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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

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TABLE OF CONTENTS

ARTICLES

ANNOUNCEMENTS

President's Message

Karen Hall
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As president of Texas Council of Teachers of Mathematics I am asked to speak before groups that would never ask Karen Hall, mathematics teacher, to speak. I consider the opportunity to represent you to be one of the best aspects of this job. Last August I was asked to speak to the Long Range Research Seminar of Exxon Research Center in Houston. Surprised at the invitation, I wondered what I would say to a room full of Ph.D mathematicians and scientists that they did not already know. I called my talk, "Mathematics for a new Century" and told these successful participants in 20th century mathematics education about the vision of NCTM for the 21st century. I was proud to tell them that the state of Texas is participating fully in the implementation of NCTM standards and I hoped they would soon see the results in their local schools.

The questions after the talk were thoughtful, but there was one that bothered me long after the seminar. That question was "Why haven't I heard anything about this before?" Well. . . Why hasn't she heard about NCTM standards. I think we are so busy teaching our students that we think someone else, surely not us, will tell the story of the standards to the community. We are the mathematics teachers and we must participate in telling the story. If each of us would give one talk a semester that would be about 7,000 talks. To make this easier for us to do, NCTM has prepared a packet of overhead masters so we can prepare professional looking overhead transparencies to accompany our talks.

What group would like to hear your message? To list a few: the PTO, principals in your community, Junior Chamber of Commerce or any other group that is civic-minded and interested in the analytical thinking skills of the future work force. If you are nervous about doing this alone, form a group in your mathematics department or your grade-level and seek speaking opportunities.

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Thoughts and Ideas about Probability and Statistics Essential Elements in Algebra II

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Probability and statistics are slowly filtering into high school Algebra I and II. This is due to an increased awareness that these topics must be taught not only to college bound but also to non-college bound students so they may function in an increasingly quantified world. The Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989) and Everybody Counts (Mathematical Sciences Education Board, 1989) are recent documents calling for increased teaching of probability and statistics. This new emphasis is also true in Texas because essential elements specifically identify probability and statistics topics to be taught in algebra. In this paper I review the essential elements dealing with probability and statistics in Algebra II and present ideas on how to implement them in the classroom.

Essential Elements

The essential elements (EEs) for probability and statistics in Algebra II are as follows (Texas Education Agency, 1989):

9.1 Recognition of the importance of unbiased sampling and valid reasoning in statistical arguments
9.2 Selection of an appropriate sampling method for a given real-world problem situation
9.3 Interpretation of probabilities relative to the normal distribution
9.4 Designing a simple statistical experiment to test a hypothesis generated by a real-world problem situation and interpretation of the results
9.5 Use of computer simulation methods to represent and solve problem situations involving uncertainty

Some EEs may not seem very clear. For example, how can a student "... recognize the importance of unbiased sampling?" Many teachers may be unsure what material to cover. I have tried to interpret these EEs in light of the NCTM Curriculum Standards, the recently released Professional Standards for Teaching Mathematics (NCTM, 1991), the

Quantitative Literacy materials, and a recent statement by a group of statisticians on teaching statistics (Hogg, 1990).

Let's examine each of these EEs in detail.

9.1 Recognition of the importance of unbiased sampling and valid reasoning in statistical arguments.

Probably the best way to recognize the importance of unbiased sampling is to examine situations in which the sampling has been biased and note how the bias can severely limit the information from the sample. Many common biases, like the common cold, are with us today even though experts have been fighting them for years. Some common sampling errors are readily apparent in perhaps the biggest statistical fiasco of all time: the 1936 Literary Digest poll.

In 1936, the Literary Digest, a popular, well-respected magazine which had successfully predicted every Presidential election since 1916, conducted the largest poll ever undertaken. The magazine sent ten million surveys to determine who people would vote for President: Franklin Roosevelt or Al Landon. The 2.4 million surveys which were returned indicated Al Landon would win with 57% of the vote, compared to 43% for Roosevelt. In the election Roosevelt received 62% of the vote and Al Landon 38%. The Literary Digest never recovered from this embarrassment and folded shortly thereafter. The questions are, "What went wrong, and how was this poll biased?"

You might let students explore ideas themselves. Most biases that occurred are fairly obvious, after a little thought. The Literary Digest poll suffered from at least the following two biases.

