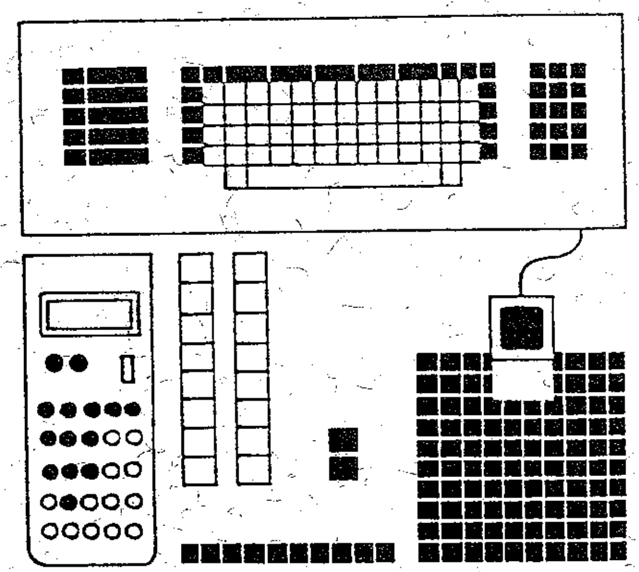
TEXAS MATHEMATICS TEACHER



Number Tricks

Numero

Equation Solving As A Process of "Undoing"

Never Say "No Slope" Again

MARCH 1990

EDITORS

TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published four times each year, in October, January, March, and May. All manuscripts and editorial correspondence should be addressed to George H. Willson, listed above.

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

March 1990

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ANNOUNCEMENTS

President's Message

Headlines from the DALLAS MORNING NEWS on January 10, 1990--BASIC SKILLS CALLED APPALLING. What mathematics teacher had to pay $0.25 to read this. Why was a federally funded survey needed to tell us that our students can not read or write. Mathematics teachers will tell everyone this for free any day. Yes, mathematics teachers can carry this much further and say without federal funds, that most American youths in public education can not compute, calculate and most definitely they can not think or reason.

Mr. Cavazos seems to have forgotten Basic Statistics 101 in several of his statements. The more that you sample, the closer the mean, median and mode tend to come together. Yes, blacks and hispanics are improving, but we are holding many students back who might do much better if we could separate them and work on their basic skills, not try to keep them all in a row and teach all the minimum skills. Just give them the opportunity to really work hard and get ahead of the pack. Colleges are forced to balance their enrollments and to recruit more and more minorities. This is not bad, but it could be better if only the very best students received scholarships according to need and ability, without regard to race or creed.

Mr. Cavazos says "Without solid literacy skills we can never expect to see improvements in mathematics or science and the cost will be staggering." We can go this statement one better and see that the technical society we are going into will be without an adequate work force. If we do not have adequately prepared workers, then someone (less prepared) must be used to keep the equipment going. This can and will lead to unsafe and unprofitable technical jobs. Making do with what one has is not sufficient. We must start to work to improve public education and mathematics must be at the very front of this charge. We have the tools - calculators, computers, technology, new and improved textbooks, and dedicated teachers who want to teach and should be able to use the above mentioned items to the best advantage. Many of our districts will not help prepare materials and tests to use the calculator with, but have their use as an after thought. This is not sufficient. We must do more to move out of the dark ages of mathematics and into the future. We must look at the book MEGATREND 1990'S and prepare our students for this society.

Recently, an article by W. A. Rock, a retired university professor of over forty (40) years experience from Sellinsgrove, Pennsylvania stated, "Our schools will not improve one whit more than they have to. If high-school graduates who can't read or grasp ideas expressed in print or who can't write coherently and solve modest mathematical problems with their shoes on are accepted by accredited colleges, such will be the product of the college-feeder mechanism. The self-serving grip of the educational establishment on school standards, which proclaim mediocrity to be excellence, will be cracked by public pressure when accredited higher education is open only to those prepared for it. People in a market economy may not get what they want, but they certainly will not get more than they demand."

Let's get together and try to turn the educational process around. We have the tools and opportunity to help. Let's do our share.

See you next issue.

