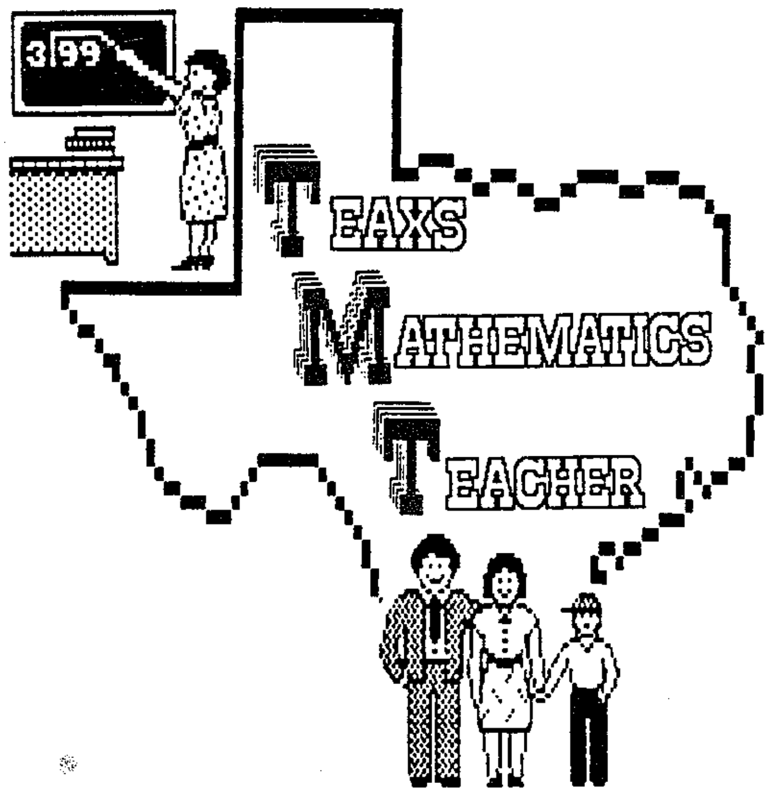

How Calculators Do It - Looking Inside Calculators

Teaching New Teachers How to Teach Math in the Elementary Classroom

Listen, Copy, Remember: Do Primary Grade Children Really Understand Mathematical Terms?

Start with the Last Section

MAY 1991

TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published four times each year, in October, January, March, and May. All manuscripts and editorial correspondence should be addressed to George H. Willson, listed above.

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council
of Teachers of Mathematics

May 1991

TABLE OF CONTENTS

ARTICLES

ANNOUNCEMENTS

President's Message

If classroom teachers were paid by the amount of energy and ingenuity expended, then we would receive the bulk of our pay for what we do the last six weeks of the school year. We are trying to complete curriculum topics while the students are dreaming of summer vacation activities. Every distraction requires extreme effort to refocus the student's attention on the subject at hand. When the school year ends, we feel as if we have been all used up.

However, something happens to us over the summer. We find new energy to start the process again. We create a new classroom environment each fall for the students who come to us and we nurture them through the year until they leave for summer vacation. The students move on to something else, and we start our process again. It is as if we are water wheels in a stream; we fill our buckets from the passing stream and carry the contents to a higher level. If we do not have the energy to turn the wheel, our charges will suffer.

Summer is the time of renewal for classroom teachers. The frustrations fade and the hope for a better year gains strength during the summer. The Conference for the Advancement of Mathematics Teaching (CAMT) should be part of your self-renewal program for this summer. CAMT stands apart as an example of teachers teaching teachers. This is a three day conference where you can find other teachers addressing the concerns you face each week in your classroom. In addition, there are outstanding textbook authors and college professors as well as our own TEA staff to inspire you.

If you attend every year, I look forward to seeing you. If you have never attended, join us this year in Houston. If your vacation plans conflict with CAMT this year, put CAMT on the next year's calendar before anyone else uses your week.

Karen S. Hall

CAMT Registration

TCTM members are needed to help with registration and TCTM information and sales booth. Two hours of your time will help smooth the registration process for participants.

I would like to help with (please circle)

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As a member of TCTM you are invited to attend a membership breakkfast on Thursday, August 8 at 7:00a.m. We need your reservations by June 15. There is no cost to members.

Name: ______________________________

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HOW DO CALCULATORS DO IT--

Looking Inside Calculators

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Introduction:

Electronic calculators can be very important helpers of mathematics teachers and despite the fact that we live in the computer age, they still have a distinct place among electronic computing devices available. The "*low technology*" in the silicon-chip family is not having the impact it should in mathematics education (Zollmann, 1990), but we may see changes. The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) are very explicit about calculators and the NCTM is presently preparing a yearbook on calculators for 1992.

Much has been written about calculators as an aid for arithmetic and "*numbercrunching*" at all levels, also about calculators as a teaching aid for exploration and concept development (ICMI, 1986: Johnson, 1978; López, 1988).

In contrast to those writings, in these notes we present calculators themselves as a study object -- it is really worthwhile to look inside calculators and think about how calculators do it. We want to move away from calculators as a black box and make the way they function more transparent (DIFF, 1980).

Do you ever wonder what happens when you press a key on your calculator?

Model of a simple calculator (without hierarchy)

In order to answer this last question, we present a model of a simple calculator without hierarchy.

