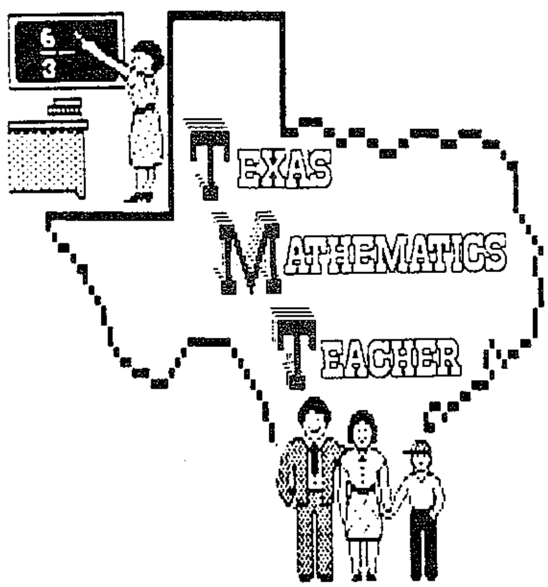
TEXAS MATHEMATICS TEACHER



Ben Jimenez, Where Are You?

Trinomial Factoring-- No More Guessing

Involving the Underrepresented in Mathematics

Cognitive Technologies with Graphing Features

OCTOBER 1990

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

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President's Message, Fall 1990

I came late to camping. I started camping when I had two preschool children and a desire to see the wilderness before it all disappeared. I bought a tent, a propane stove, sleeping bags and something brand new on the market in 1969, disposable diapers. The piece of equipment that I found most amazing was the tent, particularly the first one I owned, a cabin tent with high sides. I quickly learned that tent stability was necessary for a good night's sleep and thus the key to stability was properly placed stakes. The total number was not so important as each stake sharing the load to keep the tent upright, even against a sudden summer squall. I'm embarrassed to tell you how many times my tent fell down before I mastered the art of tent erection.

I have been thinking about stability and support as I accept the responsibilities as president of TCTM. The "stakes" in this organization are strong, stable and flexible. At the CAMT meeting in Dallas many of you told me that you wanted to become more involved in the organization and that you would like to serve on committees. The most active committee this past year is the constitution committee. We revised an outdated constitution to reflect the structure of the present organization and to remain flexible enough for growth. As the committee met to work on the constitution, the question asked about every proposed article was "How is what we are doing strengthening the purposes of TCTM?" As a result, the committee wrote a streamlined constitution and proposed a smaller executive committee.

TCTM is an affiliate of NCTM and serves a crucial role for the classroom teacher of mathematics in the state of Texas. TCTM has two representatives on this small committee, a secondary classroom teacher and a K-8 classroom teacher. This organization is your voice on the CAMT Steering Committee. Would you like CAMT to be bigger, smaller, shorter, longer? Let the committee hear from you. CAMT is for you, a person who has an active interest in mathematics. One thing we do know you want is more workshops. To have workshops, we have to have workshop leaders. Find a partner, think of a workshop and submit a proposal.

Do you need extra help convincing your school board or your principal to allow you to use calculators? At the board meeting in August, Texas Council of Teachers of Mathematics wrote the following position statement.

Calculators in the Classroom

The Texas Council of Teachers of Mathematics supports the recommendation of the National Council of Teachers of Mathematics calling for the integration of the calculator into the school mathematics program at all grade levels in classwork, homework, and evaluation. The calculator will allow students to explore mathematical ideas and to develop problem-solving strategies. Arithmetic facts and procedures continue to have a role in the mathematics curriculum, although the emphasis will shift from computation to using mathematics and solving problems.

At each grade level every student should be taught how and when to use a calculator. Assessment instruments should incorporate calculator-active questions that test understanding of mathematical concepts and their applications.

