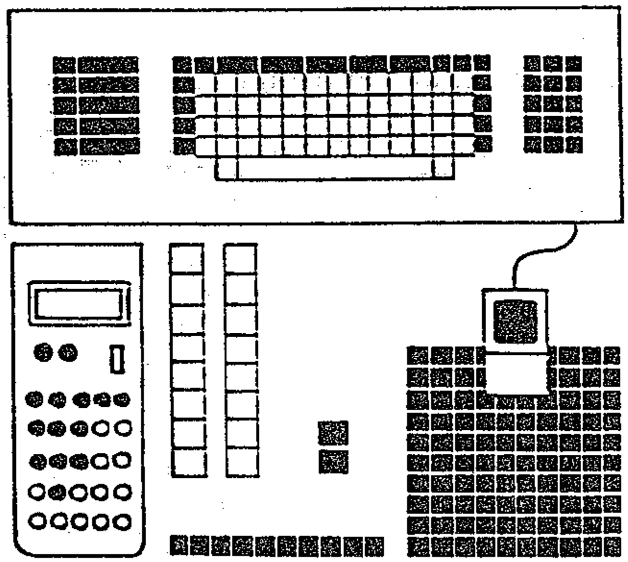
TEXAS MATHEMATICS TEACHER



STUFF "Strategic Tactics Ultimately For Fun"

Drilling For TEAMS (Without Tears)

Using A Computer Algebra System
To Help Understand The Derivative

Graphing Capabilities Of Calculators Extended

MAY 1989

TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published four times each year, in October, January, March, and May. All manuscripts and editorial correspondence should be addressed to George H. Willson, listed above.

Manuscripts should be neatly typewritten and double spaced, with wide margins, on 8 1/2 " by 11" paper. Authors should submit the signed original and four copies. Illustrations should be carefully prepared in black ink on separate sheets, the original without lettering and one copy with lettering added. As soon as possible after refereeing, authors will be notified of a publication decision. Originals of articles not accepted for publication will be returned to the authors. Two copies of the issue in which an author's manuscript appears will automatically be sent to the author.

SUBSCRIPTION and MEMBERSHIP information will be found on the back cover.

TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

Affiliated with the National Council of Teachers of Mathematics

May 1989

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ANNOUNCEMENTS

TEXAS MATHEMATICS TEACHER
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President's Message

"TEACHING IS A LONELY PROFESSION." This is usually how we appear to the new comers to our profession. We hand them a textbook or two, the curriculum guide for the district, the school improvement plan, the TEAMS objectives to be sure that their students have mastered them, the practice TAPS material, the Essential Elements for the course(s) that they are to teach and put them in a room with 35 unprepared youth. Nobody likely will appear again until it is time for TTAS.

As professionals, it is up to us to change this. We can become "big brothers/sisters" to these new teachers. We can serve as informal mentors, giving insight and advice on teaching strategies, school system policies, helps for teaching, helps for handling discipline and the like.

The best help one can receive is offered through the professional organizations; local, state and national. Invite the new teachers to go with you to these meetings. These meetings offer programs designed to offer motivational topics and enrichment ideas for both teachers and students. Many of these meetings offer Advanced Academic Training (AAT) for use on the professional growth part of TTAS.

Yes, this means that CAMT is just ahead and needs you to bring your friends, enemies, fellow teachers, and administrators to beautiful San Antonio on August 2-4, 1989 at the Henry Gonzales Convention Center.

On Monday, July 31, and Tuesday, August 1, 1989, a pre-conference will be held on the staff development training modules K-8, Algebra, Geometry, etc. SEE CAMT BULLETIN FOR REGISTRATION AND MORE INFORMATION ARRIVING AT YOUR HOME IN LATE SPRING OR EARLY SUMMER.

A breakfast for TCTM members will be held on Thursday, August 3, 1989 at 7:00 a.m. See CAMT program for location. EVERYONE MUST register for the TCTM BREAKFAST. It is free to members, but we must know before the conference how many to prepare for. We must pay for all ordered, so PLEASE DO NOT register unless you are certain that you will attend. PLEASE DO REGISTER IF YOU DO PLAN ON ATTENDING. On page four you will find the TCTM Breakfast Reservation Form. Return it to me and I will have your ticket at the registration desk. The ABSOLUTE deadline is July 20, 1989.

We will need some help at the registration desk. Please help us out be spending some time at the registration desk.

Look at your program and block off a few hours during which you can schedule working at the registration table. Look at page five and send it to me by July 15, 1989.

Thanks for all the help that many of you have given during the past year. It is so wonderful to work with such wonderful people. It sure makes one's job easy. THANKS AGAIN.

OTTO W. BIELSS, JR.
President, TCTM

TCTM BREAKFAST AND BUSINESS MEETING

It's become an annual event! This year at 7:00 a.m. on Thursday, August 3, we will breakfast at the HILTON HOTEL. See your CAMT program for the name and location of the room. The TCTM Breakfast and Business Meeting will be in the same place. Mail the reservation form below by July 11. This event is for members only and breakfast is BY RESERVATION ONLY. You will receive a ticket from the registration desk. The ticket will be taken at the door for the breakfast.

Mail by July 11 to:

Otto W. Bielss
2609 Trinity Street
Irving, Texas 75062

Name ____________________

Home Address ____________________

____

Home Telephone (   ) ____________________

Areas of Interest ______ K-5 ______ 6-8 ______ 9-12

Local Council Name ____________________

Please reserve a place for me at the TCTM Breakfast and Business Meeting, Thursday, August 3.

