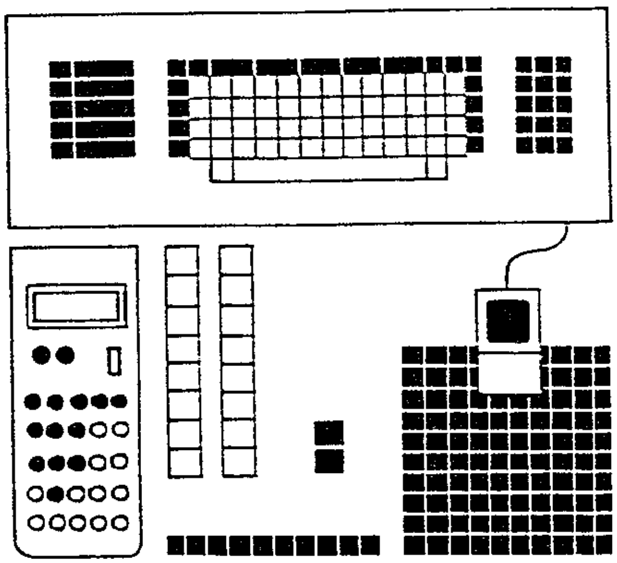
TEXAS MATHEMATICS TEACHER



Now You're Cooking With Mathematics

The Sum Of The Squares Of The Digits Of A Natural Number N Vs The Square Of The Sum Of The Digits Of N

Problem Solving With Fractions Using The Digit Fill Game

The Spreadsheet In The Mathematics Classroom

STEAM

OCTOBER 1989

EDITORS

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

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October 1989

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TEXAS MATHEMATICS TEACHER
VOL. XXXVI (4) October 1989

President's Message

We are off to an excellent start for the new school year. As my friend and great philosopher, Jerry Clower would say, WHOOOOEEEEEE!!!!

We, the mathematics teachers of Texas, are looked upon by our colleagues nationwide as trend setters in mathematics teaching. We must keep up the good, strong dedicated efforts that many have been giving to mathematics teaching in Texas. We cannot relax now. Yes, we have had several setbacks in our progress, and many more will come, but if we remain united and strong, we shall prevail.

As members of TCTM, you are leaders in mathematics education. The youth of Texas need you to help them succeed in the career of their choice. They are to be faced with many challenges in the future and need to have useable mathematics to succeed, in business and the technical areas. It is up to us as leaders and teachers to see that everyone has the opportunity that they deserve.

The new tests such as TAAS and TASP are to be a challenge to each teacher to be sure that their students know the mathematics that they teach and how to properly use that mathematics.

The officers and board of directors of TCTM are in the process of updating the organization to be useful to the membership in the 21st century. On labor day weekend, several dedicated members met and worked on revision of the TCTM constitution to update it to be useful in the future. The present constitution was very outdated and unworkable in the present age, let alone in the future. We are trying to help you and therefore need your help in determining what the membership desires.

The staff of NCTM, the members of NCTM, TCTM and your local affiliate group are working together to keep mathematics and mathematics teaching current and useful for the user. It is up to the teachers in the State of Texas to do their part and help to be sure that each and every student has the best mathematics instruction that money can buy. No, I really mean that dedication can give them. Money cannot buy dedication and we do have many dedicated teachers in the State of Texas. No part of the complex system of education, computing, comprehension, communication and commitment can be taken for granted.

Most of you are doing an excellent job. Just keep up the great work. We need you and your dedication. We need you to help TCTM grow. Encourage your fellow teachers to join the professional organizations and be active in them. We need the professionalism that they can give to the profession of mathematics teaching.

See you in Dallas for CAMT during August 7-9, 1990. Make your plans to attend now. We will have a great time as well as a wonderful program.

OTTO W. BIELSS, JR.
President, TCTM

NOW YOU'RE COOKING WITH MATHEMATICS

Donna Forbis

O'Donnell ISD
Dickens, Texas

Elementary teachers are constantly on the look out for new ideas to help them teach mathematics to their young learners. Recipes and food preparation are exciting and motivating ways of teaching not only mathematical concepts, but a fun way to incorporate science, reading, and language development into the lesson.

George Polya suggest in his Principles of Motivation that the learner should be interested in the activity of learning and find pleasure in the activity (Polya, 1981). Elementary students do get very excited about actually cooking in the classroom.

Addition, multiplication, division, fractions, and measurement of volume, time, and temperature are among the concepts that can be taught with food preparation. The kindergarten students can see the concepts, such as addition, taking place. "Isaac adds one egg to the cookie batter and Renee' adds one egg. How many eggs were put into the batter? One plus one equals two."

Teaching the yield of a recipe in problem solving can involve multiplication or division for older students. "A recipe yields twelve cupcakes. What can we do to make enough for thirty-six people?" The students will multiply the amount of ingredients by three. Students can change a recipe that yields two dozen cookies to one that yields one dozen. They can also use time in problem solving. "A cake bakes for seventy-five minutes. How many hours and minutes is this?" "A meat loaf is put in the oven at 4:15. It needs to bake for forty-five minutes. What time do you take it out of the oven?" Students can think about problems like these as they actually prepare the recipe.

