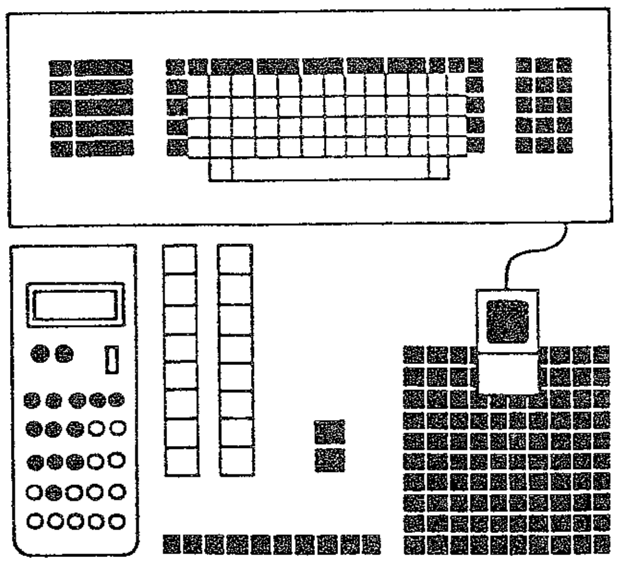
TEXAS MATHEMATICS TEACHER



STUFF "Strategic Tactics Ultimately for Fun"

Mental Math

Female Math Avoidance and Causal Attribution Theory

Fibonacci Operation Tables: Activities and Patterns

OCTOBER 1988

TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published four times each year, in October, January, March, and May. All manuscripts and editorial correspondence should be addressed to George H. Willson, listed above.

Manuscripts should be neatly typewritten and double spaced, with wide margins, on 8 1/2 " by 11" paper. Authors should submit the signed original and four copies. Illustrations should be carefully prepared in black ink on separate sheets, the original without lettering and one copy with lettering added. As soon as possible after refereeing, authors will be notified of a publication decision. Originals of articles not accepted for publication will be returned to the authors. Two copies of the issue in which an author's manuscript appears will automatically be sent to the author.

SUBSCRIPTION and MEMBERSHIP information will be found on the back cover.

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TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

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ANNOUNCEMENTS

President's Message

As the great humorist Will Rogers once said, "Even if you are on the right tract, you'll get run over if you just sit there." Well, it certainly seems that CAMT is on the right track and we are not going to sit still. The meeting in Houston during the first week in August was a tremendous success, in fact, it was overflowing in many areas. We had 3200 pre-registration; a first day total of 3700 and a final total of 3821. We had 517 sections, over 700 speakers/leaders, and some 500 presiders. Thanks to Claire Gifford for her work on the program, the presiders, the speakers, to all who attended and worked to make the CAMT program and meeting such a success. THANKS AGAIN!! The meeting was wonderful, as was the Brown Convention Center.

I do not want to miss anyone who helped make the conference a great success. Therefore, thanks to all who had any part, performed any task or just gave support to the CAMT conference. Without each and everyone of you, we could not have such a great organization and conference.

It is with great pleasure that I assume the office of President of TCTM. Please remember that everything is a sum of its parts and you are a part of TCTM. Your voice is needed and will be heard. Please let me know of any problems that you might have with TCTM, CAMT, or the JOURNAL. Please keep the officers of TCTM informed of any activities that your local organization has. We would like to publish a calendar of events in the Journal, but we need your help on events and dates so that we can perform this task.

Now back to CAMT and Houston. Yes, we did have a few problems. Long registration lines, long lunch lines, long elevator lines at the hotels, lines for the buses, many crowded sessions, etc., but wasn't it great? Most of those attending left Houston on their way to San Antonio and CAMT XXXVI on August 2-4, 1989. See you there.

The CAMT board needs your suggestions for improving any of the above mentioned bottle necks. If you have suggestions for improving any of the operations, please send them to me or just call. This is your organization and meeting. Let's work together to improve it.

We need your help in renewing your membership. See the back cover and respond as necessary. Remind your co-workers to join TCTM.

We need your help in obtaining manuscripts for the JOURNAL. If you have, or know of things that are good and work in the classroom, please write them up and let's get them in print for others to use.

We need help in reviewing and refereeing articles for the JOURNAL. If you or any of your co-workers or friends would help us perform this task, please contact the editors or me and we will forward an application for this position.

In a related article elsewhere in this journal, you will find that a revision of the Board of Directors has been voted in. We will have eight regions instead of the previous four. We need help in locating qualified personnel to staff these positions. If you have any suggestions, please call or write me ASAP. Please keep in mind that any officer or director must be a member of TCTM.

OTTO W. BIELSS, JR.