____
Signature

HELP NEEDED AT CAMT

REGISTRATION

TCTM has the very important responsibility at the Conference for the Advancement of Mathematics Teaching (CAMT) of On-Site-Registration. During the entire conference the Registration Table will be open. Working there is exciting and fun! Every participant at CAMT will want to stop there to greet friends. This conference is for teachers of mathematics, so teachers contribute some of your time to make it a success. The more volunteers we have, the fewer hours anyone will need to spend at the table. We expect more people than ever to attend CAMT this summer and, thus, there will be more people than ever coming through registration. Wednesday, August 2, will be the heaviest traffic time and therefore when we need the most help. Thursday and Friday are also important for someone must be at the table at all times and particularly on Friday as the conference is closed out.

Use this form to volunteer to work at the registration table.

Mail by July 15, 1989 to: **Otto W. Bielss**
2609 Trinity Street
Irving, Texas 75062

I will work at the Registration Desk during CAMT, August 2-4, 1989 at the Henry Gonzales Convention Center in San Antonio, Texas.

Times I will help ______________________________

Times I can not help ______________________________

Times do not matter, assign me whenever you most need me ________

Name ______________________________
Home Address ______________________________
City, State, Zip ______________________________
Home telephone ( ) ______________________________

STUFF

Strategic Tactics Ultimately For Fun

Beverly Millican

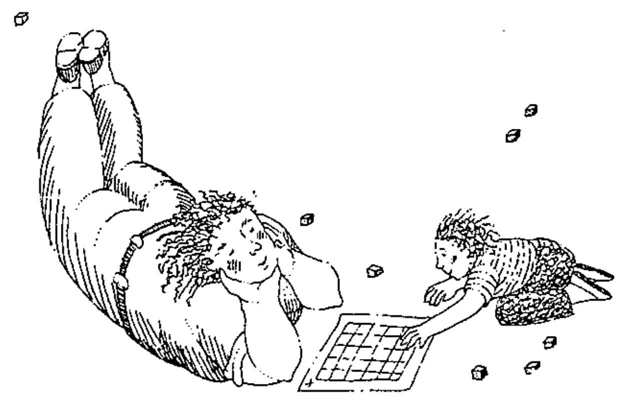
Elementary Mathematics Coordinator
Plano Independent School District

MATH GAMES TO PLAY AT HOME

How often have we as teachers had parents ask us, "Whatcan I do to help my child in math?" especially as summer time nears. Many parents are especially concerned that their children master addition, subtraction, multiplication, and division "facts," but often become frustrated when children resist efforts to drill and practice with them. Since flash cards quickly invite boredom, you may want to offer your parents alternative activities that encourage memorization of these facts while still providing an enjoyable family experience.

Although many teachers have amassed quite a collection of original games and activities that they may share with parents, a perfect source is Family Math, published by the University of California at Berkley. Besides being simple to read and understand, this book emphasizes a hands-on, problem-solving approach to mathematics. A little gentle advice is offered to parents by the Family Math authors: "Besides learning to compute, children benefit from enjoying the computations, appreciating the beauty and structure of numbers. Try to keep or regain the natural exuberance most young children have about numbers. Before these games become a drudgery, or a discussion become a quiz session, relax and stop for a while. The activities should be used to enrich your relationship with your child, and your child's relationship with mathematics, not create more stress!"

On the following pages are several games, the last of which is from Family Math, adapted for parents to use with primary age children, focusing on mastery of addition and subtraction facts.



Put those flash cards away and try these three games instead.

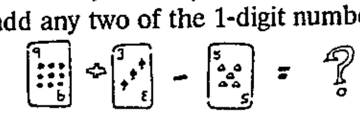
"AIM" Card Game

Tools **Deck of Cards**
Paper and Pencil

A game for 2-4 players

Practice both addition and subtraction math facts by playing this simple game with a regular deck of cards, from which all of the face cards have been removed. Leave the aces in the deck so they can represent "1." Shuffle the remaining cards, place them face down, and have each player draw three cards.

The object of the game is to "AIM" at a small number, like 8, by having each player first add any two of the 1-digit numbers



shown on the cards and then subtract the value of the 1-digit number on the remaining card.

You may have to jot down the different combinations you could get, but try to work it out in your head. The resulting numbers are compared, and the player closest to the target number wins the play and takes all of the cards. In case of a tie, the cards go in a pot. On any play, the winner also gets the cards that are in the pot. When too few cards remain for another play, the player who has taken the most cards is the winner.

The next time you play, vary the game by changing the "target" number to 6, 7, 9 or 11. Let your child choose the variation, or better yet, let him change the rules and invent new games with addition and subtraction.

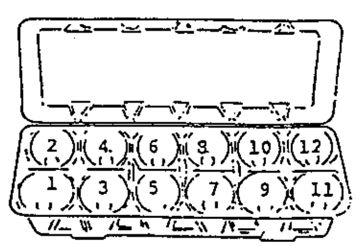
EGG CARTON TARGETS

Tools — **Egg cartons**
Ping-Pong balls or pennies, washers

A game for 2 or more players

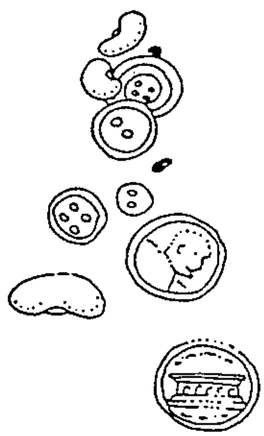
Don't throw away those empty egg cartons! Instead, save them to make a "target" for addition and subtraction practice.