The Texas Council of Teachers of Mathematics recommends that all students use calculators to:

-- concentrate on problem-solving rather than on the calculations associated with problems;
-- gain access to mathematics beyond the students' level of arithmetical and computational skills;
-- explore, develop, and reinforce concepts including estimation, computation, approximation, and properties;
-- experiment with mathematical ideas and discover patterns; and
-- perform those tedious computations that arise when working with real data in problem-solving situations.

The Texas Council of Teachers of Mathematics recommends that publishers, authors, and test writers integrate the use of the calculator into their mathematics materials at all grade levels.

KAREN HALL
President, TCTM

BEN JIMÉNEZ, WHERE ARE YOU?

Cathy L. Seeley

Director of Mathematics,
Texas Education Agency

A recent issue of *AMP Line*, the newsletter of the American Mathematics Project, contained an article that I found compelling and moving. It was about Garfield High School, the school where Jaime Escalante teaches. (You may know of Escalante as the subject of the movie *Stand and Deliver*). Jay Mathews, with the *Washington Post*, visited the school extensively and has written a book entitled, *Escalante: The Best Teacher in America*. The article presents excerpts from a talk Mathews gave in 1988. I was fascinated to read a description of Escalante's methods and philosophy, and I think every educator in Texas and the rest of the world should be familiar with them. I was even more excited, however, about Mathews' description of the classroom of Ben Jiménez, the **other** calculus teacher at Garfield High School.

Jiménez was, according to Mathews, an "ordinary mortal," complete with the compassion that marks a dedicated teacher and, unfortunately, the low expectations of underprivileged students that so often come with that compassion. Eventually, Jiménez discovered that he could succeed in his own way with some of Escalante's methods if he adopted the same kind of high expectations that Escalante communicates so clearly to his students. Jiménez' students, too, continue to reach startling levels of achievement in mathematics.

As I have worked with educators throughout Texas, I have been struck by a number of observations. The most exciting observation that I have made is that there are teachers in Texas schools going along in their everyday teaching while reaching out to students who might otherwise have been written off. They are making a difference that can last a lifetime with students we have no reason to believe will achieve. You know the students I mean-- the "yes, but..." students:

"Yes, but some students just can't handle conceptual understanding/higher level thinking/reasoning."
"Yes, but some students can't read well enough to solve problems."

"Yes, but some students don't even speak English."
"Yes, but some students hardly ever come to school."
"Yes, but some students have never learned their facts so they can never go beyond."
"Yes, but some students never study."
"Yes, but some students can only go so far."

Personally, I do not believe that there is any student who absolutely cannot learn meaningful mathematics. For many students, we may not yet have found the key to open the door to mathematical power, and in fact, we may never find the key for every student. Fortunately, there are teachers in Texas who refuse to accept a life sentence for a student simply because no one has yet discovered what works for that student. These teachers do everything they can to find something to hook their "Yes, but..." students into mathematics and into school.

At the same time, what scares me the most about my work with Texas teachers is that hundreds of *good* teachers do not believe that all of their students can learn meaningful mathematics. These are teachers who may do a beautiful job teaching some of their *average* or *enriched* or *gifted* students. They may be teachers who get a lot of *EQ*s on their appraisals. But when I hear their remarks about how little their *basic* students can do, when I hear them ask us not to increase the level of mathematics required for high school graduation because some kids can't handle it, or when I hear them lightly mention that 40 percent of the students in their Algebra class "didn't learn it even though I taught it," I get very discouraged. If teaching is not about learning, I do not know what it is about. As described so simply in *Everybody Counts*, "If more is expected, more will be achieved." This sentiment is echoed in everything we know and are learning from work with effective schools and teacher expectations (Kerman, 1979).

For me to sit in my office and write these words is relatively easy. I do not face 150 or 200 students every day as I once did. I actually do remember that there were students who frustrated me greatly both in their behavior and in their achievement. Clearly, those of you dealing with the day-to-day stresses of the classroom have the true challenge -- and the true opportunity. You have the opportunity to make that difference with that one student.