Fractions and measurement can easily be taught through recipes. Primary-grade students can use measuring cups and measuring spoons. Kindergartners can use a glass measuring cup. This makes it easier for them to see the idea of a half of a cup. For one-half cup of milk, they can fill the cup half full. Contemporary learning theories suggest that learning a concept should begin at the motor level (Abel, 1987). Measuring cups and rice can be put in a learning center for manipulative play. Through manipulative play, the child can learn to handle the equipment (Dienes, 1967). Students can measure the rice with ease. They can see that two one-half cups equal one cup or that one-half plus one-fourth equals three-fourths. The students are able to discover mathematical ideas for themselves (Bruner, 1960). Piaget's work implied that young children internalize ideas through concrete manipulation (Wirtz, 1976). Food preparation is an excellent hands-on experience for students.

Other subjects can be incorporated into the lesson. Chemical and physical changes of food during cooking can be observed and discussed. Preparing applesauce allows students to see changes taking place. The teacher can read the recipe carefully with younger students. Older students can read the recipe thoroughly for themselves.

Cooking related terms, such as boil, bake, simmer, grate, etc., may be encountered. Students can rewrite the recipe in their own words or write one of their favorite recipes. It's fun to make a class cookbook and include the recipes written by the students. This also makes a nice gift for parents.

Recipes that are prepared in the classroom can be as simple as fruit salad that requires no cooking. Cutting the fruit, measuring, and mixing are all that is involved. More elaborate recipes can be used depending on the facilities. These include recipes that require heating from a hot plate or microwave oven and the use of a mixer or blender. Close adult supervision is extremely important with all activities. This is also a good opportunity to stress safety and sanitation. Students may be assigned to bring ingredients. For example, everyone could bring a fruit for fruit salad or everyone bring a vegetable for vegetable soup. Most budgets will allow for the ingredients and supplies.

Using food preparation and recipes is a fun way to teach mathematical concepts to elementary students. Students will be interested and very motivated. They will see and taste the end result to correctly measuring and following directions. You may even have students asking for "seconds" in math.

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THE SUM OF THE SQUARES OF THE DIGITS OF A NATURAL NUMBER N VS THE SQUARE OF THE SUM OF THE DIGITS OF N

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Mathematics is a way of thinking. The discovery approach in today's mathematics and noting number patterns by performing experiments with numbers can establish the thinking process needed to become better in the mathematics field. Consider the following process of (1) squaring each of the digits of a given counting number, p_1, and adding these squares yields a new counting number, p_2. Squaring the digits of p_2 and adding these squares yields another counting number, p_3. We could obtain p_4, p_5, . . . in a similar manner. For example, start with 123. The square of the digits are 1, 4 and 9, and their sum is 14. The squares of the digits of 14 are 1 and 16, and their sum is 17. The squares of the digits of 17 are 1 and 49, and their sum is 50. Continuing this process would yield a sequence of numbers that might have some interesting properties. Now, if we (2) add the digits of a given counting number, q_1, and square the result, we obtain a new counting number, q_2. Squaring the sum of the digits of q_2 yields another new counting number, q_3. We could obtain q_4, q_5, . . . in a similar manner. For example, start with 123. The sum of its digits is 6 and the square of 6 is 36. The sum of the digits of 36 is 9, and its square is 81. Continuation of this process would yield another sequence of numbers that might also have some interesting properties. Interestingly enough, if both processes (1) and (2) are continued long enough, we will obtain a number that appears earlier in the sequence. When this happens, further continuation of either process would re-generate the same numbers over and over forming a finite "loop" of numbers. Let us first consider the sum of the squares. Given any number $\underline{n}$, decompose $\underline{n}$ into its digits $d_1, d_2, \ldots, d_k$. Square each digit and sum the results so that $d_1^2 + d_2^2 + \ldots + d_k^2 = \underline{m}$. Now repeat the process for $\underline{m}$ and for all succeeding numbers until (hopefully) a loop is found. This is achieved when a sum is obtained that has appeared anywhere earlier in the list of m-values, possibly $\underline{n}$ itself. Some interesting questions arise at this point. Does every starting number eventually "converge" to a finite loop? Is there a finite number of possible loops of numbers that are generated indefinitely by processes (1) or (2)? It would be possible to use a computer to search for all the loops if we could establish an upper bound for the number of loop generators. That is, a number $\underline{b}$ with the property that all starting numbers larger than $\underline{b}$ eventually produce a number smaller than $\underline{b}$ so that all loops will be found by examining the numbers between 1 and $\underline{b}$.

To establish an upper bound, consider the inequality $n(9^2) < 10^{(n-1)}$. The left term is the largest sum of squares of digits that can be obtained for numbers with $\underline{n}$ digits, while the right term is the smallest number with $\underline{n}$ digits. If it can be shown that $n(9^2) < 10^{(n-1)}$ for some value $\underline{k}$, then the sum of the squares of the digits of $\underline{k}$ will be a number that has less than $\underline{k}$ digits. Furthermore, if $n(9^2) < 10^{(n-1)}$ is true for all numbers with $\underline{n}$ digits where $n > k$, then we will have a decreasing sequence of numbers until we obtain a number less than $k(9^2)$. Therefore, we can let $\underline{b} = k(9^2)$, the upper bound we were seeking.