First, make your "target" by labeling the egg carton with numbers 1 through 12. If you are working on a particular set of facts, i.e., all of the addition facts using 8 (8 + 1, 8 + 2, 8 + 3, etc.), label a ping-pong ball with "8." Take turns tossing the "8" ball into the egg carton. Each time the ball lands in a cup, the child tossing the ball should tell the sum of 8 and the number written in the cup. ("8 + 4 is 12"). Award a point for each correct fact.



To practice subtraction facts involving 8, each player should toss the ball and decide whether the number in the cup is less than 8 or greater than 8. The small of the two numbers should then be subtracted from the larger. To vary the games, change the number on the ping-pong ball (to 5 or 9, etc) and play again.

In order to practice all of the basic facts of addition (or subtraction), players should toss two objects (for example, pennies or washers) into the egg carton. The numbers in the cups should then be added (or subtracted) and a point awarded for correct answers.



Since fluency in knowledge of the facts is being developed, you may soon want to emphasize speed in reciting these facts, but first go for accuracy. Also, encourage your child to develop variations of these games, whether it involves the numbers you write in the egg cartons, or even the number of egg cartons you use.

TARGET ADDITION

<u>Tools</u> **Game board**
Markers (beans, paper bits)

A game for 2-4 players

This game will encourage your child to use his addition facts while also using his problem solving skills to plan a winning strategy.

First, choose a target number between 25 and 55. Take turns placing a marker on one of the numbers on the board, each time announcing the total of all the covered numbers. For example, if the first player covered a 4, the second a 3, and the third a 2, the sum would be 4 + 3 + 2 or 9. If the fourth player covered a 4, the total would be 9 + 4 or 13.

Remember, each square may be used only once. The first player to reach the target number **exactly** wins. If a player goes over the target number, he or she is out.

5	5	5	5	5
4	4	4	4	4
3	3	3	3	3
2	2	2	2	2
1	1	1	1	1

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Lawrence Hall of Science
University of California

DRILLING FOR TEAMS (WITHOUT TEARS)

Cheryl Fowler

Deer Park I.S.D.
in conjunction with Texas Tech University

The dreaded preparation for the TEAMS! It consists primarily of drill, drill, drill. Drills and pretests are necessary, but fun activities can also prepare students for the TEAMS test.

One such activity is FRACTIONARY. It is played similarly to Pictionary. All that is needed is:

Blackboard and chalk
A paper sack
Fractionary cards, run on light-weight cardboard, and cut apart

THIRD GRADE VERSION: The cards that have 1/2, 1/3, 1/4, 2/3, and 3/4 written on them satisfy the specifications for the TEAMS at the third grade level (Figure 1). One volunteer comes to the front of the room, reaches into the paper sack and pulls out one of the fractionary cards. He or she draws a pictorial representation of the fraction written on the card on the blackboard. The student saying the correct fraction first receives a point. The fractionary card should be returned to the bag, so that as the game progresses, the process of elimination is avoided, and the students receive more drill.

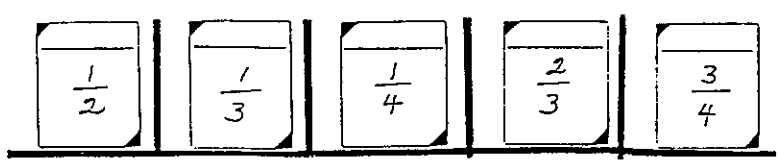

Fig. 1

FIFTH GRADE VERSION: The cards that have pictorial representations (Figure 2) and the shaded bars (Figure 3) should be used at the fifth grade level. Divide the class into two teams. A person from one of the teams comes to the front of the room, and reaches into the bag and pulls out a fractionary card. He or she then draws a pictorial representation on the blackboard of an EQUIVALENT fraction to the fraction represented on the card. The teacher should verify that it is, indeed, an equivalent fraction. If one of his or her own team members say the name of the fraction being represented by the drawing on the blackboard first, then that team gets a point, and it remains that team's turn. However, if the opposite team guesses the fraction being drawn first, no one gets a point and play (the opportunity to do the drawing) goes to the opposite team. When the time allotted for the game is up, the team with the most points wins.

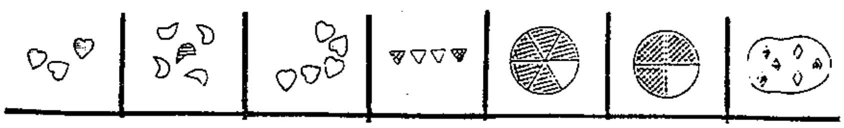

Fig. 2

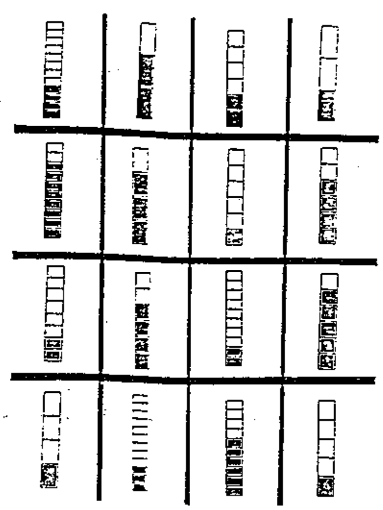

Fig. 3

SEVENTH GRADE VERSION: The fractionary cards with the addition and subtraction problems on them (Figures 4 and 5) are to be used at the seventh grade level. Divide the class into two teams. A person from one of the teams comes to the front of the room, and reaches into the bag and pulls out a fractionary card. He or she computes the answer to the problem with pencil and paper, or in his or her head, and draws a pictorial representation of the fractional answer on the blackboard. If one of his or her own team members guesses the answer represented in pictorial form on the blackboard first, the team gets a point, and it remains that team's turn. However, if the opposite team guesses the answer to the problem first, no one gets a point, and play (the opportunity to select a problem, solve it, and draw pictorial representation of the answer on the blackboard) goes to the opposite team.