Where are all the Jaime Escalantes in Texas? More important, where are all the Ben Jiménezes? Where are all the ordinary mortals making a difference for students? Are you one? Do you know one? How are you helping ensure that there are more of them and that they will be willing and able to stay in the classroom?

Isn't it time we claimed all the children, even the obnoxious kids and the troublesome teenagers? Isn't it time we claimed them all?

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TRINOMIAL FACTORING --

NO MORE GUESSING

Mozelle L. Williams

Doctoral Student
University of Houston
Mathematics Education
Houston, Texas

Do your students factor all the trinomials except those with leading coefficients greater than 1? If you answered yes to this question, try this method of factoring all trinomials. Trinomial factoring can be made easier for students if all trinomials factoring is treated in the same way. Removing the common monomial factor and group factoring both involve techniques that are understandable to most algebra students. Why not allow or show students that trinomial factoring can be done with a combination of group factoring and removing common monomial factors? Remember to give the students good examples for removing common monomial factors and then show them how to apply this concept to group factoring.

Examples of Common Monomial Factoring

Example 1:

$$a^2 + 2a = a(a + 2)$$

Example 2:

$$24n - 20n^2 = 4n(6-5n)$$

Examples of Group Factoring

Example 3:

$$y^2 + 3y + 4y + 12 = (y^2 + 3y) + (4y + 12)$$
$$= y(y + 3) + 4(y + 3)$$
$$= (y + 3)(y + 4)$$

Example 4:

$$ax^2 + ay - bx2 - by = (ax^2 + ay) + (-bx^2 - by)$$
$$= a(x^2 + y) - b(x^2 + y)$$
$$= (a - b)(x^2 + y)$$

Remember, use a common monomial factor that will allow for the same expression in the parentheses, like $(y + 3)$ in example 3, and $(x^2 + y)$ in example 4.

Let us try another trinomial that can be factored, and write it so that group factoring can be used.

$$x^2 + 9x + 20$$

-- Think of factors of 20 that add up to give the number 9, which is the coefficient of the x term.

5,4 are the correct factors. Now write the middle term or the x term 9x as 5x and 4x.

$$x^2 + 9x + 20 = x^2 + 5x + 4x + 20$$

Use group factoring

$$x(x+5) + 4(x+5) = (x+4)(x+5)$$

Take the trinomial $a^2 - 10a + 25$.
What are the factors of 25 that will add up to equal -10? Answer -5 and -5. Now write the middle term as -5a and -5a.

$a^2 - 5a - 5a + 25$, Factor as group factoring.
$a(a-5) - 5(a-5) = (a-5)(a-5)$.

Let us move to trinomials with a leading coefficient greater than 1.

Examples of Trinomial Factoring

$$12a^2-7a+1$$

(1) Arrange the trinomial in descending order.

$$12a^2-7a+1$$

(2) Multiply the coefficient of the squared term times the constant term.

$$(12)(1) = 12$$

(3) Write factors of 12 that will add up to equal -7.

$$(-3) + (-4) = -7$$

(4) Use these factors to rewrite -7a.

(5) Factor using group factoring

$$4a(3a-1)-1(3a-1)=(3a-1)(4a-1)$$

Note: Don't forget the minus 1 with the second monomial factor.

The trinomial has been successfully factored without trial and error or guess work.

Another Example with Trinomial Factoring

$$24x^2-2-47x$$

(1) Arrange in descending order

$$24x^2-47x-2$$

(2) Multiply the coefficients of the squared term by the constant term.

$$(24)(-2)=-48$$

(3) Write factors of -48 that will add up to equal -47.