1. Convenience sampling: The Literary Digest used addresses from phone books (There were only 11 million residential phones in 1936.), occupational data, its subscription list, and club memberships. People sampled from these groups were not a random sampling of voters.
2. Non-response bias: Only one survey in four was returned. Even if the original ten million were a random sample, the people who expended the effort to return the survey probably would not represent a random sample of American voters. Volunteers usually do not represent the population sampled.

In general, the larger the sample, the more accurate the estimate. However, the Literary Digest poll dramatically illustrates that a large sample does not "save you" if the sample is biased.

Another common sampling bias is response bias in which responses are affected by the phrasing of a question, the order of questions, or the tone, attitude, and/or physical characteristics of the interviewer. A 1990 New York Times/CBS News poll provides a good illustration of response bias. When asked, "Do you think there should be an amendment to the Constitution prohibiting abortions?" 29% were in favor and 62% opposed. When the same group of people were later asked, "Do you believe there should be an amendment to the Constitution protecting the life of the unborn child?" 50% were in favor and 39% opposed (Moore, 1991).

Identifying biases is important so students can make more intelligent judgments of data in the media and so students know what can go wrong if they design an experiment themselves (see EE 9.5).

Two good references are the Quantitative Literacy series Exploring Surveys and Information from Samples (Landwehr, Swift, & Watkins, 1987), and Statistics: Concepts and Controversies (Moore, 1991). Both references have projects and problems appropriate for Algebra II students. Moore includes many media reports which illustrate biased sampling. Some suggested projects are as follows.

Suppose you want to conduct a poll of juniors about a sensitive subject. Describe how you would conduct a poll which (1) is a convenience sample, (2) has no response bias, or (3) has response bias (illustrate several types).

Imagine you are the leader of a small organization with a political agenda. Describe some activities you could do to make it look as if your ideas were held by many Americans.

9.2 Selection of an appropriate sampling method for a given real-world problem situation.

Now that many sampling problems have been identified, how do you take an unbiased sample? At this level, the best method is a simple random sample. To take a simple random sample, identify a population to be sampled and then select a predetermined number from the population so that each object or person has the same chance of being selected. This can be done by simple mechanical means (e.g., drawing numbers from a hat) or by using a random generating device (e.g., rolling a die).

Essential elements 9.1 and 9.2 are very closely related. EE 9.1 states that students should see the importance of unbiased sampling, and EE 9.2 states that students should be able to take a simple random sample of a real-world population. If a student conducts an experiment (EE 9.5), finding a simple random sample may be the most important part of the entire project.

The references for EE 9.1 are also appropriate for EE 9.2.

9.3 Interpretation of probabilities relative to the normal distribution.

The normal distribution is often called "the bell curve" and is characterized by its mean, mu (μ) and standard deviation, sigma (σ). These indicate the location and "spread" of the curve, respectively. Although probability tables of the standard normal distribution (μ is zero and σ is one) are readily available, it is important students know that if you have n independent observations of a normal population, then about:

68% of the observations lie within $\mu \pm 1\sigma$;
95% of the observations lie within $\mu \pm 2\sigma$; and
99.7% of the observations lie within $\mu \pm 3\sigma$.

Most observations lie within two standard deviations from the mean, so the "bell" is approximately 4 standard deviations wide. This provides a "quick and dirty" estimate of sigma. Since the range of data is about 4 standard deviations wide, then the standard deviation is approximately the range divided by four. [Ed.: The expected value of the range is approximately 4σ when the sample size is 25 to 35 but increases to 6σ when the sample size increases to 400.]

If there are enough observations, usually thirty or more, we can replace μ with the average (denoted by $\overline{x}$) and replace σ with its estimate (denotes by s), which is the square root of the average squared deviation from the mean:

$$s = \sqrt{\frac{1}{n}\Sigma_{i=1}^{n} (x_i - \overline{x})^2}$$

Although the usual estimate of s has n-1 in the denominator instead of n, the difference is minor and it makes more intuitive sense to divide by n. In a pinch, you could even use the range estimate of the standard deviation: take 1/4 the range of the data as an estimate of s.