$\frac{5}{6} - \frac{1}{3} =$	$\frac{1}{3}$ $- \frac{1}{4}$	$\frac{9}{12} - \frac{8}{12} =$	$\frac{3}{4} - \frac{4}{6} =$
$\frac{3}{4}$ $- \frac{1}{6}$	$\frac{4}{12}$ $- \frac{1}{6}$	$\frac{2}{3}$ $- \frac{4}{12}$	$\frac{5}{6} - \frac{2}{6} =$
$\frac{3}{4}$ $- \frac{2}{3}$	$\frac{2}{3}$ $- \frac{1}{2}$	$\frac{1}{2}$ $- \frac{1}{6}$	$\frac{1}{2}$ $- \frac{4}{12}$

Fig. 4

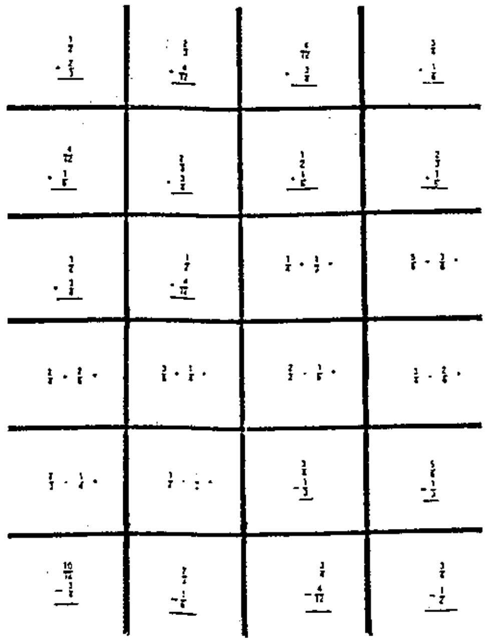

Fig. 5

The format can be carefully designed to be non-threatening while competitive. At the fifth and seventh grade levels, the class is divided into teams which makes it fun and exciting for the competitive student in nature; but, the whole team works together which gives it the feature of being unintimidating. To eliminate the shy, or not-so-sure-of-himself (or herself) student from having to come to the front of the room and solve the problem, ask for volunteers. If time is taken to discuss each problem, and to give explanation both verbally and visually as to why an answer is right or wrong, learning can take place, and concepts reinforced without every student coming to the front of the room. Sometimes just telling students before a review of fractions to listen carefully and to take notes because a game will follow, is sufficient motivation for students to pay closer attention. The handling of this issue is up to the individual teacher, who knows his or her students best. In closing, one last recommendation: <u>tell</u> students that this is a review for the TEAMS test. Sometimes students pay closer attention and try to apply themselves more if they know the reason they are doing an activity.

Activities are not substitutes for drill worksheets, but rather supplements. They are a different way to do the same thing - prepare students for the TEAMS test. Competency testing is probably here to stay, so, for the student's sake and the teacher's, let's find some creative ways to prepare for it!

*The inspiration for this activity came from my two children, ages 8 and 15, and my husband who knew I was looking for new, exciting ways to drill the aspects of fractions that appear on the TEAMS test, and who love Pictionary.

SCHOLARSHIPS
FOR FIRST-TIME CAMT GOERS

Are you a TCTM member who has never attended the Conference for the Advancement of Mathematics Teaching? Is your trip to CAMT this summer not being funded by your district or local math council? Do you have a position for the Fall of 1989 in which you teach math or influence math education? If your answer to all questions was "yes," then you are eligible to apply for one of the two TCTM $100 scholarships for CAMT. Winners will be notified by the end of June and will be introduced and checks presented at the TCTM Breakfast, August 3. Be sure to register for the Breakfast.

On this form, get the appropriate signature and mail it with the recommendation and your statement **by June 15 to:**

Otto W. Bielss
2609 Trinity Street
Irving, Texas 75062

Name ______________________________
Home Address ______________________________
Home Telephone ( ) ______________________________
Local Math Council ______________________________
Position (Be Specific) ______________________________

1) Verification: I verify that ______________________________
   a) Has not previously attended CAMT.
   b) Is not receiving district or math council funds for the trip this August 2-4.
   c) Has a position teaching mathematics or influencing math education for the Fall of 1989.

____
Signed by Principal or Supervisor

2) Include a brief recommendation from your principal or supervisor.
3) Include an account by you of how the ideas you get at CAMT will be shared with colleagues and how those ideas may influence your job performance.

