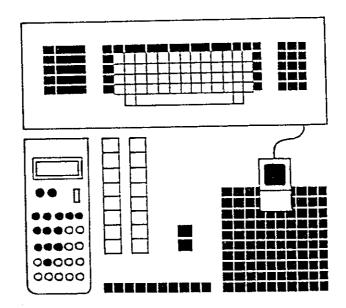
TEXAS MATHEMATICS TEACHER



Some Interesting Disguises for 0 and 1

Is It In Lowest Terms?

Getting A Grip on Math Manipulatives

MARCH 1988

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President's Message

One afternoon on a streetcar a tourist sat down by the theologian Karl Barth. This particular tourist was a fan of Barth and proceeded to talk to the stranger on the streetcar about how exciting it was to be in Barth's own city, his great desire to meet the man, etc. He went on and on, not allowing his new acquaintance a word. Finally, the tourist said, "I don't suppose you've ever met him?" "Well, actually," replied Barth, "I shave him every morning." At this the tourist jumped off at the next corner and rushed back to the hotel to tell his wife his good fortune, "I met Karl Barth's barber!"

This Bill Forbes story spoke to the educator in me. How often do we talk when we should ask questions and then listen? How often do we settle for a partial answer and rush on rather than take the time to pursue with deeper questions? How much are we missing because we don't take time to know who people really are?

Now, to get on with current events . . .

There are also some things you ought to know. Are you up to date on changes being discussed for mathematics teaching? At the Sam Houston Mathematics Conference, Cathy Peavler shared with us the work of the NCTM Commission on Standards (of which she is a member) and the committee to write Texas Essential Elements proposed for use in 1990 for Algebra I, Geometry, Algebra II, and Trigonometry. There will be some changes in Texas, for the Essential Element Committee studied the Standards Committee Report and the Mathematical Sciences Education Board Recommendations for the year 2000 before writing. Emphasis on statistics, mathematical modeling, and the use of technology (the computer and the calculator) in the mathematics classroom show up in the proposed Essential Elements. Just last Tuesday I attended an input session at Region IV for the revision of the Exit Level TEAMS objectives. There seemed to be unanimous agreement among those present that the calculator should be a tool used freely in the classroom and on the TEAMS Exit Level test.

If you would like to read the report of the NCTM Commission on Standards, send for a copy to Dept. E, NCTM, 1906 Association Drive, Reston, Virginia, 22091.

The January High School Mathematics conference at Sam Houston State University was even better than usual for it was expanded to include all of high school math, including calculus. During the conference, the TCTM officers who were present discussed a plan to increase the number of regions in Texas and have more regional Saturday conferences. The first of these regional conferences was the Southeast Regional Conference, held at Alvin Community College last fall. Thanks to Charles Reinauer, Conference Director, and John Huber, Program Chairman, for an excellent secondary math conference on October 24.

It's almost election time! As a matter of fact, it <u>is</u> time to send in your suggestions to the nominating committee. Are <u>you</u> interested in being an officer? Would someone in your district or school be an asset on the Executive Committee of TCTM? Please share your ideas with us. Send name, address, school, and home phone number of potential candidates to Cathy Rahlfs, Humble ISD, P. O. Box 2000, Humble, Texas 77347.

Bettye Hall and Judy Tate have done the "Stuff" column in the Journal for several years. At this time other responsibilities prevent them from continuing. Bettye and Judy, we appreciate your hard work and creativeness. Beverly Millican will be taking over this column. Thanks so much, Beverly!

Remember that CAMT will be at the new Brown Convention Center in downtown Houston, August 2 - 4. TCTM will offer CAMT scholarships again this year. No one applied for these last year. Considering that we have many new members, there should be many applications this year. Scholarship information will be in the May journal. During CAMT, August 2 - 4, I will need your help at registration. There will be a volunteer form in the next journal. We expect more people this year, so we'll need more help at registration.

Watch for more information on CAMT. There will even be a pre-conference meeting on Monday, August 1.

For the morning of August 3, TCTM is planning to have breakfast and our annual meeting. Breakfast is by advanced registration only for we have to commit well in advance of the conference for a specific number of breakfasts to be served. The May issue of the <u>Texas Mathematics Teacher</u> will contain this registration form. Look for it, mail it to me, and remind your friends to do so too.