If we type $7 * 4 =$

on a calculator, the display window shows the procduct 28. Therefore the calculator must have at least two memories for the numbers 7 and 4 and a register for the operation *.

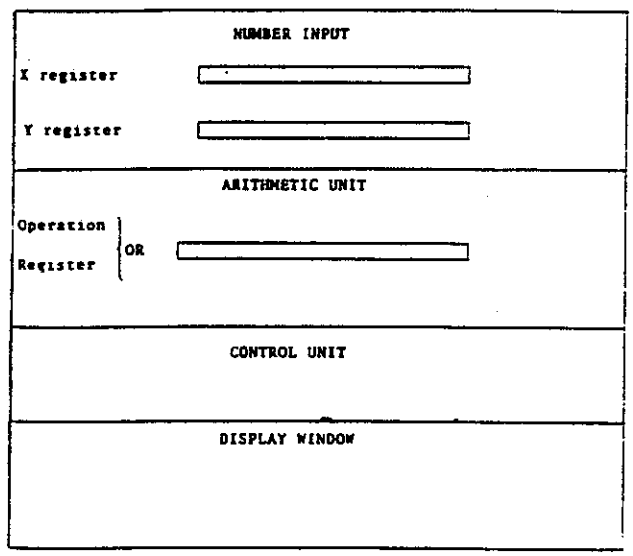

Figure 1.

Figure 1 shows a simple model which can be drawn on the blackboard to illustrate the basic elements of a calculator.

There are two registers for number input, called X and Y, one register for operations (OR), an arithmetic unit, a control unit and a display. According to this model we can devise a simulation game and learn about the internal logic calculators use to perform basic arithmetic.

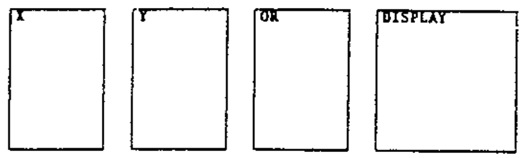

Figure 2.

Simulation Game

The game begins by making a set of playing cards that match Figure 2. Each card represents a register or the display window. A complete set of cards should include several number register cards, operation cards for each of the four basic operations and display cards.

A large cardboard or real demonstration calculaor is also needed and four boxes with name tags (X-register, Y-register, Operation register [OR], Display).

The game involves six students. They assume the role of a certain key on the calculator or an internal component, and act out what happens if it is activated.

One student, Calculator, presses the keys on the large caluclator. A second student is the control unit and gives orders to the arithmetic unit, the number registers and the display.

Another student is the X-register, a fourth the Y-register. A fifth student represents the arithmetic unit and controls the operation cards. The last student acts as display window.

Now we define the rules of the game and describe the actions taken by the different elements of the calculators.

1. Any number input will be stored in X and displayed in the window. Action taken: write the number on an X card and on a display card. Deposit the cards in the respective boxes.

2. Pressing an operation key causes the operation to be stored in OR. At the same time, the contents of register X are copied into Y. Action taken: write the operation on an OR card. Write the number on the X-card on a Y-card. Deposit the cards in the boxes after you remove any card which was deposited before. No box can contain more than one card.

3. If we press the " = " key, the operation in the OR register will be carried out. The numbers on the X-card and the Y-cards are used. Y is the first operand, X the second. The result is stored in X and displayed in the window. The contents of Y are changed to 0 and the contents of OR to " + ". Action taken: write the result of the operation on a X card and a display card. Write 0 on a Y-card. Write + on a OR-card.

4. The "clear" key changes the contents of the X register and the window to 0. Action taken: write 0 on a X-card and a display card.

Example Game

As an example we play $7 * 4 = 28$

The tables illustrate the contents of the four boxes (X, Y, OR, display).

1. Calculator presses 7.

The Control-Unit tells the X-register to write 7 on a X-card. The Y register to write 0 on a Y-card and the display to write 7 on a display card. In this and the following steps, the cards are deposited in the respective boxes.

Table 1.

KEY	7
Register X	7
Register Y	0
Regiser OR	+
Window	7

2. Calculator presses *.

The Control-Unit tells the arithmetic unit to write * on an OR-card and the Y-register to replace the 0 by a 7. The new or corrected cards are deposited in the boxes, old cards are taken out - each box can have only one card at a time.

Table 2.

KEY	7	*
Register X	7	7
Register Y	0	7
Register OR	+	*
Window	7	7

3. Calculator presses 4.

The Control-unit tells the X-register and the display window to replace 7 by 4.

Table 3.

KEY	7	*	4
Register X	7	7	4
Register Y	0	7	7
Register OR	+	*	*
Window	7	7	4

4. **Calculator presses =.**

The arithmetic unit receives orders from the control unit to carry out the operation (7 * 4), to change the contents of OR from * to + and to communicate the result of the operation to the X-register and the display. They write the answers on their cards and deposit them in the boxes. The Y-register is told by the control unit to replace the 7 by 0.

Table 4.

KEY	7	*	4	=
Register X	7	7	4	28
Register Y	0	7	7	0
Register OR	+	*	*	+
Window	7	7	4	28

5. **Calculator presses "Clear".**

The Control-unit tells the X-register and the display to replace their contents by 0.

Table 5.

KEY	7	*	4	=	Clear
Register X	7	7	4	28	0
Register Y	0	7	7	0	0
Register OR	+	*	*	+	+
Window	7	7	4	28	0

Table 5 represents the complete operation which was simulated. The tables are useful to keep track of the contents of the registers and the display window. They are somewhat like the script of the simulation game.