The inequality is false for $n = 1$, $n = 2$ and $n = 3$. However $4(9^2) = 324 < 1{,}000 = 10^3 = 10^{(4-1)}$. Thus we have found a beginning number. We need to show the inequality is true for all $\underline{n}$ greater than 4. Suppose it is true for $\underline{m}$, so $m(9^2) < 10^{(m-1)}$.

$$
\begin{aligned}
\text{Then } (m+1)(9^2) &= m(9^2) + 81 \\
&< 10^{(m-1)} + 81 \\
&< 10^{(m-1)} + 100 \\
&< 10^{(m-1)} + 10^{(m-1)} \quad (\text{Since } m > 3) \\
&= 2(10^{(m-1)}) \\
&< 10(10^{(m-1)}) \\
&= 10^{((m-1)+1)} \\
&= 10^{((m+1)-1)}
\end{aligned}
$$

Therefore, when the inequality holds for $\underline{n}$, it holds for $\underline{n+1}$. So by mathematical induction, the inequality holds for all $\underline{n}$ greater than or equal to 4. Thus, if we search all the numbers between 1 and 324 ($324 = 4(9^2)$), we will find all possible loops. Note, by the way, that the existence of $\underline{b}$ answers the two questions above in the affirmative - there is only a finite number of possible loops and every number must converge to one of them.

The following BASIC computer program was written for a TRS 80 Model III, but, since it contains only standard BASIC statements, it will also run on an Apple, IBM, etc. The program examines each number by means of a FOR - NEXT loop from 1 to 324, to find what loop it determines.

PROGRAM 1. LISTING

```
5 DIM S(100), T(100), F(100)
10 CLS : FOR N = 1 TO 324
12 M = N
15 D = 0
20 C = 0 : X = 0
30 L = INT(M/10) * 10
40 C = C + 1
50 F(C) = M - L
60 IF L = 0 THEN 80
70 M = L/10 : GOTO 30
80 FOR I = 1 TO C
90 X = X + F(I)*F(I)
95 NEXT I
100 D = D + 1
110 S(D) = X
120 FOR I = 0 TO D - 1
130 IF S(I) = X THEN PRINT "FOR ";N;", A LOOP OF LENGTH ";D-I;" FOUND
            IN ";D;" STEPS" : GOTO 170
140 NEXT I
150 M = X
160 GOTO 20
170 FOR Q = I TO D : PRINT S(Q); : NEXT Q : PRINT : PRINT : NEXT N
180 END
```

The output of only the distinct loops is given in the following table.

TABLE 1.

FOR 1, A LOOP OF LENGTH 1 FOUND IN 2 STEPS
1 1

FOR 2, A LOOP OF LENGTH 8 FOUND IN 9 STEPS
4 16 37 58 89 145 42 20 4

Note that there are only two distinct loops in the output. Therefore we can claim that if a sequence is formed by summing the squares of the digits of one term to produce the next term, then it will end in one of the loops above in a finite number of steps. The loop lengths and the number of steps needed to find the loop may be interpreted as follows: The loop length is the number of distinct numbers that are repeated as the process continues. The numbers of steps includes the loop length and the number of numbers needed to get from the starting number to the first number in the loop.

Now let's consider the square of the sum of the digits of a number. This means we will use the digits of a number n to form a new number m $= (d_1 + d_2 + \ldots + d_k)^2$ where $d_1, d_2, \ldots, d_k$ are the digits of n. We can use the computer again to find all possible loops if we can establish an upper bound b with the property that all numbers larger than b will eventually produce a number smaller than b. The existence of this number will also establish that there are only a finite number of loops and every number must converge to one of them.

To find an upper bound, consider the following inequality: $(9n)^2 < 10^{(n-1)}$. The left term is the largest square of the sum of the digits of an n-digit number, while the right term is the smallest n-digit number. If it can be shown that $(9n)^2 < 10^{(n-1)}$ for some value k, then the square of the sum of the digits of every k-digit number will have less than k digits. In addition, if the property holds for all numbers with more than k digits, then we can let $b = (9k)^2$ and know that every number will eventually produce a number less than b, so all loops can be found by examining the numbers between 1 and b.

The inequality is false for n = 1, n = 2, n = 3 and n = 4. for n = 5, we have $((5x9))^2 = 2025 < 10{,}000 = 10^4 = 10^{(5-1)}$, so we have found a starting number. Suppose the inequality holds for m, so $(9m)^2 < 10^{(m-1)}$.

