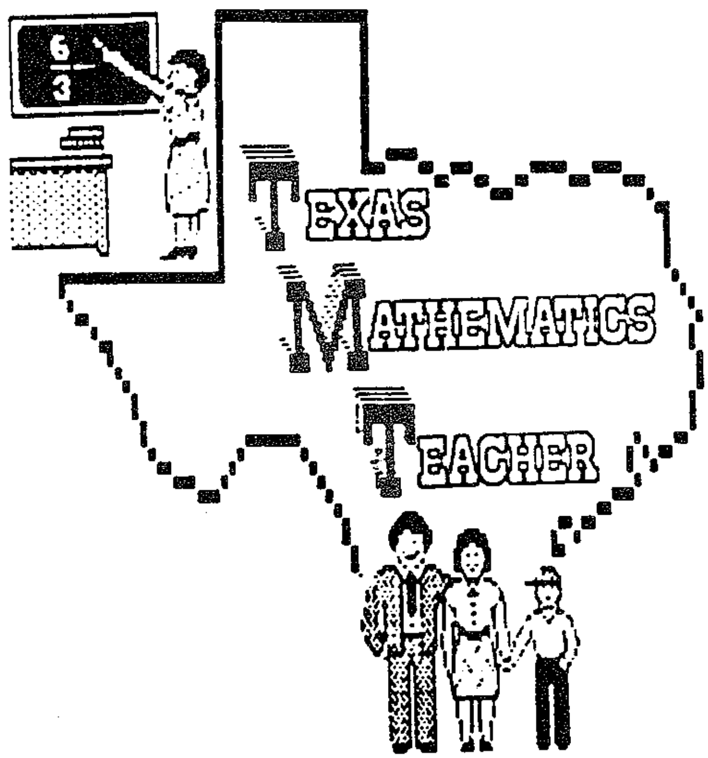

Using an Algebraic/Geometric Approach to Solving Equations and Inequalities

On a Three Dimensional Pythagorean Theorem and the Distance Between a Point and a Plane

Inflection Points and Roots: A Calculus Connection

Algorithms for Base Two Addition

JANUARY 1991

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

January 1991

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President's Message

In the days since the Educational Testing Service announced that students will be using calculators on the SAT, I have had the same conversation with several parents and friends. It always starts with the question "What do you think about using calculators on the SAT?" and quickly leads to talk about mastering arithmetic facts. The parent is concerned that elementary children will not pass through the same ritualistic learning of arithmetic facts that each of us experienced. Usually the parent expresses a concern that if her child can't do arithmetic, access to mathematics will be denied. The parent's concern is valid, since, in too many cases, mathematics educators have barred the door to algebra until the student demonstrates mastery of arithmetic.

At this point in the conversation, I start talking about the NCTM Standards and the changes in mathematics education in the state of Texas as reflected in the Essential Elements for each elementary grade level or secondary course. I try not to use too many acronyms nor too much education vocabulary. I emphasize that the use of the calculator is a reflection of our concern as mathematics educators that access to mathematics topics not be limited by arithmetic skills, in the same way that access to writing exercises is not limited by spelling skills. This analogy holds up in that we still value rapid calculation skills but in the same way that we value spelling skills.

I am writing this on my word processor and will use the spell-checking feature and grammar-checking package when I finish writing. If my spelling is not almost correct, even the spell-checker is useless. If I could not spell at all, writing would be even more painful than it already is. I find that my own tolerance for incorrectly spelled words is about 5%. When I returned to school to earn a degree in mathematics, that same 5% error rate showed up as arithmetic mistakes in my examinations. Is this something that each of us develops? My students have a personal grade tolerance; they will do whatever work I require to make an acceptable grade. That acceptable grade is different for each student. One particularwhy he didn't strive for grades that more closely reflected his abilities, say a 98 instead of an 88, his reply was that he had too many other things that interested him and that an 85 average in math was good enough. Maybe an elementary student "decides" on an acceptable skill level and no amount of coercion or number of work sheets improves that skill level? Access to mathematical topics may motivate improvement in arithmetic skills or it may not. In the meantime, students have been doing mathematics.

Somehow in the public mind, rapid calculation is the sign of understanding mathematics so the decision to use calculators on the SAT is a catalyst for discussions about mathematics education. I am pleased to participate in these conversations, to listen to concerns about speed and accuracy in calculations and to offer examples of mathematics lessons that go beyond routing calculations. If I am having this conversation several times a month, the three thousand plus members of the Texas Council of Teachers of Mathematics also must be discussing this crucial issue. I hope you enjoy the opportunity to participate in mathematics education conversations with non-educators and have prepared yourself with thoughtful and informed responses.