OTTO W. BIELSS, JR.
President, TCTM

NUMBER TRICKS

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Number "tricks" provide a way of promoting interest in mathematics. One of our favorites is to ask each student to write down any three-digit number greater than 111, to multiply this number by nine, and, finally, to lightly cross out any nonzero digit in this four-digit product. If the student tells you the remaining three digits (in any order), you can "guess" what digit was crossed out. This is because the sum of the four digits must be 9, 18, or 27. For example, if the student tells you that the remaining digits are 5, 3, and 2, then you know that the fourth digit must be 8. If another student tells you that the remaining digits are 1, 2, and 6, then you know that the fourth digit must be either 0 or 9. However, since only a nonzero digit can be crossed out, the fourth digit must be 9. After you "guess" several correctly, some of the students will begin to see how the "trick" works. This can then lead to discussion about divisibility by 9, casting out nines, etc.

A related number trick is to ask each student to write down any three-digit number where the left digit is at least two greater than the right digit. Then, reverse the digits, subtract, and add to this difference the three-digit number obtained by reversing its digits. For example, beginning with the number 672, we would have the following computation:

$$\begin{array}{r} 672 \\ \underline{-\,276} \\ 396 \end{array}$$

$$\begin{array}{r} 396 \\ +\ 693 \\ \hline 1089 \end{array}$$

Students will be surprised to discover that the final number is always 1089. This, of course, should motivate an investigation into why 1089 is always obtained. However, a more profound question that generates deeper insight is to investigate what happens if this number trick is performed in some number base other than ten.

Let $Lb^2 + Mb + R$ represent any three-digit number in base b. As above, we assume that the left digit L is at least 2 greater than the right digit R. To subtract the number obtained by reversing the digits, we rewrite $Lb^2 + Mb + R$ as $(L\text{-}1)b^2 + (b+M\text{-}1)b + (b+R)$, which amounts to "borrowing." The computation is as follows:

$$\begin{array}{lcccc} \{(L-1)b^2 & + & (b+M-1)b & + & (b+R)\} \\ -\ \{Rb^2 & + & Mb & + & L\} \\ \hline (L-R-1)b^2 & + & (b-1)b & + & (b-L+R). \end{array}$$

Now, reverse the digits of this last number and add:

$$\begin{array}{lcccc} \{(L-R-1)b^2 & + & (b-1)b & + & (b-L+R)\} \\ +\ \{(b-L+R)b^2 & + & (b-1)b & + & (L-R-1)\} \\ \hline (b-1)b^2 & + & 2(b-1)b & + & (b-1). \end{array}$$

This final number can be written as $b^3 + (b\text{-}2)b + (b\text{-}1)$ or, more simply, $(1,0,b\text{-}2,b\text{-}1)_{\text{base } b}$. Notice that for $b = 10$ we get $(1,0,8,9)_{\text{base } 10}$. For $b = 8$ we get $(1,0,6,7)_{\text{base } 8}$ and for $b = 12$ we get $(1,0,t,e)_{\text{base } 12}$, where t and e represent the digits in base twelve having values ten and eleven, respectively.

This investigation provides motivation for further study of arithmetic in other bases and, at the same time, strengthens students' understanding of the decimal system. It also shows that the manipulation of numbers need not remain mystical. What begins as a trick evolves into a generalized mathematics problem. First, the trick captures the students' interest and, second, the magic is uncovered.

We conclude with the following number trick. Begin with any two different natural numbers (preferably small) and add them to get a third number. Continue generating a total of ten numbers by adding the two must recent numbers. The trick is to announce the sum of ten numbers before or as the tenth number is found. As shown by Gannon and Converse (1987), this sum is simply 11 times the seventh number. Students should certainly not have difficulty adding ten numbers, but discovering the shortcut (the trick) is only the beginning of their investigation.

Reference

Gannon, G. E. & Converse, C. (1987, December). Extending a Fibonacci Number Trick. Mathematics Teacher, 80, 744-747.

REFEREES WANTED

Manuscripts published in the Texas Mathematics Teacher are reviewed by at least three mathematics educators. Members of TCTM who are regular readers of the journal - classroom teachers, supervisors, and teacher educators - who would like to review manuscripts should write to the TCTM indicating their willingness to serve and the level of interest (Elementary, Secondary, College). Contact George H. Willson, Box 13857, University of North Texas, Denton, Texas 76203-3857. The Editorial Panel will review the responses and make the final selection.