The coin-flipping (binomial) model is very common (e.g., polls follow this model), and you may want to mention how to find the estimate of the mean and the standard deviation of this model. If you flip a coin n times, where the probability of heads is p on each flip, then the mean, μ, will be np and the standard deviation, σ, will be $\sqrt{np(1-p)}$. These can then be used in the Central Limit Theorem. Here are some examples of real-world problems which may be interesting. In order to solve these, it helps tremendously to sketch the normal curve, shade the part of the curve desired, and refer to the Central Limit Theorem.

1. The SAT has a mean of 950 and a standard deviation of 100. Suppose you score a 1200. Estimate your percentile ranking.
2. You are a civil engineer. Your assignment is to come up with a plan to replace light bulbs in traffic lights before they burn out. Suppose each bulb has an average lifetime of 5000 hours with a standard deviation of 300 hours. How often should you change each bulb so that, on the average, only 2% of the bulbs burn out before they get changed?
3. Suppose you are the campaign manager of a Texas gubernatorial candidate In 1994, and the race is close. You want to conduct a poll. How many people must you poll to be reasonably sure to get an answer within 3 of the correct proportion of voters who favor your boss?
4. In 1938, a famous experiment was conducted at Duke University. A student was shown one of five randomly chosen cards and concentrated on the pattern on the card, and another student was to guess which of the five cards was being watched. The students were correct 12,489 times out of 60,000 attempts. Give a cogent, objective argument whether this experiment supports or does not support the existence of ESP.

9.4 Designing a simple statistical experiment to test a hypothesis generated by a real-world problem situation and interpretation of the results.

In order to perform a statistical experiment, a student must know how to take a random sample (EE 9.2) and perhaps use the normal distribution (EE 9.3). In order to better present and communicate the data, graphical techniques, such as line plots, stem and leaf plots, boxplots, histograms, and scatterplots may be necessary (the best guide can be found in Exploring Data, by Landwehr & Watkins, 1987). Plotting techniques are supposed to be covered in Algebra I, but of course it might take a few years to "trickle down." Some ideas for experiments follow. Although statistical projects could be assigned, encourage each student or team of students to select their own experiment. An experiment does not have to be elaborate; rather, attention should be concentrated on the care of selecting the sample, formulating a result, and clearly communicating the result to others.

Ideas for Experiments and Reports

1. Ratings for pop records
2. Hours watched of TV per week
3. Students' "Nielsen's Ratings"
4. Food items bought from vending machines
5. Strength of rubber bands
6. Rubber band shooting distance
7. Human memory experiments
8. Paper clip endurance (how many bends till it breaks ?)
9. Polls of all kinds (see EEs 9.1 and 9.2. In particular, students could study the effect of biases when conducting a poll.)
10. Study time vs. test grades
11. Stock market analysis
12. Fast food nutrition
13. Historical population trends (see The Texas Almanac)
14. Injury ratings of cars (see Consumer Reports)
15. Sports analysis
16. Testing the strength and absorbency of paper towels

EE 9.5: Use of computer simulation methods to represent and solve problem situations involving uncertainty.

Although the EE specifically states computer simulation, students must have a feel for simulation using common random generating devices such as coins, dice, cards, slips of paper, or random digit tables before graduating to the use of a computer simulation (see NCTM Standards, p. 139). The computer is a device which can allow you to flip a coin very fast and to keep accurate tabs on the results, and cannot do anything a student couldn't do with a concrete random generating device, given a lot of time and patience. The computer must not be viewed as a black box possessing occult powers.

The computer requirement in this EE is troubling for another reason: few mathematics classrooms have access to class sets of computers; most likely there is a single computer available for demonstration. The new NCTM Teaching Standards state that "Tasks that promote the active involvement of all students should be selected," (NCTM, 1991, p. 139), which indicates it is better to have all students involved with simple materials than have students observe a computer demonstration in which many had no direct involvement. The best resource for simulation is The Art and Techniques of Simulation

(Gnanadesikan, Scheaffer, & Swift, 1987). Many of the sample problems below are based on similar problems in The Art and Techniques of Simulation. They are arranged from easy to more difficult. The first simulation uses the simplest random generator: a coin flip.