(4) Use these factors to rewrite -47x. Factors to use: (-48) and (-1).

$$24x^2-48x+x-2$$

(5) Factor using group factoring

$$\begin{aligned} 24x^2-48x+x-2 &= 24x(x-2)+1(x-2) \\ &= (x-2)(24x+1) \end{aligned}$$

More Examples of Trinomial Factoring Using Group Factoring and Removing Monomial Factors Procedure

$$20a^2-23a+6$$

(1) $(20)(6) = 120$

(2) $120 = (-15)\ (-8)$

Note: With difficult or large numbers, use prime factors to determine the correct combination of factors.

$$(-2)(-2)(-3)(-5)(-2)$$

Now group the factors as odd plus even whose sum is equal -23.

$$\begin{aligned} (-3)(-5)+(-2) &= (-15) + (-8) \\ &= -23 \end{aligned}$$

$$\begin{aligned} 20a^2-23a+6 &= 20a^2-15a-8a+6 \\ &= 5a(4a-3)-2(4a-3) \\ &= (4a-3)(5a-2) \end{aligned}$$

Next Example

$$2t + 5t^2 + 2t-3 = 5t^2 + 2t-3$$

Multiply $(5)\ (-3) = -15$

The factors are (5) and -3) since these add up to give the coefficient of the t term.

Rewrite as $5t^2 + 5t - 3t - 3 = 5t(t+1) - 3(t+1)$
$= (t+1)(5t-3)$

Other Examples to Practice this Method:

A). $3x^2-10x+8$

B). $6y^2-5y-25$

Answers:

A. Factors (-6)(-4) since (3)(8) = 24
$3x^2-6x-4x+8 = 3x(x-2)-4(x-2)$
$= (x-2)(3x-4)$

B. Multiply (6) and (-25) and get -150.
$(-150) = (-15)(10)$
$6y^2-15y+10y-25 = 3y(2y-5) + 5(2y-5)$
$= (2y-5)(3y+5)$.

Try this method with all trinomial factoring and students will have little or no trouble with factoring more difficult trinomials with leading coefficients greater than 1.

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INVOLVING THE UNDERREPRESENTED IN MATHEMATICS

A. Cheryl Fowler

Deer Park Independent School District
Deer Park, Texas

Charles P. Geer

Texas Tech University
Lubbock, Texas

Education is changing as Texas teachers prepare students for successful careers in the twenty-first century. From pre-kindergarten programs through graduate school, educational institutions need to assist all students acquire the knowledge and skills necessary for success in an increasingly technologically oriented society. No where will the changes be more far reaching and significant than in elementary school mathematics.

Texas is already planning for these changes by developing a new set of essential elements and a state-wide testing program that emphasizes the mathematical concepts and skills needed for the next century. In addition to these curriculum changes, Texas schools must do a better job of teaching mathematics to students from the diverse populations attending school in the 1990's.

Currently, black and Hispanic students are minority populations in most of the state's school districts. By the year 2000, more than half of the students enrolled in Texas' elementary schools will be from these groups. While developing mathematics programs that successfully involve black and Hispanic students, school districts must also make sure that the girls have an equal opportunity to excel and specialize in mathematics.

All too frequently, students from these groups score lower on the mathematics section of the Texas Educational Assessment of Minimum Skills (TEAMS) test, take fewer mathematics courses in high school, and are less likely to be employed in jobs requiring advanced mathematical skills than are white males. In the 1990s these students need to receive high-quality instruction in mathematics and this instruction must begin in the elementary school. These students need to participate in programs that encourage them to study, enjoy, and appreciate mathematics. Students who have these experiences will be more likely to develop the basic foundation in mathematics necessary for later success in high school and throughout their chosen careers.

In attempting to provide information to school districts on successful techniques for involving students form underrepresented populations in mathematics, the American Association for the Advancement of Science (AAAS) summarized data collected from 168 special programs for these students. Their data indicates that if students from underrepresented populations are provided with early, excellent, and sustained instruction in mathematics, then their achievement levels parallel those of white males (American Association for the Advancement of Science, 1984). Additionally, in reviewing characteristics of successful programs, the AAAS found that the programs were similar in many ways and identified 16 characteristics that were common to most of the programs. Unfortunately, many of these characteristics involve funding sources, program leadership, or staff recruitment and are beyond the control of the classroom teacher. Five of the characteristics of these exemplary programs can be incorporated into existing classroom situations with minimal effort and expense. According to AAAS data, teachers can help students from underrepresented populations achieve greater success in mathematics by incorporating the suggestions listed below into their daily programs.