NUMERO

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The following activity describes a number relationship game that can be used with students of any age, and/or adults in small groups of three or four or with hundreds. The purpose of the activity is to help participants understand place value. Each number symbol, whether it be 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9 has its actual value and also has a place value. Its position or where it is in place in a numeral determines that place value. The author has used this activity with elementary classes on every level, at children's parties, with teachers in a workshop setting and with an entire school who had to spend the lunch break in the gymnasium because of inclement weather. The best part is that the slower and/or non-mathematically oriented students have as much chance of winning as the brighter ones and a fifth grader has the same chance as a high school senior.

The materials needed are listed below. They will be explained and a sketch is provided to show what the spinner should look like.

For the Overhead Projector

- A circle of any size (divided into 3 to 10 equal segments) made on a transparency
- A cardboard spinner fastened in the center of the circle with a paper brad. Vary the size of the hole in the spinner until it moves easily.
- Number the segments accordingly

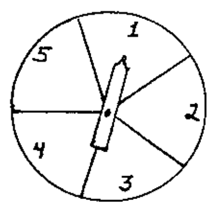
For Small Groups

Do the same as for the overhead projector except make the circle smaller and make it on a file folder or cardboard.

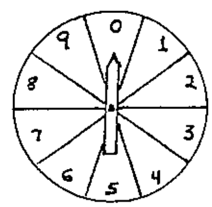
The following information may help when dividing the circle into different segments:

Number of segments	3	4	5	6	7	8	9	10
Number of degrees in each Central Angle of the circle	120	90	72	60	51+	45	40	36

Sketch:



or



The number of segments that the circle is divided into will be determined by the age of the participants. For Pre-K and K students, use 5 segments and the numbers 1, 2, 3, 4 and 5. For grade 1 and higher grades, increase the segments and include the numeral zero. The most segments needed is ten with the numbers (0,1,...9).

The beginning game should be simple so that the students understand what is happening. Once they catch on, which is very soon, then variations can be tried.

A simple example (maybe for a second grade class):

1. Have the students mark off four spaces on a piece of paper to indicate a 4-digit number. i.e., __ __ __ __
2. Tell them you are going to spin the overhead projector pointer 4 times very fast and that all students will be able to see. Each time the pointer stops at a number, they are to place that digit in one of

EQUATION SOLVING AS A PROCESS OF "UNDOING"

Sandra McCune

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Nacogdoches, Texas

Equation-solving is at the core of the study of algebra. Many beginning algebra texts initiate the process of linear equation-solving for one variable in a formal manner, emphasizing the transformations (e.g., adding or subtracting the same quantity from both sides of the equation) which yield equivalent equations. The students are instructed to perform a sequence of such transformations to achieve an equivalent equation of the form

variable = numerical value

or

variable = mathematical expression (that does not contain the given variable)

Very often the sequence of transformations is formalized into a written procedure similar to the following:

1. In equations involving parentheses, use the distributive property to remove all sets of parentheses and combine similar terms.
2. If fractions are involved, multiply each and every term on both sides of the equation by the least common denominator of all the fractions to eliminate the fractions.
3. Undo all indicated addition or subtraction to obtain all terms containing the variable on one side of the equation and all other terms on the other side.
4. Combine similar terms.

their four spaces immediately and not to change any previously placed number.

3. The student with the largest number wins. (I always gave prizes - stars, stickers, pennies, candy, paper crowns, name on board, pencils, etc. Don't have the prize be too substantial -- children will want to play this many times and ties are possible.)
4. Spin the spinner and call out the number. Suppose it points to a 7. The students must place the 7 somewhere in their four spaces. There is no right place. Continue until four numbers have been called. The person with the largest number wins. Someone always wins. Sometimes there are ties. The teacher should keep track of all the digits called.
5. If the pointer points to the line between two numbers on the spinner, use the smaller number.

Variations:

1. Mark off any number of spaces; for larger groups, use more than seven digits. Spin for the smallest or largest number.
2. Mark off 4 places, 2 on top and 2 below and make it an addition problem with the sum.

   __ __
   \+ __ __
   \-\-\-\-\-\-\-\-\-\-\-\-\-\-
3. Mark off 5 places, 3 on top and 2 below and make it an addition or subtraction problem. Be specific about what wins -- the largest answer or the smallest (non-negative) answer.
4. Mark off 3 or 4 places, 2 or 3 on top and 1 below for a multiplication problem. Cover over the segments on the spinner for multipliers not yet studied.
5. Try all math operations with as few spaces or as many spaces as children can handle or want.
6. Ask the students to come up with their own variations and allow them to be the one who spins. If it turns out to be too difficult -- it can be written down with the originator's name and put up in a special section of the bulletin board with an appropriate heading for a solution at a later time.
7. Use the game with parents at open house to show them one of the fun ways to teach place value.