Maggie Dement

Note From the Editors

As you have probably noticed from past editions of the journal, there have been many more articles dealing with secondary mathematics than elementary. In the January issue there were two elementary articles which is quite rare. It would be ideal to have a balance of elementary and secondary topics but so few elementary articles are submitted. Elementary topics are needed! Therefore, would you consider writing one for the journal? In fact, articles are needed at all levels but especially for elementary. Keep in mind that the articles go through a review process, and in some cases, a rewrite which takes quite a bit of time until publication. KEEP THE MANUSCRIPTS COMING!

Some Interesting Disguises for 0 and 1

Joe F. Allison

Eastfield College Mesquite, Texas

Students will use disguises of 0 and 1 in almost all their mathematics and a sequence of several of the following examples could be used to sensitize students to anticipating timely applications of disguises of 0 and 1.

Basic to students' training are the statements: "Zero is the additive identity element" and "One is the multiplicative identity element." Instruction introduces the notational relationships:

$$a + 0 = a;$$

$$a \cdot 1 = a$$
.

Both 0 and 1 "go to work" in the following disguises:

$$0 = a - a;$$

$$1=\frac{a}{a}$$
.

For an example of using disguises of 1, we add:

 $\frac{1}{2}$ to $\frac{1}{3}$:

$$\frac{1}{2} + \frac{1}{3} = (\frac{1}{2})(1) + (\frac{1}{3})(1) = (\frac{1}{2})(\frac{3}{3}) + (\frac{1}{3})(\frac{2}{2}) = \frac{3}{6} + \frac{2}{6} = \frac{5}{6}.$$

Here, we entered "one" twice and then used two different "disguises" for one.

Ways to add, say, 147 and 16 would be to include zeros in disguises:

$$147 + 16 + 4 - 4 = 147 + 20 - 4 = 167 - 4 = 163$$
.

Or,
$$147 + 16 + 3 - 3 = 150 + 16 - 3 = 166 - 3 = 163$$
.

We will show other zero disguises presently as we may be accused of forcing the issue for "throwing in zeros." But, now we will show again how "pulling a 'one' out of the air" can offer another convenience. For example, consider the arithmetic "rule" that most students are able to recall as they progress to algebra: that of trading division by a fraction for multiplication by the fraction's reciprocal.

$$\frac{a}{\frac{b}{c}} = \frac{a}{\frac{b}{c}} (1) = \frac{a}{\frac{b}{c}} \left(\frac{\frac{c}{b}}{\frac{c}{b}}\right) = \frac{a(\frac{c}{b})}{\frac{bc}{bc}}$$

$$= \frac{a\left(\frac{c}{b}\right)}{(1)(1)} = \frac{a\left(\frac{c}{b}\right)}{1} = a\left(\frac{c}{b}\right)$$

Notice that since $\frac{b}{b}$ and $\frac{c}{c}$ may be replaced by ones, the "rule" follows:

At higher levels, it becomes necessary to have equivalences for expressions such as the examples:

$$\frac{x}{x+2}$$
 and $\frac{x^2}{x^2+1}$.

Of course, long division could be employed. However, entering zeros and then properly disguising the zeros can offer conveniences. These examples continue:

$$\frac{x}{x+2} = \frac{x+0}{x+2} = \frac{x+2-2}{x+2} = \frac{x+2}{x+2} - \frac{2}{x+2} = 1 - \frac{2}{x+2}.$$

$$\frac{x^2}{x^2+1} = \frac{x^2+0}{x^2+1} = \frac{x^2+1-1}{x^2+1}$$

$$= \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1} = 1 - \frac{1}{x^2+1}.$$

These ostensible "school-house curiosities" come into their own in second semester calculus as instructors and students alike will testify. At the same level of development occurs another demonstration which will now follow that uses disguises for a one and a zero.

Theorem. If a function f is differentiable at a point, it is continuous there.

Proof: Let y = f(x) and let f be differentiable at x = a. Now, add zero to the expression f(a + h):

f(a + h) = f(a + h) - f(a) + f(a); and, multiply, on the right, the difference f(a + h) - f(a) by one disguised as h/h:

$$f(a + h) = [f(a + h) - f(a)] (h/h) + f(a), or$$

$$f(a + h) = (\frac{f(a + h) - f(a)}{h}) (h) + f(a)$$

If we take the limit as h approaches zero on both sides and use the hypothesis and properties about limits of products and sums (not proved here) we have:

 $\begin{array}{l}
\text{limit } f(a + h) \\
h > 0
\end{array}$

=
$$\begin{pmatrix} \lim_{h>0} \frac{f(a+h)-f(a)}{h} \end{pmatrix}$$
 $\begin{pmatrix} \lim_{h>0} (h) \end{pmatrix} + \lim_{h>0} f(a)$.

