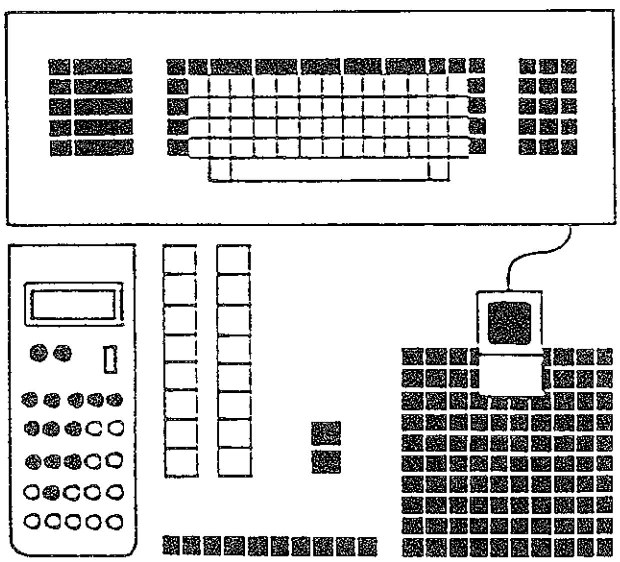
TEXAS MATHEMATICS TEACHER



Extra Lotto Entries: How Much Should They Cost
Least Common Multiples and the Euclidean Algorithm
The Sum and Product of the Zeroes of a Polynomial

OCTOBER, 1987

TEXAS MATHEMATICS TEACHER, the official journal of the Texas Council of Teachers of Mathematics, is published four times each year, in October, January, March, and May. All manuscripts and editorial correspondence should be addressed to George H. Willson, listed above.

Manuscripts should be neatly typewritten and double spaced, with wide margins, on 8 1/2 " by 11" paper. Authors should submit the signed original and four copies. Illustrations should be carefully prepared in black ink on separate sheets, the original without lettering and one copy with lettering added. As soon as possible after refereeing, authors will be notified of a publication decision. Originals of articles not accepted for publication will be returned to the authors. Two copies of the issue in which an author's manuscript appears will automatically be sent to the author.

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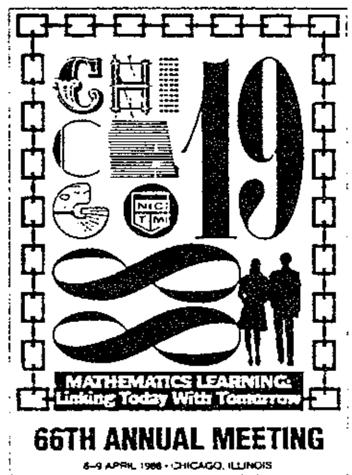

Membership Dues Date Change

If your expiration date on the mailing label is March 1st or later, your dues must be paid by September 1, 1988. Otherwise they are due NOW.

Note From the Editors

The editors are pleased to present this first issue of the Texas Mathematics Teacher in its new format. We hope that it meets with your approval. If you have any suggestions or ideas for improvement, please write to us at the address given on the previous page. We look forward to hearing from you.

TEXAS MATHEMATICS TEACHER

TEXAS COUNCIL OF TEACHERS OF

MATHEMATICS

Affiliated with the National Council
of Teachers of Mathematics

October, 1987

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President's Message

When Thomas Jefferson was president, he was quite a horseman. One of his outings was after a hard rain while the streams were still swollen. A hobo (modern day street person) stood at the edge of a stream, wishing to cross. It was too deep for a man. However, a horse would have no trouble. Along came the president's party, and the hobo watched the horseback group cross. When Thomas Jefferson came along, the hobo asked for a ride across the stream. "Surely," said Jefferson, and took him upon his horse. When safely across, the hobo dismounted. Members of Jefferson's party asked the man if he knew who he'd asked for the ride. "No," was the reply. "He is the president of the U. S. Why did you ask <u>him</u> and none of us?" Responded the hobo, "He had a 'yes' face."

