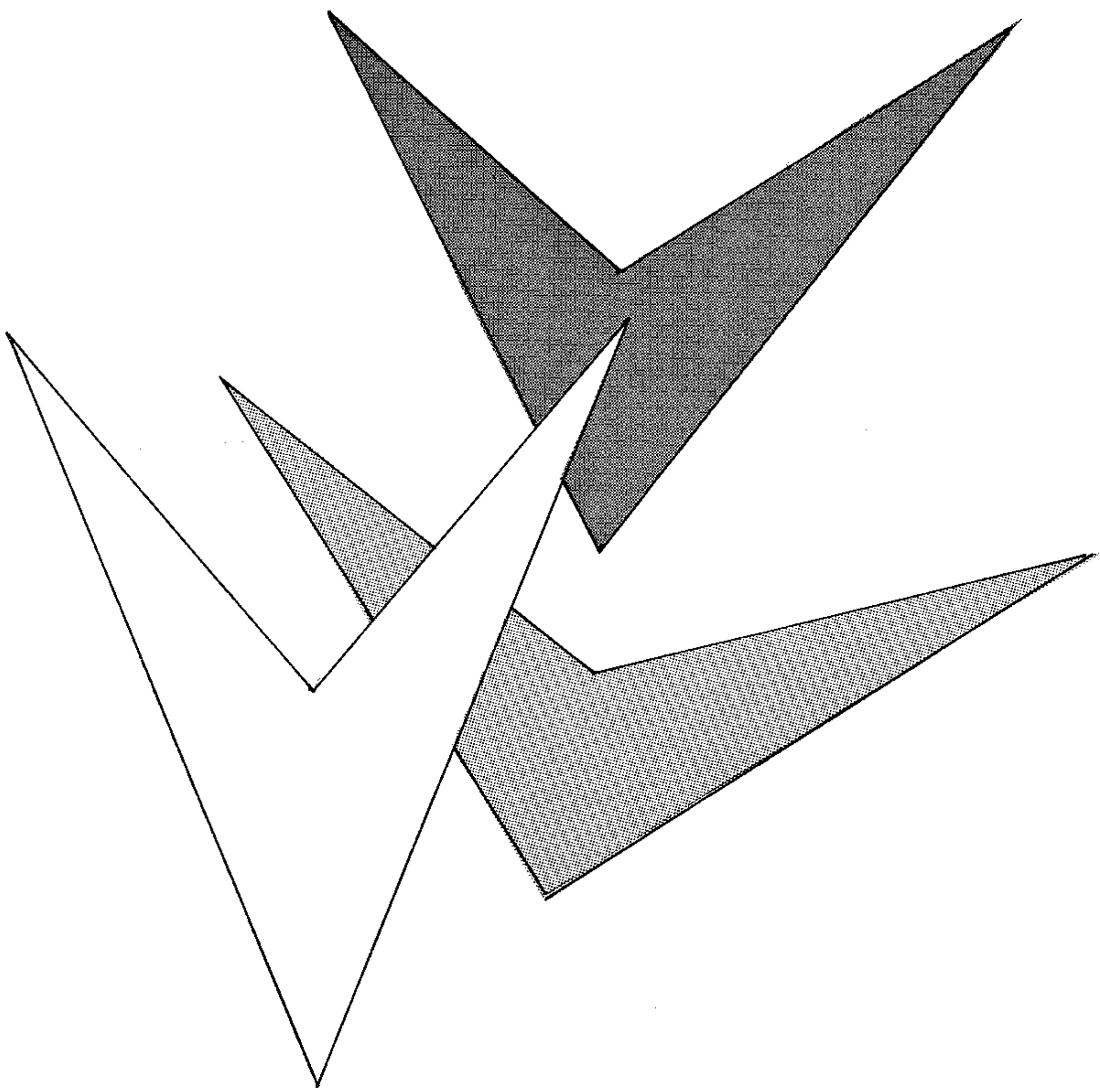
T.C.T.M.