Therefore,

limit
$$f(a + h) = (f^{1}(a)) (0) + f(a) = f(a)$$
.
h>0

so f is continuous at a.

This demonstration of "Cauchy continuity" is itself useful but the point here was to demonstrate other disguises for "zero" and "one."

As another example let us now concern ourselves with complex numbers. The usual convention is that $i^2 + 1 = 0$ or $i^2 = -1$. With this convention it follows that

$$i^3 = i^2i = (-1)i = -i$$
; $i^4 = i^2i^2 = (-1)(-1) = 1$; $i^5 = i^4i = +i$, etc.

Our attention here will be restricted to $-i^2$ or +1 since $-i^2$ = -(-1) = +1 (another disguise for one).

Now to solve, say, $x^2 + 25 = 0$, we multiply the 25 by "one;" here, $1 = -i^2$:

$$x^{2} + 25 = 0;$$

 $x^{2} + (1)(25) = 0;$
 $x^{2} + (-i^{2})(25) = 0;$
 $x^{2} - 25i^{2} = 0;$
 $(x - 5i)(x + 5i) = 0;$ so,
 $x - 5i = 0 \text{ or } x + 5i = 0,$ and
 $x = 5i \text{ or } x = -5i.$

Thus we see useful 0 and 1 disguises and hopefully we see that students can be sensitized to anticipate some of the useful disguises that these identity elements can take on.

Is It In Lowest Terms?

Earl Hines

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The following are representative of the types of statements made by a student who is frustrated by the task of changing a fraction to lowest terms. "I can't find a number that goes into both the numerator and denominator an even number of times." "It can't be reduced." "It's already in lowest terms." "I tried five numbers, it must be in lowest terms." "The numerator and denominator are both odd numbers: therefore, the fraction is in lowest terms."

What is the source of this unending difficulty with an operation which should be very easy to perform? Are the types of statements presented in the previous paragraph descendants of faulty teaching strategies? Are we teaching students reliable techniques for changing a fraction to lowest terms? The purpose of this article is to present a technique that could offer some relief for students who experience difficulty when confronted with the problem of changing a fraction to lowest terms.

Fractions such as $\frac{2}{4}$, $\frac{6}{8}$, or $\frac{5}{10}$ do not seem to present much of a challenge. But as the numerals in the numerator and denominator get larger, as in $\frac{34}{51}$, $\frac{39}{91}$, $\frac{187}{233}$, or $\frac{351}{208}$, students begin to experience some difficulty in their efforts to change them to lowest terms. Odd numbers in the numerator and/or denominator seem to present a greater problem than even numbers. This is probably due to the fact that all even numbers are multiples of 2. Although the last two fractions ($\frac{187}{233}$ and $\frac{351}{208}$) are not typical of those encountered in a general mathematics class; as one advances to more specialized areas, fractions with large numerators and denominators are encountered. For example, in geometry where the student solves problems involving circumference and arc length, fractions with large numerators and denominators. In

probability theory, especially for problems involving conditional probability and independence, the student has to contend with fractions of this type. Even in general mathematics classes, fractions with large numerators and denominators are sometimes presented for enrichment. The value of the technique presented here makes designing problems that demonstrate its usefulness as a method for changing fractions to lowest terms a worthwhile undertaking.

Two false beliefs that need to be dispelled immediately are: (1) Fractions such as $\frac{13}{5}$, in which the numerator is larger than the denominator, are not in lowest terms. Many students and teachers share this misconception. They think that writing $\frac{13}{5}$ as 2 $\frac{3}{5}$ is a step in changing $\frac{13}{5}$ to lowest terms. (2) Any fraction in which the numerator and denominator are both odd numbers is in lowest terms. Outrageous as it may seem, there are those who are plagued by this misconception also.

The following are true statements related to the task of changing a fraction to lowest terms. (1) A fraction is in lowest terms when the numerator and denominator are relatively prime. That is, their greatest common factor (GCF) is 1. (2) A fraction may be changed to lowest terms by dividing the numerator and denominator by their greatest common factor. The technique presented here only requires an awareness of these two statements and the ability to perform long division with whole numbers.