<u>Note From the Editors</u>

Please check the expiration date on your mailing label, your address and zip code. If any are incorrect, send your correction to John Huber, Sam Houston State University, P. O. Box 6768, Huntsville, Texas 77340. The data base for TCTM is kept by John.

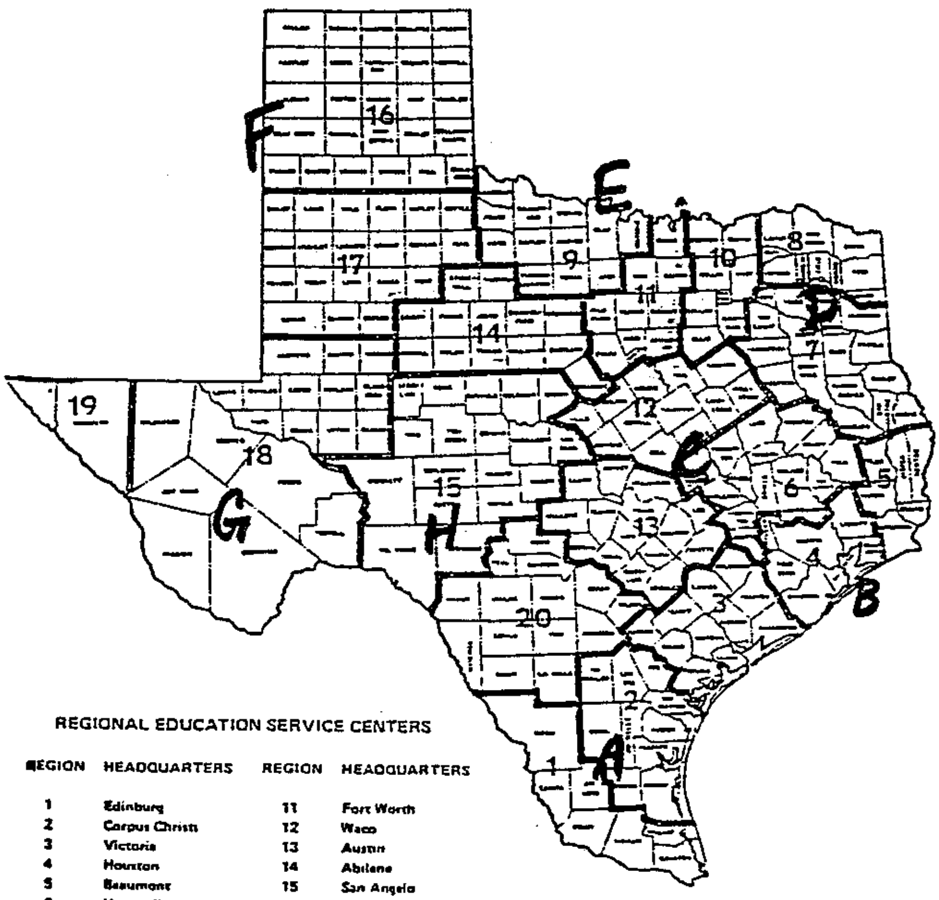
When submitting journal articles, please send the signed original and four copies. Figures should be drawn on separate sheets and in black ink. The masthead page of the journal has complete directions.

TCTM to Try Eight Regions

The TCTM Executive Board has decided to try dividing the state into eight geographical regions defined along the boundaries of the Education Service Centers. We had previously been using four geographical regions defined by TSTA regions. It was felt that through more regions, the TCTM could play a greater role building, strengthening, and sustaining the local affiliated councils. Regional directors will vary depending on the needs of the region. The regional director will serve as a liaison between TCTM and the local councils. This plan will be tried for a two year period and reevaluated at the end.

Names have not yet been assigned to the regions: in the interim, the regions will be identified by the letters A through H. Along with a map, a complete list of the regions, education service centers, and local affiliated councils has been prepared.

TCTM Region	ESC'S	Local Affiliated Councils	Director
A	1,2	Rio Grande Valley Coastal Bend	vacant
B	3,4,5	Fort Bend Gulf Coast Houston Sabine Area San Jacinto Area Spring Branch 1960 Area	Charles D. Reinauer
C	6,12,13	Austin Area Trinity Brazos River Valley	vacant
D	7,8,10	East Texas Greater Dallas	Bob Mora
E	9,11,14	Fort Worth Area North Texas	vacant
F	16,17		vacant
G	18,19	Greater El Paso West Texas Frontier	Sally Ann Rucker
H	15,20	Alamo District, San Angelo	vacant