TEXAS MATHEMATICS TEACHER



Texas Council of Teachers of Mathematics

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TEXAS MATHEMATICS TEACHER

Texas Council of Teachers of Mathematics

Volume XXXIII MARCH, 1986 No. 2

PRESIDENT'S MESSAGE

As this is written many conflicting feelings are swirling around and through your TCTM president, and I am sure the same may hold true for many of you. The tragedy of the exploding space shuttle Challenger has made us all aware of the potential dangers hidden among some of our most exciting activities. Our sorrows at the deaths of the crew members are felt along with the feelings of pride that a teacher had been selected to participate in that fateful flight. I have been shocked by the heart attack suffered by my colleague Charles Lamb, known to many of you for his work with teachers in Texas and his directorship of the University Interscholastic League Number Sense contests, but I am gladdened by the knowledge that he has progressed well after multiple bypass surgery and is going home from the hospital soon. Some of you may be concerned about the upcoming TECAT tests, but others may be welcoming the opportunity to show the public and the politicians that you do have abilities in the verbal areas. (Wouldn't it have been nice to have had some chance to demonstrate quantitative and spatial abilities, too?). For you I wish that the bright days outweigh the gloomy ones, the successes outweigh the failures, and the feelings of accomplishment outweigh those of frustration and disappointment.

There are several things happening in our profession this spring that should be interest to us

There are several things happening in our profession this spring that should be of interest to us. I hope that many of you made the NCTM meeting in Dallas February 27 – March 1, and I would like for us to have a good representation at the Annual Meeting of NCTM in Washington, D.C., April 2 – 5. The restrictions placed upon absence from the classroom may make it more difficult to participate in some of these meetings, but if you have the opportunity to attend an Annual Meeting of NCTM, it will be worth the effort, the time, and perhaps the money. The sessions on the program, the fellow teachers from other states and countries, the leaders in the field, textbook authors – – these and other professional factors plus the vacation-like atmosphere of the trips themselves all make for a very positive experience.

Finally, let me remind you to take note of what is happening in your own local council. While it may not be possible to participate in national, regional, or even state meetings very readily, the activities and programs of your local council are easily accessible to you. Take advantage of their offerings. If you do not know what is happening locally, try to find out. If you are not impressed with what is happening, see what you can do to improve the situation. If you find that little is being done, take it upon yourself to initiate something. If you need some help, contact an officer of TCTM in your area or correspond with one of the active local councils. Effective programs and activities for teachers of mathematics are most likely to come from teachers of mathematics, and that means YOU.

– RALPH W. CAIN

PROPOSED DATES FOR CAMT

1986: October 9 – 11	**1989: June 22 – 24**
1987: August 3 – 5	**1990: June 20 – 22**
1988: August 2 – 4	**1991: June 19 – 21**

Note your mailing label for renewal date of TCTM membership!

STUFF Strategic Tactics Ultimately For Fun

Dear T.C.T.M. Member,

Stuff is stumped. We don't understand why we are not hearing from you. We are convinced you are either awfully dull, awfully lazy or awfully busy. Why don't you send in materials to this column? We really are glad to get anything you are willing to send. What more can we say? Send your material to Judy Tate, H.C.D.E., 6208 Irvington Boulevard, Houston, Texas, 77022. Remember all the freebie offers. If you don't remember check your old issues. If you do not have old issues send your $5.00 membership to Bettye Hall, Houston I.S.D., 3830 Richmond, Houston, Texas, 77027. We are a <u>multi purpose</u> column!

Not much love
Stuff Staff
Bettye and Judy

PRIMARY
WHAT'S THE DIFFERENCE?

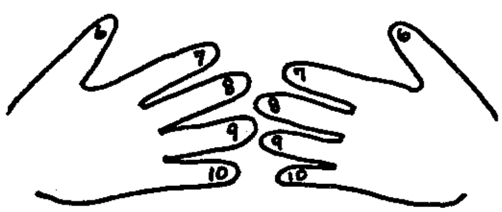
Grade levels: 1-3
Number of Players: 2
Topic: Subtraction facts
Material: Prepared 3 x 5 cards of two colors, paper and pencil for each player.