Finding the greatest common factor for two numbers by use of the Euclidean algorithm (Dolciani, Wooten, Beckenback, & Chinn, 1967, p. 240) is by no means a newly developed technique, but its value as a tool in changing fractions to lowest terms has been overlooked. Many textbooks present material on number theory to be used in subsequent work with fractions. Use of the Euclidean algorithm does not require such prior knowledge. The student only needs to know that he is finding the number to be used in changing the fraction to lowest terms. Even if he does not know that the number is called a greatest common factor, the fraction can still be successfully changed to lowest terms.

The procedure is to divide the larger of the two numbers (numerator or denominator of the fraction) by the smaller and perform repeated divisions until the remainder is zero. At this point, the last nonzero remainder or last divisor is the number to be used in changing the fraction to lowest terms. Consider the following examples:

1) To change $\frac{12}{42}$ to lowest terms, we divide 42 by 12 in the manner shown below.

$$\begin{array}{c}
 \frac{3}{42} \\
 \frac{36}{612} \\
 \hline
 \frac{12}{42} = \frac{12 \div 6}{42 \div 6} = \frac{2}{7}
\end{array}$$
CCF.

Notice that each nonzero remainder is used as a divisor and the previous divisor becomes the dividend in the construction of repeated divisions. As stated above, the last divisor (6 in this case) is used to change the fraction to lowest terms.

The remainder is zero on the first division. This example demonstrates the ability of this method to save time in certain cases.

3) 提

$$\begin{array}{c}
143)167 \\
143 & 5 \\
\hline
24)143 \\
& 120 & 1 \\
\hline
23)24 \\
& 23 & 23 \\
\hline
& 1 &)23 \\
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Whenever the last divisor (or last nonzero remainder) is 1, the fraction is already in lowest terms.

4) $\frac{351}{208}$

$$\begin{array}{c}
 \frac{1}{208)351} \\
 \frac{208}{143)208} \\
 \frac{1}{65)143} \\
 \frac{130}{13)65} \\
 \text{GCF} \qquad \frac{65}{65}
\end{array}$$
... $\frac{351}{208} = \frac{351 \div 13}{208 \div 13} = \frac{27}{16}$

When used as demonstrated here, the Euclidean algorithm can serve as a useful tool for changing fractions to lowest terms. Supplied with a definite and easily performed technique, one can be confident in his ability to find the lowest term fraction. Increased confidence could ultimately lead to reduction in frustration and anxiety.

Reference

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Getting A Grip On Math Manipulatives

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Despite the fact that teachers don't use them often, manipulatives are valuable teaching tools. Well chosen and properly used manipulative materials enhance children's learning, promote problem-solving and computational skills (Kennedy 1986, 7).

Manipulatives are objects of quantity, shape, mass, or other relevant characteristics used to demonstrate mathematical ideas. Manipulatives can be rods, blocks, marbles, poker chips, cardboard cutouts--almost anything. They can be used to give students "hands-on" experiences and help them learn mathematical concepts. Objects that students can look at and hold also help the student understand by visualizing. Studies of mathematics achievement at different grade and ability levels show that children benefit when real objects are used as aids in learning mathematics. The student must be actively involved in using that manipulative. enough for the student to observe a demonstration of the use of a The manipulating of the concrete objects allow manipulative. students to experience the patterns of the abstract mathematical concepts (Hynes 1986, 11). The cognitive development of children and their ability to understand ordinarily move from the concrete to the abstract. Learning from concrete objects takes advantage of this fact and provides a firm foundation for the later development of skills and concepts (U. S. Department of Education 1986, 29).

The use of manipulatives is a mandate from the State of Texas. Proclamation No. 60 of the State Board of Education includes requirements for mathematics textbooks submitted for adoption. These adoption requirements stress the use of manipulatives in the student's books and teacher's editions. TAC Chapter 75 includes the seven essential elements for elementary school mathematics originally contained in HB 246. These essential elements stress the

use of a variety of manipulatives (Geer 1986). In addition to their use in such topics as counting, place value, the basic operations, measurement, and geometry, their usefulness in promoting achievement in problem solving and with fractions has received attention (Suydam 1976, 10). The National Council of Teachers of Mathematics, as well as HB 246, recommends more problem solving techniques and the use of calculators and computers. changes in our curriculum will better equip students for life and working in an age of technology. Data from a new international assessment in January, 1987 indicated United States math students Researchers from the University of Illinois at are mediocre. Champaign--Urbana call for a complete overhaul in teaching methods. This study compares United States students to students One of their recommended revisions from 20 other countries. includes effectively using computers and calculators (McKnight, et.al., 1987, 558). Papert believes in a computer culture the child is in control. The child can be builder of their own intellectual structures (Papert 1980, 12). Dienes believes real math is built up in children's minds by actually building concepts concretely. They are working towards abstractions and can manipulate their own learning environment.