1. Present a mathematics curriculum that focuses on enrichment rather than remediation.

2. Be a highly competent teacher who believes that all students can learn mathematics.

3. Emphasize applications of mathematics and present information on careers that require mathematics.

4. Use an integrative approach to mathematics that incorporates all subject areas, hands-on opportunities, and computers.

5. Provide students with opportunities for in-school and out-of-school learning experiences.

Based on these recommendations, the authors developed six suggestions and then designed activities to integrate cultural activities with mathematics and assist teachers to successfully involve students from underrepresented populations in this important subject. In addition to providing students from these populations with mathematical experiences related to their cultural heritage, the activities also help all students become aware of the important contributions other cultures have made to mathematics. Teachers who use these and similar activities to enrich, integrate, and make more relevant their mathematics curriculum will find that all students benefit from these learning experiences.

1. Eliminate the perception of students that mathematics is a white, male prerogative by including information on the many representatives of underrepresented populations who have made significant contributions to mathematics or who have achieved success in careers requiring expertise in mathematics. Among the most famous of these are Benjamin Banneker who edited an almanac famous for its mathematics puzzles and constructed the first clock made in the United States, Ada Lovelace who programmed the world's first computer, and Emmy Noether, Einstein's colleague, who was the founder of modern algebra. Currently, mathematics teacher Jaime Escalante and astronauts Guion Bluford, Franklin Chang-Diaz, Bonnie Dunbar, and Sally Ride are well-known representatives of underrepresented populations who have used mathematics to gain prominence in their careers.

2. Integrate art, music, and physical education with mathematics emphasizing cultural activities from these areas that require the use of numbers, shapes, and designs. These subjects are especially motivating for students in the elementary school and involve many applications of mathematics. Representatives from the black and Hispanic populations have made significant contributions in all three areas. Many prominent artists, musicians, and athletes are members of these populations and excellent role models for students in the elementary school. One of them is Akeem

Olajawon, the Houston Rockets all-star center. Akeem studied mathematics as a business major and excels at basketball, a game of numbers and shapes. His advice to students is: "Stay in school and study hard, it's very important."

3. Use cooperative learning and other classroom grouping procedures which result in diverse groups. An excellent initial experience for students with cooperative learning is "The Problem of Four Choices." This problem requires groups to make four choices by selecting four different numbers from 1-6 (Example: 2, 3, 5, and 6) and then use all four numbers and any of the four basic operations to make as many numbers from 0-50 as possible. When doing this, groups must use each digit once and only once, but may combine digits to form a two-digit number (2 and 3 may be combined to form 23 or 32). Groups are then allowed to work 10-15 minutes a day for one week on the problem and see how many different numbers they can form. Regardless of the four numbers selected, it is possible to form most, but not all, numbers from 0 to 50.

4. Introduce students to ancient numeration systems such as those used by the Babylonbians, Egyptians, and Mayans. These systems provide students with information on the mathematics used by ancient cultures and give them a better understanding of their own numeration system. The numeration system developed by the Mayans about 2500 years ago is especially interesting and relevant for students in Texas. This system was one of the first to use zero and positional notation. In many ways it was more advanced than the Roman or Hindu-Arabic Systems. The Roman System never contained a symbol for zero and the Hindu-Arabic System added this symbol 1000 years after it was first used by the Mayans. Many Hispanic students will be interested in learning about their ancestors and proud of this ancient culture's mathematical skills.

5. Play mathematical games from other cultures to develop a variety of problem-solving skills. An excellent game to teach your students is Kalah, the ancient African stone game (Haggerty, 1964 and Zaslavsky, 1976). In addition to its interesting history and use by a variety of cultures throughout the world, Kalah involves lots of logic and provides students with valuable experiences in developing patterning skills at the motor and visual levels. Other board games that are excellent for promoting problem-solving

skills and relating mathematics to culture are Patolli, a game of the ancient Aztecs, Senet, King Tut's favorite game, and Konana, a Hawaiian game very similar to checkers (Krause, 1988).