Now $(9(m + 1))^2 = 81(m^2 + 2m + 1)$
$= 81m^2 + 162m + 81$
$< 10^{(m-1)} + 162m + 81$
$< 10^{(m-1)} + 200m + 200$
$= 10^{(m-1)} + 200(m + 1)$
$= 10^{(m-1)} + 2(10)^2(m + 1)$
$< 10^{(m-1)} + 2(10)^2(10)^{(m-3)}$ since $m > 4$ and $6 < 100$, $7 < 1000$, $8 < 10{,}000$, $9 < 100{,}000$, $10 < 1{,}000{,}000$, etc.

Thus $(9(m + 1))^2 < 10^{(m-1)} + 2(10)^{(m-1)}$
$= 3(10^{(m-1)})$
$< 10(10^{(m-1)})$
$= 10^{((m-1) + 1)}$
$= 10^{((m+1) - 1)}$.

Therefore, the inequality holds for $(n + 1)$ when it holds for n, so we can claim by mathematical induction, that it holds for all n greater than or equal to 5. Thus we will be able to find all possible loops by examining the numbers from 1 to 2,025. Such a search would take a lot of computer time and produce a rather lengthy table. To shorten the search, notice that the number less than 2,025 that has the largest digit sum is 1,999. Its digit-sum squared is 784, so no number less than 2,025 will have digit-sum over 784. Thus we could confine our search to the interval 1 to 784. Can we do even better? The number less than 784 with the largest digit-sum is 699. Its digit-sum squared is 576. The number less than 576 with the largest digit-sum is 499. Its digit-sum squared is only 484. The number less than 484 with the largest digit-sum is 399. Its digit-sum squared is 441 which is larger than 399, so we can not guarantee a search to only 399 will find all the loops. However, we can use the number 441 as an upper bound for our search to find all possible loops.

The BASIC program listed below is almost identical to the one listed above with the exception of 90 and 96 in the FOR - NEXT loop that compute the square of the sum instead of the sum of the squares.

PROGRAM 2. LISTING

```
5 DIM S(100), T(100), F(100)
10 CLS : FOR N = 1 TO 484
12 M = N
15 D = 0
20 C = 0 : X = 0
30 L = INT(M/10) * 10
40 C = C + 1
50 F(C) = M - L
60 IF L = 0 THEN 80
70 M = L/10 : GOTO 30
80 FOR I = 1 TO C
90 X = X + F(I)
95 NEXT I
96 X = X * X
100 D = D + 1
110 S(D) = X
120 FOR I = 0 TO D - 1
130 IF S(I) = X THEN PRINT "FOR ";N;", A LOOP OF LENGTH ";D-I;" FOUND
              IN ";D;" STEPS" : GOTO 170
140 NEXT I
150 M = X
160 GOTO 20
170 FOR Q = I+1 TO D : PRINT S(Q); : NEXT Q : PRINT : PRINT
180 NEXT N
190 END
```

The output of only the distinct loops is given in the following table.

Table 2.

```
FOR 1, A LOOP OF LENGTH 1 FOUND IN 2 STEPS
1

FOR 2, A LOOP OF LENGTH 2 FOUND IN 6 STEPS
256  169

FOR 3, A LOOP OF LENGTH 1 FOUND IN 3 STEPS
81
```

Note that there are only three distinct loops. A loop of 1 was expected for each process since $1^2 = 1$ and, for a single-digit number, the sum of squares is the same as the square of sums. The other loops are different from the ones obtained for the sums of squares by Program 1. The loop lengths and the number of steps to find them were determined in the same manner used for Table 1.

The following exercises should challenge the thinking and programming ability of the reader:

1. Alter the programs to find the loop generated by a specific number.
2. Alter the programs to find how many numbers generate a particular loop.
3. Alter the programs to print out only distinct loops the first time they are generated rather than all the loops for all the numbers.
4. Do loop patterns also appear when the digits of a number are cubed, then added?

REFEREES WANTED

Manuscripts published in the Texas Mathematics Teacher are reviewed by at least three mathematics educators. Members of TCTM who are regular readers of the journal - classroom teachers, supervisors, and teacher educators - who would like to review manuscripts should write to the TCTM indicating their willingness to serve and the level of interest (Elementary, Secondary, or both). Contact George H. Willson, Box 13857, University of North Texas, Denton, Texas 76203-3857. The Editorial Panel will review the responses and make the final selection.

PROBLEM SOLVING WITH FRACTIONS USING THE DIGIT FILL GAME

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In recent years there has been considerable support in the mathematics education community for an increased emphasis on estimation, exploring relationships, and problem solving. Since much of the mathematics curriculum focuses on paper and pencil computation and algorithms, a teacher may often find that he/she must look beyond the textbook in order to find activities that can be used to provide students with estimation and problem solving experiences.

The following discussion will focus on four versions of a computer game called **DIGIT FILL** that requires a student to estimate using fraction addition. This game would be suitable for students at grades 5-8. Problem solving extensions of the games will also be presented. Finally, suggestions will be made as to how these games and related activities could be used in the classroom.

Description of the Game

Each of the four versions of the game focuses on estimation, fraction concepts and relationships, and problem solving using fraction addition. The following are goals for using the game with students.