REGIONAL EDUCATION SERVICE CENTERS

REGION	HEADQUARTERS	REGION	HEADQUARTERS
1	Edinburg	11	Fort Worth
2	Corpus Christi	12	Waco
3	Victoria	13	Austin
4	Houston	14	Abilene
5	Beaumont	15	San Angelo
6	Huntsville	16	Amarillo
7	Kilgore	17	Lubbock
8	Mount Pleasant	18	Midland
9	Wichita Falls	19	El Paso
10	Richardson	20	San Antonio

STUFF

Strategic Tactics Ultimately For Fun

Beverly Millican

Elementary Mathematics Coordinator
Plano Independent School District

The last two issues of TMT provided you with activities for place value instruction based on the developmental levels of mathematics learning. The first two levels, "concept" and "connecting," were addressed in the March and May, 1988 issues. The third and final "abstract" level will be the focus of this issue. (A substantial amount of time needs to be spent at the "concept" and "connecting" levels before these abstract level activities are begun.)

Since the key word in "STUFF" is "FUN," the format for the previous place value lessons has been one of games, incorporating manipulative materials into them. Likewise, the following abstract level activities will continue with this same format but utilize only simple student made gameboards and number cubes or spinners.

Both of these games have been adapted from Marilyn Burn's, The Math Solution.

"REJECT"

MATERIALS: Scratch paper for gameboards
Number cube or spinner

This game can be played as a whole group, small group, or partner activity. Before the game begins, each student sketches on his scratch paper a number of empty blanks to make the gameboard as follows:

_____, _____ _____ _____

reject —> _____

Each student takes a turn throwing the number cube (or spinning the spinner). Whatever number comes up must be recorded by all the students in any one of the blanks on their gameboard. Once a number is recorded, it cannot be changed. Play continues until all the blanks are filled. The student with the largest number wins, provided he can read it aloud.

(The number of blanks used depends upon the level of place value that is being taught. For older children decimal numbers can be added, as well as, extra blanks for ten thousands, hundred thousands, etc.)

The second place value game is also at the abstract level in that it uses only symbols and no concrete materials

ROLL TO WIN

MATERIALS: Scratch paper (for gameboards)
Three number cubes or three 0-9 spinners

Roll to Win is best played in pairs, with each student recording on his own gameboard, as follows:

START
WIN

The first player tosses all three number cubes (or spins all three spinners) and chooses any two of the three numbers to form a two-digit number that he records under the START box. The second player does likewise on his first turn. On each subsequent turn, players roll or spin all three, and use any two of the numbers to make a two-digit number that is greater than the previous one on their list. If they are unable to make a number that is greater, or choose not to, they skip that turn. The first player to fill in all of the boxes is the winner.

Mental Math

Francis J. Gardella

30 Dexter Road
East Brunswick, NJ

One of the major goals of a mathematics program is to have the students develop a sense of number. Students with this sense acquire a flexibility with the concepts involved so they can perform computation more intelligently. Two groups of skills that promote number sense are mental math and estimation, skills that are receiving more emphasis as we move toward a mathematics curriculum of the 1990's.

This article outlines one set of mental math procedures involving addition, subtraction, and multiplication. However, mental math can be very personal. As you read, there may be points at which you will say, "But I can do it another way," or "I feel more comfortable doing it in another way." Fine, as long as the method is based on mathematical logic and yields the correct response, alternatives to the suggested procedures may be developed.

ADDITION

When we approach mental addition, there are at least two ways to view the operation. In each, the basic technique involves understanding place value as much as it involves accuracy with basic facts.

Add 37 + 45.

Method 1: 37 + 45 ----> 30 + 40 = 70; 7 + 5 = 12; 70 + 12 = 82.

Method 1 focuses on gathering place values, dealing with the sum of each, and then combining the subtotals.

Method 2: 37 + 45 ----> 37 + 40 = 77; 77 + 5 = 82

Method 2 focuses on one of the numbers as a starting point (37), and then adding the second number in a place-by-place fashion, beginning with the higher place value.

Although both methods seem to work nicely, the first becomes somewhat more cumbersome when adding multi-digit numbers since the subtotal for each place must be remembered and then combined with the other subtotals. When applied to multi-digit numbers, the second method, as is shown below, entails focusing on one place value at a time, dealing with it, and then incorporating it into the running total.

Add 457 + 285 ----> 457 + 200 = 657; 657 + 80 = 737; 737 + 5 = 742

As this exercise proceeds, one sees, especially in the second step, the number/place value sense that develops. When dealing with the tens' place, it is not just a matter of adding 657 and 80. It is the impact of the addition of 50 and 80 on not only the tens' but also the hundreds' place. As you begin to practice, you may simply add 50 and 80, treating the hundreds' place accordingly. However, with more experience, a number sense view gives you a situation similar to an odometer, wherein the 50 and 80 automatically generate in the mind of the user an additional hundred.