Karen S. Hall

ELEMENTARY MANUSCRIPTS NEEDED

Manuscripts should be neatly typewritten and double spaced, with wide margins, on 8 1/2" by 11" paper. Authors should submit the signed original and four copies. Illustrations should be carefully prepared in black ink on separate sheets, the original without lettering and one copy with lettering added. As soon as possible after refereeing, authors will be notified of a publications decision. Originals of articles not accepted for publication will be returned to the authors. Two copies of the issue in which an author's manuscript appears will automatically be sent to the author.

USING AN ALGEBRAIC/GEOMETRIC APPROACH TO SOLVING EQUATIONS AND INEQUALITIES

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Too frequently Algebra teachers tend to teach students to become competent algebraic manipulators in solving equations and inequalities. Rarely are students taught to see the geometry of the solution. Solving such problems tends to be a sterile experience for the student, lacking in any intuitive basis. Some students, unfortunately, never see the connection between the algebraic and geometric views of the solution. The coordinate plane is one of the best vehicles to help students understand and visualize basic algebraic notions. Perhaps, consideration should be given to introducing the coordinate plane at the beginning of Algebra I and use it as a concrete model.

It is the purpose of this paper to suggest ways in which a geometric view can be included to affirm, and supplement the algebraic approach to problem solving. This approach employs one of the most useful problem-solving strategies, that of making a sketch, and allows the student to use geometric intuition to estimate the regions of solution sets and to check the feasibility of algebraically determined solutions. Using algebraic and geometric experiences simultaneously reinforce their mutually supportive roles and can result in deeper understanding by the student. Although finding the points that determine the exact region of a solution may require algebraic (or other) methods, this additional geometric approach should deepen the students' insight by indicating where to look for solutions and where to eliminate some incorrect solutions.

To facilitate the use of the geometric view in these solutions, the following curve sketching technique is presented. For the equation $2x-3=5$, the solution, $x=4$, is easily explained by simple algebraic manipulation. The key to the geometric approach to this solution is as follows: for the equation $2x-3=5$, we define $f(x)=2x-3$ and $g(x)=5$. Then the graphs of f and g intersect at $x=4$, as in Figure 1. Thus, it is possible to translate this algebraic question to the geometric question "where does the graph of the linear function $f(x)=2x-3$ intersect the graph of the constant function $g(x)=5$?"

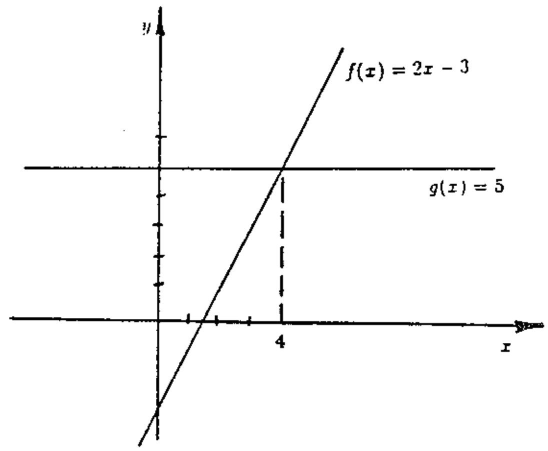
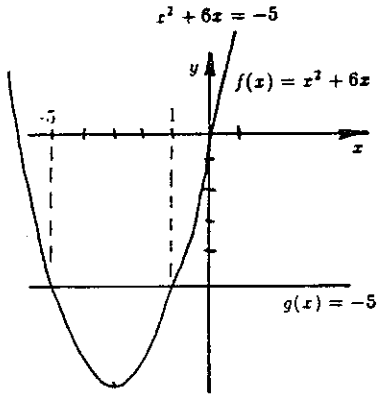
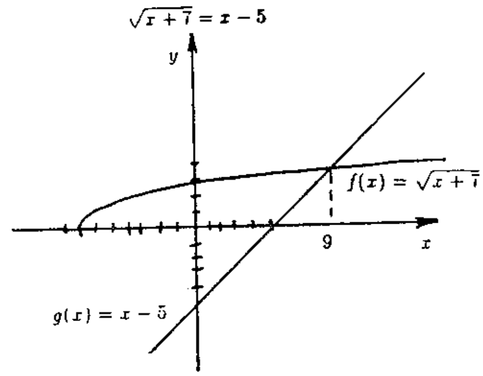