1. A quarterback completes 50% of his passes. Suppose he throws 10 passes in a game. Using simulation, determine the probability that at least five are completed. (Note: after running the simulation with coin flips, repeat the simulation using different devices, such as dice and a random digit table. Repeat the above but use a different percentage completion rate, such at 45% or 60%.)
2. Two baseball teams in the World Series are evenly matched. Using simulation, find the probability that the series lasts 4, 5, 6, and 7 games. (Repeat with an unevenly matched series. Use several kinds of random generating devices.)
3. You must go through three traffic lights each day to school. The probability that any one of them is green is 0.3, and they work independently. Use simulation to determine the probability that all the lights will be green on a particular day. Compare with the theoretical value.
4. McDonald's has six different toys in its "Happy Meals." Using simulation, find how many boxes you would expect to buy to get all six toys.
5. You have $10,000 and a casino has $5000. You play a game where the probability you win is 0.4, and you make $1000 bets. What is the probability that you break the bank? (Variation: change the probability of winning. Variation: change the size of the casino's reserves. Variation: Change the size of the bets.)
6. You are a contestant on a game show. There are three curtains: behind one of them is a new car, behind the other two are goats. You pick a curtain. The host, who knows where the car is, opens one of the other curtains and reveals a goat. He asks, "Do you want to change curtains or stick with your first choice?" What should you do? Use simulation to determine your strategy. (See the recent "Ask Marilyn" article.)

CONCLUSION

Probability and statistics are here to stay, and that should be a good thing. Students who do not respond very well to standard Algebra II material may find this material fresh, interesting, and relevant. Though the curriculum is crowded already, some of these topics can be dovetailed with standard Algebra II fare or conducted outside of class. For example, simulations can be done at home. The function $\exp(-x^2)$ can be graphed and used to illustrate an even function, an exponential function, and horizontal asymptotes. The function $\exp(-(x-\mu)^2)$ can be used to illustrate translation of function. Modeling scatterplot data can reinforce equations of lines, parabolas, exponential curves, and illustrate the concepts of asymptote and slope. (See Schultz & Rubenstein, 1990, for more ideas of how to integrate probability and statistics into algebra.)

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Applying Place Value Concepts and Logical Thinking Strategies

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We all know children who avoid using mathematics, because they do not feel comfortable with multi-step computations and multi-step problems (even when the problems are realistic applications of mathematics). Lack of understanding of place value is a great source of difficulty for older children when performing multi-step computations and when solving everyday problems involving place value in various components.

Alternative approaches to practice of basic place value concepts, rather than additional computation exercises, may enhance understanding and increase competence in computation. Here are four interesting non-computation puzzles in which children have to make decisions based on simple place value concepts and related logical thinking.

Puzzle 1: Complete the table to find each child's number.

Clues for Puzzle 1:

1. Amy's number is not the same as three tens.
2. Steve's number is not greater than five tens.
3. Rebecca's number is more than three tens and less than seven tens.

	Steve	Rebecca	Amy
30			
50			
70			

Puzzle 2: Complete the table to find each pet's I.D. tag number.

Clues for Puzzle 2:

1. Rover's tag does not have the smallest number.
2. Spot's number is not the largest.
3. Polly's number has a seven in the ten's place.

	7	67	75
Spot			
Polly			
Rover			

Puzzle 3: Complete the table to find the number of baseball cards for each person.

Clues for Puzzle 3:

1. Ado's number does not have a zero in the ones place.
2. Avon's number has fewer than seven tens.
3. Alicia's number has zero tens.

	7	37	70
Ado			
Alicia			
Avon			

Puzzle 4: Complete the table to find each person's birthday.

Clues for Puzzle 4:

1. Cash's birthday is not the largest number.
2. Buzz' birthday has more than one ten.
3. Amos' birthday has the smallest number of tens.

	13th	27th	31th
Amos			
Buzz			
Cash			

Extensions

As extensions of the previous type problems, consider the following suggestions:

- Ask students to explain how each clue selects some alternatives and/or eliminates others.
- Ask students to mark the table to keep track of the information given in each clue. (For example, use an "N" to mark alternatives eliminated and "Y" to mark alternatives selected.)
- Ask students to explain how they checked that each alternative is matched to one and only one name. (For example, there is only one "Y" in each column and each row.)
- Let students create their own puzzles. What are the criteria for a "good" puzzle?