1. To give students opportunities to explore concepts and relationships related to magnitude of fractions and estimation of fraction sums. These would include the relationship between numerator and denominator, rounding fractions, using some fractions as reference numbers, and using an interval for estimation.

2. To encourage students to use a variety of problem solving strategies in order to help them develop a "winning strategy" for the game. Students would be asked to look for patterns, investigate relationships, consider alternatives, test hypotheses, and organize data.

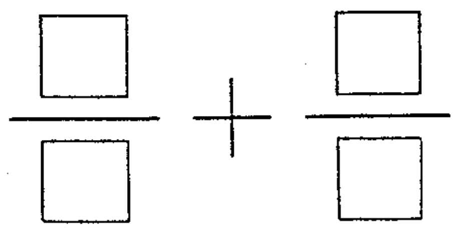
When starting the game, the student must choose from among four versions of the game. These are:

1. Largest sum - In this version the student attempts to use four one-digit numbers to form two fractions whose sum is the largest possible sum, using those four digits. The four digits are generated by the computers one at a time. As each digit is generated, the student must decide where that digit should be placed in the model shown in Figure 1. The computer places the digit in the selected numerator or denominator and then generates the next digit. The student must focus on the goal of the game (largest possible sum) in order to decide where a given digit should be placed.

2. Smallest sum - This version is identical in format to the Largest Sum version. The goal here is to use the four digits to form two fractions whose sum is the smallest possible sum, using those four digits.

3. Sum closest to 1 - This version has the format previously described. The goal in this version is to use the four digits to form two fractions whose sum is closest to 1.

4. Range game - Again, the format here is the same as in the other versions. In this version, the student attempts to use the four digits to form two fractions whose sum is in a specified range.



YOUR NUMBER IS 1

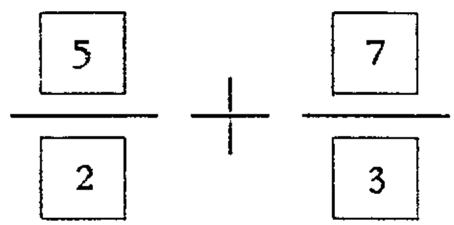
IN WHICH BOX WILL YOU PUT THE NUMBER?

1 - UPPER LEFT   2 - UPPER RIGHT
3 - LOWER LEFT   4 - LOWER RIGHT

FIGURE 1

After the fourth number has been placed in the model the computer reports the sum of the two resulting fractions and the largest sum, smallest sum, or sum closest to 1 using the four generated digits. Note that a student does not get to rearrange the numbers after he/she has placed the numbers in the model.

As an example, let us suppose that the computer has generated the numbers 2, 5, 7, 3 (in this order) and the student has placed them in the model as follows for the game "Largest Sum."



YOUR NUMBER IS 3

IN WHICH BOX WILL YOU PUT THE NUMBER?

1 - UPPER LEFT 2 - UPPER RIGHT
3 - LOWER LEFT 4 - LOWER RIGHT

FIGURE 2

The computer would report that this gives a sum of 29/6 or 4.83. It would also report that the largest possible sum using these digits is 31/6 or 6.16. The format is similar for the "Smallest Sum" and "Sum Closest to 1" options.

The fourth option is somewhat different in format from the others. Here, the student will generate five sums using the procedure previously described. Each sum will receive a score dependent upon the interval in which a sum falls. The intervals and point values are:

RANGE	POINT VALUE
0 TO 1/2	3
1/2 TO 1	2
1 TO 1 1/2	1
1 1/2 TO 2	1
2 TO 2 1/2	2
2 1/2 TO 3	3
Greater than 3	0

Thus, if the student generates a sum of 26/30 he/she receives 2 points for that sum. The computer reports the number of points received for each sum generated and the total number of points for the 5 rounds combined.

<u>Discussion of the Game</u>

In the "Largest Sum" and "Smallest Sum" games the student must focus on several specific concepts related to addition of fractions. Among these are:

1. With numerator larger than denominator, the larger the difference between the numerator and denominator of the fraction, the larger the fraction will be. With numerator smaller than the denominator, the smaller the fraction will be.

2. If possible, the largest of the four numbers should be the numerator and the smallest of the four numbers should be the denominator in the same fraction to generate the largest sum. To get the smallest sum, the largest digit should be a denominator and the smallest digit a numerator, but in different fractions.

Students should be encouraged to search for some of these generalizations as they play the game. For students who do not "see" these rules it may be necessary for the teacher to ask appropriate questions or provide additional activities designed to help the students make these discoveries. Some of these will be suggested later in this discussion.

In the "Sum Closest to One" the students will find that there are no such "nice" generalizations that can be made. There are some observations that should be encouraged by the teacher. These are stated as questions as follows.

1. What sums are possible if the four digits we get are equal?
2. What sums are possible if three of the four digits are equal?
3. What sums are possible if two of the four digits are equal?
4. What sums are possible if no two of the four digits are equal?
5. Do you see a strategy that you think will improve your chances of getting a high score?
6. What fractions serve as good reference points for trying to get a sum close to 1?