The idea for Numero was suggested by a colleague, the late Dr. Ernest Ranucci, from the State University of New York at Albany who tried it out while chaperoning a bus full of students in South America many years ago.

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Very often the sequence of transformations is formalized into a written procedure similar to the following:

1. In equations involving parentheses, use the distributive property to remove all sets of parentheses and combine similar terms.
2. If fractions are involved, multiply each and every term on both sides of the equation by the least common denominator of all the fractions to eliminate the fractions.
3. Undo all indicated addition or subtraction to obtain all terms containing the variable on one side of the equation and all other terms on the other side.
4. Combine similar terms.

5. If necessary, factor the variable from the side of the equation containing the variable.
6. Divide both sides of the equation by the coefficient of the variable (provided the coefficient is nonzero) to obtain a coefficient of 1 for the variable.
7. Check by substituting the possible solution in place of the variable in the initial equation and verifying that the resulting statement is true.

The students commit the procedure to memory and then proceed mechanically through the process to the desired form of the equation. My complaint with this approach is that the students are often unaware of the underlying motivation of the process -- that is, that they are trying to "undo" what has been done to the variable. The procedure is intended to accomplish this task.

To help students come to this realization, one might introduce equation-solving with a set of simple exercises as follows:

Solve and check.

1. Take a number. Multiply by 3. Add 5. The result is 29. What is the number?
2. Take a number. Add 4. Multiply by 3. The result is 42. What is the number?
3. Take a number. Subtract 5. Divide by 2. The result is 6. What is the number?
4. Take a number. Multiply by 30. Add 5. The result is 20. What is the number?
5. Take a number. Divide by 5. Multiply by -3. The result is -63. What is the number?

Because the students have not yet learned to write an equation and formally solve it; to find the solution, they must work backward, undoing operations as they go, to the original number. To solve this set of problems the students might reason as follows:

Problem 1. We have 29. We need to subtract 5. That gives 24. Now we should divide by 3. That gives 8 for the answer.

We can check it: Take 8. Multiply by 3. That gives 24. Add 5. The result is 29. Our solution is correct.

Problem 2. We have 42. We should divide by 3. That givers 14. Now we should subtract 4. That gives 10 for the answer.

We can check it: Take 10. Add 4. That gives 14. Multiply by 3. The result is 42. Our solution is correct.

Problem 3. We have 6. We must multiply by 2. That gives 12. Now we add 5. That gives 17 for the answer.

We can check it: Take 17. Subtract 5. That gives 12. Divide by 2. The result is 6. Our solution is correct.

Problem 4. We have 20. We should subtract 5. That gives 15. Now we divide 15 by 30 to obtain 1/2 as the answer.

We can check it: Take 1/2. Multiply by 30. That gives 15. Add 5. The result is 20. Our solution is correct.

Problem 5. We have -63. We should divide by -3. That gives 21. Now we multiply by 5 to obtain 105 as the answer.

We can check it. Take 105. Divide by 5. That gives 21. Multiply by -3. That gives -63. Our solution is correct.

After completing the set of exercises, the class should discuss this process of "undoing in reverse order" and its usefulness for discovering the original number. Next, to help reinforce the concept further, students should make up similar exercises for their peers to solve. The exercises should be worked in class. This will usually lead to lively discussion when, as will invariably happen, a student-written exercise is too difficult to work using the undoing in reverse order approach. An example might be as follows:

Take a number. Add 3. Multiply by -6. Subtract 8 times the number. The result is 10. Find the number.

When this occurs, the teacher should explain that the problem posed here is of such a complicated construction that attempting to solve it by undoing in reverse order would lead to frustration. It should be clear to the students that there is a need for a more dependable tool for accomplishing the process of undoing. At this point, the formal procedure for solving linear equations should be introduced and validated by applying it to the previously solved exercises. For each problem, the set of sentences should be translated into mathematical symbolism and solved as follows.