This story was told during "Moment with the Children" at St. Philip Presbyterian Church by my minister, Bill Forbes, the Sunday before school began. At its conclusion, he asked if the children knew what a "yes" face was. Their answer: "Some teachers have 'yes' faces and some have 'no' faces."

School's been in session now for several weeks. How do your students perceive you? Is your face a "yes" or a "no?" Does it matter? You bet it does! Think about it!

I saw many "yes" faces at CAMT, August 3-5. There were some people who had said, "Teachers won't go to CAMT during the summer on their own time." Teachers went! Teachers said, "Yes! Yes, I want to learn more about math and teaching it." Like the stream, CAMT attendance swelled to 2600, 400 more than ever before.

Special thanks from TCTM go to TEA math consultants Bill Hopkins, Barbara Montalto, and Ramona Jo DeValcourt. Bill included membership to TCTM on the registration form. He or his wife, Anita, was always at the registration desk. This was an incredible help to the TCTM members working there. While Barbara was making all the physical arrangements for CAMT, she also arranged the TCTM breakfast. Ramona Jo has organized a section of TCTM especially for elementary teachers, STEAM.

Can we ever say enough thanks to John Huber of SHSU for his technological work for CAMT and TCTM? To those of you who helped at the registration desk, the TCTM membership table, the NCTM materials booth, or the STEAM booth, an enormous thank you for your time.

The TCTM Breakfast and Business meeting was attended by 101 members. The highlight of this meeting was J. William Brown's remarks about our history when he was presented Honorary Membership. He was recognized for many years of service as member and editor of the Texas Mathematics Teacher. We appreciate you, J. William!

Maggie Dement

THANK YOU
For a lovely breakfast feast TCTM says thank you to
Houghton Mifflin
Holt, Rinehart, Winston
Scott Foresman

Extra Lotto Entries: How Much Should They Cost

Bonnie H. Litwiller and David R. Duncan

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Many states have introduced a game called Lotto in an attempt to raise money. These games are almost all organized in a similar way. A player pays money (usually $1.00) to select 6 numbers out of a larger set, (sets from 30 to 44 are commonly used). After selecting the 6 numbers, a player receives a ticket as a record of his/her selected numbers.

At the end of a given period of time, 6 winning numbers are selected in some way by the Lottery Commission. If a player has selected these same 6 numbers on a ticket, he/she wins the grand prize, the amount of which is determined by the number of players and the number of winners.

Some states employ an extra "wrinkle" in this game, that is, a player can either select 6 or 7 numbers. If a player chooses 7 numbers and if this set of 7 numbers contains the 6 winning numbers, the player wins the grand prize. Clearly these two tickets (with 6 and 7 numbers selected) for the same game, should not cost the same. A player who chooses 7 numbers has a better chance of winning (by matching the 6 winning numbers) than does the player who chooses only 6 numbers. Consequently, it should cost more to choose 7 numbers than to choose 6. The question is to determine relative costs for these two types of tickets which are competing for the same prize. If the 6-number ticket sells for \$1.00, how much should the 7-number ticket cost?

To answer this question we must compute the probability of a player winning with a 6-number or with a 7-number ticket. Suppose that there are x numbers in the larger set from which the winning numbers are selected. There are then $\binom{x}{6}$ ways of the player's selecting a subset of 6 numbers from the set of x numbers. The player purchasing a 6 number ticket then has one chance out of $\binom{x}{6}$ to match the winning numbers exactly. The probability that a given 6-number ticket will win is thus $\frac{1}{\binom{x}{6}}$.