Conclusion:

The actual functioning of a simple calculator can be experienced by students in a very tangible and concrete way. The same simulation game can be devised for more complex operations on a calculator with or without heirarchy. Unfortunately, the simulation gets complicated very fast and becomes too cumbersome to be of any methodological value for students. The simple model and "game" are enough to let students experience hands on what is going on inside their calculators. They begin to appreciate the speed and accuracy of their computing device and catch a glimpse of the architecture and internal logic that make electronic calculators what they are - a great helper for mathematics students and teachers.

Bibliography

Abel, H. et al. Elektronische Taschenrechner. Deutsches Institut fur Fernstudien. Tubingen, 1980.

ICMI Study Series: School mathematics in the 1990's Cambridge University Press, 1986.

Johnson, D.C.: Calculators Abuses and Uses. Mathematics Teaching, 85, 50-56, 1978.

López, J.M.: Manual para la utilización de la calculadora, Departamento de Instrucción Pública, Hato Rey, Puerto Rico, 1988.

National Council of Teachers of Mathematics: Curriculum and Evaluation Standards for School Mathematics, 1989.

Wenzelburger, E.: Guía práctica para la calculadora en la enseñanza de la matemática. UNAM, México City, 1989.

Zollmann, A.: Low Tech, 198 and the Geomery of the calculator keys. Arithmetic Teacher, 37, 5, 1990.

TEACHING NEW TEACHERS HOW TO TEACH MATH IN THE ELEMENTARY CLASSROOM

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The National Council of Teachers of Mathematics asserts that "...it will no longer do for teachers to teach as they were taught in the paper-and-pencil era" (1989). At the University of Houston's Gregory Lincoln Academy School, one powerful intervention strategy, substituting concrete and pictorial aids for written drill, appears to be working. In Math Methods for Elementary Teachers ELED (4315), pencils have been replaced with Cuisenaire Rods, and paper traded in for place-value mats.

Preservice math education courses offered at the University of Houston are ordinarily made up of students who have taken six to nine semester hours of mathematics. These students will have completed College Algebra, Mathematics for Elementary School Teachers and, usually, Finite Mathematics. Their knowledge of the subject is assumed to be adequate for teaching math to elementary school children.

Surprisingly, these same education majors are often uncertain of their ability to perform adequately in the elementary mathematics classroom. This lack of confidence is transformed into self-reliant delight as they discover, through the use of instructional aides, especially manipulatives, how to get their young charges actively and successfully involved in mathematics.

Program Structure

In this program, University of Houston education majors spend each Tuesday attending their elementary math methods class on the Gregory Lincoln Academy School campus. Here they learn, step-by-step, the proper use of manipulatives such as Cuisenaire Rods and Deines Blocks. They gain further practice by working as math tutors in kindergarten and elementary classrooms. These students are concurrently enrolled in a generic teaching strategies course on the university campus. This class teaches instructional design, classroom management and reflective inquiry, and dovetails nicely with the students's tutorial activities across town at Gregory Lincoln.

In Everybody Counts: A Report to the Nation on the Future of Mathematics Education, we are told that "teachers themselves need experience in doing mathematics." What the tutors are doing at Gregory Lincoln relates directly to several NCTM standards by: 1) Developing meaning for operations by modeling and discussing a rich variety of problem situations; 2) Relating mathematical language and symbolism of operations to problem situations and informal language; 3) Recognizing that a wide variety of problem structures can be represented by a single operation; and 4) Developing operation sense (41).

This math methods class is concerned with increasing the university students' self-confidence as well as their levels of competence. This confidence building is achieved by showing the class members first how to do something such as finding differences with Cuisenaire Rods, and then giving them the opportunity to try it out under supervised conditions. The objective is to make certain the university tutors understand the concepts and operations, and are able to get them across to their own students.

The model of instruction in ELED 4315 seems relatively simple: 1) Model what is being taught and provide guided practice with peers; 2) Use the strategies in a classroom setting with a small group of elementary students; 3) Analyze and evaluate the teaching performance and its effect on elementary students. In reality however, the application of this model may be complex. Implementing the model requires effective communication, willingness to adjust when unpredictable events occur, and professional commitment from everyone involved in the program. Placing some twenty-five university students, a graduate student coordinator, and a professor in fourteen elementary classrooms with teachers and their students is not an uncomplicated management problem.

Model and Practice

Instruction in the two and a half hour math methods workshop concentrates on the use of manipulatives and visual aids, emphasizing the correct transition from one to the other, and introducing abstract representation. Inductive and deductive strategies utilizing discovery and problem solving are modeled and taught by the professor.

The math methods students are initially encouraged to play around with the manipulatives in the same manner they will, hopefully, allow their elementary students to experiment with them. Then they gain expertise with manipulatives through practice in teaching and reteaching their use to small groups of peers.

Since the main thrust of elementary math instruction should be to enable learners to think creatively in solving problems, it is imperative that teachers find ways to involve students in the discovery of the solutions. In this course, the professor asks leading questions, explaining as he goes, their pedagogical relevance. Class members are then given the opportunity to ask such questions of one another. This technique is designed to foster similar, however individualized, instructional patterns among the students when they become tutors or teachers themselves.