Support from learning theorists began in 1930 by William This theory is based on the belief children must understand basic concepts underlying what they are learning if learning is to be permanent. His theory of learning generated interest in having students use manipulatives to form concepts necessary in learning mathematics. Piaget and Skemp conclude all individuals pass through stages as they mature. Manipulatives are significant in all stages. Students mental images and abstract ideas are based on their experiences. Students who see and manipulate a variety of objects have clearer mental images and can represent abstract ideas more completely than those students who have not had these experiences. Dienes advocated the use of manipulatives to build children's understanding of numbers. Each of the manipulatives gives a concrete representation of a concept. also advocated the use of multiple embodiments to teach the concept, e.e., three beans, three rods, three blocks. Only after the student has progressed through much play with representatives of a

concept will they move to symbols. Teachers must facilitate this transfer of concepts. Learning theories suggest children whose mathematical learning is firmly grounded in manipulative experiences will be more likely to bridge the gap between the world in which they live and the abstract world of math (Kennedy 1986, 6).

Researchers claim children should use manipulative materials as they learn math, and most believe the materials are worthwhile. Fennema summarized research on Cuisenaire rods versus the traditional approach. There is some indication children learn better when the learning environment includes a predominance of experiences with models suited to the child's level of cognitive She also summarized studies about other materials development. and stated "these data tend to support the hypothesis that a learning environment embodying representational models should be suited to the development level of the learner." She supports the use of many materials at the early levels of learning, with a gradual decrease in their use as children are able to handle concepts more symbolically. Manipulatives are more widely used in primary grades, however research supports their value at all school Manipulatives not only help children in the intermediate grades develop new concepts but also can be used to provide remedial help. "If there is any risk related to the use of manipulatives in these grades, it derives from their being ignored or abandoned too quickly" (Kennedy 1986, 7).

Although researchers advocate strongly the use of manipulatives, they are not frequently used. One survey reported that nine percent of elementary school classes (K-6) never used manipulatives and 37 percent used them less than once a week (Fey 1979, 12). Suydam reported that although most teachers indicate a belief in the importance of using manipulatives, this belief is not always translated into classroom behavior (Suydam 1984, 27).

There are several reasons why teachers do not use manipulatives. One reason is cost. Another reason is classroom management and control. Achievement in math is largely determined by student ability to compute on standardized tests. Use of manipulatives would seem almost counterproductive. Some teachers do not

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STUFF

Strategic Tactics Ultimately For Fun

Beverly Millican

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Marilyn Burns, in her book <u>The Math Solution</u>, describes two games that many teachers have used successfully in their classrooms. The two games, Race for a Flat and Clear the Board, are appropriate for instructing place value concepts at the concrete level for both primary and intermediate elementary levels.

A brief description of these two popular games, emphasizing whole number place value up to hundreds, is given. Following these, two variations for use in instructing place value with decimals at the concrete level are explained.

The two original games are played with Base 10 materials: "units"

"longs"
and "flats"

The gameboard can easily be made for each player by using a legal size piece of paper, divided into three columns and labeled at the top, as shown.

F	L	U

This "FLU" (for Flats, Longs and Units) board helps students organize their materials, facilitates the exchanges that have to be made when regrouping, and helps provide the connection with numbers that must be made when recording at the symbolic level is begun.

RACE FOR A FLAT

MATERIALS: FLU boards

Base 10 materials

2 dice or number cubes

Race for a Flat. Two (or three) players take turns tossing the dice, finding the sum and taking that number of units from "the bank" to place on his/her own FLU board in the correct column, as pictured. The rules parallel those of place value in that no more than 9 "units" or 9 "longs" can remain in each column - exchanging needs to take place when possible.

The winner is the first one to get a flat.