Now calculate the probability that a player with a 7-number ticket will win the grand prize. There are $\binom{x}{6}$ ways in which a 7-number ticket can be selected. In how many ways can it be a winner? If the 7-number ticket contains the 6 winning numbers, its <u>seventh</u> number can be any of the remaining x - 6 <u>non</u>-winning numbers. There are thus x - 6 distinct 7-number tickets that will win the grand prize. The probability that a single 7-number ticket wins the grand prize is

$$\frac{x - 6}{\binom{x}{7}}$$

$$= \frac{x - 6}{\frac{x(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}$$

$$= \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{x(x-1)(x-2)(x-3)(x-4)(x-5)}$$

$$= 7\left[\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{x(x-1)(x-2)(x-3)(x-4)(x-5)}\right]$$

$$= 7 \cdot \frac{1}{\binom{x}{6}}$$

The probability that a 7-number ticket wins the grand prize is thus 7 times the probability of winning with a 6-number ticket. Since the 7-number ticket is 7 times as likely to win as the 6-number ticket, it should reasonably cost 7 times as much. If a 6-number ticket costs $1.00, a 7-number ticket should cost $7.00.

What would be a fair price for an 8-number ticket in the same game for which the 6-number ticket costs $1.00? Should it cost $8.00 or eight times the cost of a 6-number ticket?

The 8-number ticket can be selected in $\binom{1}{8}$ ways. For such a ticket to win, it must contain the 6 winning numbers <u>and</u> two <u>non</u>-winning numbers. These two <u>non</u>-winning numbers can be selected in $\binom{x-6}{2}$ ways. There are thus $\binom{x-6}{2}$ winning 8-number tickets. The probability that a given 8-number ticket wins is thus

$$\frac{\binom{x-6}{2}}{\binom{x}{8}} = \frac{\dfrac{(x-6)(x-7)}{2\cdot 1}}{\dfrac{x(x-1)(x-2)(x-3)(x-4)(x-5)(x-6)(x-7)}{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}}$$

$$= \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3}{x(x-1)(x-2)(x-3)(x-4)(x-5)}$$

$$= \frac{8\cdot 7}{2\cdot 1}\left[\frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{x(x-1)(x-2)(x-3)(x-4)(x-5)}\right]$$

$$= \binom{8}{6}\cdot\frac{1}{\binom{x}{6}}$$

$$= 28\cdot\frac{1}{\binom{x}{6}}\ .$$

The probability that a player with an 8-number ticket will win is thus 28 times the probability of winning with a 6-number ticket. Since an 8-number ticket is 28 times as likely to win as the 6-number ticket, it should reasonably cost 28 times as much or $28.00.

In general, what should be the price of an r-number ticket $(r < x)$? There are $\binom{x}{r}$ ways of selecting the r numbers on the ticket. Our reasoning then proceeds as in the earlier cases. We must <u>count</u> the number of possible winning r-number tickets. In addition to the 6 winning numbers, a winning r-number ticket must contain r-6 <u>non</u>-winning numbers. These non-winning numbers may be selected in $\binom{x-6}{r-6}$ distinct ways. There are thus $\binom{x-6}{r-6}$ winning r-number tickets. The probability that a given r-number ticket wins is thus

$$\frac{\binom{x-6}{r-6}}{\binom{x}{r}} = \frac{\dfrac{(x-6)(x-7)\cdots(x-r+1)}{(r-6)(r-7)\cdots 3\cdot 2\cdot 1}}{\dfrac{x(x-1)(x-2)\cdots(x-r+1)}{r(r-1)(r-2)\cdots 3\cdot 2\cdot 1}}$$

$$= \frac{r(r-1)(r-2)\cdots 3\cdot 2\cdot 1}{x(x-1)(x-2)\cdots(x-r+1)} \cdot \frac{(x-6)(x-7)\cdots(x-r+1)}{(r-6)(r-7)\cdots 3\cdot 2\cdot 1}$$

$$= \frac{r(r-1)(r-2)(r-3)(r-4)(r-5)}{x(x-1)(x-2)(x-3)(x-4)(x-5)}$$

$$= \frac{r(r-1)(r-2)(r-3)(r-4)(r-5)}{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} \cdot \frac{6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{x(x-1)(x-2)(x-3)(x-4)(x-5)}$$

$$= \binom{r}{6}\cdot\frac{1}{\binom{x}{6}} = \binom{r}{6}$$ times the probability of winning with a 6-number ticket.