Figure 1

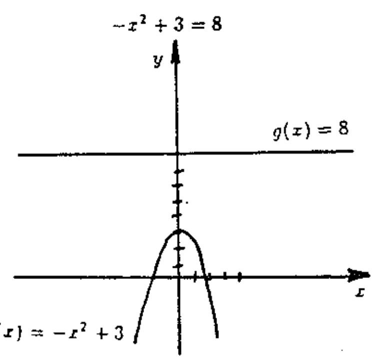
A few more examples of this algebraic/geometric approach follow in Figures 2-4.



Solutions found algebraically are $x=-1, x=-5$. It is obvious neither is an extraneous root.
Fig. 2

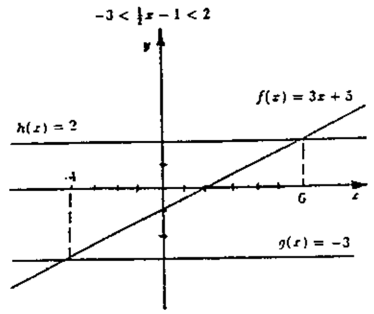


Solutions found using algebra are $x=2, x=9$. It is easy to identify the extraneous root.
Fig. 3



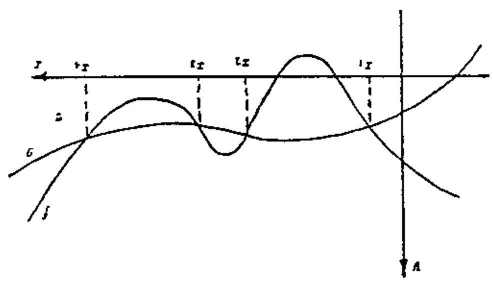
The algebraic solutions turn out to be complex conjugates. A good visual representation of no solution (no real roots).
Figure 4

This method also supports the solutions of inequalities. In the study the conjunction $-3 < \frac{1}{2}x - 1 < 2$, it is possible to translate the solution to "where the graph of the linear function $f(x) = \frac{1}{2}x - 1$ lies between the graphs of the constant functions $g(x) = -3$ and $c(x) = 2$", as in Figure 5.



The geometric solution set is (-4,6).
Fig. 5

More generally, we can apply this additional powerful geometric view to the solution of $f(x) = g(x)$ and/or $f(x) < g(x)$, as in Figure 6.



$f(x) = g(x)$ when $x = x_1, x_2, x_3, x_4$ and is visually translated to those points where the two functions intersect.

$f(x) < g(x)$ when $x \in (x_1, x_2) \cup (x_3, x_4)$ or, geometrically, the region of the x axis where the graph of f is below the graph of g.

Figure 6

This concept can be extended to absolute value equations. Take, for example $|x+3| = |\frac{1}{2}x-2|$. Solving algebraically the equation has two solutions, $x = -\frac{2}{3}$ and $x = -10$. These solutions are expected, because of the intersections of the graphs, as shown in Figure 7.

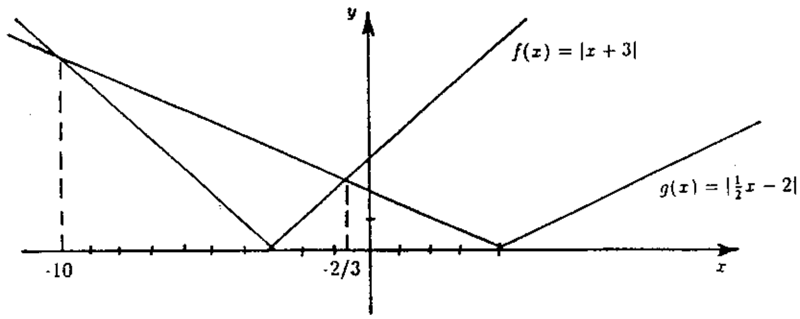


Figure 7

The solution sets of absolute value inequalities can be geometrically interpreted also. For $|x+1|+|x-1|\leq 3$, moving one of the absolute value expressions to the other side, as $|x+1|\leq 3-|x-1|$, simplifies graphing. The solution, found algebraically to be [-1.5, 1.5], translates geometrically to "where the graph of $f(x)=|x+1|$ is below or on the graph of $g(x)=3-|x-1|$," as in Figure 8.