Problem 1.

$$3n + 5 = 29$$
$$3n + 5 - 5 = 29 - 5$$
$$3n = 24$$
$$\frac{3n}{3} = \frac{24}{3}$$
$$n = 8$$

Check:

$$3(8) + 5 \stackrel{?}{=} 29$$
$$24 + 5 \stackrel{?}{=} 29$$
$$29 = 29$$

Problem 2.

$$3(n+4) = 42$$
$$3n + 12 = 42$$
$$3n + 12 - 12 = 42 - 12$$
$$3n = 30$$
$$\frac{3n}{3} = \frac{30}{3}$$
$$n = 10$$

Check:

$$3(10+4) \stackrel{?}{=} 42$$
$$3(14) \stackrel{?}{=} 42$$
$$42 = 42$$

Problem 3.

$$\frac{n-5}{2} = 6$$
$$\frac{2}{1}\left(\frac{n-5}{2}\right) = \frac{2}{1}(6)$$
$$n - 5 = 12$$
$$n - 5 + 5 = 12 + 5$$
$$n = 17$$

Check:

$$\frac{17-5}{2} \stackrel{?}{=} 6$$
$$\frac{12}{2} \stackrel{?}{=} 6$$
$$6 = 6$$

Problem 4.

$$30n + 5 = 20$$
$$30n + 5 - 5 = 20 - 5$$
$$30n = 15$$
$$\frac{30n}{30} = \frac{15}{30}$$
$$n = 1/2$$

Check:

$$30\left(\frac{1}{2}\right) + 5 \stackrel{?}{=} 20$$
$$15 + 5 \stackrel{?}{=} 20$$
$$20 = 20$$

Problem 5.

$$\frac{-3n}{5} = -63$$
$$\frac{5}{1}\left(\frac{-3n}{5}\right) = \frac{5}{1}(-63)$$
$$-3n = -315$$
$$\frac{-3n}{-3} = \frac{-315}{-3}$$
$$n = 105$$

Check:

$$\frac{-3(105)}{5} \stackrel{?}{=} 63$$
$$-63 = -63$$

It should be noted to the students that, for a given problem, the sequence of steps used in the formal procedure and the sequence of steps used in the undoing in reverse order approach may or may not coincide. For example, in Problem 1 the sequences for the two procedures coincide. On the other hand, in Problem 2, the students using the formal procedure multiply n + 4 by 3, subtract 12, and then divide by 3 to obtain the answer of 10; while students using the undoing in reverse order approach, divide by 3, then subtract 4 to obtain the same answer. The point to make with the students is that both procedures accomplish the same goal -- they undo what has been done to the variable.

Now the students should be sufficiently motivated to appreciate the efficiency and usefulness of the formal procedure for undoing what has been done to the variable in the problem posed in the student-written exercise. The set of sentences would be written symbolically and solved as follows:

$$-6(n+3) - 8n = 10$$

$$-6n - 18 - 8n = 10$$

$$-14n - 18 = 10$$

$$-14n - 18 + 18 = 10 + 18$$

$$-14n = 28$$

$$\frac{-14n}{-14} = \frac{28}{-14}$$

$$n = -2$$

Check:

$$-6(-2+3) - 8(-2) \stackrel{?}{=} 10$$

$$-6(1) + 16 \stackrel{?}{=} 10$$

$$-6 + 16 \stackrel{?}{=} 10$$

$$10 = 10$$

The lesson could then continue in the traditional, more formal, manner; but now the students will be more aware of the concept of equation-solving as a process of "undoing," rather than a mechanical sequence of steps.

NEVER SAY "NO SLOPE" AGAIN

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The concept of the slope of a line can cause some confusion for students of mathematics, especially when they study the slopes of horizontal lines and vertical lines. Some students think a horizontal line has "no slope" because it is "level" and/or they equate "no" with "zero" possibly because when their bank balance is zero, they have no money. Others think that a vertical line has "no slope" because they equate the concept of "undefined" or "not defined" with non-existence. Algebra textbooks are not consistent in their terminology concerning vertical lines. Several textbooks I examined had a variety of ways of discussing the "slope" of vertical lines. The following table is a summary of my findings.

AUTHOR	NO SLOPE	UNDEFINED	NOT DEFINED
Welch and Peter	X	X	
Larson and Hostetler			X
Tobey, Nanney and Cable	X	X	
Cohen			X
Gustafson and Frisk			X
Christy		X	
Barnett and Ziegler			X
Schoen and Marcucci	X	X	
Fleming and Varberg	X		
Bryant and Saltz	X		
Zuckerman		X	
Barros and Neti	X		
Kolman and Shapiro	X		
Johnson and Miller	X	X	

Table 1.