Using the same line of reasoning as before it should cost $\binom{r}{6}$ dollars.

Table 1 reports the fair prices of an r-number ticket for various values of r, assuming that a 6-number ticket in the same game costs \$1.00.

Table 1

r	Fair Price of r-numbered ticket
6	$\binom{6}{6}$ = \$ 1.00
7	$\binom{7}{6}$ = \$ 7.00
8	$\binom{8}{6}$ = \$ 28.00
9	$\binom{9}{6}$ = \$ 84.00
10	$\binom{10}{6}$ = \$210.00
11	$\binom{11}{6}$ = \$462.00
12	$\binom{12}{6}$ = \$924.00

Do you think that many people would buy an 8, 9, 10, 11, or 12-number ticket even though the price is fair? Note that the solutions to all the questions raised in this article are independent of the value of x as long as the player of the r-number ticket selects only a proper subset of the original x numbers. The reader and his/her students are encouraged to find other probability questions related to Lotto games.

Least Common Multiples and the Euclidean Algorithm

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The greatest common factor (gcf) of two natural numbers is used to reduce a fraction to lowest terms. For example, to reduce the fraction 28/70 to lowest terms, we observe that 14 is the gcf of the numerator and denominator and write

$$\frac{28}{70} = \frac{(2)(14)}{(5)(14)} = \frac{2}{5}$$

There are several ways to compute the greatest common factor of two natural numbers. One method involves listing all factors of each number. Another is to use the prime factorization of each number. Both of these methods involve a certain amount of trial and error. A third, the Euclidean algorithm, is unique in that it provides a direct procedure for computing the greatest common factor of two numbers.

We begin the Euclidean algorithm by dividing the larger of the two numbers by the smaller. Then the most recent divisor is divided by the most recent remainder. Since the remainders form a decreasing sequence, eventually we will get a zero remainder, at which point the process is terminated. The last divisor is the greatest common factor.

For example, to find the greatest common factor of 680 and 153, begin by dividing the larger by the smaller:

$$680 = (4)(153) + 68.$$

The quotient is 4 and the remainder is 68. We then divide the divisor, 153, by the remainder, 68:

$$153 = (2)(68) + 17.$$

The most recent divisor, 68, is then divided by the most recent remainder, 17:

$$68 = (4)(17) + 0.$$

Since a zero remainder was obtained, the process is terminated and we conclude that the last divisor, 17, is the greatest common factor.

The least common (lcm) multiple of two natural numbers is used to add or subtract two fractions having different denominators. For example, to add

$$\frac{13}{12} + \frac{7}{30}$$

we use the fact that 60 is the least common multiple of 12 and 30 to write

$$\frac{13}{12} + \frac{7}{30} = \frac{65}{60} + \frac{14}{60} = \frac{79}{60}$$

To compute the least common multiple of two numbers, we also can call upon several "trial and error" methods. These include prime factorization and the listing of multiples of each number. However, we rarely teach that there is a direct method for computing the least common multiple of two numbers. Specifically, the Euclidean algorithm can be used in conjunction with the following:

The product of two numbers is equal to the product of their greatest common factor and least common multiple.

Although the proof of this last statement is rather abstract, we can provide some insight into it with an example:

Since $90 = 2^1 3^2 5^1$ and $24 = 2^3 3^1$, we see that gcf(90,24)

$= 2^1 3^1$ and lcm(90,24) $= 2^3 3^2 5^1$. Thus,

gcf(90,24)lcm(90,24) $= 2^4 3^3 5^1 = (90)(24)$.