On each white 3 x 5 card write one of the numerals from one through ten. On 3 x 5 cards of another color write one of the numerals from eleven through nineteen. Shuffle the cards and deal them to the two players. The first player turns over a card and so does his opponent. If the cards are the same color the player showing the lower number of the two cards scores the differences between the two cards (ex. 6 and 10 would mean the player having the "six" card would write down four points on his scoree). Each player turns over another card. If the cards are not the same color, the player having the higher number receives the differences as his score.

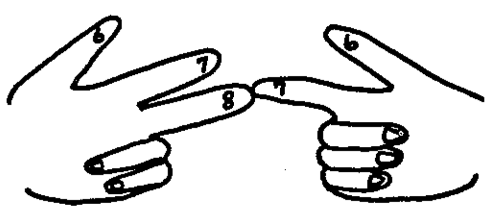
After all cards are used the sums are added together for a total for each player. The largest sum is the winner.

ELEMENTARY
FINGER MULTIPLICATION

Finger multiplication takes three simple steps. First students hold their hands up as shown. Notice that each finger represents a number from 6 to 10.



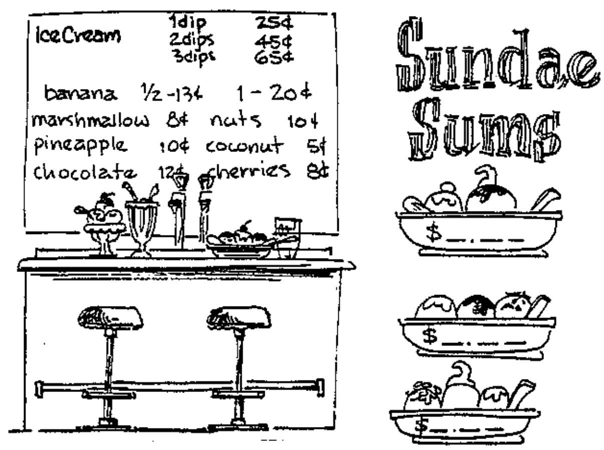
Now if we wish to multiply 8 x 7, we would let finger 8 touch finger, 7, and bend all the fingers below these touching fingers forward.



8 × 7

Finally, we need only to read the results. The tens digit of the product is found by counting the number of extended fingers. Each of these fingers is worth 10. The units digit is found by multiplying the number of bent fingers in the left hand by the number of bent fingers on the right hand.

In our example there are five extended fingers, so the tens digit is 5. There are two bent fingers on the left hand and three bent fingers on the right hand. We multiply 2 times 3 to get 6, the units digit. Our product is 50 + 6 = 56. It is just as we expected, 8 x 7 = 56.



SUNDAE SUMS

PURPOSE

Students get practice in adding money and choosing the correct amounts of money needed to buy various items.

MATERIALS

Use poster paper for the counter and stools. Make several signs of costs of items so they can be changed. Design the sundaes from construction paper, making the ice cream separate from the dishes for ease in changing.

ACTIVITIES

Have students practice adding amounts of money and then finding correct amounts needed to buy various things. Use the activity sheets after students have had adequate practice. Place the correct costs of the sundaes on the sundaes after students have finished working on the activity sheets.

EXPANDED ACTIVITIES

Use grocery store ads and magazine pictures to change the bulletin board to grocery buying. Have your students figure the savings on sale items as compared with the regular prices. They can also compare per ounce costs of differentsized containers of the same product.

Make sundaes. Students could earn the amounts of toppings by a designated number of perfect papers in math, spelling or other subject areas. Each could figure the cost of the sundae he/she has earned and then figure the cost for the entire class. This cost could then be compared with actual cost of a sundae.

INTERMEDIATE
OPERATION COVER UP

Purpose: To reinforce basic number operations.
Directions:

Two to four students can play this game. Each player has one roll per turn with the dice. He must use one or two operations with the three numbers revealed by the dice. That number is then covered with a position marker on the game board. For example, if player one rolled a 3, a 5, and a 4, he would have several options -- he might add 3 and 5 and multiply by 4 giving him 32 to cover on the game board -- or -- he might multiply 3 times 5 and subtract 4 giving him 11 to cover on the game board. Player one covers his number and cannot score on his first turn. Subsequent players continue in the same fashion. In order to score in this game, players must cover a number on the board which is adjacent to another covered number either vertically, horizontally, or diagonally. A player scores one point for each adjacent covered number.