Note that a few authors have two entries in the table. This means they equate the two concepts in their texts. Also note that some use "undefined" while others use "not defined." These two concepts are not the same. We say that points are "undefined," but they do exist. We say division is "not defined" because the quotient does not exist. Therefore, we should say that the slope of a vertical line is "not defined" instead of "undefined."

All of the above textbooks said horizontal lines had "zero slope." None of them equated the concept of having "zero slope" to having "no slope." However, the variety of terms used with vertical lines gives rise to the following question: What would students do if they were asked to graph a line with no slope? To find out, a short test was given to 394 students in several mathematics classes at XYZ University that asked them to do just that. A copy of the test appears at the end of this article. Readers may wish to use it or something similar to test their students. If you do, I would like to know your results to compare them with mine.

On the test, each student was asked to work three problems on slope. The first two questions were designed to determine the students' knowledge of slope and to set the tone for the test. The third question asked them to graph a line through the origin with no slope. A BASIC computer program was written for a TRS 80 Model III to analyze the data. It appears at the end of this article for readers to use to analyze their data. Line 699 is a DATA statement that identifies the various courses by number that the students were enrolled in when they took the test. Line 700 is a typical DATA statement containing the data on four tests. DATA 141,1,0,1,1 means that the student was in College Algebra 141, was a freshman, missed question 1, got question 2 correct and thought a line with no slope was vertical. I used 1, 2, 3, 4, 5, and 6 for classification numbers to indicate freshman, sophomore, junior, senior, graduate and post graduate. I used zero for wrong and one for right on the first two questions and zero for horizontal, one for vertical and two for neither on question three. The program revealed the following results.

First, the program counted the various responses to question three and printed the following:

WHEN 394 STUDENTS WERE ASKED TO GRAPH A LINE WITH NO SLOPE, 226 OR 57% GRAPHED A HORIZONTAL LINE, 91 OR 23% GRAPHED A VERTICAL LINE AND 77 OR 20% DID NEITHER.

Note that well over half the students think a line with "no slope" is horizontal. None of the textbooks I examined said that a horizontal line had "no slope." I would be most interested if any readers know of such a text. All the textbooks I examined that mentioned "no slope" at all said a line with "no slope" was vertical, but only 23% of the students think this way.

Second, the program tallied the responses by course to see if any trends were apparent. It printed the following table.

COURSE	HORIZONTAL		VERTICAL		NEITHER	
Intermediate Algebra	36	9%	17	4%	20	5%
College Algebra	71	18%	25	6%	28	7%
Business Math. I	15	4%	6	2%	6	2%
Business Math. II	40	10%	14	4%	13	3%
Calculus II	14	4%	11	3%	1	0%
College Geometry	16	4%	4	1%	3	1%
Math. for Elem. Teachers	10	3%	0	0%	3	1%
Essentials of Statistics	23	6%	8	2%	3	1%
Complex Analysis	1	0%	6	2%	0	0%

Table 2

Notice that even as the level of the courses increase, most of the students think a line with "no slope" is horizontal with the exception of those in the graduate course in complex analysis. The percentages in the table are rounded to the nearest whole percent and are of the total number of students tested. For example, 36 students taking intermediate algebra think a line with "no slope" is horizontal which is 9% of the 394 students tested.

Third, the program tallied the responses by classification to see if experience made any difference. It printed the following table:

CLASSIFICATION	HORIZONTAL		VERTICAL		NEITHER	
Freshman	103	26%	49	12%	41	10%
Sophomore	41	10%	11	3%	10	3%
Junior	41	10%	13	3%	14	4%
Senior	35	9%	11	3%	11	3%
Graduate	6	2%	6	2%	1	0%
Post Graduate	0	0%	1	0%	0	0%

Table 3.

Here we can see that most of the undergraduate students think a line with "no slope" is horizontal, while the few graduate students who took the test are about evenly split.

Lastly, the program checked to see if the student making the response knew anything about slope int he first place. It tallied the responses for those students who answered both questions one and two correctly. Then it tallied the responses for those students who missed exactly one of those questions. Finally, it tallied the responses of those students who missed both questions one and two, and printed the following table.