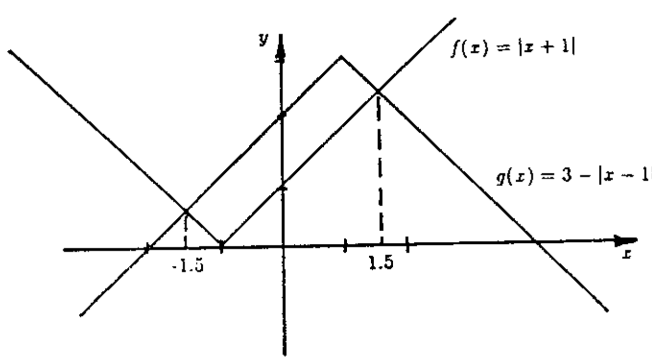


Figure 8

An easy method to graph $|x+1|$ is to graph $f(x)=x+1$ and rotate each point where $y<0$ directly above (symmetric with respect to the x-axis) (Figure 9). the graph of $3-|x-1|$ is easily plotted using compositions of functions as follows:
a.) plot $g(x)=x$-1; **b.)** fold to get |x-1|; **c.)** rotate the entire graph around the x-axis to form $-|x-1|$; and **d.)** raise the entire graph 3 units up to obtain $3-|x-1|$ (Figures 10-11).

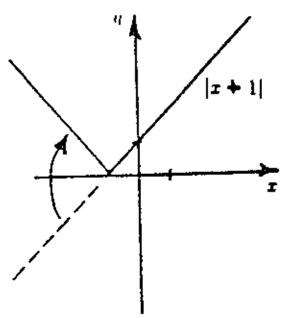


Figure 9

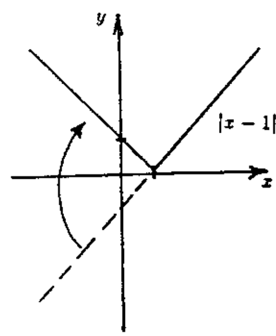


Figure 10

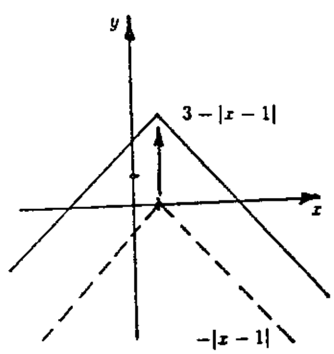


Figure 11

In extending this graphical method to solutions of inequalities such as $|x^3\text{-}2x|$ - $|x\text{-}1| < 0$, the solver determines the regions, and thus the equations, which are to be solved to locate the exact boundaries of the intervals in the solution set. Adding $|x\text{-}1|$ to both sides yields $|x^3(x\text{-}2| < |x\text{-}1|$. The solution, $(x_1,\ x_2) \cup (x_3,\ x_4)$ translates to "where the graph of $f(x) = |x^2(x\text{-}2)|$ is below the graph of $g(x) = |x\text{-}1|$."

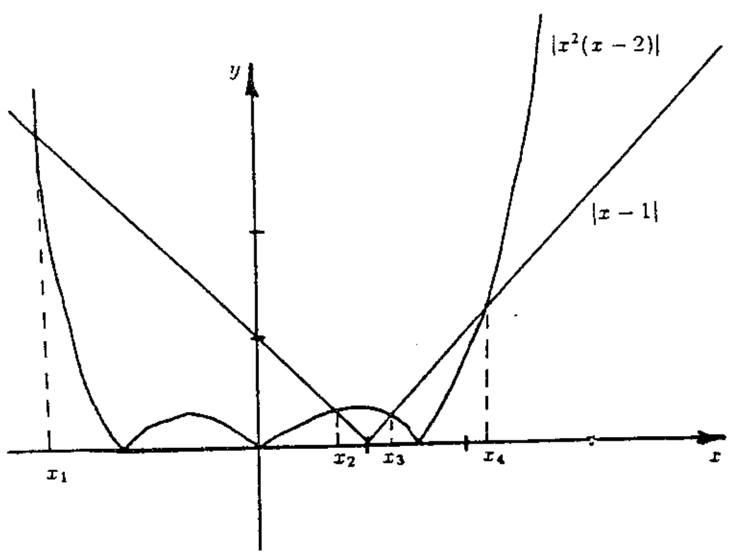


Figure 12

This approach to viewing solutions to equations and inequalities has been used in the author's Algebra II Honors and Precalculus classes. Students have readily accepted it and are able to quickly sketch the graph and determine if algebraically found solutions are correct. It is this author's opinion that this method of viewing solutions to equations and inequalities could be started and used successfully as early as Algebra I. A sense of the geometry of the solution, will benefit students as they continue their studies in mathematics. As the calculus reformation movement develops, students will be expected to increase their use of calculus in problem-solving situations, to interpret their solutions, and to draw informed conclusions. With the use of graphing calculators and computers with sophisticated calculus software, the interpretations of graphically presented solutions and the opposite activity, graphing numerical and functional solutions, will become more important.