If a player is unable to compute a number which has not already been covered, he loses his turn and the game progresses on to the next player. After a designated period of time, the player with the highest score is the winner. An egg timer may be used to limit the amount of time a player may take on any one turn.

OPERATION COVER UP

1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32
33	34	35	36	37	38	39	40
41	42	44	45	48	50	54	55
60	64	66	72	75	80	90	96
100	102	120	125	144	150	180	216

MIDDLE SCHOOL

Number sense shortcut-working with series of numbers adding series of numbers:

$$4 + 5 + 6 + 7 \ldots\ldots + 29 = \_\_\_\_$$

Take the last number of the series, add it to the first, multiply by the number of numbers in the series, then divide by 2.

$$\frac{(29 + 4)\ (26)}{2} = 429$$

$$1 + 2 + 3 + 4 + 5 + 6 + \ldots\ldots + 27 = \_\_\_\_$$

$$\frac{(1 + 27)\ (27)}{2} = 378$$

If the series is made up of counting numbers beginning with one, another way of doing the same thing is

$$1 + 2 + 3 + 4 + 5 + 6 \ldots\ldots + 67 = \_\_\_\_$$

Take the last number of the series and multiply it by the next number in the series and divide by 2.

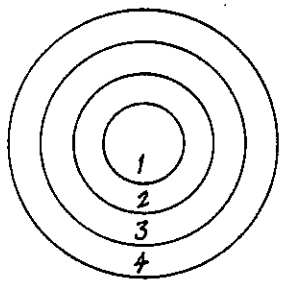
$$\text{So,} \quad \frac{67 \times 68}{2} = 4556$$

Nancy Lee
Campbell Jr. High
Cypress - Fairbanks I.S.D.
Houston, Texas

MIDDLE SCHOOL
DART GAME

1. A group of students were playing a game of darts after school one day using a game board of concentric circles drawn with radii whose measurements were 1 unit, 2 units, 3 units and 4 units.

   Draw a game board like theirs after deciding what you want to use for a unit of measure (one inch, 3 centimeters, 10 centimeters, ?).



Scoring
1 = 30 pts.
#2 = 23 pts.
#3 = 17 pts.
#4 = 9 pts.

2. One of the students said that the scoring on this board was unfair because the region in which a hit could score the greatest number of points was the smallest of the four regions, and that as the regions became larger the number of points you could score with a hit became less. Is it true? Compute the areas of the four regions. Record below.

   #1 ____ #2 ____ #3 ____ #3 ____

3. How much larger is the area of region $4 than #1? ____

   How much larger is the area of region #3 than #2? ____

   How much smaller is the area of region #1 than #2? ____

4. If you throw five darts all of which hit in the scoring areas, in which regions would you have to hit to score a total of

   67 points
   96 points?
   123 points

5. can you score 100 points in 5 throws? In any number of throws? What is the greatest total you can score in five hits on this dart board?

MANUSCRIPTS NEEDED!!!!! Send them to 100 S. Glasgow, Dallas, Texas, 75214.

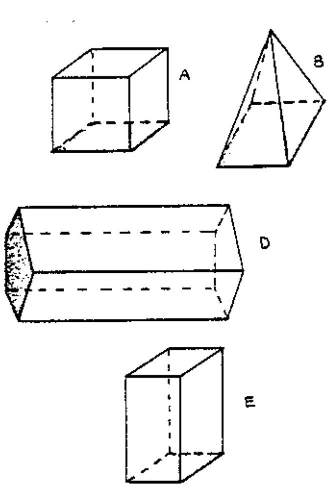
SENIOR HIGH
TIC TAC INTEGERS

Topic: Drill on the addition of integers
Grade level: F.O.M.
Number of players: 2
Materials needed: a 4 x 4 blank grid for each game and 2 pencils