Questions 1 through 4 above should help students to organize their investigations by considering various special cases. Question 5 is designed to encourage students to formulate a "strategy" that will maximize their chances of winning the game. Question 6 can helps students to focus on key fractions that will help them estimate sums close to 1. Such fractions are 1/4, 1/3, 1/2, 2/3, 3/4, etc.

Once again, the teacher will need to provide appropriate supplemental activities to help the students organize their strategies and processes in order to answer questions such as those above. Example activities will be discussed later.

In the "**Range Game**" students will again find that this version is more open-ended than the "**Smallest Sum**" or the "**Largest Sum.**" Questions to investigate would include the following.

1. In which intervals do the most number of sums seem to fall?
2. In which intervals do the least number of sums seem to fall?
3. Do you see any strategy that you think will improve your chances of getting a high score?

Investigating questions 1 and 2 will help students make various hypotheses about the intervals used in the game. For example, they may decide that it is more likely to generate a sum greater than 3 than a sum between 2 1/2 and 3. This and similar hypotheses can help students develop a strategy in which they do not simply attempt to get a sum in an interval with the largest point value, but take into account the likelihood of generating a sum in that interval.

<u>Implementing the Game in the Classroom</u>

Several activities will be suggested here that could be used to help students focus on the mathematics and problem solving found in these games. The teacher is to be seen as one resource that the students might use, while the computer allows the students to investigate these games on their own. Organized guess and test is a strategy that the students can use rather well since the computer can be used as a "scorekeeper." The students are more free to explore many number combinations, estimation techniques, and strategies.

An activity sheet for either the "Largest Sum" or "Smallest Sum" games is found in Figure 3. This activity is designed to help the students focus on the relationships among the numerators and denominators of the fractions.

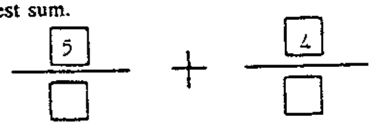
FIND THE LARGEST SUM

Use the game "Largest Sum" from the DIGIT FILL progr... help you answer the following questions.

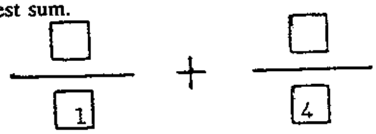
1. Play the game 5 times and fill in the blanks below.

GAME	MY SUM	LARGEST POSSIBLE SUM
1	________	________
2	________	________
3	________	________
4	________	________
5	________	________

2. Answer the following questions.
   a. For the digits 1, 2, 3, 4 which two digits should be placed in the denominators? ________________
   b. For the digits 2, 4, 7, 9 which two digits should be placed in the numerators? ________________
3. Suppose you have the digits 2, 3, 4, 5 and the 4 and 5 have been placed as shown below. Put the 2 and 3 in the drawing so you will get the largest sum.



4. Suppose you have the digits 1, 4, 6, 8 and the 1 and 4 have been placed as shown below. Put the 6 and 8 in the drawing so you will get the largest sum.



5. Suppose you have placed the digits 7 and 9 as shown below. The next digit the computer gives is 6. Place it in the drawing. Tell why you placed it where you did.

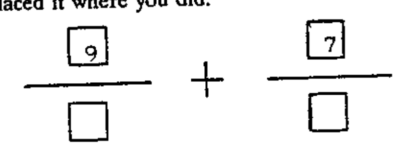


FIGURE 3

Students might work individually or in small groups on this activity. A teacher might also choose to have this be a class discussion activity, perhaps using the overhead to record results.

Figure 4 is an activity sheet to be used with the Range Game option of the game. Here, students are asked to consider the possible results of various choices in these games. There is no "best" strategy for this game, but students should be encouraged to consider the possibilities and then choose so as to "maximize" their chances of winning. Students will find that this activity helps them to organize their search for a "winning" strategy. By looking at a sample distribution of sums a student may hypothesize that there is a greater likelihood of a sum in the greater than 3 interval than in the 0 to 1/2 interval.

THE RANGE GAME

Use the game "**Range Game**" from the DIGIT FILL program to help you answer the following questions.

1. For each of the following collections of four digits show how they should be arranged to get the sum that will give you the most points.

   2,5,7,6 8,6,1,9 8,8,6,1 1,2,1,2 1,3,5,7

2. Play the game 10 times. For each time played use the following chart to help you list the number of sums you get in each interval.

Game Number	1	2	3	4	5	6	7	8	9	10	Total
0 to 1/2											
1/2 to 1											
1 to 1 1/2											
1 1/2 to 2											
2 to 2 1/2											
2 1/2 to 3											
Greater than 3											

a. Which interval had the most sums? ______
b. Which interval had the fewest sums? ______
c. Based on these results, into which interval(s) should you try to get your sums? ________

FIGURE 4 **(continued on next page)**

3. Suppose you have placed the digits 3 and 7 as shown below. The next number the computer gives is 4. Place it in the drawing. Tell why you placed it where you did.

$$\frac{\boxed{3}}{\boxed{7}} + \frac{\square}{\square}$$

4. Suppose you have the digits 1,4,5,8. Find all the possible sums you could get using these digits. Show the sums below.