SUBTRACTION

Mental subtraction does not deal with each place separately. (However, with paper and pencil, the procedure with subtraction may be an interesting application of integers.)

With mental subtraction, we begin with the larger number, and then subtract the value of each place in turn, working left to right.

Subtract 82 - 47 ----> 82 - 40 = 42; 42 - 7 = 35.

Subtract 722 - 437 ----> 722 - 400 = 322; 322 - 30 = 292; 292 - 7 = 285.

Again, notice the number/place value sense which is involved. When the use of such a sense occurs, the thinking moves more into the realm of understanding and utilizing the relationship of numbers and the place value system, and away from the mere rote subtraction of quantities.

One last type of exercise in mental subtraction is worthwhile to investigate.

Subtract 700 - 437 ----> 700 - 400 = 300; 300 - 30 = 270; 270 - 7 = 263.

Because of the multiple regroupings, especially involving zero, many students tend to have difficulty with this type of computation. It is noteworthy that this type of subtraction becomes much clearer and more direct when the mental math processes are applied.

MULTIPLICATION

A major feature of mental multiplication is that it displays the use of the distributive property of multiplication over addition in its relation to the place value system, and analogous to its application in an algebraic framework.

Multiply 3 x 27 ----> 3 x 20 = 60; 3 x 7 = 21; 60 + 21 = 81.

The unique feature of this process for multiplication is the direct correlation it creates between the whole number and fraction operations.

Multiply 3 x 2 1/3 ----> 3 x 2 = 6; 3 x 1/3 = 1; 6 + 1 = 7.

Multiply 4 x 2 1/3 ----> 4 x 2 = 8; 4 x 1/3 = 1 1/3; 8 + 1 1/3 = 9 1/3.

Very interestingly, this process of mental multiplication now becomes more than a way of computing without paper and pencil. Mental math offers a continuity of process, applying the distributive property to both whole numbers and fractions in a similar fashion. Also, the stage for its application in an algebraic setting is clearly set.

SUMMARY

Mental math is the process of computing without the assistance of, or need for paper and pencil. Mental math consistently calls for the understanding and application of place value as well as basic fact knowledge. Although several processes for mental math have been delineated, there are certainly other ways in which a person can work with the place values and facts involved to develop a personal system. Mental math puts control of the process in the hands of the person doing the computing. It not only allows but encourages the user to think, create, and experiment with number.

So, to all of you, try some mental math, whether you share someone else's style or develop your own. Have your students try mental math. Using some of the examples posed, all of you can begin to look for new ways of being flexible with number. Such flexibility is a key ingredient in the teaching of number sense, a skill certainly needed as we continue through the technological age.

STEAM

STEAM is the new elementary mathematics section of the TCTM. Its newsletter, "STEAM across Texas," will be mailed to each TCTM member with the Journal starting in January. The newsletter will announce a new teaching award to recognize the Elementary Mathematics Teachers of the Year. Each issue of "STEAM across Texas" includes articles about elementary mathematics happenings across the state and nation, student contests, and helpful hints from other elementary mathematics teachers. It's coming in January. Watch for it, and hop aboard!

Female Math Avoidance and Causal Attribution Theory

H. Earl Hines

Math Coordinator
University of Houston
Houston, Texas

In career education meetings, students are often told to be anything they want to be. One thing very few girls seem to want to be is mathematically talented or at least employed in professional careers in mathematics or science (Brush, 1980; Fox, 1976; Olson & Kansky, 1981). Females tend not to study, as much as do males, the most advanced math courses (Fennema, 1981; Fennema & Sherman, 1977, 1978; Fennema, Wolleat, Pedro & Becker, 1981). As early as grades seven and eight, boys and girls appear to differ with respect to their perceptions of the importance of mathematics for their future (Brush, 1980; Fox, 1976; Pedro, Wolleat, Fennema, & Becker, 1981). The failure of many females to take mathematics courses beyond the minimum requirements has been labeled "math avoidance" (Tobias, 1978). This label conveys the affective or motivational aspects of the female enrollment problem. The purpose of this article is to focus attention on causal attribution theory as a possible source of knowledge that may be applied to the problem of female math avoidance.

Males have been more visible than females in mathematics-related activities (Wolleat, Pedro, Becker, & Fennema, 1980). Certainly, if one observes adult use, many more males than females use mathematics daily, especially advanced mathematics beyond arithmetic (Fennema, 1979). Four general (Brush, 1980, p. 9) types of explanations for the problem of math avoidance are (a) lack of ability, (b) negative attitudes toward the subject, (c) perceived lack of usefulness of mathematics in future life, and (d) a discouraging social milieu.