KNOWLEDGE	HORIZONTAL		VERTICAL		NEITHER	
Got Both Right	130	33%	56	14%	21	5%
Missed One	68	17%	22	6%	38	10%
Missed Both	28	7%	13	3%	18	5%

Table 4.

Here we can see that well over half of the students who got both questions one and two correct think a line with "no slope" is horizontal. A little over half of the students who got question one or two right think such a line is horizontal. Those who missed both questions one and two favor "horizontal" over "vertical" if they answered at all.

What conclusions can be made from the above results? It would seem that even though the test was given to just a few students at only one university, there is confusion among students as to the meaning of "no slope." Most think "no" means "zero," but several others think "no" means "non-existent." Rather than try to decide which meaning of "no slope" to adopt and then try to convince everyone to use it, perhaps teachers and textbook authors should agree to say a horizontal line has a slope of zero, the slope of a vertical line is not defined and never say "no slope" again.

A QUIZ ON THE SLOPE OF A LINE

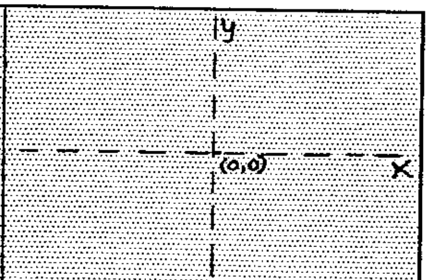
Name: ____

Class: Math. ____________________ Section: __________________

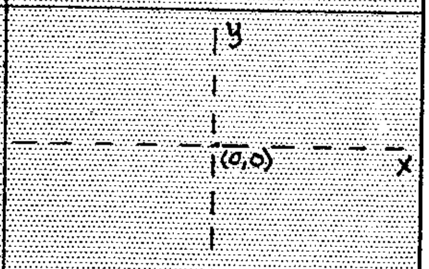
Teacher: ______________________ Classification: ____________

Please graph the following lines so that they will all pass through the origin.

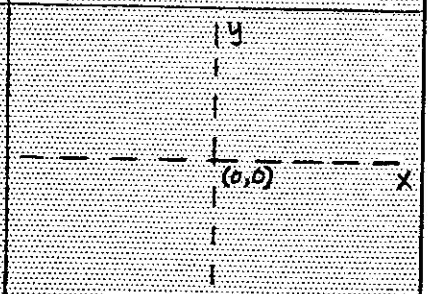
1. A line with slope +1:



2. A line with slope -1:



4. A line with no slope:



SLOPE TEST ANALYSIS PROGRAM LISTING

```
5   DIM C(50), M(1000,5)
10  CLS : PRINT "READING AND ANALYZING DATA - PLEASE WAIT"
15  FOR Z = 1 TO 50
16  READ C(Z)
17  IF C(Z) = 999 THEN M = Z - 1 : GOTO 20
18  NEXT Z
20  FOR I = 1 TO 1000
30  FOR J = 1 TO 5
40  READ M(I,J)
50  IF M(I,J) = 99 THEN N = I - 1 : GOTO 70
60  NEXT J,I
70  H = 0 : V = 0 : A = 0
80  FOR I = 1 TO N
90  IF M(I,5) = 0 THEN H = H + 1
100 IF M(I,5) = 1 THEN V = V + 1
110 IF M(I,5) = 2 THEN A = A + 1
115 IF M(I,5) < 0 OR M(I,5) > 2 THEN PRINT I, "ERROR IN DATA" : END
120 NEXT I
140 CLS : PRINT "'NO SLOPE' EXPERIMENT"
150 PRINT
160 PRINT " WHEN ";N;" STUDENTS WERE ASKED TO GRAPH A LINE WITH NO
    SLOPE,"
161 PRINT H;" OR ";INT(100*(H/N+.005));" % GRAPHED A HORIZONTAL LINE
162 PRINT V;" OR ";INT(100*(V/N+.005));" % GRAPHED A VERTICAL LINE AND"
163 PRINT A;" OR ";INT(100*(A/N+.005));" % DID NEITHER."
170 PRINT:PRINT:INPUT "PRESS <ENTER> TO CONTINUE";C
175 CLS : PRINT "ANALYSIS BY COURSE" : PRINT
180 PRINT " COURSE"," HORIZONTAL"," VERTICAL"," NEITHER"
185 PRINT
190 FOR I = 1 TO M
200 H = 0 : V = 0 : A = 0
210 FOR K = 1 TO N
220 IF M(K,1) <> C(I) THEN 260
230 IF M(K,5) = 0 THEN H = H + 1
240 IF M(K,5) = 1 THEN V = V + 1
250 IF M(K,5) = 2 THEN A = A + 1
260 NEXT K
270 PRINT C(I),H;" (";INT(100*(H/N+.005));"%)",V;" (";INT(100*(V/N+.005));"%)",A;"
    (";INT(100*(A/N+.005));"%)"
280 NEXT I
290 PRINT:INPUT "PRESS <ENTER> TO CONTINUE";C
300 CLS:PRINT " ANALYSIS BY CLASSIFICATION"
305 PRINT:PRINT " CLASSIFICATION"," HORIZONTAL"," VERTICAL"," NEITHER"
307 PRINT
310 FOR I = 1 TO 6
320 H = 0 : V = 0 : A = 0
330 FOR K = 1 TO N
340 IF M(K,2) <> I THEN 380
350 IF M(K,5) = 0 THEN H = H + 1
```

```
360  IF M(K,5) = 1 THEN V = V + 1
370  IF M(K,5) = 2 THEN A = A + 1
380  NEXT K
390  IF I = 1 THEN C$ = "FRESHMAN"
400  IF I = 2 THEN C$ = "SOPHOMORE"
410  IF I = 3 THEN C$ = "JUNIOR"
420  IF I = 4 THEN C$ = "SENIOR"
430  IF I = 5 THEN C$ = "GRADUATE"
440  IF I = 6 THEN C$ = "POST GRAD"
450  PRINT " ";C$,H;" (";INT(100*(H/N+.005));"%)",V;" (";INT(100*(V/N+.005));"%)"
     ,A;" (";INT(100*(A/N+.005));"%)"
460  NEXT I
470  PRINT:INPUT "PRESS <ENTER> TO CONTINUE";C
480  CLS:PRINT " ANALYSIS OF SLOPE KNOWLEDGE":PRINT
485  PRINT " KNOWLEDGE"," HORIZONTAL"," VERTICAL"," NEITHER"
490  H0=0:H1=0:H2=0:V0=0:V1=0:V2=0:A0=0:A1=0:A2=0
500  FOR I = 1 TO N
510  IF M(I,3) <> 1 OR M(I,4) <> 1 THEN 550
520  IF M(I,5) = 0 THEN H0 = H0 + 1
530  IF M(I,5) = 1 THEN V0 = V0 + 1
540  IF M(I,5) = 2 THEN A0 = A0 + 1
545  GOTO 620
550  IF M(I,3) <> 0 OR M(I,4) <> 0 THEN 590
560  IF M(I,5) = 0 THEN H1 = H1 + 1
570  IF M(I,5) = 1 THEN V1 = V1 + 1
580  IF M(I,5) = 2 THEN A1 = A1 + 1
585  GOTO 620
590  IF M(I,5) = 0 THEN H2 = H2 + 1
600  IF M(I,5) = 1 THEN V2 = V2 + 1
610  IF M(I,5) = 2 THEN A2 = A2 + 1
620  NEXT I
625  PRINT : PRINT " GOT BOTH RIGHT" ,H0;" (";INT(100*(HO/N+.005));"%)",V0;"
     (";INT(100*(VO/N+.005));"%)",A0;" (";INT(100*(A0/N+.005));"%)"
627  PRINT
630  PRINT "  MISSED ONE",H2;" (";INT(100*(H2/N+.005));"%)",V2;" (";INT(100*(V2
     /N+.005));"%)",A2;" (";INT(100*(A2/N+.005));"%)"
635  PRINT
640  PRINT " MISSED BOTH",H1;" (";INT(100*(H1/N+.005));"%)",V1;" (";INT(100*(V1
     /N+.005));"%)",A1;" (";INT(100*(A1/N+.005));"%)"
650  PRINT
660  PRINT " THIS CONCLUDES THE ANALYSIS"
670  PRINT:PRINT: END
699  DATA 131,141,175,176,192,321,372,453,520,999
700  DATA 141,1,0,1,1,141,1,1,1,0,141,1,1,1,0,141,2,1,1,0
```

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