentation of a given concept should progress from the concrete to the pictorial to the abstract.

10. Calculators should be used extensively. A computer is also a valuable tool for some statistical applications.

These principles seem to be both mathematically and instructionally sound, but some questions seem to be appropriate to raise at this point.

Before the questions are presented, some selected finding from the study by Vaquiax Alvarado (1991) suggest some concerns that Texas teachers have expressed. His results were based on responses from teachers, primarily department chairs, from 145 secondary schools responding out of 211 selected by stratified random sampling with optimum allocation. For example, he found that 70% of the mathematics teachers in his sample felt that present teacher education programs do not prepare teachers adequately to teach statistics and the related area of probability as described in the Texas essential elements (Texas Education Agency, 1990). Only 46% felt it was realistic to expect to teach statistics in secondary schools as described in the Standards. Not surprisingly, the likelihood that a course, or courses, in statistics and the related field of probability were offered and a qualified teacher available was related to the size of the school, indicating that smaller schools would be less likely to be able to offer such courses. One of the major reasons given for not including statistics in their other courses was shortage of time. Finally, while 69% of the respondents reported that their schools had calculators or computers available, only 40% of them had personal computers or computer terminals for use by students. These findings suggest that some changes will need to be made in teacher training and use of technology, among other things, if the Standards for statistics are to be implemented to any great extent in Texas schools.

Questions that have been raised as a result of the reported study, conversations with teachers and others in mathematics education, and published reports include the following:

1. What about the **time** required to gather, organize, and analyze statistical data?

2. What about the **space** requirements and the need for **flexibility** of classroom organization to carry out activities in small groups?

3. What about the **cost** of calculators and computer hardware and software that seem to be necessary to carry out some of the activities and projects suggested by the Standards?

4. How are statistical topics to be **integrated** into the existing curriculum, and what is to be removed to make room for them?

5. How are **teachers** to be trained to teach statistics at the level suggested by the Standards?

6. Are the suggestions for the secondary level based on an **assumption** that students will have gone through a "Standards-based" K-4 curriculum?

These questions represent serious concerns that must be dealt with if the Standards for statistics are to impact the secondary school mathematics curriculum in any significant manner.

While this paper has focused on the Standards and its suggestions for statistics at the middle school (Grades 5-8) and high school (Grades 9-12) levels, one should also be aware of what the essential elements for Texas mathematics curriculum (Texas Education Agency, 1989) include in this area. They embody many of the same ideas as in the Standards but are more prescriptive than the Standards, which is intended to be more suggestive than prescriptive. The Essential Elements include collecting, organizing, and interpreting data in application problems and measures of central tendency at Grade 5 and similar content for Grade 6 plus basic ideas related to the generalization of sample data to populations. They also prescribe some graphing which has possible statistical applications. For Grade 7 graphing continues, including box-and-whisker graphs and histograms, and drawing inferences, constructing convincing arguments, and dealing with misuses of statistical information are included. Grade 8 includes selecting appropriate formats for presenting data and evaluating arguments based on data analysis.

At the high school level most of the statistics included in the Essential Elements occurs outside the mainstream courses. Fundamentals of Mathematics contains statistical content similar to that for Grades 5-8 with more emphasis on problem solving. Consumer Mathematics focuses on interpretation of charts and graphs, appropriate uses of measures of central tendency, and evaluation of claims based on statistical data, all in the context of using gathered information to make decisions in real-life situations. Only in Algebra II of the mainstream courses is statistics mentioned, and the section on data handling and analysis includes recognizing the importance of unbiased sampling and valid reasoning in statistical arguments, selecting appropriate sampling methods, interpreting probabilities relative to the normal distribution, designing and interpreting the results of a statistical experiment, and using computer simulation methods to solve problems. It would appear, then, that students who take a traditional college preparatory curriculum sequence of courses would have little exposure to statistics at the high school level. A course in statistics (or probability and statistics) could easily overcome that problem.

It seems clear that there are many issues to be raised and settled, questions to be asked and answered, and problems to be posed and solved related to the implementation of the Standards in the secondary school curriculum for statistics. Some are content related--misconceptions of statistical concepts, simple lack of knowledge of statistical principles and procedures, and lack of knowledge of the concepts and procedures involved in inferential statistics--while others are related to instruction--use of technology, unfamiliar assessment and evaluation procedures, and incorporation of group activities. while it is true that efforts are underway in several quarters to respond to the questions and concerns presented in this paper (e.g., the Texas Staff Development Program), it is still true that there is much work left to be done in many areas--teacher education, curriculum development, instructional materials, and school finance, to name a few. How to speak to the issues, answer the questions, and solve the problems will be the focus of activities in mathematics education and teacher education for years to come if the standards are to be implemented for statistics in the secondary schools of Texas and throughout the United States.

References

Burrill, Gail (1990), "Implementing the Standards: Statistics and Probability," The Mathematics Teacher, 83:2, February 1990, 113-118.

National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics, Reston, VA.

Texas Education Agency (1989). "Revised Essential Elements" of State Board of Education Rules for Curriculum, Austin, TX.

Vaquiax Alvarado, Alfredo (1991). "Probability and Statistics in Secondary Schools: Problems and Issues" unpublished doctoral dissertation, The University of Texas at Austin.

-CORRECTION-

TEXAS MATHEMATICS TEACHER
VOL. XXXVIII Fall 1991

Page 12, Puzzle 3, Clue 1, should read as follows:

Ado's number does not have a zero in the tens place.

Will the author of the poem
Math Class 90-91
please send name and address
to the editor.
(See address on inside front cover.)
Thank you.

TEXAS MATHEMATICS TEACHER

George H. Willson, Editor
Texas Council of
Teachers of Mathematics
P. O. Box 13857
University of North Texas
Denton, TX 76203-3857

BULK RATE

U.S. Postage
PAID
Denton, Texas
Permit #237

MEMBERSHIP

Texas Council of Teachers of Mathematics

Affiliated with the National Council of Teachers of Mathematics

Annual Membership Dues for Teachers $8.00

USE THIS CARD FOR MEMBERSHIP

Cut on dotted line

Cut on dotted line

Texas Council of Teachers of Mathematics

________ Last Name ________ First Name ________ School ________ (Leave Blank)

________ Street Address ________ City ________ State ________ Zip

Dear Teacher,

To ensure continuous membership, please print your name, zip code, and school above. Enclose this card with your check for $8.00 for one year payable to T.C.T.M., and mail to:

Lynn Walcher
Treasurer
5711 Sugar Hill #21
Houston, TX 77057

___ Renewal ___ New ___ Change of Address

Circle area(s) of interest: K-2 (STEAM) 3-5 (STEAM) 6-8 9-12 College