Females, as compared to males, have been found to be less confident in their ability to learn mathematics, to underestimate their ability to solve mathematical problems, and to believe to a lesser degree that mathematics will be useful (Crandall, Kathovsky, & Crandall, 1965; Fennema, 1981; Fennema & Sherman, 1977, 1978; Hilton & Berglund, 1974). These attitudinal variables may interact with yet another factor, the attribution of causation, to produce the math avoidance syndrome (Wolleat et al., 1980). According to attribution theory, personal explanations of causality (attributions) fall into one of four categories: ability, effort, task difficulty, or luck. These four categories are derived from two factors, each of which has two levels: locus of causation (internal/external) and degree of stability (stable/unstable) (Weiner, 1974a, 1974b; Wolleat et al., 1980).

Attribution theory predicts that performance which is consistent with expectations will be attributed to a stable cause, whereas outcomes contrary to expectations will be attributed to unstable causes. These attributions hold whether the outcome is success or failure (Wolleat et al., 1980). If an outcome is attributed to a stable cause, the same outcome is expected in the future. If an outcome is attributed to an unstable cause and there is sufficient doubt as to whether a prior success will be repeated, the same outcome is not expected in the future. The importance of causal attribution theory lies in its relationship to expectancy of success. The two-by-two matrix shown below, taken from Wolleat et al. (1980), gives the classification scheme for explanations of causality.

		LOCUS OF CAUSATION	
		Internal	External
STABILITY	Stable	Ability	Task
	Unstable	Effort	Luck

Lack of motivation has been often cited as a reason for the failure of students to study mathematics beyond the required courses. Cognitive theories of motivation generally state that the intensity of aroused motivation is determined by (a) the expectation that a given response will lead to goal attainment and (b) the attractiveness of the goal object (Weiner, 1974a, 1974b). The greater the expectancy of success in attaining a goal and the greater the incentive value of the goal, the more intense is the motivation toward goal attainment. Goal expectations have been found to be influenced by the stability of the perceived causes of success and failure (Weiner, 1974b).

The attribution of success to unstable factors and failure to stable ones has been linked to a pattern of behavior called "learned helplessness." Learned helplessness is a condition in which failure is viewed as inevitable. This condition leads to lowered motivation to persist (Wolleat et al., 1980). Females are more likely than males to exhibit learned helplessness (Dweck, Davidson, Nelson, & Enna, 1978).

Wolleat et al. (1980) conducted a study involving male and female secondary school students in which the principles of attribution theory and learned helplessness were applied to the problem of math avoidance. The results showed that females when compared to males, display more of the learned helplessness condition in their attribution of success and failure in mathematics. They more strongly than males use effort (unstable) to explain their success and less strongly then males attribute their success to ability (stable). To explain failure, they more strongly than do males point to ability and task difficulty (both stable). These results are consistent with the theory of causal attribution, especially as it relates to gender differences in mathematics performance.

According to Weiner (1974b), locus of causality influences the affective consequences of achievement behaviors. Pride and shame are maximized when success and failure are attributed to internal causes and minimized when success and failure are ascribed to external causes. That is, success attributed to high ability or hard work creates more pride than success that is attributed to good luck or the ease of a given task. Similarly, failure ascribed to low ability or a lack of effort creates greater shame than failure that is ascribed to bad luck or a hard task.

A number of studies have identified gender differences in mathematics achievement (Armstrong, 1981; Benbow & Stanley, 1980; Callahan & Clements, 1984; Moore & Smith, 1987; Mundy, 1987; Pattison & Grieve, 1984; Swafford, 1980; Wolleat et al., 1980). Typically, no gender differences in achievement have been found through middle school. By the end of high school, however, males have higher achievement scores and are frequently reported as doing better on higher level cognitive tasks as well (Armstrong, 1981). While concluding that cognitive gender differences were disappearing, Feingold (1988) found that gender differences (favoring males) at the upper levels of performance in high school mathematics have remained constant.

Reports by Armstrong (1981) and Fennema (1979) suggest that gender differences in achievement might be explained by differential participation in mathematics. This possibility receives support from those who claim that when the number of years of studying mathematics is controlled, gender differences in achievement are negligible (Fennema, 1974, 1979; Fennema & Sherman, 1977, 1978). Could it be that these reports of differences in mathematics achievement by gender are actually reports of differences in achievement between individuals who differ mainly on the basis of their course-taking decisions?