Teaching Suggestions

Some suggestions for adapting these type problem to your own class follow:

- Substitute children's names in the class, names of their pets, or names of important people. For example, famous Black Americans can be used especially during Black History Month. Create new puzzles using dates of important holidays or school events.
- Create new puzzles using three-digit numbers and clues involving the hundreds place.
- Let children work in teams to solve puzzles and to create new ones.

Answer Key

Puzzle 1. Steve 30; Rebecca 50; and Amy 70.
Puzzle 2. Spot 7; Polly 75; and Rover 67.
Puzzle 3. Ado 70; Alicia 7; and Avon 37.
Puzzle 4. Amos 13; Buzz 31; and Cash 27.

Making Connections in Algebra: Problem Solving and Multiple Representations

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Too often in algebra instruction, topics are treated in isolation through independent practice on specific objectives. The NCTM Standards (1989) calls for real-world applications which facilitate the construction of connections among topics in algebra. It is important that students develop an "intuitive" feel for mathematics through multiple representations, including concrete/pictorial representations, and through exploration and problem solving before they are inundated with abstract, symbolic notation. The following is a discussion of a method for taking a simple problem situation and examining much of the "linear" part of beginning algebra in a way that allows students to make connections necessary to build a strong conceptual framework of algebra. The problem is called the Birdhouse Problem.

First the use of tables and guess and check strategies gives rise to the notion of introducing and using a variable to solve the problem. The same problem situation is then used to explore the idea of slope, the notion of the line, systems of equations, and functions. The problem is simple enough so that it can be altered to suit the reader's particular teaching situation. Some of the sequencing is optional since the topics are, in fact, connected.

Problem Solving and the Notion of Variable

Algebra students typically have difficulty with variables, particularly when the concept of variable is introduced without connecting it to student experiences. Usiskin (1988) offers a comprehensive view of five ways in which variables are used in algebra. Variables can be used to demonstrate an arithmetic pattern as in $a+b = b+a$. A variable can represent an "unknown" - something to be solved for. A variable can take on different values as a parameter in the truest sense of varying. Variables can be manipulated, and variables can be use for storage, as in computer programming. It is important that students be aware of the role which a variable plays in a particular situation. Here problem solving is used to develop the concept of variable by investigating how variables take on values, are used as unknowns, and are manipulated. Consider the following problem.

The Birdhouse Problem: A couple, in an effort to alleviate some of the boredom in their lives, decides to build birdhouses in their spare time and sell them at the local flea market. After some experimentation with the project, they decide to buy some power equipment (saws, drills, etc.) at the local Shears store. Their monthly payment for the equipment is $60, and the monthly rental for the booth at the flea market is $20 for a total of $80 per month. They estimate that the materials for building each birdhouse cost $3. They also estimate that they can build an average of one birdhouse per day. They determine that they can sell the birdhouses for $8 each. They are interested in investigating monthly costs and monthly revenue.

This situation can be represented by using problem-solving strategies which include making a table or chart. Students would be asked to complete a table. (Only a completed table is shown.)

The couple can build at most 30 or so birdhouses per month. Students would complete the table in small groups. The teacher would summarize the students' results would as follows to allow for the transition to the "use a variable" strategy.

Number of Birdhouses	Cost = $3 Each + $80	Revenue = $8 Each
0	Cost=$3(0)+$80=$80	Rev. = $8(0)=$0
5	Cost=$3(5)+$80=$95	Rev. = $8(5)=$40
10	Cost=$3(10)+$80=$110	Rev. = $8(10)=$80
15	Cost=$3(15)+$80=$125	Rev. = $8(15)=$120
20	Cost=$3(20)+$80=$140	Rev. = $8(20)=$160
25	Cost=$3(25)+$80=$155	Rev. = $8(25)=$200
30	Cost=$3(30)+$80=$170	Rev. = $8(30)=$240

After some time the couple begins to feel that spending all their spare time on this project is for the birds. They decide to build and sell just enough birdhouses each month to break even. Determine how many birdhouses they must build and sell to break even.

Here students will use a guess and check strategy until they discover that 16 birdhouses cost $128 and also sell for $128. The table provides an opportunity to introduce the idea of using a variable to represent different values. Notice in the chart that the cost depends on the number of birdhouses. The cost "varies" according to how many birdhouses are built. We can replace "$3 Each" with the expression $3x. Notice that "$3 Each" means that we multiply $3 times the number of birdhouses. So $3x means $3 times x. This provides us with a second representation of the problem situation.