FIGURE 4

Students may use a variety of strategies for estimating sums. Some students will round fractions to more convenient fractions such 1/4, 1/2, 3/4, 1/3, 2/3, etc. Other students may estimate one or both fractions using a common denominator and then mentally calculate the sum.

In general, a student must make some sort of estimate before he/she decides where to place a given digit. Students must also consider what possible outcomes could arise from a particular choice. In a similar way, students would need to consider the likelihood of other numbers that could be generated. A teacher may need to help students use a variety of estimation techniques. Asking questions to help the students focus on important ideas is one way a teacher might do this. Consider the student activity sheet shown in figure 5. This activity is designed to help students focus on some important estimation strategies. These estimation strategies can be useful in playing the other options of the game.

SUMS TO ONE

Use the "**Sum Closest to 1**" from the DIGIT FILL program to help you answer the following questions.

1. Play the game 5 times and fill in the blanks below.

GAME	MY SUM	SUM CLOSEST TO 1
1	______	______
2	______	______
3	______	______
4	______	______
5	______	______

FIGURE 5 (continued on next page)

2. Round each of these fractions to a fraction whose denominator is 2, 3, or 4.

3/7 = ________	5/9 = ________
2/5 = ________	4/7 = ________
5/7 = ________	3/8 = ________

3. Estimate to place a fraction in the box so that the sum will be close to 1. Your estimate must have a numerator of 1. Then add the two fractions and place the sum in the blank. You may want to round the given fraction.

5/8 + [ ] = ________	2/5 + [ ] = ________
7/9 + [ ] = ________	7/8 + [ ] = ________
6/7 + [ ] = ________	4/5 + [ ] = ________

4. Estimate to place a fraction in the box so that the sum will be close to 1. Your estimate must have a denominator of 2, 3, or 4. Then add the two fractions and place the sum in the blank. You may want to round the given fraction.

2/7 + [ ] = ________	3/8 + [ ] = ________
4/9 + [ ] = ________	2/9 + [ ] = ________
7/9 + [ ] = ________	5/7 + [ ] = ________

5. Suppose you have placed the digits 3 and 7 in the model as shown below. The next digit the computer gives is 4. Place the 4 in the model and tell why you placed it there.

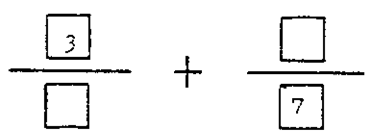


Figure 5

<u>**Summary**</u>

Fractions and fraction operations often prove to be difficult for many students. Further, the curriculum places emphasis on computational proficiency with fractions with often little emphasis given to estimation with fractions. The games and related activities presented here may provide students with opportunities to develop estimation skills with fractions.

Additionally, a teacher can help students focus on important fraction concepts, relationships, and extensions.

Students also need to have opportunities to investigate various strategies, possible outcomes, and alternatives. Students need to participate in mathematical thinking beyond basic computations. They need to "think" mathematics as well as "do" mathematics. These games can help a teacher provide an environment in which students are encouraged to think and explore.

Try these games with your students. Encourage them as their skills improve and their insight increases. Allow them to be creative - they may make their own versions of the games (sum closest to 1/2 or 2, change the intervals on the range game, etc.). Certainly, these games could be played without using a computer - a spinner would do quite well. The students would then have to find the largest possible sum, smallest sum, sum closest to 1, etc. Whatever the format, the games can provide students with opportunities to refine their estimation skills, their understanding of fractions, and engage in some important mathematical behaviors.

A copy of this game can be obtained by sending a blank disk to Terry Goodman, Department of Mathematics and Computer Science, CMSU, Warrensburg, MO 64093.

Illinois Institute for Statistics Education

The Illinois Institute for Statistics Education (IISE) will hold a three-week summer workshop on the teaching of statistics for middle school and high school teachers, July 2-20, 1990, at the University of Illinois, Urbana-Champaign. The purpose of IISE is to encourage excellence in American secondary school statistics education.

The IISE summer program, funded by the National Science foundation, is open to school district teams of 3-5 teachers. The workshop is designed to introduce teachers to modern statistical concepts and how to teach them. The workshop features a "hands-on," data-based approach to teaching statistics (Monte Carlo method) that has been shown to be effective with students at a wide variety of grade and ability levels. The workshop staff is made up of UIUC faculty in statistics and mathematics education, classroom teachers, and computer specialists. While it is expected that most team members will be mathematics teachers, the inclusion of a teacher on the team in a field that makes use of statistics, such as biology or social science, is encouraged.

The Illinois Institute plan calls for each school district team to have a coordinator who works at the school district level (for example, a mathematics supervisor or resource teacher). Applications to attend the Institute are initiated by this team coordinator, who identifies the membership of the school district team and assists in the preparation of the application materials. During the 1990-1991 school year, the team coordinator plays a key role in assisting the teachers in implementing the ideas and materials from the summer workshop.