USING A COMPUTER ALGEBRA SYSTEM TO HELP UNDERSTAND THE DERIVATIVE

John H. Mathews

California State University
Fullerton, California

Computer Algebra systems (CAS's) such as DERIVE, MAPLE, muMATH and MATHEMATICA can be used as a powerful assistant for performing symbol manipulations in algebra and calculus (see ref. [1,2,3,5]). High school and undergraduate students in mathematics can benefit from these interactive software packages to keep track of equations during complicated manipulations (see ref. [1,2,3,4,5]). This article shows the step-by-step process in translating mathematical theory into the symbolic-manipulation setting. The product muMATH has been illustrated because of its widespread availability. It is available on IBM P.C. compatible computers which use the MS-DOS operating system. Examples include the COMPAQ, Eagle, Columbia, Corona, Zenith, Tandy, and TI professional. A new version, muMATH-80, has been released which runs on the APPLE II family of computers and is available through the Educational Technology Center, College of Education, University of South Carolina at Columbia.

Background

The symbol dy/dx as the notation for the derivative was first used by the German mathematician Gottfried Wilhelm Leibniz (1646-1716). Leibniz envisioned small changes dx and dy and the derivative as the ratio dy/dx as dx and dy become small. This concept can be concretely illustrated with a computer and the symbol manipulation program muMATH. We shall go through the process step by step and discover how to define dy in terms of x and dx and take the limit of the difference quotient.

Rate of Change on a line.

Consider the function

(1) $y = f(x) = b + mx.$

The rate at which f is changing with respect to x, is the change in y per unit change in x. When x changes, by the amount dx, from x to x + dx then y changes, by the amount dy, from f(x) to f(x + dx). This gives us a definition for the symbol dy, that is

(2) $dy = f(x + dx) - f(x).$

The slope of the graph of (1) at the point (x,y) is defined as the difference quotient dy/dx. Moreover, since (1) is a linear function this quotient is constant

(3) $$\frac{dy}{dx} = \frac{f(x + dx) - f(x)}{dx} = \frac{b + m(x + dx) - b - mx}{dx} = m.$$

The symbol manipulation program muMATH will perform all the above tasks. First we must define the a function f(x) which we will use in our examples.

```
FUNCTION F (X),
 EVAL (Y)
ENDFUN$
```

Next, we must place the mathematical formula b + mx in the variable Y:

```
? Y: B + M X;
```

MuMath will respond and let us know that we have successfully entered the formula:

```
@: B + X M
```

Next, use muMATH to define dy. Here the colon ":" is muMATH's replacement symbol

```
? DY: F(X + DX) - F(X);
```

The muMath calculation for this difference is

```
@: M DX
```

Then we can ask muMATH to find the difference quotient, that is

```
? DY/DX;
```

and the muMATH answer is

```
@: M
```

The above dialogue with the computer is straightforward and shows how to algebraically manipulate the quantities dx, dy and dy/dx. Also, for the linear function $y = b + mx$, it should be observed that the quotient dy/dx is independent of the choice of x and dx.

<u>Rate of change for a nonlinear function</u>.

In general, let us consider a nonlinear function

(4) $y = f(x)$.

The rate at which f is changing with respect to x is not constant at all points. The number

(5) $dy = f(x + dx) = f(x)$

measures the net change dy in y corresponding to a change dx in x. Thus, the difference quotient

(6) $\frac{dy}{dx} = \frac{f(x + dx) - f(x)}{dx}$

measures the average rate of change of y with respect to x over the interval (x,x + dx).

The limit of the ratio dy/dx as dx approaches 0 is called <u>the instantaneous rate of change of y with respect to x</u> and is denoted f'(x). The quotient in (6) is approximately equal to f'(x) when dx is small, and the approximation becomes better and better as the interval width dx becomes smaller. Thus, we take the limit:

$$(7)\quad f'(x) = \lim_{dx \to 0} \frac{dy}{dx} = \lim_{dx \to 0} \frac{f(x + dx) - f(x)}{dx}.$$

These concepts can be reinforced with a computer algebra system which has a built-in symbolic limit procedure.

EXAMPLE 1. Let $f(x) = a + bx + cx^2$. Use definition (6) to find f'(x).

Solution. The formula for f(x) must be placed in the variable Y:

```
? Y: A + B X + C X^2;
@: A + B X + C X^2
```

The difference dy is defined in muMATH with the statement

```
? DY: F(X + DX) - F(X);
@: B DX - C X^2 + C*(X + DX)^2
```

muMATH displays the expanded form of dy if we issue the command

```
? DY: EXPAND(DY);
@: B DX + 2 C X DX + C DX^2
```

This last quantity is the change in $f(x) = a + bx + cx^2$ over the interval [x,x + dx] and depends on both x and dx. The difference quotient corresponding to equation (6) is obtained by typing

```
? DY/DX;
@: B + 2 C X + C DX
```

Finally, the derivative of is found by taking the limit of dy/dx as dx -> 0. This is accomplished with the muMATH command

```
? LIM( DY/DX , DX , 0 );
@: B + 2 C X
```

Therefore, we have been assured (by our computer) that f'(x) = b + 2cx.

The advantage of a computer algebra system is not obvious from the previous example. When f(x) is a higher degree polynomial, the assistance from a CAS can be appreciated. For simplicity, the next example involves a simple power of x.

EXAMPLE 2. Let $f(x) = x^9$. Find f'(x).