Cost = $3x + $80

and Revenue = $8x.

Students can create the following table.

Number of Birdhouses	Cost = $3x + $80	Revenue = $8x
0	Cost=$3(0)+$80=$80	Revenue=$8(0)=$0
15	Cost=$3(15)+$80=$125	Revenue=$8(15)=$120
16	**Cost=$3(16)+$80=$128**	**Revenue=$8(16)=$128**
17	Cost=$3(17)+$80=$131	Revenue=$8(17)=$136

Before proceeding to the next activity, it is important that students investigate the rules for solving simple equations, ideally, using concrete models to allow for the connection among the concrete, pictorial, and symbolic representations of equation solving. The Birdhouse Problem can be used to show students "why in the world would we ever want to solve for x." We found that 16 birdhouses cost $128 to build and generate $128 in revenue. Is there another way to solve the problem? In the "looking back" step of problem solving, we are interested in making a generalization, if possible, that will allow us to solve problems which have the same structure. Students will immediately see the benefits of using a variable in problem situations like this to reduce the amount of work necessary to solve the problem.

Now we are interested in the break-even point so we have the following analysis:

$$
\begin{aligned}
\text{Revenue} &= \text{Cost}\\
\$8x &= \$3x + \$80\\
\$8x - \$3x &= \$3x + \$80 - \$3x\\
\$5x &= \$80\\
\$5x/\$5 &= \$80/\$5\\
x &= 16\\
\text{Check: } \text{Cost} &= \$3(16) + \$80 = \$128\\
\text{Revenue} &= \$8(16) = \$128
\end{aligned}
$$

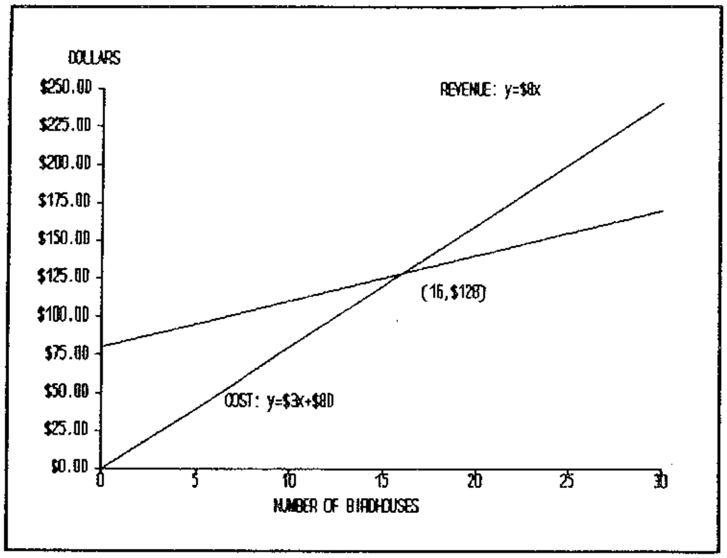
By solving for x, the student experiences using a variable as an unknown and manipulating a variable.

The Geometric Representation

The Birdhouse problem can be used again to introduce the Cartesian Plane and the equation of the line. In our effort to "find x" we essentially solved a system of linear equations problem by substitution. The groundwork has been laid here for the student to make the connection between this simple problem and the more complex notion of solving systems. Here we make a necessary link. It is often interesting to graph relations like cost and revenue on a coordinate grid which provides us with a third representation of the problem situation. The number of birdhouses will again be called x and the cost or revenue will be called y so that we can graph ordered pairs of the form (x,y). Students should be familiar with plotting points in the first quadrant before this activity.

Students can make a table similar to the previous tables to show the ordered pairs to be plotted. Graph the COST points and connect them, and on the same grid, graph the Revenue points and connect them. See the graph on the following page.

From the following graph it appears that all points on both graphs lie on a straight line, and the point of intersection is precisely the bread-even point. This gives us yet another representation, a geometric representation. The problem provides a simple example of solving a system graphically, a topic that would be invested in more depth in a subsequent lesson.