The tutors also practice changing their questions in order to change the anticipated student response. Using Cuisenaire Rods, a tutor might, for example, give the addends and have the students provide the sums, or vice versa. "If I put yellow and green rods end to end, they will be the same length as what color rod?" Or, "What combination of two rods is the same length as the yellow rod?" Or, "If I put the red rod before the green rods, are they the same length as when I put the green rod before the red rod?"

The professor then guides the tutors through a series of exercises with Deines Blocks and assigns simple addition or subtraction algorithms for each tutor to "teach" the others in their group. As soon as the instructor feels that the tutors are comfortable directing a mini lesson using the blocks, he introduces place value mats to use with them. At this point the tutors are taught how to solve increasingly difficult addition and subtraction problems using both base 10 blocks and mats.

It is essential for teachers to give immediate meaning to any symbolization by demonstrating, illustrating or explaining what is happening within the algorithm itself. While working through exercises with manipulatives, the university students are shown how to move from the concrete teaching aid to an illustration or picture, to symbolic abstraction, and finally to a word problem. The abstraction is often presented in conjunction with the manipulative or the illustration, but never presented before the other two. Symbols are explained as they are used in pictorial representation of the manipulatives. Color or letter representations of Cuisenaire Rods, for instance, are always given in the same order as the rods themselves.

The University of Houston math education tutors are required to symbolically record what they have done with a manipulative in order to better visualize it. The use of math manipulatives breaks down if teachers are unable to manage the transition between the concrete and abstract representations, and for many student tutors these transitions are very difficult to master.

Classroom Tutors

The students are given tutorial assignments in elementary or kindergarten classrooms, two or three tutors to a class. The classroom teachers assigns each of them from one to three children to tutor for approximately one hour per week.

Their first task is to administer and analyze a diagnostic test of their own design, to assess a child's achievement in a single area of math (i.e. addition of whole numbers). Such diagnostic tests used properly have the potential to facilitate numeracy, and measure readiness for the introduction of new concepts.

Tutors are given the following rules for constructing a diagnostic test: 1) Test one concept at a time; 2) Start out easy and get progressively difficult; 3) Avoid arbitrary time frames. The tutors are shown how to select prediagnostic test items from their diagnostic tests so that they can tell which levels should actually be administered.

This testing procedure determines an elementary student's level of math competence in the shortest time and with the least pain for all concerned-- it constitutes a quick check to ascertain a student's readiness for a specific type of algorithm. The NCTM warns that "Symbolic tasks with numbers should not be presented in isolation and should not be emphasized until the numerals have been carefully linked to concrete materials and children understand the major concepts" (38). The utilization of carefully constructed diagnostic tests, thus, facilitates numeracy by providing teachers with a dependable tool for determining which concepts have or haven't been mastered by individual students.

Analysis and Evaluation

While tutoring is taking place, the instructor visits classrooms to watch his university students practice their newly learned skills and methods. Classroom teachers are encouraged to make comments and offer suggestions to the professor during these observational visits, as well as during their scheduled meetings with him. Classroom teachers continually assess the tutorials as do the tutors themselves. Tutors evaluate their own performance in journals as well as by discussing their successes and failures with their professor and peers.

The positive aspects of the program have been demonstrated by a marked improvement in elementary student math test scores, as well as by enhanced levels of performance and confidence on the part of the university tutors. Beginning teachers invariably learn by emulating successful colleagues. This math methods class at Gregory Lincoln gives the university students an invaluable opportunity to work alongside experienced teachers.

An additional benefit of the arrangement has been that the classroom teachers have, themselves, been able to observe lessons being presented to their students, and have found their own methods and ideas creatively enhanced.

Initial participant evaluations of this program have been highly positive. Such misgivings as have been voiced, have focused on the possibility of better communication regarding the university's expectations, and difficulties inherent in adjusting elementary classroom schedules to the needs of the math methods student tutors.

The university students have discovered themselves to be much more comfortable teaching mathematics than they ever thought possible, and as a result, their ability to understand and work with children has shown a marked improvement. One tutor noted in her class journal,

> "By focusing on my questioning skills, I noticed that I allowed the students to find their own answers to questions. This seemed to be more effective than my giving the answers. My skill also appeared to improve, since by the end of the lesson, the students had fewer questions about my questions. I learned to condense my sentences and only use the words that were important to the current task. This also helped me to add just one dimension of a problem at a time. I will continue to practice this skill in my effort to become a more effective teacher."

One final, though perhaps not readily evident, benefit of a class such as this is that the university professor is better able to bridge the gap between theory and practice, while, at the same time, keeping himself current with regard to the realities of the elementary school environment. This model, with slight modification, could be used with other groups of math educators. Such an approach has, in fact, been used in a summer program for inservice teachers with positive results (Hollis, 1985).

As a cautionary note, it should, perhaps, be pointed out that any such introduction of new math methods should ideally involve a small number of learners. This reduces the risk of failure and subsequent retreat to the same old pencil and paper math methodology.

As Prof. Lynn Arthur Steen of St. Olaf College has so tongue-twistingly put it, "Since teachers teach much as they were taught, the task for universities must be to teach teachers as we would have them teach." The University of Houston's ELED 4315 has young teachers utilizing a hands-on approach to teaching mathematics, laying aside their pencils and paper in favor of manipulatives.