Solution. First, place the formula x^9 in the variable Y:

```
? Y: X^9;
@: X^9
```

Then the difference dy is obtained

```
? DY: F(X+DX) - F(X);
@: -X^9 + (X + DX)^9
```

Which can be expanded

```
? DY: EXPAND(DY);
@: 9 X DX^8 + 36 X^2 DX^7 + 84 X^3 DX^6 + 126 X^4 DX^5
+ 126 X^5 DX^4 + 84 X^6 DX^3 + 36 X^7 DX^2 + 9 X^8 DX + DX^9
```

Notice how easily muMATH found the binomial coefficients 9, 36, 84, 126, 126, 84, 36, 9, 1 for the terms in the expression DY. Next, the difference quotient DY/DX is determined

```
? DY/DX;
@: 9 X DX^7 + 36 X^2 DX^6 + 84 X^3 DX^5 + 126 X^4 DX^4
+ 126 X^5 DX^3 + 84 X^6 DX^2 + 36 X^7 DX + 9 X^8 + DX^8
```

Then the derivative of $f(x) = x^9$ is found by taking the limit

```
? LIM( DY/DX , DX, 0 );
@: 9 X^8
```

Therefore, muMATH has helped prove that $f'(x) = 9x^8$.

The full power of a CAS is available for more complicated expressions. The derivation in the next example would take a student at least a half page of work, and is prone to human error.

EXAMPLE 3. Find the derivative of $f(x) = 1/x^4$.

Solution. Place the formula $1/x^4$ in the variable Y:

```
? Y: 1/X^4;
@: 1/X^4
```

Compute the difference dy

```
? DY: F(X+DX) - F(X);
@: -1/X^4 + 1/(X + DX)^4
```

This quantity can be expressed over a common denominator if we issue the command

```
? DY: EXPD(DY);
@: (-4 X DX^3 - 6 X^2 DX^2 - 4 X^3 DX - DX^4) /
   (X^4 DX^4 + 4 X^5 DX^3 + 6 X^6 DX^2 + 4 X^7 DX + X^8)
```

The difference quotient is

```
? DY/DX;
@: (-4 X DX^2 - 6 X^2 DX - 4 X^3 - DX^3) /
   (X^4 DX^4 + 4 X^5 DX^3 + 6 X^6 DX^2 + 4 X^7 DX + X^8)
```

Finally, the derivative of $f(x) = 1/x^4$ is found by taking the limit

```
? LIM( DY/DX , DX , 0 );
@: -4/X^5
```

Therefore, muMATH has shown that $f'(x) = -4/x^5$.

In conclusion, we have shown how the limit definition for the derivative can be implemented symbolically on personal computers. This illustrates the computer's role as an "expert system" in calculus and can be viewed as an important role of "artificial intelligence." There are literally hundreds of other ways for computer algebra to be used in mathematics. The reader is encouraged to use muMATH or some other CAS and discover its capabilities.

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GRAPHING CAPABILITIES OF CALCULATORS EXTENDED

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During mathematical training, an important and convenient skill for one to acquire is that of being able to rapidly "picture" functions and relations. For example, if quantitative position data for a projectile "fit" the algebraic representation $y = bx - ax^2$, then the "picture" in the mind's eye is that of a "parabola opening 'down'". And, if graphed, the geometrical representation is indeed a "downward opening" parabola. Another example might be that of a "supply-demand" situation algebraically expressed a $apq - b = 0$. When graphed as $p = f(q)$ one sees a "rotated" hyperbola that "says" as quantities increase, prices decrease.

It may be that the important skill of picturing quantitatively related data, both in the mind's eye and in graphs, has gained a valuable aid - the relatively inexpensive handheld calculators with graphing capabilities. Some models are convenient to use as they have built-in GRAPH keys. When the GRAPH key is pressed, the screen of the calculator's display shows: Graph y = . The user may enter $4x - x^2$, touch the "carriage return," that is, the EXEcute key, and then see a parabola plotted on the display screen. If the user again presses the Graph key, Graph y = , appears. By entering x^{-1} and pressing EXE (the execute or return key), the two branches of an equilateral hyperbola are plotted on the display "over" the parabola already there. One immediately sees significant aids in physics, economics and mathematics classrooms.

However, the "Graph y = " restricts one to functions. How would one graph the members of the families of limacons, lemniscates, and rose-curves to name just a few polar curves? These are the questions this paper intends to answer.

A first step is for the user to become familiar with parameterizations. Parameterizations for circles, ellipses, hyperbolas and cycloids are treated as standard topics in most analytic geometry courses.

For example, the circle $x^2 + y^2 = 9$ may be parameterized as $x = 3\cos(T)$ and $y = 3\sin(T)$, a usual inclusion by almost all texts.

Most texts do not offer parameterization techniques to handle members of the families of rose-curves, $r = A\sin(B\Theta)$ and $r = A\cos(B\Theta)$. A procedure to parameterize the particular rose-curve $r = 2\sin(3\Theta)$ follows.

One recalls that $r^2 = x^2 + y^2$ (a "polar" transformation) and that $\cos^2(T) + \sin^2(T) = 1$. Then,

$$r^2 = 4\sin^2(3T),$$
$$r^2 = (4\sin^2(3T)) \cdot 1,$$
$$x^2 + y^2 = 4\sin^2(3T)\cos^2(T) + 4\sin^2(3T)\sin^2(T).$$

Thus, a convenient parameterization could be:

$$x = 2\sin(3T)\cos(T), \qquad y = 2\sin(3T)\sin(T).$$

A generic "BASIC" program to plot this 3-petal rose curve is:

```
10  Set to RADIAN measure
20  T = φ
30  2sin(3T)cos(T) = X
40  2sin(3T)sin(T) = Y
50  PLOT X,Y
60  T + pi/k = T  (one selects k arbitrarily)
70  If T < 2pi GOTO 30
80  END
```

For a handheld calculator (e.g., the Casio fx-7000G) this program is entered in the "WRITE" mode. (An appendix containing the key stroke operations follows at the end of this article to further explain the programming steps described here.)