6. Show Stand and Deliver to your students. This film shows the true-life experiences of Jaime Escalante, a high school mathematics teacher, as he motivates Hispanic students from an inner city high school to prepare for the Advanced Placement Test in calculus. The film (PG-rated) is available from most education service centers in Texas. It is an entertaining and meaningful learning experience for students in the upper elementary grades and above.

Teachers following the American Association for the Advancement of Science's suggestions and including activities similar to the ones described in this article in their classroom mathematics' programs will play an important role in providing students from underrepresented populations with the early, excellent, and sustained instruction in mathematics necessary for future success. Students participating in these programs will find mathematics more interesting and understandable, realize the many connections that exist between mathematics and culture, and become aware of the vital role mathematics will play in their future.

As Texas prepares for a new century, it will be essential to involve the underrepresented in mathematics and provide all students with a quality educational environment. As this is accomplished and barriers to achievement in mathematics based on culture or sex are eliminated, all students will have the opportunity to become intelligent and productive citizens of Texas, a culturally rich and diverse democracy preparing for the twenty-first century.

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EDITOR'S NOTE:

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COGNITIVE TECHNOLOGIES WITH GRAPHING FEATURES

William Tate

University of Maryland

> Investigating graphical representations and their relationships to algebraic representations can give students a real sense of the dynamic relationship between the variables. Such problem settings allow students to reason directly to, and hypothetically from, graphs (NCTM, 1989, p. 83).

Yet, research indicates more than half of middle and high school students see graphs as pictures not as mathematical representations (Bell, Brekke, & Swan, 1987; Clement, 1985; Janvier, 1978; Kerslake, 1977). This graph-as-picture error also effects as many as 50% of college level students (Mokros & Tinker, 1987; Barclay, 1985; Clement, 1985, McKenzie & Padilla, 1984).

It has been argued that constant exposure to literal "picture" graphs (e.g., bar graphs and pie charts) of numerical information during elementary mathematics and science instruction provides an adequate framework for future graphical analysis (Mokros & Tinker, 1987; Janvier, 1978). This misconception is especially alarming given that graphs provide a 'visual argument' allowing for dynamic illustrations rather than symbolic or linguistic solutions to problems (Winn, 1987). The ramifications of such inadequate instruction affects not only mathematics and science achievement but destroys true understanding of a subject such as economics or any quantifiable curriculum.

Previous research efforts have successfully reformulated children's graphing concepts through teaching experiments (Bell, Brekke, & Swan, 1981; South Notts Project, 1986; Bell & Janvier, 1981; Janvier, 1978). These experiments centered on questioning techniques which facilitated extensive discussion of strategies for interpreting graphic representations.

The availability of new cognitive technologies has improved students' power in overcoming the graph-as-picture error. The microcomputer base laboratory (MBL) and function grapher environments promote 'conceptual fluency' (e.g., Pea, 1987) by helping students become more fluent in performing routine graphing or plotting tasks that could be laborious and counterproductive to scientific thought. This environment can potentially maximize students' mental resources for problem solving and concept development.

The intent of this work is to discuss efforts to increase student understanding of graphs and their relationship to mathematical applications via electronic cognitive technologies. More specifically to examine the following:

1. The impact of technology motivated projects on content/process recommendations in the 7-12 mathematics curriculum.
2. What roles do teachers and students play in these new microcomputer learning environments?

I will discuss major problem/opportunity areas and give examples of some of the more striking components of work within the framework previously described.

CONTENT/PROCESS OBJECTIVES

According to Fey (1988) these are two major areas of technology motivated developments with respect to content/process goals:

1. Creating projects which decrease time and effort on mathematical work that can be done by machines and thus increase time for concept development.

2. Developing curriculum centered on applications of greater complexity than attainable via traditional classroom strategies.