* * *

References

Hollis, L.Y. (1985). A summer professional growth experience. *Arithmetic teacher.* 32, 16-17.

Mathematical Sciences Education Board, Board on Mathematical Sciences, Committee on the Mathematical Sciences in the Year 2000, National Research Council (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, D.C.: National Academy Press.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: The National Council of Teachers of Mathematics, Inc.

Steen, L.A. (1988). A "New Agenda" for mathematics education. *Education week,* May 11, 28, 21.

LISTEN, COPY, REMEMBER: DO PRIMARY GRADE CHILDREN REALLY UNDERSTAND MATHEMATICAL TERMS?

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Do primary grade children really understand the mathematical terms used by their teachers during their mathematics class? If understanding is defined as solving a paper and pencil problem, then the answer to the question is probably yes. If, however mathematical understanding is defined as the ability to use information presented and to describe verbally the strategies used to arrive at the solution to the problem, then the answer to this question is probably no.

In our role as supervisors of elementary preservice teachers, it has become increasingly apparent as we observe seven- and eight-year-olds that a discrepancy exists between their ability to solve problems and their ability to explain the process used to arrive at the answer. Indeed, for many children explaining what they did and why, is a frustrating, if not an impossible task. Yet, these same children are able to solve paper and pencil problems.

Why is there a discrepancy between the ability to solve a problem by rote and the ability to explain what they did and why they did it? From our discussions with primary grade children several possible reasons for this discrepancy were revealed. First, the children's obvious reluctance to "talk through" the problem suggested unfamiliarity with the terminology to use, and indeed some children said "we don't do it this way, we just write down the answer." Second, seven- and eight-year-olds vary greatly in their level of language ability and their conceptual knowledge of mathematics. Third, when asked to interpret words with more than one meaning, they tend to restate the word with the meaning most frequently used in everyday conversation.

Fourth, there has been "a long-standing preoccupation with computation and other traditional skills..." (Curriculum Evaluation Standards for School Mathematics, 1989, p. 15). Consequently, the K-4 curriculum "...fails to foster mathematical insight ... and emphasizes rote activities" (Curriculum Evaluation Standards ..., 1989, p. 15). The response to the question may lay in the mismatch that exists between the children's ability to solve problems by rote and the mathematical comprehension needed to explain what was done. If this is the case, primary grade teachers then assume that just because children solve a problem and can read the mathematical term, they understand the process involved and the meaning of the term. The children fail to develop the belief that "...learning mathematics is a sense making experience" (Curriculum Evaluation Standards, ..., 1989, p. 15).

Crockcroft (1982) cites an excellent example to illustrate this point. There was a visitor who came to a British school and asked the following question, "What is the difference between ten and seven?" One child answered, "ten is even and seven is odd" instead of "three" which the visitor had expected. For this child the word difference meant unlike and not amount by which one number exceeded another.

In this example, it is obvious that the child recognized these terms but did not understand them in a mathematical context. The degree of language competency that a child brings to the classroom and the mathematical language used in oral or printed materials are significant determiners in the children's ability to understand the mathematical information presented in class as well as in the child's ability to read mathematical information in the text and on worksheets (Knight and Hargis 1977).

A journal article by Cox and Wiebe (1984) described an informal inventory for assessing primary students' knowledge for various mathematical terms. The Mathematical Vocabulary Reading Inventory consisted of four parts. Part 1 required the students to listen to a mathematical term read aloud and then to locate the read-aloud word in a row of words (e.g., five: five, fever, field, find). Part 2 required the students to match a mathematical term to a picture that illustrated the meaning of the term (e.g., cent, 1¢). Part 3 required the students to read a word from among several options similar in meaning (e.g., equal: even, equip, largest, special, same as). Part 4 had the students read a sentence with a word deleted (e.g., I can _______ to one hundred. Choices: equal, even, count, money, cents) and they select a word that correctly completed the sentence.

The authors of this article used the Mathematical Vocabulary Reading Inventory to investigate informally the knowledge of seven- and eight-year olds. We administered the inventory to 61 seven-year-olds and 43 eight-year-olds. The results of our informal assessment suggested that there is a discrepancy between their ability to solve whole number operations and their ability to explain how they arrived at the solution.

Some children clearly had difficulty in recognizing and comprehending oral and written mathematical terms. Children experienced less difficulties with concrete terms such as missing, shape, eleven, first, and count in parts 3 and 4. The more specialized the term, the more random the responses and the higher incidence of errors. Mathematical terms such as equal, multiplication, quotient, and adjacent, posed problems for the majority of children.

Some terms are more difficult than others because seven- and eight-year-olds have limited experiences in applying their mathematical knowledge and have limited experiences with word-attack skills. The degree of familiarity with the term in everyday conversation and in its use in mathematics appears to have influenced the final scores in part 3.

From looking at the information generated from our informal assessment, several possible reasons come to mind for the children's poor performance on the following items, in part 3, less than, number sequence, difference, etc. and in part 4, product, equivalent, adjacent, etc. The first is that the children may not have covered the material being tested since the seven-year-olds (compared to eight-year-olds) have not yet fully encountered the operations of multiplication and division in their math curriculum. The second is that the teaching of mathematical concepts and terminology may be overlooked or underplayed because children are not yet ready to incorporate mathematical terms into their oral language background. So the emphasis in mathematics class seems to be on rote learning (Salmans, 1987). Third, comprehension of mathematical terms may be assumed if children can recognize and read terms and can solve computational problems. But, it takes more than just having children demonstrate that they recognize the mathematical term, can read it, and can solve a problem. Fully comprehending mathematical terminology means that children can verbally explain not only what they have read but also what they have done to solve the problem. What could primary grade math teachers do to help children develop an understanding of math concepts and terms? The next section of this article will present three instructional strategies for teaching meaning and comprehension of mathematical concepts to children.