From the above it follows that the least common multiple of two numbers is equal to the product of the two numbers divided by their greatest common factor. Therefore, to find their least common multiple, the Euclidean algorithm can be used to find the greatest common factor, which is then divided into the product of the two numbers.

For example, to find the least common multiple of 420 and 750, first use the Euclidean algorithm:

		Quotient Divisor		Remainder
750	=	(1)(420)	+	330
420	=	(1)(330)	+	90
330	=	(3)(90)	+	60
90	=	(1)(60)	+	30
60	=	(2)(30)	+	0

Since 30 is the last divisor, gcf(420,750) = 30. Therefore,

$$\text{lcm}(420,750) = \frac{(420)(750)}{30} = 10500$$

We conclude by listing a BASIC program which uses the Euclidean algorithm to compute both the greatest common factor and the least common multiple of two numbers:

```
100 INPUT  "A,B (A>B)";A,B
110 R1 = B
120 R2 = A
130 R = R2 - INT(R2/R1)*R1
140 If R = 0 THEN 180
150 R2 = R1
```

```
160 R1 = R
170 GOTO 130
180 PRINT
190 PRINT  "THE GCF IS";R1;"."
200 PRINT
210 PRINT  "THE LCM IS";(A*B)/R1;","
220 END
```

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The Sum and Product of the Zeroes of a Polynomial

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Many students of algebra are familiar with a procedure for checking the roots of a quadratic equation after they have solved it. Given the quadratic equation $ax^2 + bx + c = 0$, the sum and product of its roots must be $-\frac{b}{a}$ and $\frac{c}{a}$, respectively. Not so well known is the fact that these formulas are a special case of a more general pair of formulas which give the sum and product of the zeroes of any polynomial of positive degree in terms of certain coefficients of the polynomial. (In this paper, the term <u>polynomial</u> refers to a polynomial function.) The proof of these formulas requires only elementary algebra and is thus appropriate for a secondary school algebra class.

<u>THEOREM 1</u>: Let $f(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$ be a polynomial with complex coefficients a_i, $0 \leq i \leq n$, and $a_n \neq 0$. If $r_1, r_2, \ldots, r_n$ denote the n complex roots of the equation $f(x) = 0$, then

$$r_1 + r_2 + \ldots + r_n = -\frac{a_{n-1}}{a_n}$$

and

$$r_1\, r_2 \ldots r_n = (-1)^n \frac{a_0}{a_n}.$$

Proof: Since $r_1, r_2, ..., r_n$ are the n complex zeroes of $f(x)$ we must have

$$a_n x^n + a_{n-1}x^{n-1} + ... + a_1 x + a_0 = a_n(x - r_1)(x - r_2) ... (x - r_n).$$

Dividing both sides of this equation by a_n and multiplying out the right side, this becomes

$$x^n + \frac{a_{n-1}}{a_n} + ... + \frac{a_1}{a_n} x + \frac{a_0}{a_n}$$

$$= x^n - (r_1 + r_2 + ... + r_n)x^{n-1} + ... + (-1)^n r_1 r_2 ... r_n.$$

Equating coefficients of x^{n-1} on the right and left sides of this equation yields the desired sum-of-zeroes formula, and equating constant terms gives the product-of-zeroes formula.

Example 1: $6x^4 + 13x^3 - 16x^2 - 28x + 16 = 0$ has sum of roots $-\frac{13}{6}$ and product of roots $(-1)^4 \frac{16}{6} = \frac{8}{3}$, according to Theorem 1. Applying the Rational Roots Theorem, one finds the roots of the equation to be $\frac{1}{2}$, $\frac{4}{3}$, -2, and -2, and these four numbers have the correct sum and product.

Example 2: $2x^5 + 3x^4 - 7x^3 + 9x^2 + 23x - 30 = 0$ has sum of roots $-\frac{3}{2}$ and product of roots $(-1)^5 - \frac{30}{2} = 15$, according to Theorem 1. Using the Rational Roots Theorem and the quadratic formula, one finds the roots of the equation to be -2, $-\frac{5}{2}$, 1, $1 + \sqrt{2i}$, and $1 - \sqrt{2i}$, and these five numbers have the correct sum and product.