The algebraic/geometric approach to solutions of equations and inequalities emphasizes the fundamental connection between the algebraic (symbolic) and the geometric (intuitive) views of the same problem, yielding a more powerful problem-solving method.

ON A THREE DIMENSIONAL PYTHAGOREAN THEOREM AND THE DISTANCE BETWEEN A POINT AND A PLANE

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Inspiring students to think, to reason and to hypothesize can be accomplished by encouraging them to generalize. For example, they could be challenged to find non-trivial analogs of the well known Pythagorean theorem. Some generalizations are well known, others are too easy to be of interest, and others still remain beyond the horizon of present knowledge, as Fermat's last theorem. Of course, the law of cosines provides a powerful generalization to non-right triangles, whereas formulas involving semi-circles or similar polygons constructed on the sides of the original right triangle can be routinely verified.

Of greater immediate interest, students possessing algorithmic intuition may suggest increasing the dimension of the ambient space. In three dimensional space, a tetrahedron having three right-triangle faces may be seen as a natural analogue to a right triangle in the plane. The purpose of this note is to prove a three dimensional analogue of the Pythagorean theorem for such a tetrahedron. Although other proofs of this formula have been given before (see [2] for example, where a proof based on vector products is presented), we believe our proof to be very elementary. In fact, it only uses the notion of distance between two points in space. As a consequence, the formula for the distance between a point and a plane is easily obtained, provided one knows how to find the volume of a tetrahedron.

1. A Three Dimensional Pythagorean Theorem

Consider the tetrahedron depicted in Figure 1. Without loss of generality, assume the right-angle vertex is located at the origin O and the other three vertices P, Q and R have coordinates (h, O, O), (O, k, O) and (O, O, ℓ) respectively, where $hk\ell \neq 0$; thus, they lie on the coordinate axes.

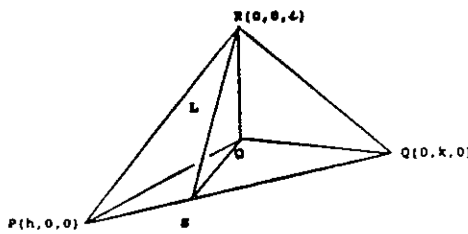


Figure 1

It follows that on the xy-plane, the line through P and Q has equation

$$kx + hy - hk = 0,$$

whereas its perpendicular through the origin has equation

$$hx - ky = 0.$$

Solving this linear system, one gets

$$x=\frac{hk^2}{h^2+k^2}, y=\frac{h^2k}{h^2+k^2}.$$

In figure 1, S is the point with coordinates

$$\left(\frac{hk^2}{h^2 + k^2}, \frac{h^2k}{h^2 + k^2}, 0\right)$$

and L is the distance between R and S. Therefore,

$$L = \sqrt{\left(\frac{hk^2}{h^2 + k^2}\right)^2 + \left(\frac{h^2k}{h^2 + k^2}\right)^2 + \ell^2}$$

$$= \sqrt{\frac{h^2k^4 + h^4k^2 + (h^2 + k^2)^2 \cdot \ell^2}{(h^2 + k^2)^2}}$$

$$= \frac{1}{h^2 + k^2} \sqrt{(h^2 + k^2)(h^2k^2 + h^2\ell^2 + k^2\ell^2)}\,.$$

Therefore,

(1) $$L = \frac{1}{\sqrt{h^2 + k^2}} \sqrt{h^2k^2 + h^2\ell^2 + k^2\ell^2}\,.$$

It follows that the area A of the "front-face" of the tetrahedron is given by

$$A = \frac{L}{2} \sqrt{h^2 + k^2};$$

consequently, it follows immediately from (1) that

$$(2) \qquad A = \frac{\sqrt{h^2k^2 + h^2l^2 + k^2l^2}}{2}.$$

Let the remaining faces, having areas B, C and D, be chosen so that

$$B=\frac{hk}{2}, C=\frac{kl}{2}, D=\frac{hl}{2}.$$

We have just proved the following result:

THEOREM (Three dimensional Pythagorean theorem) Let a tetrahedron have three pairwise perpendicular edges. If A is the area of the face opposite the right angle and B, C, D are the areas of the other faces, then

$$A^2=B^2+C^2+D^2.$$

2. The Distance Between a Point and a Plane

In [1], the formula for the distance between a point and a line in the plane was obtained by computing the area of a right triangle in two different ways. (A similar approach was presented in [3]). This time we shall use the corresponding three dimensional approach to write the formula for the distance between a point and a plane.

Consider a plane Π with cartesian equation

$$Ax+By+Cz+D=0$$

and a point P having coordinates (x_0, y_0, z_0). Without loss of generality, assume that P is not in Π and that $ABC \neq 0$, because otherwise the problem is trivial.

Let h, k and l be non-zero real numbers such that Π is the plane determined by $(x_0 - h, y_0, z_0)$, $(x_0, y_0 - k, z_0)$ and $(x_0, y_0, z_0 - l)$. This means that the distances between P and the points $(x_0 - h, y_0, z_0)$, $(x_0, y_0 - k, z_0)$ and $(x_0, y_0, z_0 - l)$ are $|h|$, $|k|$ and $|l|$ respectively, and also that

$$(3) \qquad \begin{cases} A(x_0 - h) + By_0 + Cz_0 + D = 0 \\ Ax_0 + B(y_0 - k) + Cz_0 + D = 0 \\ Ax_0 + By_0 + C(z_0 - l) + D = 0. \end{cases}$$

In order to simplify the notation, let $\Gamma = Ax_0 + By_0 + Cz_0 + D$.

Then

$$(4) \qquad \Gamma = Ah = Bk = Cl,$$

an immediate consequence of (3).

Consider now the tetrahedron with vertices P, $(x_0 - h, y_0, z_0)$, $(x_0, y_0 - k, z_0)$ and $(x_0, y_0, z_0 - l)$. Let d be the distance between P and Π and let A be the area of the face not containing P; see Figure 2.

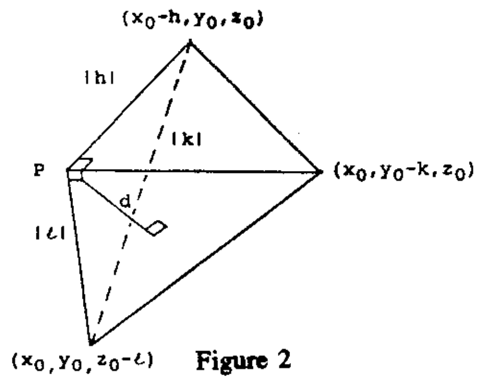


Figure 2

Since the volume of a tetrahedron equals one third the product of the area of the base times the height, then

$$\frac{1}{3}Ad=\frac{1}{3}[\frac{|hk|}{2}]|l|,$$

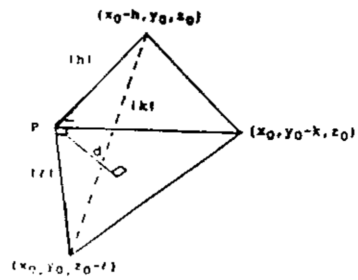
by computing the volume of the tetrahedron in two different ways. Clearly, this implies that

$$d=\frac{|hkl|}{2A}.$$

Using (2) and (4), one gets that

$$d = \frac{\dfrac{|r|^3}{|ABC|}}{\sqrt{r^4\left(\dfrac{1}{A^2B^2}+\dfrac{1}{A^2C^2}+\dfrac{1}{B^2C^2}\right)}}$$

Consequently,



which is the formula for the distance between the point (x_0, y_0, z_0) and the plane with equation $Ax + By + Cz + D = 0$.

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INFLECTION POINTS AND ROOTS: A CALCULUS CONNECTION

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In studying any mathematics idea, it is important not only to understand the separate concepts but to look for connections between concepts. Two important polynomial concepts involve roots of equations and inflection points of graphs.

All calculus students are aware that a cubic equation has exactly three roots (real or imaginary) if repeated roots are counted. They are also aware that the graph of a cubic function has exactly one point of inflection at which the concavity changes from positive to negative or from negative to positive. Many calculus students are not aware of the connection between these two concepts. The purpose of the article is to develop this connection.