Instructional Strategies

According to Steen (1989), teachers to prepare students for the future, "...must change their curriculums [and] teaching methods ..." (p. 18). One teaching strategy that teachers can use to improve children's understanding of mathematical concepts is to preteach mathematical vocabulary that the child will encounter during oral instruction and in the text. Cooper (1986) provides a convincing case for teachers to devote instructional time to preteaching vocabulary in written and oral contexts before the math lesson begins. The most effective way to preteach vocabulary according to Gipe (1978-79), is to have teachers take time to discuss word meanings in depth and give children sufficient examples to make sure that they understand the words in different contexts (Jenkins, Pany, and Schreck 1978). Without fully comprehending the meaning of mathematical terms, children begin to accumulate skill deficiencies that can handicap them as they move into the intermediate grades.

Preteaching of vocabulary should attempt to link previous knowledge of the mathematical term with the new meaning being presented. Returning to the example of the meaning of difference, (question 19, part 3), the following scenario illustrates how teachers can preteach vocabulary.

Question: What is the difference between ten and seven?

Child: One is an even number, and one is odd.

Teacher: That's one way of looking at it. Let's look at another example using the same meaning for the word "difference." What is the difference between winter and summer? (Pictures are shown.)

Child: One is cold, and one is hot.

Teacher: Now let's look at another meaning for difference. John is ten years old and Jenny is seven years old. What is the difference in their ages? (Counting out paper candles for each to be placed on a paper cake.)

Child: John is three years older than Jenny.

Teacher: That's correct, but how did you find the answer?

Child: I took seven away from ten.

Teacher: You found the difference in their ages. So "difference" in mathematics means to take away. Other examples may be in amount of allowance you get and what someone else gets, your age and height and someone elses. Now you can see that "difference" has more than one meaning.

In this example the teacher, instead of just telling the child that the first response was not the answer expected, spent time reinforcing the original concept. Also, the teacher illustrated the mathematical meaning of the word difference by showing the relationship between two measures and by using manipulatives and pictures. This type of discussion can help to expand the child's comprehension in mathematics. The amount of instructional time spent preteaching is time well spent.

A second strategy that may prove helpful is to encourage children to discuss their ideas about how they solved a problem or why they used a particular method to arrive at a solution. Providing opportunities for students to communicate helps them clarify their thinking about mathematical ideas (Curriculum Standards Evaluation for School Mathematics, 1989). According to Hard (1982), this strategy is used frequently in language arts classes but is seldom applied to mathematics. Too often, mathematics instruction in the primary grades is confined to rote learning exercises (Salmans, 1987). When verbal exchanges occur, they are limited to such statements as "What is the answer?" "Can you think of another way?" or "Is this right?" Such a verbal interchange develops "number sense"; according to the report, Everybody Counts, compiled by the National Research Council (1989); number sense builds on arithmetic as words build on the alphabet" (p.46). Hart (1982) encourages teachers to create a new climate "in which language plays an important part in the child's understanding of the processes involved."

Through verbal exchanges, a child's level of understanding and "misunderstanding" can be discovered. This discovery can help the teacher to answer the question, "Does the child lack reading comprehension skills or mathematical experiences and understanding?" As children exchange ideas and translate thoughts into words, sentences, and stories, they are testing their mathematical knowledge. Children experience greater success in learning new terms if they "discuss words and practice using them in meaningful communication" (Johnson and Pearson 1984).

Let's take an example of row and column from Part 2 of the Inventory which requires the student to match the term to a picture to demonstrate how to use verbal exchanges. A teacher could present a diagram like the one that follows. The children are required to distinguish between row and column.

		Columns			
		A	B	C	D
Rows	1.	.	.	.	.
	2.	.	.	.	.
	3.	.	.	.	.

After viewing this diagram, the children could discuss the meaning of the terms row and column. In addition, the teacher could ask such questions as "How many dots in row 1? How many dots in row 3? How many dots in column A? How many dots in column C?" After asking several children these questions, the children are ready to define the terms, row and column, in their own words. Two approaches are outlined here. On the one hand, the teacher asked the children to use a visual image to seek and find the answer. On the other, the teacher asked the children to form a generalization or definition. Both these approaches are essential to the child's understanding of the mathematical terms. At a later date as a follow-up activity, the teacher could use the same diagram minus numerals, letters, and arrows to assess the children's continued understanding of the concepts presented.

By using verbal exchanges along with visual thinking experiences as described in the previous example, the children learn to clarify the terms introduced and begin to build their own mathematical language. Kaufman (1971) believes that the use of visual imagery improves the flexibility with which data can be transformed. In addition, Threadgill-Sowder and Sowder (1982) found that presenting problems visually along with a verbal explanation was clearly more effective than a word-only presentation for children.