Note that, as in Example 2, if the original equation has rational coefficients, it may have complex roots, but Theorem 1 guarantees that the sum and product of the roots will be rational. This is reasonable in light of the fact that complex zeroes of polynomials with real coefficients occur in conjugate pairs. Since $(a + bi) + (a - bi) = 2a$ and $(a + bi)(a - bi) = a^2 + b^2$, the conjugate pairs "cancel out" each other's imaginary parts in the total sum and product.

Example 3: $x^4 - 21x^2 + 20x = 0$ has sum of roots $-\frac{0}{1} = 0$ and product of roots $(-1)^4 \frac{0}{1} = 0$ by Theorem 1. (The roots can be found to be -5, 0, 1 and 4.) Note that, by Theorem 1, $a_nx^n + a_{n-1}x^{n-1} + ... + a_1x + a_0 = 0$ will have sum of roots 0 exactly when $a_{n-1} = 0$, and product of roots 0 exactly when $a_0 = 0$. Of course, the product of roots is 0 if and only if 0 is a root, which occurs if and only if the polynomial has constant term 0.

After solving an n^{th} degree polynomial equation, one can add and multiply the n roots of the equation and see if the results agree with the sum and product guaranteed by Theorem 1, serving as a check on the solution. A natural question about this procedure is whether a positive result on such a check guarantees a correct solution; phrased differently, do there exist two different polynomials (i.e., not constant multiples of each other) of the same degree having the same sum and product of zeroes? For polynomials of degree n with $n \geq 3$, the answer is yes. Such polynomials have more than three coefficients, and the sum and product of the zeroes are completely determined by the coefficients of x^n and x^{n-1} and the constant term. Thus, varying the other coefficients will change the zeroes while leaving the sum and product of zeroes constant.

Example 4: $x^3 - 3x^2 - 22x + 24$ and $x^3 - 3x^2 - 10x + 24$ are different polynomials and hence have different zeroes, but the three coefficients that determine the sum and product of zeroes are identical; thus, both polynomials have sum of zeroes 3 and product of zeroes -24. (The zeroes of the two polynomials are 1, 6, -4 and 2, 4, -3, respectively.)

What about the cases n = 1 and n = 2? For a linear polynomial, the sum and product of zeroes is simply the single zero, and this zero uniquely determines the polynomial (up to a constant multiple). This case is somewhat trivial; however, a similar result holds for quadratic polynomials, which we present as a theorem.

<u>THEOREM 2</u>: If s and p are complex numbers, there is a unique quadratic polynomial (up to a constant multiple) with sum of zeroes s and product of zeroes p.

<u>Proof</u>: By Theorem 1, $x^2 - sx + p$ has the desired sum and product of zeroes. To demonstrate uniqueness of this quadratic polynomial, it suffices to show that any complex numbers r and q satisfying (1) $r + q = s$ and (2) $rq = p$ must be roots of $x^2 - sx + p = 0$. (1) implies that $q = s - r$. Substituting into (2), we obtain $r(s-r) = p$, or $r^2 - sr + p = 0$. Thus, r is a solution of $x^2 - sx + p = 0$, and, by the symmetry of (1) and (2) in r and q, q is also a solution.

The above result guarantees that, in the case that the roots r and q found in the solution of the quadratic equation $ax^2 + bx + c = 0$ satisfy $r + q = -\frac{b}{a}$ and $rq = \frac{c}{a}$, one is assured of having found the correct roots. Although such guarantees cannot be made for polynomials of degree three or higher, agreement of the sum and product of roots with the results obtained by applying Theorem 1 does indicate a high probability of accuracy. The formulas of Theorem 1 thus serve as a quick, simple, highly reliable check on the roots found in the solution of a polynomial equation.

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