Knowledge of behavior patterns that cause students, particularly females, to quit mathematics as soon as possible, could lead to success in reversing the process. By avoiding mathematics, females are limiting their chances for success; especially as success relates to preparation for the job market and/or further studies in mathematics-related areas. Points made in this article suggest that females' lack of confidence in their mathematical ability and their failure to persist in mathematics study may indeed be affected by the attributional patterns they learn to use. If this is the case, then causal attribution theory could be quite useful when applied to the problem of math avoidance. That is, effective development of appropriate attributions for success and failure in mathematics could conceivably be accompanied by a decrease in the tendency of students, especially females, to avoid the study of mathematics beyond required levels.

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REFEREES WANTED

Manuscripts published in the Texas Mathematics Teacher are reviewed by at least three mathematics educators. Members of TCTM who are regular readers of the journal - classroom teachers, supervisors, and teacher educators - who would like to review manuscripts should write to the TCTM indicating their willingness to serve and the level of interest (Elementary, Secondary, or both). Contact George H. Willson, Box 13857, University of North Texas, Denton, Texas 76203-3857. The Editorial Panel will review the responses and make the final selection.

Fibonacci Operation Tables: Activities and Patterns

David R. Duncan and Bonnie H. Litwiller

Professors of Mathematics
University of Northern Iowa
Cedars Falls, Iowa

Interesting number patterns can be discovered when the question, "What happens if . . . ?" is considered. We asked, "What would addition and multiplication tables which use Fibonacci Numbers look like?"

The notion of Fibonacci numbers is credited to Leonardo of Pisa, who was nicknamed Fibonacci. In 1202, he wrote a book which first described the so-called Fibonacci numbers.

Fibonacci supposed that on the first day of January, a pair of rabbits (Pair 1) is born. He also supposed that each litter will be a pair of rabbits consisting one female and one male. After maturing for two months, Pair 1 then gives birth to Pair 2 on the 1st of March.

Then, Pair 1 gives birth to another pair of rabbits on the first of every month; also, all other pairs give birth to their first pair of offspring two months after they themselves are born, and to another pair the first of each month thereafter. How many rabbits exist the first of every month? Do not count the parents of the first pair and assume to pairs die.

On January 1, there is 1 pair.
On February 1, there is still 1 pair.
On March 1, there are 2 pairs; the original pair and a newborn pair.
On April 1, there are 3 pairs; the original pair has given birth again.
On May 1, there are 5 pairs since both the original pair and the pair born in February give birth.

As this reasoning continues, the Fibonacci numbers are generated, yielding the sequence, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, . . .

The defining rule for the sequence of Fibonacci number (called a_1, a_2, a_3, . . .) is:

$$a_1 = a_2 = 1$$

$$a_{n+2} = a_n + a_{n+1}$$

Figures 1 and 2, denoting sums and products of Fibonacci numbers, were constructed in the same way that "regular" addition and multiplication tables were constructed.

Figure 1

+	1	1	2	3	5	8	13	21	34	55	89	144 ...
1	2	2	3	4	6	9	14	22	35	56	90	145
1	2	2	3	4	6	9	14	22	35	56	90	145
2	3	3	4	5	7	10	15	23	36	57	91	146
3	4	4	5	6	8	11	16	24	37	58	92	147
5	6	6	7	8	10	13	18	26	39	60	94	149
8	9	9	10	11	13	16	21	29	42	63	97	152
13	14	14	15	16	18	21	26	34	47	68	102	157
21	22	22	23	24	26	29	34	42	55	76	110	165
34	35	35	36	37	39	42	47	55	68	89	123	178
55	56	56	57	58	60	63	68	76	89	110	144	199
89	90	90	91	92	94	97	102	110	123	144	178	233
144	145	145	146	147	149	152	157	165	178	199	233	288
⋮												

Figure 2

x	1	1	2	3	5	8	13	21	34	55	89	144 ...
1	1	1	2	3	5	8	13	21	34	55	89	144
1	1	1	2	3	5	8	13	21	34	55	89	144
2	2	2	4	6	10	16	26	42	68	110	178	288
3	3	3	6	9	15	24	39	63	102	165	267	432
5	5	5	10	15	25	40	65	105	170	275	445	720
8	8	8	16	24	40	64	104	168	272	440	712	1152
13	13	13	26	39	65	104	169	273	442	715	1157	1872
21	21	21	42	63	105	168	273	441	714	1155	1869	3024
34	34	34	68	102	170	272	442	714	1156	1870	3026	4896
55	55	55	110	165	275	440	715	1155	1870	3025	4895	7920
89	89	89	178	267	445	712	1157	1869	3026	4895	7921	12816
144	144	144	288	432	720	1152	1872	3024	4896	7920	12816	20736
⋮												

<u>Activity 1</u>

What happens if squares of varying sizes are drawn upon the Fibonacci addition table (Figure 3), and the numbers which represent the pairs of opposite vertices of the squares are summed? Is there a pattern?