Rules:

1. Decide on the number of games to be played to determine a winner. Five is a suggested number. Decide who will go first.
2. The players take turns writing an integer in any one of the cells. The object of the game is to put the last integer in a row, column, or diagonal so that the sum of the four integers in that row, column or diagonal is zero.
3. The player who completes the sum of zero for the four integers scores 2 points. If in doing this, the player also completes another row, column, or diagonal with a sum of zero, that player scores 5 points, two for each sum, and an extra one for being so clever.
4. The player scoring the greatest number of points in one game is the winner of that game. The overall winner is the person with the largest score after the set amount of games have been played.

Contributed by: Michelle Rohr, Houston I.S.D.



POLYHEDRON	F	V	E
A			
B			
C			
D			
E			
F			
G			

FACES, VERTICES, AND EDGES OF POLYHEDRON

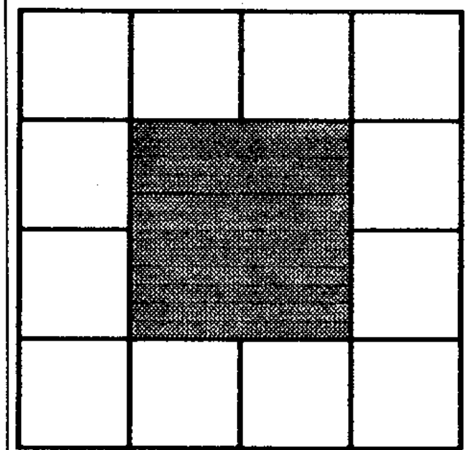
1. Count the number of faces, vertices and edges in poylhedrons A - E. Record these tallies in the chart.
2. Use your ruler to draw two polyhedron F and G; then record the number of faces, vertices, and edges in the chart.
3. Can you find a relationship between the number of faces, the number of vertices, and the number of edges in the polyhedrons? Write a formula to express the relationship.

____

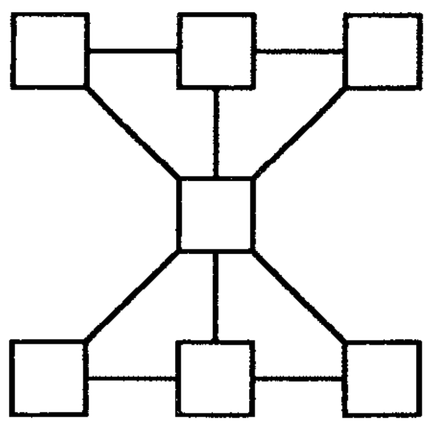
This formula is known as EULER'S FORMULA.

NUMBER PUZZLES

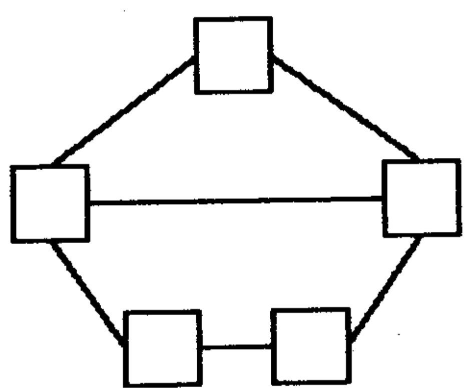
Place the numbers 1 through 12 in the squares so that the sum of the numbers on each side is 25.



Place the numbers 1 through 7 in the squares so that the sum of any three numbers in a straight line is 12.



Place each of the numbers 6 through 10 in the squares so that no number is connected directly to a number that is exactly one larger or one smaller than it.