Let a, b, and c be the three roots of the cubic function f(x), having real coefficients. Then, f(x) can be written as f(x) = k(x-a)(s-b)(x-c). To find the x coordinates of the inflection point, find the second derivative of f(x), set it equal to 0, and then solve for x.

Thus,

$$f(x) = k(x-a)(x-b)(x-c)$$

$$f'(x) = k[(x-a)\frac{d}{dx}(x-b)(x-c) + (x-b)(x-c)\frac{d}{dx}(x-a)]$$

$$= k[(x-a)[(x-b)\frac{d}{dx}(x-c) + (x-c)\frac{d}{dx}(x-b)] +$$

$$(x-b)(x-c)\frac{d}{dx}(x-a)]$$

$$= k[(x-a)(x-b) + (x-a)(x-c) + (x-b)(x-c)].$$

$$f''(x) + k[(x-a)\frac{d}{dx}(x-b) + (x-b)\frac{d}{dx}(x-a) +$$

$$(x-a)\frac{d}{dx}(x-c) + (x-c)\frac{d}{dx}(x-a) +$$

$$(x-b)\frac{d}{dx}(x-c) + (x-c)\frac{d}{dx}(x-b)]$$

$$=k[(x-a)+(x-b)+(x-a)+(x-c)+(x-b)+(x-c)]$$

$$=k[6x-2a-2b-2c]$$

$$=2k[3x-a-b-c].$$

Solving f"(x) = 0:

$$(3x-a-b-c)=0$$

$$x=\frac{a+b+c}{3}.$$

The x-coordinate of the inflection point is, therefore, the mean of the three roots of f(x) = 0.

Ask your students whether this would yield the possibility of an imaginary x coordinate of the inflection point, if the originial cubic function had complex roots? Will they recognize that, if a cubic has complex roots, it has exactly two which are conjugates? The sum is, therefore, real.

Also ask them what happens if the original cubic had repeated roots? Since a, b, and c are not necessarily distinct, the same formula should apply. To illustrate this, consider:

$$f(x)=(x-2)^2(x+1)$$

$$=x^3-3x^2+4$$

$$f'(x)=3x^2-6x$$

$$f''(x)=6x-6$$

Solving 6x - 6 = 0 yields x = 1. Thus the inflection point has x-coordinate 1. Using the formula which we developed earlier, we could predict that the x-coordinate of the inflection point is

$$\frac{2+2+(-1)}{3}=\frac{3}{3}=1.$$

You and your students are encouraged to look for other calculus connections.

ALGORITHMS FOR BASE TWO ADDITION

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Base two arithmetic can prove to be particularly tedious for even the best students. The following discussion provides a method which simplifies the process and, if taught properly, can aid in students' understanding of operations in other bases.

To write 100101_{two} as a base ten numeral, for example, the following place value system is employed.

2^6	2^5	2^4	2^3	2^2	2^1	2^0
64's	32's	16's	8's	4's	2's	units
	1	0	0	1	0	1

Thus we have:

$$100101_{two} = 1(32)+0(16)+0(8)+1(4)+0(2)+1(1) = 37_{ten}$$

When performing arithmetic in base two, the set of allowable digits is $S=\{0,1\}$, so that