Recent research efforts (Linn, Layman, and Nachmias, 1987; Mokros and Tinker, 1987) have utilized laboratory interface hardware and software that allows real-time graphing of data from science experiments. Mokros and Tinker (1987) suggest four reasons MBL provides a powerful environment for learning graphing skills: MBL promotes multiple learning modalities; it represents genuine scientific endeavor; it provides a real-time link between a hands-on experience and its graphic representation; it reduces the painstaking process of plotting graphs.

Function graphers provide productive methods of illustrating mathematical relationships of quantifiable variables. Specifically, function grapher software allows the user to enter functions or relations, choose their domain and range, observe the resulting graph, and then opt for changes of scale. Utilizing this cognitive technology removes the artificial restrictions of typical mathematics problems. The computer graphing approach provides geometric insight or a visual argument for various mathematical representations. According to Wachsmuth and Becker (1986) the use of technology to support visual thinking of abstract mathematical situations facilitates students ability to understand the global picture of mathematics.

To this point the discussion has centered around positive commentary concerning the use of cognitive technologies. Yet, other research implies this approach to developing conceptual understanding may be counterproductive or at best speculative. Wavering (1987) argues the increased use of computers to produce graphs for students, reduces the opportunity for the user to develop their own graphing skills. Linn (1986) calls for additional research on mechanisms which govern changes in conceptual understanding. She refers to "mechanisms of change as procedures or characteristics of the learning environment that lead to changes in the learner's conceptions of scientific phenomena" (p. 174).

Wenger (1988) notes algebra and analysis textbooks emphasize methods to graph functions, rather than methods to solve applications or promotion of conceptual fluency. The major resource of the curriculum (i.e., textbooks) do not fully promote the potential of cognitive technologies. This argument parallels Fey's (1988) second major area of technology motivated projects. The need for a curriculum which utilizes technology to solve problems of greater complexity. In response to this problem, mathematics educators (e.g., Fey and Good, 1985; Wenger, 1988) have called for a new content and reorganization of the mathematics curriculum. Wenger (1988) predicts that technology will have a catalytic effect on rethinking what we are trying to do in mathematics.

Summary- Cognitive technologies offer students visual arguments of important mathematical concepts. However, research on how students learn mathematics and development of appropriate curriculum must continue at a rapid pace to fully utilize the potential of these tool environments.

TEACHER/STUDENTS ROLES

In typical classroom, teachers demonstrate various methods to produce graphs. Cognitive technologies shift the teacher's role from presenting methodologies to promote conceptual understanding of the mathematical connections between graphs and the algebraic expression or situations it represents (Fey, 1988).

Tall (1985) provides a theoretical model for the relationship between teacher, students, and the graphing tool. The graphing tool is a *generic organizer* which enables the student to manipulate examples of a concept. The term generic infers that the learners attention is centered on properties of the example which embody important concepts. The teacher and curriculum serve as the agencies to guide students to conceptual understanding of important course objectives. The graphing tool is a microworld where the student controls the presentation of exemplars with guidance from the *external organism agents*.

Heid and Sheets (in press) report that within the context of their algebra curriculum, tool environments facilitated: students engaging in small group decision-making, and utilizing fellow students as primary resources for understanding the concept of the day. The function grapher was a vehicle for resolving assigned problems, provided a stimulus for student's questions, and enticed group exploration (Heid and Sheets, in press).

Summary- The literature which describes the use of cognitive technologies with graphing capacity contributes very little to the topic of new teacher and student roles. It would appear from those who have written on this topic, that teachers will move from traditional roles to facilitators of conceptual understanding, and students will become explorers of mathematics and its applications.

FUTURE IMPLICATIONS

The tool environments described in this work and the present projects represent powerful interventions which could enhance the way we teach school mathematics in the years to come. This research potentially could extend the capacity of human understanding of scientific phenomena. Collaborative research efforts by educators, mathematicians, scientists, and other disciplines which require quantitative analysis is required to fully exploit our golden opportunity.

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