COMPUTER DRILLS

by Jo Anne Askins, Damon Means and Jimmy Green
Harmony High School, Gilmer, Texas

```
3 REM   THIS PROGRAM GENERATES DISTRIBUTIVE PRO-
   PERTY WORKSHEETS.
5 RANDOMIZE TIMER
10 LPRINT CHR$ (27) CHRS (69)
20 LPRINT "DISTRIBUTIVE PROPERTY
   NAME________________________"
25 LPRINT "BY DAMON AND JIMMY
30 LPRINT:LPRINT
40 INPUT "HOW MANY QUESTIONS WOULD YOU LIKE ON THIS
   WORKSHEET";Q
50 FOR I=1 TO Q STEP 6
55 RANDOMIZE TIMER
57 A=INT (RND*8)+1:B=INT(RND*8)+1:C=INT(RND*8)+1:
   D=INT(RND*8)+1:E=INT(RND*8)+1:F=INT(RND*8)=1:
   G=INT(RND*8)=1:H-INT(RND*8)=1:J=INT(RND*8)+1
58 K=INT(RND*8)+1:L=INT(RND*8)+1:M=INT(RND*8)+1:
   O=INT(RND*8)+1:P=INT(RND*8)+1:R=INT(RND*8)+1:
S=INT(RND*8)+1:T=INT(RND*8)+1:U=INT(RND*8)+1
60 LPRINT I "." A "("B"x +"C"y )" TAB(25) I+1 "." D"("E"x
   -"F")" TAB(50) I+2 . " G "("H"a +"J")":LPRINT:LPRINT
70 LPRINT I+3 "." K "("L"c -"M"d )" TAB(25) I+4 ". ("O"x
   +"P"y )"R TAB(50 I+1 "." S "("T"c +"U")":LPRINT:LPRINT
80 NEXT
85 LPRINT CHR$(27) "F"
90 END
```

DISTRIBUTIVE PROPERTY   NAME________________________
BY DAMON MEANS AND JIMMY GREEN

1. 6(2x+2y)	2. 7(2x-5)	3. 8(1a+5)
4. 5(4c-8d)	5. (8x+3y)5	6. 7(2c+7)
7. 7(1x+5y)	8. 8(6x-3)	9. 5(3a+5)
10. 2(3c-6d)	11. (8x+7y)1	12. 8(6c+5)
13. 4(1x+3y)	14. 3(7x-4)	15. 7(5a+2)
16. 1(1c-6d)	17. (4x+3y)7	18. 1(6c+6)

```
305 REM  COMPUTER DRILL FOR ADDING SIGNED NUMBERS
310 CLS
315 RANDOMIZE TIMER
320 PRINT"Drill: Add rules. To quit, write 777"
330 X=0:Y=0
340 A=INT(RND(1)*21)-10
350 B=INT(RND(1)*21)-10
360 IF B<0 THEN 390
370 PRINT A; "+";B "=?
380 INPUT C:GOTO 410
390 PRINT A;B;"=?
400 INPUT C
410 IF C = 777 THEN 500
420 R=A+B
430 IF R=C THEN 470
440 PRINT"Sorry; the correct answer is";R
450 Y=Y+1
460 GOTO 340
470 PRINT"Right!
480 X=X+1
490 GOTO 340
500 PRINT"Total:";"right;";Y;"wrong
510 END
```

```
RUN
Drill: Add rules. To quit, write 777
 9+5 = ?
? 14
Right!
 9+3 = ?
? 7
Sorry; the correct answer is 12
-7 -2 = ? rrect answer is -9
? 9
Sorry; the correct answer is -9
 10-1 = ?
? 9
Right!
 10 -3 = ?
? -7
Sorry; the correct answer is 7
 -1 + 4 = ?
? 3
Right!
6 + 7 = ?
? 13
Right!
 3 -4 = ?
? 777
Total: 4 right; 3 wrong
Ok
```

```
5    REM  COMPUTER DRILL FOR MULTIPLYING SIGNED
     NUMBERS
10   CLS
15   RANDOMIZE TIMER
20   PRINT"DRILL ON MULTIPLY RULES. TO QUIT WRITE 777
30   X=0:Y=0
40   A=INT(RND(1)*21)-10
50   B=INT(RND(1)*21)-10
60   PRINT A;"X";B;"=?
70   INPUT C
80 IF C=777 THEN 170
90   R=A*B
100 IF R=C THEN 140
110 PRINT"SORRY; THE CORRECT ANSWER IS";R
120 Y=Y+1
130 GOTO 40
140 PRINT "RIGHT!
150 X=X+1
160 GOTO 40
170 PRINT"TOTAL:";X;"RIGHT;";
180 PRINT Y;"WRONG
190 END
```

```
RUN
DRILL ON MULTIPLY RULES. TO QUIT WRITE 777
-1 X-9 = ?
? 9
RIGHT!
-3 X-2 = ?
? -6
SORRY; THE CORRECT ANSWER IS 6
5 X 7 = ?
? 36
SORRY; THE CORRECT ANSWER IS 35
-3 X 3 = ?
? -9
RIGHT!
2 X 7 = ?
? -14
SORRY; THE CORRECT ANSWER IS 14
6 X -7 = ?
? -42
RIGHT!
6 X -1 = ?
? -6
RIGHT!
7 X 1 = ?
? 777
TOTAL: 4 RIGHT; 3 WRONG
Ok
```