Figure 3

+	1	1	2	3	5	8	13	21	34	55	89	144...
1	2	2	3	4	6	9	14	22	35	56	90	145
1	2	2	3	4	6	9	14	22	35	56	90	145
2	3	3	4	5	7	10	15	23	36	57	91	146
3	4	4	5	6	8	11	16	24	37	58	92	147
5	6	6	7	8	10	13	18	26	39	60	94	149
8	9	9	10	11	13	16	21	29	42	63	97	152
13	14	14	15	16	18	21	26	34	47	68	102	157
21	22	22	23	24	26	29	34	42	55	76	110	165
34	35	35	36	37	39	42	47	55	68	89	123	178
55	56	56	57	58	60	63	68	76	89	110	144	199
89	90	90	91	92	94	97	102	110	123	144	178	233
144	145	145	146	147	149	152	157	165	178	199	233	288
⋮												

Table 1 reports the results of our computations for squares A through J.

<u>Table 1</u>

Square	Sums of pairs of Opposite Vertices
A	7 (3 + 4 and 4 + 3)
B	16 (6 + 10 and 9 + 7)
C	26 (10 + 16 and 11 + 15)
D	29 (8 + 21 and 16 + 13)
E	188 (36 + 152 and 146 + 42)
F	80 (22 + 58 and 24 + 56)
G	217 (60 + 157 and 68 + 149)
H	152 (29 + 123 and 55 + 97)
J	330 (42 + 288 and 165 + 165)

Observe that the sums of the numbers which represent the opposite pairs of vertices are equal in each case.

To see why this pattern holds, consider Square E. Since the sum of any two consecutive Fibonacci numbers yields the next Fibonacci number, we can write:

$$\begin{aligned} 8 &= 3 + 5 \\ &= 3 + 2 + 3 \\ &= 2 + 2(3) \end{aligned} \qquad \begin{aligned} 144 &= 55 + 89 \\ &= 55 + 34 + 55 \\ &= 34 + 2(55) \end{aligned}$$

Consequently:

$$\begin{aligned} 36 + 152 &= (34 + 2) + (144 + 8) \\ &= 34 + 2 + 34 + 2(55) + 2 + 2(3) \\ &= 2(34) + 2(2) + 2(3) + 2(55) \end{aligned}$$

$$\begin{aligned} 146 + 42 &= (144 + 2) + (34 + 8) \\ &= 34 + 2(55) + 2 + 34 + 2 + 2(3) \\ &= 2(34) + 2(2) + 2(3) + 2(55) \end{aligned}$$

These two sums are equal. The same procedure can be used for any of the other squares, and can also be used for squares of arbitrary dimensions and placement.

Activity 2

Suppose rectangles of varying sizes are drawn upon the Fibonacci addition table. Are the sums of the opposite pairs of vertices equal?

Figure 4

+	1	1	2	3	5	8	13	21	34	55	89	144...
1	2	2	3	4	6	9	14	22	35	56	90	145
1	2	2	3	4	6	9	14	22	35	56	90	145
2	3	3	4	5	7	10	15	23	36	57	91	146
3	4	4	5	6	8	11	16	24	37	58	92	147
5	6	6	7	8	10	13	18	26	39	60	94	149
8	9	9	10	11	13	16	21	29	42	63	97	152
13	14	14	15	16	18	21	26	34	47	68	102	157
21	22	22	23	24	26	29	34	42	55	76	110	165
34	35	35	36	37	39	42	47	55	68	89	123	178
55	56	56	57	58	60	63	68	76	89	110	144	199
89	90	90	91	92	94	97	102	110	123	144	178	233
144	145	145	146	147	149	152	157	165	178	199	233	288
⋮												

Table 2 displays the results of the computations for rectangles A through J.

Table 2

Rectangle	Sums of Pairs of Opposite Vertices
A	7 (2 + 5 and 3 + 4)
B	31 (5 + 26 and 23 + 8)
C	49 (10 + 39 and 13 + 36)
D	147 (56 + 91 and 57 + 90)
E	247 (90 + 157 and 145 + 102)
F	84 (16 + 68 and 63 + 21)
G	301 (68 + 233 and 178 + 123)
H	64 (6 + 58 and 8 + 56)
J	181 (24 + 157 and 34 + 147)

Again, observe that the sums of the pairs of opposite vertices are equal.

<u>Activity 3</u>

What happens if squares of varying sizes are drawn upon the Fibonacci multiplication table as in Figure 5? Will the products of the opposite vertices be equal?