The calculator must first be turned on by sliding the ON-OFF switch to the ON position. The display screen will become occupied by a display telling that the machine is in "RUN" mode.

The following is entered in WRITE (i.e., programming) mode. The user pressed the MODE key then the 2 key to commence programming. These two key strokes put the calculator into "WRITE" or programming mode. Explanations follow the program lines that are now to be entered into memory.

RAD	(Setting the machine to calculate in radian measure is done by pressing the MODE key; then the 5 key; finally, press the EXE key.)
ϕ -> T	(Press ϕ and press lower case ? for the right-pointing "assignment" arrow which is an evaluation operation; then press the ALPHA key then the division key for alphabetic T which is embossed in red on the template; finally, press the EXE key.)
Lb1 1	(Press SHIFT then the left arrow edit key for "Lb1;" then press 1; finally, press EXE.)
2sin(3T)cos(T) -> X 2sin(3T)cos(T) -> Y	(To get the four T's and the X and the Y on these two lines, press the ALPHA key before the red alphabetical T, X and Y.)
Plot X,Y	(Press the SHIFT key and the MDisp key to get "Plot;" then press ALPHA before red X; then SHIFT and left parenthesis key to get the comma; now press the ALPHA before red Y; finally, press the EXE key.)

T + π/45 -> T	(ALPHA ; T ; + ; SHIFT EXP for pi; division operation key; 4 ; 5;, lower case ? key; ALPHA ; T ; finally, press the EXE key.)
T < 2π => Goto 1	(ALPHA; T; SHIFT; 3 for "less than;" 2 ; SHIFT ; EXP for pi; SHIFT ; 7 for the conditional implication "two-barred" right arrow; press SHIFT key and Prog key for "Goto;" 1 ; finally, press the EXE key.)

To "run" this program, press MODE and press 1 (for "RUN" mode); press PROG key and press ϕ (if this program has been named program number zero); finally, then, press the EXE key.

The machine will blink repeatedly until the "Do-Loop" is satisfied. When it stops blinking, first press the key AC, the "all clear" key. This clears any text message and possibly a syntax error message from the screen. If an error message is displayed, the user may disregard it.

Pressing the key G<->T gets the graphics display to appear on the screen. (What was desired.) A three-petaled rose curve will occupy the screen. See Figure 1.

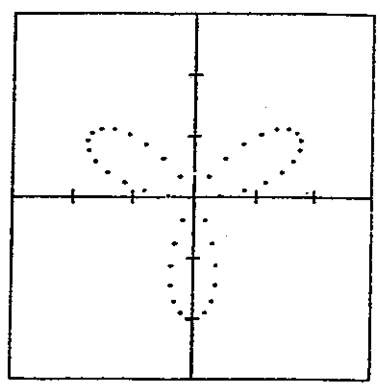

Fig. 1 3-Petal Rose Curve

This program can be made more sophisticated by utilizing the capabilities of the calculator described in its documentation and programming $r = A\sin(B\Theta)$ or $r = A\cos(B\Theta)$ where A and B are selected arbitrarily during execution to produce other rose-petal curves.

The number of iterations a program uses is arbitrary and may be determined in a manner similar to the above. Of course, an "extra" line in a program would allow the user internally to select fewer iterations to get more rapid execution, or more iterations to get slower execution but more "complete" pictures.

Generalizing, consider two types of polar equations:

(1) $r = f(\Theta)$, and (2) $r^2 = g(\Theta)$.

If (1), then a parameterization could be:

$x = \cos(T)f(T)$, $y = \sin(T)f(T)$ as was exhibited above.

This technique would also be used for all members of the families of limacons, $r = b + a\sin(\Theta)$ and $r = b + a\cos(\Theta)$. The limacon $r = 1 + 2\cos(\Theta)$ is shown in Figure 2 and a detailed program for it is included in the appendix.

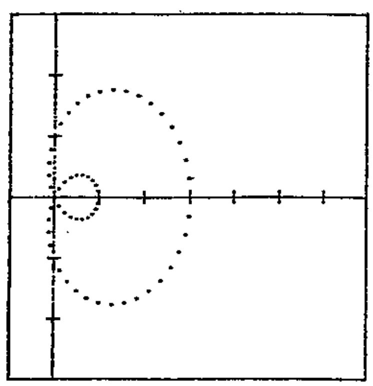

Fig. 2 Limacon

```
sin(2T) > φ =>  Goto 1
pi -> T
Lb1 2
3cos(T) √sin(2T) -> X
3sin(T) √sin(2T) -> Y
Plot X, Y
T + pi/45 -> T
sin(2T) > φ =>  Goto 2
```

"Running" this program is executed as described as above. A detailed program of key-strokes is in the appendix.

There are many valuable considerations available from just these few examples. Some of them deal with topics of domains and ranges; others deal with points of horizontal and vertical tangent lines; and still others deal with estimations of areas enclosed by the loops of particular curves.

These items should be of interest to high school students preparing for courses in calculus.

APPENDIX

PROGRAM #1

FIRST: Slide the OFF-ON switch to ON.

SECOND: Press MODE key; press 3 key; press SHIFT key; press DEL key. (This series of keystrokes clears all programs from memory. The user may choose to ignore this SECOND step.)

THIRD: Press MODE key; press 2 key; press EXE key. (These keystrokes put the calculator into "WRITE" or programming mode.)