F	L	U
		9 9 9 9 9 9



CLEAR THE BOARD

MATERIALS: FLU boards

Base 10 materials 2 dice or number cubes

<u>Clear the Board</u>. The "opposite" of Race for a Flat, this game begins with each player having a flat on his/her FLU board.

F	L	U

Taking turns, each player tosses the dice and <u>removes</u> that amount of units from the gameboard. Again, each player has to make exchanges with the materials, just like the regrouping (or "borrowing," as some of us learned it!) that must be done when subtracting abstractly with numerals.

The winner is the one to first clear his/her FLU board.

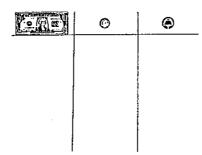
Variations:

You may want to let your students amend some of the rules to make the games more interesting. For example, if "doubles" are thrown, that player may have another throw as in Monopoly. You also may allow students to choose to use 1 die only when they are getting close to getting a flat or clearing their boards so they can roll exactly what they needed to win.

Penny McAdoo has adapted these first two games to use money (pennies, dimes, and dollars) instead of Base 10 materials as a place value model. While Base 10 blocks are a clear, proportional model, the money model has the strength of being directly connected to the real world.

The gameboard is similar to the FLU board in that it has three columns, labeled with pennies, dimes, and dollars.

Although imitation "dollars are used, real pennies and dimes, supplied by the teacher, or even better, brought by students from home, add excitement, but demand firm management.



One baggie per pair of students is required and needs to contain:

- 25 pennies
- 20 dimes
- 4 dollar bills
- 2 dice or number cubes

ROLL-A-DOLLAR

MATERIALS: Money gameboard

1 baggie of money and dice per pair of participants.

Roll-a-Dollar: This game parallels Race for a Flat in that partners take turns rolling the dice, finding the sum, taking that many pennies from the "bank" and placing them on their gameboard. Again, exchanges need to take place when possible.

The first one to get \$1.00 wins!

GO-FOR-BROKE

MATERIALS: Money gameboard

1 baggie of money and dice per pair of participants.

Go-for-Broke. Like Clear the Board, Go-for-Broke begins with \$1.00 on the gameboard, with the roll of the dice designating the amount to be removed until all the money is gone. The game is more interesting if the players are allowed to choose either 1 or 2 dice to roll when they have only a few pennies left so that exactly \$1.00 total will be removed.

All of the above games are played only with concrete materials until students have a clear understanding of the place value concepts involved. The next step in a developmental mathematics process would be to record symbolically what the students are doing with the concrete materials. In the next issue of TMT, games at this next "connecting" level will be described. Stay tuned.......

Membership Dues Date Change

If your expiration date on the mailing label is March 1st or later, your dues must be paid by September 1, 1988. Otherwise they are due \underline{NOW} .

CAMT DATES

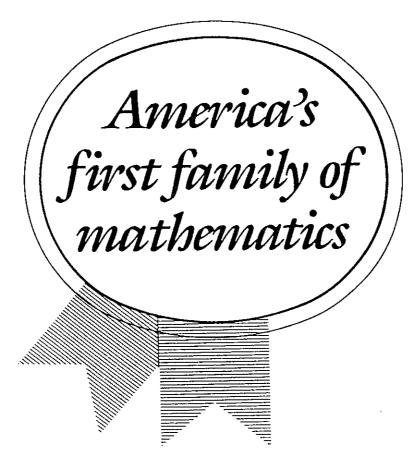
AUGUST 2 - 4, 1988 HOUSTON, TEXAS

AUGUST 2 - 4, 1989 SAN ANTONIO, TEXAS

CAMT BOARD MEETINGS

The CAMT Board has met three times this year. This is highly unusual! Last year it met only once, but this has been a very busy year for the CAMT Board. The success of the summer conference There was a clear mandate from teachers was overwhelming! attending the first August CAMT in 1987, that the switch to summer is a good idea, but now larger facilities are needed. Hence, the Board moved the 1988 conference to the Brown Convention Center in Houston. On February 8 the Board met again to plan its reorganization and to set dates and location for the 1989 conference. It was voted that in 1989 CAMT will be held AUGUST 2 - 4 at the convention center in SAN ANTONIO! If you went to the NCTM National Conference in San Antonio three years ago, you'll remember how grand those facilities were and the convenience and fun of staying in one of the hotels along the riverwalk.

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