We now consider more closely the cost graph and investigate the notion of slope.

Slope and the Equation of a Line

This problem provides students an opportunity to explore and develop an understanding of slope. Consider the Cost equation:

Cost: y = \$3x + \$80.

(Graph this equation, and on the graph show the right triangles involved in the following discussion.) When the number of birdhouses increases from 5 to 15, the cost increases by \$30 (\$140 - \$125). The ratio of the cost increase to the production increase is:

\$15/3 = \$5

per birdhouse. When the number of birdhouses increases from 5 to 15, the cost increases by \$30. The ratio of cost increase to production increase in this case is

\$30/10 = \$3

per birdhouse. Students can see that here slope is the increase in cost per birdhouse. In fact, we can take any two points and show that the ratio of the change in cost to the change in the number of birdhouses is always \$3 per birdhouse. In this way, students can develop an intuitive sense of slope and an understanding of the idea that on a line the slope between any two points is a constant. This is the slope of the line. The slope is seen, naturally enough, as the coefficient of x when the equation is of the form y = \$3x + \$80.

From this point, the students will be able to develop a fuller understanding of the various forms of the equation of the line. This simple example can be used to demonstrate the point-slope, slope-intercept, and general forms of the equation of a line.

Representation as Functions

Finally, we consider function notation. Notice that for each number in the first column, we get exactly one cost value and one revenue value. Notice also that the cost of 5 birdhouses is \$95. We will use the symbol C(5) or "C of 5" for the cost of 5 birdhouses. C(5) = \$95. Similarly R(5) = \$40. In the table x represents the number of birdhouses.

x	C(x)=\$3x+\$80	(x,y)	R(x)=\$8x	(x,y)
0	C(0)=\$3(0)+\$80 =\$80	(0,\$0)	R(0)=\$8(0) =\$0	(0,\$0)
5	C(5)=\$3(5)+\$80 =\$95	(5,\$95)	R(5)=\$8(5) =\$40	(5,\$40)
10	C(10)=\$3(10)+\$80 =\$110	(10,\$110)	R(10)=\$8(10) =\$80	(10,\$80)
15	C(15)=\$3(15)+\$80 =\$125	(15,\$125)	R(15)=\$8(15) =\$120	(15,\$120)

We call C the cost function and R the revenue function. The table of x and y values is a good real world example to introduce the ideas of domain and range. The number of birdhouses which we are interested in lies between 0 and 30.

We call the set {O,1,2,3,... ,30} the **domain** of both C and R. Here is a good example of the need for a restricted domain. The range of C is the set of cost values or {80,83,86, ..., 170} and the **range** of R is the set of revenue values {0,8,16, ..., 240}. Algebra of functions can be investigated here as well when the student considers the profit function, P, as P=A-C.

Conclusion

A problem situation which can be represented in a variety of ways provides an excellent vehicle for students to develop an understanding of the concepts of algebra by allowing them to make connections among the various representations. Here we have seen such a situation approached using a guess and check strategy with a chart representation, followed by solving for an unknown using a variable representation. Then the geometric representation provided a link to slope, the equation of the line, and systems of equations. Finally we represented the problem with function notation. It is certain that students will develop a fuller understanding of algebra when we avoid treating topics in isolation and allow students to make the connection among the different topics by facilitating students' construction of algebraic concepts using problem situations which can be represented in a variety of ways.

References

National Council of Teachers of Mathematics. (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: National Council of Teachers of Mathematics.

Usiskin, Z. (1988). "Conceptions of school algebra and uses of variables." In A. F. Coxford (Ed.), The ideas of algebra, K-12 (pp. 8-19). Reston, VA: National Council of Teachers of Mathematics.

ExCET Tests and Calculators

Beginning with the February 15, 1992 administration of the ExCET tests, examinees taking the Secondary Mathematics test (field 17) and the Elementary Comprehensive test (field 04) will be provided with a calculator for use during the test. Examinees taking the Secondary Mathematics test will be provided with a Texas Instruments TI-81 Graphics calculator. Those taking the Elementary Comprehensive test will be provided with a 4-function calculator. Examinees **cannot** bring their own calculators.