A third strategy is the use of manipulatives. Manipulatives help children better understand terms and concepts presented during mathematics class. Also, the use of manipulatives helps children to fix the mathematics concept in their minds. Understanding the concept of trading and renaming, for example, is central to the operation of subtraction. Children need opportunities to "trade" bundles of one set of ten for ten ones. When children manipulate concrete objects, they have opportunities to find the difference and to experience the trading process.

For example when given a mathematical sentence, 22 - 5 = ____, the children can assemble a set of 22 craft stick and a set of 5 craft sticks to find the difference. Once the quantities are arranged, labels with appropriate terminology can be matched to the symbols. For example "22 is equal to 2 tens and 2 ones." In addition, a visual, the place-value frame can be introduced to help children label and arrange the appropriate quantities in the ones and tens columns.

tens	ones
2	2
	5

The next step is to rename 2 ones to 12 ones by trading 1 set of ten craft sticks for 10 individual craft sticks.

tens	ones
1	12
	5
1	7

Now we have 12 ones leaving 1 ten in the tens column. "We have <u>renamed</u> one ten and 2 ones to 12 ones, leaving one ten in the tens column." So 22 - 5 + = 17.

In the foregoing example, children experience the quantities of 22 and 5. These are not meaningless symbols but symbols that translate into specific quantities. To achieve the abstract level of thinking, children need to have experiences that require handling such materials as counters, Cuisenaire rods, and other physical objects. The manipulatives encourage children to solve problems in a number of different ways. The inventive child may find alternative solutions to the problem through trial and error. Manipulatives can be housed in a "mathematics corner" so that children always have access to them. The materials can include straws, craft sticks, and rods for handling; an abacus for counting; a place-value frame and squared paper for finding the differences; and a balance for weighing. The list of materials is endless, but it is important for teachers to use concrete objects as they introduce new mathematics topics.

Conclusions

From the preliminary findings generated from our study, it appears that many primary grade children do not really understand terms used during their mathematics class. It appears that questions children are asked in mathematics class frequently require them to remember facts and follow routines rather than to understand and use the number operations. In addition, when children respond correctly they are seldom questioned about the strategies or methods they used to solve the problem. Changes are needed in the curriculum and teaching methods if mathematics education in the primary grades is to be restructured and improved. Teachers indeed can make a difference in primary-grade math instruction by having students explain math, by preteaching vocabulary and concepts, and by using visuals and manipulatives.

There are several assumptions that can be drawn from the results of our informal assessment using the Wiebe/Cox Mathematical Vocabulary Reading Inventory. First, when children are given mathematical problems they are often able to solve them, but have little mathematical knowledge about what they are doing. Second, many teachers refrain from teaching precise mathematical terminology when specific whole number operations (e.g., addition, subtraction, multiplication, and division) are presented, provided the children can cognitively assimilate this information. Third, children may need to be taught synonymous mathematical terms or phrases encountered in everyday conversation, in order to give concrete meaning to abstract mathematical concepts. From our assessment, it seems that primary-grade children appear to have little understanding of the terminology typically included in the primary-grade mathematics curriculum. Understanding terms and the ability to explain how a problem is solved and why it was solved that way should be as much a part of the primary-grade mathematics program as rote learning. Based upon the preliminary findings of this study, we have offered three instructional strategies that primary-grade teachers can employ to help children better comprehend mathematical processes and increase their mathematical understandings.

References

Cockcroft, Wilfred Halliday, Mathematics Count, London: HMSO, 1982.

Cooper, James David, Improving Reading Comprehension, Boston: Houghton-Mifflin Co., 1986.

Cox, Juanita, and James H. Wiebe, "Measuring Reading Vocabulary and Concepts in Mathematics in Primary Grades," The Reading Teacher, vol. 37 (Jan. 1984), 402-11.

Curriculum Evaluation Standards for School Mathematics, Washington, D.C.: National Teachers of Mathematics, 1989.

Everybody Counts: A Report to the Nation on the Future of Mathematics Education, Washington, D.C.: National Research Council, 1989.

Gipe, Joan P., "Investigating Techniques for Teaching Word Meanings," Reading Research Quarterly, vol. 14 (1978-1979), 624-44.

Hart, Kathy M., "Tell Me What You Are Doing," Mathematics Teaching, (London) (June 1982), 99.

Jenkins, Joseph Richard, D.Pany, and J. Schreck, Vocabulary and Reading Comprehension: Instructional Effects, Urbana-Champaign: University of

Illinois, Center for the Study of Reading, 1978, (ERIC Document Reproduction Service, 1978, ED 160 999).

Johnson, Dale D., and P. David Peason, Teaching Reading Vocabulary, New York: Holt, Rinehart, & Winston, 1984.

Kaufman, Geir, Visual Imagery and Its Relation to Problem Solving, Oslo, Norway: Universitets of Laget, 1971.

Knight, Lester N., and Charles H. Hargis, "Math Language Ability: Its Relationship to Reading in Math," Language Arts, vol. 54 (April 1977), 423-28.

Salmans, Sandra, "Learning by Rote: A New Appraisal," in Education Life Section, The New York Times, (April 12, 1987), 47-50.

Steen, L.A., Teaching mathematics for tomorrow's world, Educational Leadership, (1989), 47, 1, 18-22.

Threadgill-Sowder, Judith, and Larry Sowder, "Dawn Versus Verbal Formats for Mathematical Story Problems," Journal for Research in Mathematics Education, vol. 13 (November 1982), 324-31.