Figure 5

x	1	1	2	3	5	8	13	21	34	55	89	144 ···
1	1	1	2	3	5	8	13	21	34	55	89	144
1	1	1	2	3	5	8	13	21	34	55	89	144
2	2	2	4	6	10	16	26	42	68	110	178	288
3	3	3	6	9	15	24	39	63	102	165	267	432
5	5	5	10	15	25	40	65	105	170	275	445	720
8	8	8	16	24	40	64	104	168	272	440	712	1152
13	13	13	26	39	65	104	169	273	442	715	1157	1872
21	21	21	42	63	105	168	273	441	714	1155	1869	3024
34	34	34	68	102	170	272	442	714	1156	1870	3026	4896
55	55	55	110	165	275	440	715	1155	1870	3025	4895	7920
89	89	89	178	267	445	712	1157	1869	3026	4895	7921	12816
144	144	144	288	432	720	1152	1872	3024	4896	7920	12816	20736
⋮												

Table 3 shows the results of our calculations.

Table 3

Square	Products of the opposite pairs of vertices	
A	28,730	(65·442 and 170·169)
B	9,147,600	(1155·7920 and 3024·3025)
C	72,891	(63·1157 and 267·273)
D	130	(2·65 and 13·10)
E	6,052	(68·89 and 34·178)
F	24,192	(144·168 and 21·1152)
G	240	(10·24 and 15·16)
H	3,495,030	(1155·3026 and 1870·1869)
J	48	(2·24 and 8·6)

Observe that the products of the opposite vertices are equal.

Activity 4

Suppose that rectangles of varying sizes were drawn upon the Fibonacci multiplication table as in Figure 6. Does the same pattern hold?

Figure 6

x	1	1	2	3	5	8	13	21	34	55	89	144 ...
1	1	1	2	3	5	8	13	21	34	55	89	144
1	1	1	2	3	5	8	13	21	34	55	89	144
2	2	2	4	6	10	16	26	42	68	110	178	288
3	3	3	6	9	15	24	39	63	102	165	267	432
5	5	5	10	15	25	40	65	105	170	275	445	720
8	8	8	16	24	40	64	104	168	272	440	712	1152
13	13	13	26	39	65	104	169	273	442	715	1157	1872
21	21	21	42	63	105	168	273	441	714	1155	1869	3024
34	34	34	68	102	170	272	442	714	1156	1870	3026	4896
55	55	55	110	165	275	440	715	1155	1870	3025	4895	7920
89	89	89	178	267	445	712	1157	1869	3026	4895	7921	12816
144	144	144	288	432	720	1152	1872	3024	4896	7920	12816	20736
⋮												

Table 4 reports the computations.

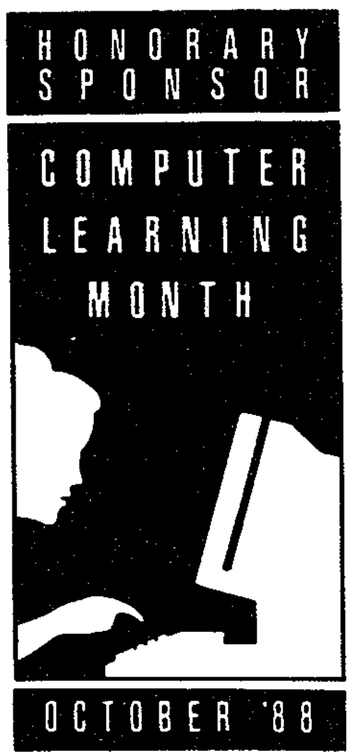
Table 4

Rectangle	Products of the Opposite Pairs of Vertices
A	18 (2·9 and 3·6)
B	1,560 (15·104 and 24·65)
C	313,992 (168·1869 and 712·441)
D	9,792 (34·288 and 68·144)
E	1,346,400 (275·4896 and 1870·720)
F	65 (5·13 and 13·5)
G	3,575 (13·275 and 55·65)
H	14,826,240 (7920·1872 and 715·20736)
J	64,080 (89·720 and 144·445)

Challenges:

1. Find other patterns on the Fibonacci addition and multiplication tables.

2. Verify some of these patterns by making use of the summing property of Fibonacci numbers, as we did in the first activity for a special case.



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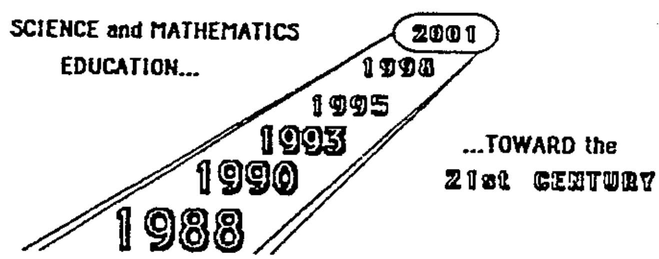

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