Texas Education Agency January 1991

Degrees in C and F

George Wheeler
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Fahrenheit and Celsius are two confusing terms.
They're in my mind I know not where, like squirming, slippery worms.
Somehow I must remember them for science and math.
As sure as I am in this school, I know they'll cross my path
These concepts are important one, yes, ones I need to know.
When flakes are falling from the sky, they'll help me know they're snow.
In the middle of the summer when the noon sun's shining bright.
And the mercury's heading upward, I'll know it isn't night.
Celsius is known by C, the alphabet's third letter,
While Fahrenheit is know as F, for sure no better.
C at zero is the point, where water's going to freeze.
F at thirty-two will do the same with ease.
If I look at two thermometers, the top is where I aim,
In F two-twelve is boiling, in C, one hundred is the same.
I know these are related, but now I must confess,
From Celsius to Fahrenheit causes me distress.
If I choose to go from one because I want to know the other,
I usually have to ask my dad, my sibling, or my mother.
I'm tired of asking and I know that I must learn the rule.
From C to F is simple from this rule I learned in school.
Nine-fifths times C plus thirty-two will tell me what I need,
From zero C to thirty-two is Fahrenheit indeed.
If I reverse the order, from Fahrenheit to C,
Five-ninths less thirty-two degrees is zero if you please.
A formula is writing, with symbols, lines and letters,
To use them right will give insight, and will my mind unfetter.
To learn these two, as I will do, what more could teacher ask.
I'll switch degrees with perfect ease, it's such a simple task.
The formulas are here for me, I'll study and I'll learn,
With just a little practice, an A is what I'll earn.

$C = (F - 32) \cdot (5/9)$ $F = (9/5) \cdot C + 32$

Editor's Note

George H. Willson has given generously of his time as Editor of the Texas Mathematics Teacher. The Texas Council of Teachers of Mathematics as an organization and we as members have been enriched by his professional service and contribution. As the new editors, I also appreciate very much his assistance in the transition process.

This journal is the primary method of communicating with Texas mathematics educators. In my opinion, a very important part of this communication process is for the members of the Texas Council of Teachers of Mathematics to communicate professionally with each other about issues of common interest. As such, the journal requires members willing to participate in the communication process. If you have teaching techniques which you have tried in your classes or ideas which you have developed or learned, please share them with your colleagues. This means speaking and also taking the time to write up your professional contributions for others to read. Then please submit your papers to me. (See the inside of the front cover for procedural details.)

Another important way to participate in the communication process is to review manuscripts submitted for possible publication. If you would be willing to read several manuscripts a year, please let me know. While reviewing is a less visible part of the publication process, it is the only way we have of knowing what you think should be shared with your fellow members.

Finally, if you have suggestions about the journal, please send them to me or to the President. We want your input.

This first issue is somewhat delayed because of difficulties producing the more than 3,000 printing labels. Lynn Walcher and John Huber are to be commended for completing this rather difficult task. Also much appreciation is due Anne Papakonstantinou for proofing manuscripts, suggesting numerous changes, and helping figure-out how to print sideways on a previously exclusively vertical word-processing system.

Joe Dan Austin
Editor

Conference for the Advancement of Mathematics Teaching

Pathways to Mathematical Power

The Conference for the Advancement of Mathematics Teaching - CAMT - is the annual Texas state conference for mathematics education at elementary, secondary, and college levels. the 39th annual conference - CAMT 1992 - will be held from August 12-14, 1992 in the Henry B. Gonzales Convention Center in the fiesta city of San Antonio, Texas.

If you would be willing to serve as a presider volunteer or assist with this meeting, please contact:

George Nattrass
Northside I.S.D.
5900 Evers
San Antonio, Texas 78238

This three-day professional enrichment experience attracts between 4,000 and 5,000 participants and brings together outstanding speakers in mathematics education in approximately 600 sessions/workshops.

As we implement the National Council of Teachers of Mathematics Standards, it is important that mathematics teachers continue to forge connections in and empower our students with mathematics. Join us in a mathematics fiesta!

George Nattrass
CAMT 1992 Program Director

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National Council of Teachers of Mathematics Annual Meeting
Nashville, Tennessee — April 1-4, 1992

Conference for the Advancement of Mathematics Teaching
San Antonio, Texas — August 12-14, 1992

If you have scheduled a professional meeting for mathematics teachers which you would like listed here, please notify the editor.

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