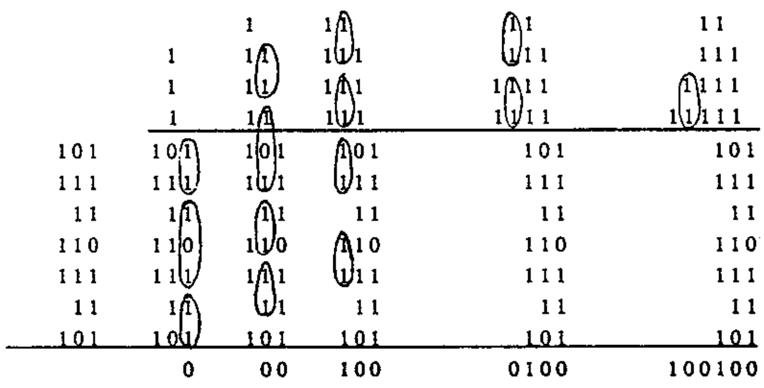
i) $1_{two}+1_{two}=10_{two}=2_{ten}$

ii) $10_{two}+10_{two}=100_{two}=4_{ten}$

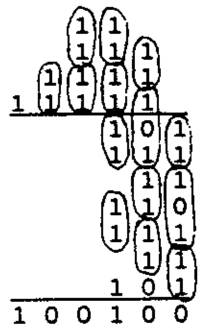
and so on. These two examples provide the rationale for the algorithm that will follow. But first, let's examine the "usual" approach to base-two addition. After polling several instructors of mathematics for elementary teachers it was determined that students generally learn to add base two numbers two addends at a time. Consider the following example:

```
   101 \        101       10101
   111 |       +111        +111
    11 |       1100       11100
   110 >        +11         +11
   111 |       1111       11111
    11 |       +110        +101
   101 |      10101      100100
------ |
100100 /
```

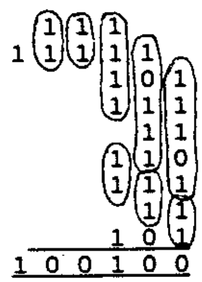
Now, let's proceed with the algorithm. Noting that in the units column each pair of "1's"will sum to 10_{two}, we may carry "1"to the "2's"column for each pair circled. The "1's"carried lie above the solid line. In a similar manner, then group by two's in the "2's"column and carry "1"for each pair of "1's"circled since $10_{two}+10_{two}=100_{two}$. Continue with the third column, the "4's"place and so on.



The computation can be done all at once as shown on the next page.



The algorithm can be extended by noting that $1_{two} + 1_{two} + 1_{two} + 1_{two} = 100_{two}$. Thus grouping by four's allows one to carry a "1" two columns as shown.



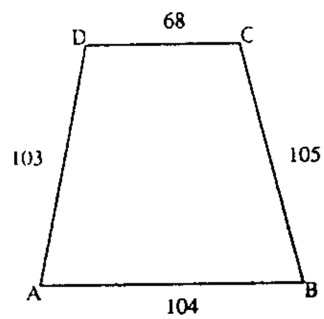
An examination of the properties of base two led to this rather obvious method for addition in base two. A careful development of the algorithm can provide for students additional insight into arithmetic in other bases.

DR. BOYD'S LAKE LOT

James R. Boone and
Bruce Treybig

Texas A & M University
College Station, Texas

This note contains a geometric exercise which we hope your students will find rewarding and challenging. Dr. Boyd owns a lot on a lake which has the shape of a trapezoid with bases 104 feet and 68 feet and sides of 103 feet and 105 feet as shown below.



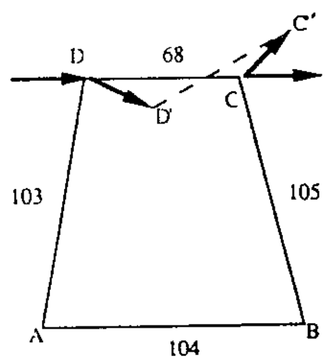
The problem is that the Tax Assessor has billed him for taxes computed for more square feet than he thinks he owns.

Problem 1.) How large is this lot? (This is a neat little exercise in the use of Heron's Formula to find the altitude of the trapezoid.)

Problem 2.) Since you are given only the lengths of the sides are you sure the shape is unique? That is, since quadrilaterals with fixed sides, say a parallelogram can be tilted, thus changing their area, is this trapezoid unique? This is a question of rigidity, such as a triangle is a rigid figure.

The fact that trapezoids with unequal bases are rigid figure follows from the discussion below: Consider the trapezoid ABCD below, where the bases satisfy $CD < AB$. Secure the base AB and move point D to the right.

Since AD is fixed, D must move down. With D moving to the right, C must move to the right also. the Causes C to move up. Thus D moving down and C moving up implies the C'D' is no longer parallel to AB. Thus, the parallel property provides the rigidity property for trapezoids with unequal bases.



MATH-E-MATICS*

Chorus

Math-e-matics,
Math-e-matics,
Take it every year.
If you have (insert teacher's name),
You'll have nothing to fear.
Oh Math-e-matics,
Math-e-matics,
Take it every year.
If you have (insert teacher's name),
You'll have nothing to fear.

Verse

She can teach you trig. or geometry,
You will learn to factor in algebra, you'll see.
So list'n to what I say,
'Cause what I say is true.
You can use MATH in your life,
And MATH will work for you!

Repeat chorus.

*sung to the tune of "Jingle Bells

Contributed by William Babcock, Senior at Vilonia High School, Vilonia, AR

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