Graduate credit, housing, meals, transportation, and stipends are provided for summer participants. For details, contact: Dr. Janny Q. Travers, IISE Program Coordinator, UIUC Department of Statistics, 725 S. Wright Street, Champaign, IL 61820. (217) 244-7284. **Deadline for Application - March 1, 1990.**

THE SPREADSHEET IN THE MATHEMATICS CLASSROOM

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A spreadsheet is a rectangular array, or matrix, into whose individual cells one can electronically enter numbers, strings (for example, letters, words, phrases) and mathematical formulas. Spreadsheets such as Excel, Microsoft Works, Supercalc, Appleworks, and Lotus 1-2-3 are readily available with at least one brand for each brand of microcomputer. Their most common uses are in the financial and administrative areas (balance sheets, mortgage payment and amortization schedules, baseball statistics, class record books, etc.). Unfortunately, there is one valuable use for the spreadsheet which has been largely overlooked, and that is in the teaching of pre-algebra and algebra.

There are numerous ways that the microcomputer can strongly supplement the teaching of mathematics. For example, a spreadsheet can be used to teach the concept of "function." Consider $f(x) = 2x + 5$. Using a spreadsheet, once the formula 2*A1 + 5 is entered into cell B1, the student need only enter different values of x into cell A1 to immediately see the function value in cell B1. For example, as indicated below, entering a 7 into cell A1 will result in the immediate appearance of the function value 19 in cell B1:

	A	B	C	D
1	7	19		
2				
3				

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In the following illustration, the slope and y-intercept of a line appear as soon as the coordinates of two of its points are entered into cells B1, C1, B3, and C3:

	A	B	C	D	E
1	FIRST POINT	2	4		
2					
3	SECOND POINT	1	5		
4					
5	SLOPE =	-1			
6					
7	Y-INT =	6			
8					
9					

Cell B5 contains the formula (C1-C3)/(B1-B3) and cell B7 contains the formula C1-(B5*B1).

The only prerequisite skill required is that of moving the cursor and writing formulas, which, of course, is what is supposedly being taught in the class. Advantages of using a spreadsheet to supplement the teaching of pre-algebra and algebra include the following:

1. Instant gratification (in the first example, the 19 appears in cell B1 immediately upon entering the 7 into cell A1

2. The opportunity to literally <u>see</u> the variable (cell A1 in the first example) as a <u>placeholder</u>

3. Familiarization with a two-dimensional coordinate system, namely, the spreadsheet itself

4. Access to the introduction of elementary programming strategies

We illustrate four more examples: The Fibonacci sequence, rounding off to an arbitrary number of decimal places, an arithmetic sequence, and the quadratic formula.

Fibonacci Sequence (cell display)

	A	B	C
1	1		
2	1		
3	2		
4	3		
5	5		
6	8		
7	13		
8	21		
9	34		
10	55		

Cell A3 contains the formula A1+A2, cell A4 contains the formula A2+A3, cell A5 contains the formula A3+A4, cell A6 contains the formula A4+A5, etc. The user enters the first two terms of the sequence, beneath which the subsequent terms immediately appear.

Rounding Off (cell display)

	A	B	C
1	.149528	ROUNDED TO	
2			
3	5	DECIMAL PLACES IS	
4			
5	.14953		
6			

Cell A5 contains the formula INT((10^A3)*A1+.5)/(10^A3). The user enters the number to be rounded and the desired number of decimal places. The rounded number appears in cell A5.

Arithmetic Sequence (cell display)

	A	B	C	D
1	FIRST TERM =	12		
2				
3	COMMON DIFFERENCE =	5		
4				
5	TERM NUMBER	27	IS	142
6				

Cell D5 contains the formula B1+((B5-1)*B3). The user enters the sequence's first term, the common difference, and the subscript of the desired term. The desired term appears in cell D5.

Quadratic Formula (cell display)

	A	B	C	D
1	1	-7	8	
2				
3	DISCRIMINANT =	17		
4				
5	ROOTS ARE	5.561553	AND	1.438447

Cell B3 contains the formula B1^2-(4*A1*C1), cell B5 contains the formula (-B1+SQRT(B3))/(2*A1), and cell D5 contains the formula (-B1-SQRT(B3))/(2*A1). The user enters the coefficients of the quadratic function in cells A1, B1, and C1. The discriminant appears immediately in cell B3 and the roots (if real) appear immediately in cells B5 and D5. If the discriminant is negative, an error message appears in cells B5 and D5.

There are numerous ways the spreadsheet can be used in the mathematics classroom to reinforce the standard curriculum. The reader is invited to add to the following list.

Functions
Area of squares, rectangles, triangles and circles
Computation of sales tax and total cost with sales tax
Solving proportions
Arithmetic, geometric and Fibonacci sequences
Testing for divisibility
Dividing fractions
Computing monthly loan payments
Metric conversion
Converting Celsius to Fahrenheit
Rounding off
Computing batting averages to 3 decimal places
Finding the slope and y-intercept of a line
The Pythagorean Theorem
The distance formula
Solving systems of linear equations
Computing the inverse of a matrix
Finding the center and radius of a circle

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