Line	MODE 2						Program #1 for $r = 2\sin(3\theta)$										
1	MODE	5	EXE														
2	Ø	→	ALPHA	$\div_T$	EXE												
3	SHIFT	Lbl ⇦	1	EXE													
4	2	sin	(	3	ALPHA	$\div_T$	)	cos	(	ALPHA	$\div_T$	)	→	ALPHA	$+_x$	EXE	
5	2	sin	(	3	ALPHA	$\div_T$	)	sin	(	ALPHA	$\div_T$	)	→	ALPHA	$-_y$	EXE	
6	SHIFT	Plot	ALPHA	$+_x$	SHIFT	, (	ALPHA	$-_y$	EXE								
7	ALPHA	$\div_T$	+	SHIFT	π EXP	÷	4	5	→	ALPHA	$\div_T$	EXE					
8	ALPHA	$\div_T$	SHIFT	< 3	2	SHIFT	π EXP	SHIFT	⇒ 7	SHIFT	Goto Prog	1	EXE				

To "RUN" this program, press the MODE key; press the 1 key; press the PROG key; press the ϕ key; press the EXE key. This will cause the machine to execute program number zero. To have chosen to have numbered the program differently, one through nine, in the THIRD step above, after pressing MODE 2, the right arrow edit key could be pressed to move the flashing cursor to another number, 1 - 9, chosen for this program before pressing the EXE key.

To view the graph that has been plotted:

FIRST: Allow the screen to quit blinking.

SECOND: Press the AC key. (Ignore any text display even if there is an error or syntax message displayed.)

THIRD: Press the G <-> T key. The picture will occupy the screen. Pressing G <-> T again "toggles" back from G (i.e., GRAPHICS) display to T (i.e., TEXT) display.

PROGRAM #2

FIRST: Slide the OFF-ON switch to ON.

SECOND: Press MODE key; press 3 key; press SHIFT key; press DEL key. (This series of keystrokes clears all programs from memory. The user may choose to ignore this SECOND step.)

THIRD: Press MODE key; press 2 key; press EXE key. (These keystrokes put the calculator into "WRITE" or programming mode.)

Line	MODE 2							Program #2 r = 1 + 2cos(θ)										
1	MODE	5	EXE															
2	0	→	ALPHA	÷ T	EXE													
3	SHIFT	Lbl	1	EXE														
4	COS	(	ALPHA	÷ T	)	(	1	+	2	COS	(	ALPHA	÷ T	)	)	→ ALPHA	+ X	EXE
5	SIN	(	ALPHA	÷ T	)	(	1	+	2	COS	(	ALPHA	÷ T	)	)	→ ALPHA	− Y	EXE
6	SHIFT	Plot	ALPHA	+ X	SHIFT	;	ALPHA	− Y	EXE									
7	ALPHA	÷ T	+	SHIFT	π EXP	÷	4	5	→	ALPHA	÷ T	EXE						
8	ALPHA	÷ T	SHIFT	< 3	2	SHIFT	π EXP	SHIFT	⇒ 7	SHIFT	Goto Prog	1	EXE					

To "RUN" this program, repeat the instructions at the bottom of Program #1 with the following added changes:

Press the SHIFT key; press the G <-> T key; press the EXE key. (This is done to remove the previous picture that was plotted. It may be that the user may want to see both graphics plotted together. If so, then this clearing operation may be ignored.)

PROGRAM #3

FIRST: Slide the OFF-ON switch to ON.

SECOND: Press MODE key; press 3 key; press SHIFT key; press DEL key. (This series of keystrokes clears all programs from memory. The user may choose to ignore this SECOND step.)

THIRD: Press MODE key; press 2 key; press EXE key. (These keystrokes put the calculator into "WRITE" or programming mode.)

Line	MODE 2						Program #3 for $r^2 = 9\sin(2\theta)$											
1	MODE	5	EXE															
2	Ø	→	ALPHA	÷ T	EXE													
3	SHIFT	Lbl	1	EXE														
4	3	cos	(	ALPHA	÷ T	)	√	(	sin	(	2	ALPHA	÷ T	)	)	→ ALPHA	+ X	EXE
5	3	sin	(	ALPHA	÷ T	)	√	(	sin	(	2	ALPHA	÷ T	)	)	→ ALPHA	− Y	EXE
6	SHIFT	Plot	ALPHA	+ X	SHIFT	,	ALPHA	− Y	EXE									
7	ALPHA	÷ T	+	SHIFT	π EXP	÷	4	5	→	ALPHA	÷ T	EXE						
8	sin	(	2	ALPHA	÷ T	)	SHIFT	≥	Ø	SHIFT	⇒	SHIFT	Goto Prog	1	EXE			
9	SHIFT	π EXP	→	ALPHA	÷ T	EXE												
10	SHIFT	Lbl	2	EXE														
11	Repeat all of Line 4 above here.																	
12	Repeat all of Line 5 above here.																	
13	Repeat all of Line 6 above here.																	
14	ALPHA	÷ T	+	SHIFT	π EXP	÷	4	5	→	ALPHA	÷ T	EXE						
15	sin	(	2	ALPHA	÷ T	)	SHIFT	≥	Ø	SHIFT	⇒	SHIFT	Goto Prog	2	EXE			

To "RUN" this program, repeat the instructions at the bottom of Program #1 with the following added changes:

Press the SHIFT key; press the G <-> T key; press the EXE key. (This is done to remove the previous picture that was plotted. It may be that the user may want to see both graphics plotted together. If so, then this clearing operation may be ignored.)

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