Correction Notice:

January 1991 Journal,

Page 7, Figure 6 - rotate so it is right side up.

Page 10, line 6, and upper right hand corner Figure 12 should read: $|x(x^2-2)|$.

START WITH THE LAST SECTION

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In a time when educators are looking for a solution to what should be done in an honors program, presented here is a viable alternative. Too long the average to gifted students have had to sit in a classroom where mathematical concepts have been repeated and repeated again until the students say, "Oh, No! Not again!" The prevailing pedagogical methods of presenting mathematical material are based on the building process. Students learn some concept and then build on that to learn another new concept. But, for the average to gifted student, there is a much better teaching approach.

For a practical example from college algebra consider the chapter on polynomials. It basically is to teach students to solve polynomial equations and graph polynomial functions of degree greater than two. The textbooks start with the solutions of a polynomial equation, and progress through the remainder theorem, the factor theorem, the fundamental theorem of algebra, Descartes' Rule, the rational root theorem, the number of turning points theorem, and the concept of x-intercepts to reach a climax where the students graph polynomial functions using each of the concepts listed above.

The author's propose that the teacher start first with the graphing of these polynomial functions, and in doing so, teach each of the other concepts. It is agreed that sometimes the instructor may have to generate his own examples to show all of the concepts, or use more than on example, but two examples are better than ten leading to the same result.

Using the idea of determining the graph of a polynomial function, the teacher might start as follows. Graph the particular polynomial function, $P(x) = 4x^3 + 8x^2 - 9x - 18$, of degree three, finding each of the following:

1. The total possible complex roots by the fundamental theorem of algebra when $P(x) - 0$;

2. The number of turning points using the turning point theorem;

3. The possible number of positive and negative real roots using Descartes' Rule for $P(x) = 0$;

4. The possible rational solutions by the rational root theorem of $P(x) = 0$;

5. Which possible rational roots can be eliminated by finding the upper and lower bounds on the roots;

6. Which possible rational roots are actual roots by synthetic division and the remainder theorem;

7. The x-intercepts from the actual roots and the polynomial function written as factors using the x-intercepts; and

8. The graph of the polynomial function, using each of the concepts listed above and filling in the graph with the x, y chart where needed.

The solution to the given example is as follows:

1. From the fundamental theorem of algebra we find $P(x)$ has at least one at most 3 distinct solutions which are complex roots.

2. Since the number of turning points is one less than the degree of the polynomial, $P(x)$ has 2 turning points.

3. The polynomial equation, $P(x) = 0$, has one positive and two or zero negative real solutions.

4. Using the rational root theorem, the possible factors of the constant term divided by the possible factors of the leading coefficient are:

   -18, -9, -6, -9/2, -3, -9/4, -2, -3/2, -1, -3/4, -1/2, -1/4, 1/4, 1/2, 3/4, 1, 3/2, 2, 9/4, 3, 9/2, 6, 9, 18.

5. The least integral upper bound and greatest integer lower bound using the rule for upper and lower bounds are, -3 and 2 respectively. Marking off the extra possible roots in (4) we obtain:

   -9/4, -2, -3/4, -1/2, -1/4, 1/4, 1/2, 3/4, 1, 3/2.

6. Using synthetic division on the remaining possible roots, we find the roots are -2, -3/2, 3/2 for they give a remainder of zero.

7. The polynomial P(x) written as its factors from the roots, 3/2, -3/2, and -2 is:

   $$P(x) = [x + 3/2][x - 3/2][x + 2].$$

8. Using synthetic division and the remainder theorem we find points between the x-intercepts, 3/2, -3/2, -2, and form an x, y chart.

X	Y
-3	-27
-1	-5
0	-18
1	-15
2	28
-7/4	13/16

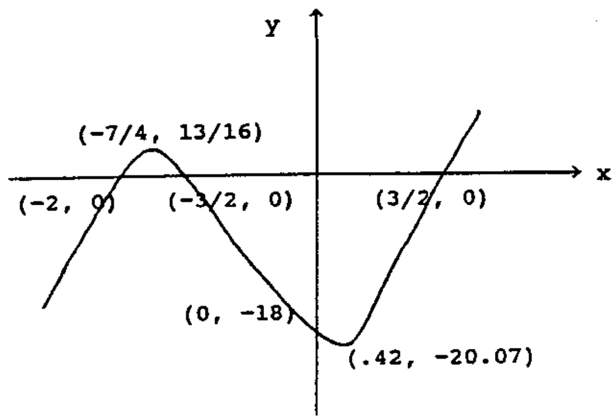

Figure 1

Using the information on the preceding page the graph of the function is shown in Figure 1.

Using this method of teaching the material, the students can immediately see the whole picture of what they are doing. Students can also visualize the polynomial function. They no longer have to wonder where all the repetition is leading. By presenting the total package they know.

We strongly recommend this type of program for honors programs. With this approach there should be ample time for enrichment material, using the software on a computer.

Some honors programs use the computer software alone in the instructional process. The students are able to see these functions graphically using the computer, but they fail to gain the manual ability to generate these functions on a graph for themselves when no computer is available. The computer and production is necessary. With the concept introduced here, the teacher would have the time to pursue both the manual and computer aspects of the investigation of polynomial functions. An easy method for doing this is to start with the crowing concept generally found in the last section of the chapter and synthesize it with all other concepts.

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