MAKING (-x) MEANINGFUL

David R. Johnson, President, National Council of Supervisors of Mathematics
Nicolet High School, Glendale, Wisconsin

How do your algebra students read the symbol "-x"? Common responses are "negative x", "minus x", "the opposite of x" or "the additive inverse of x". The most common response is "negative x". But are all these responses meaningful? Definitely not! In fact the first two responses are very misleading, if not incorrect. We are quite careful in the mathematics classroom to name a real number less than 0 (or to the left of 0 on the real number line) a negative number. Students quite easily grasp the meaning of the phrase "negative number." But all of a sudden we bring out the expression "-x" and read it "negative x"! Trouble begins. Students immediately assume that this symbol stands for a number less than zero simply because its verbal name contained the word negative.

If you don't believe the name ("negative x)" is a misleading name, I suggest that you write the symbol "(-x) on the board and ask your students how it relates to zero on the number line. You will hear a resounding cry "to the left of zero"!

Too many students do not understand that, if x is a real number, "-x" could be a positive, negative, or zero, and that more information is needed before a decision can be made. Students are so overpowered by the use of the word negative and the definition of a negative number, that they do not appreciate the meaning of the expression (-x). Students do appreciate, however, that additive inverses or opposites do not have to be negative. We must encourage our students to read the expression (-x) as the opposite of x.

If we do not read (-x) as "the opposite of x" the problem is complicated even more when we teach the definition of absolute value. Though there are many homespun ways of defining the concept of absolute value, some are confusing and often incorrect. For example: "The absolute value can be found by dropping off the sign." That idea is deadly. If b is less than zero, then b in this case equals -b. No sign was chopped off here. In fact, one was added. When it comes to the definition of words such as absolute value, it is necessary to use a mathematically sound definition:

---

a R
If a = 0, a = 0 (a remains unchanged)
If a 0, a = a (a remains unchanged)
If a 0, a = -a (the result becomes the opposite of a)

This definition, however, demands real understanding. First the student must know the size of the real number in relationship to zero. Secondly, the student must understand that "-x" is simply a symbol for the "inverse of x" and obeys the property of trichotomy. That is, (-x) could be positive, negative, or zero. If, for example, a student is asked to define the absolute value of -b where b is less than zero, it follows that the b equal -b. But for students who believe that a negative sign must be dropped to take the absolute value of a number, or for the student that does not appreciate that "-x" is really a positive number in this case, it will be difficult to apply the definition of absolute value to this expression correctly. Again, reading the (-x) as the "opposite of x" will help the student to apply the definition of absolute value correctly.

Students will do well on the examples that follow if they have a good understanding of the definition of absolute value and if they understand that the symbol "-x" represents the inverse of x.

Simplify:

a) $-b^3$ if b 0 — ans: $-b^3$, because $(-b^3)$ 0
b) -3 -x , if x 0 — ans: -(-3x), because (-3 -x) 0
c) -b + -b , if b 0 — ans: -2b because (-b) 0
d) 3b , if b 0 — ans: (-3b), because (3b) 0

Expressions with variables should be introduced when teaching the concepts of absolute value in the first year algebra course. Using only constants may lead to poor techniques in simplifying absolute value expressions. That is, students may be able to get the correct answer and yet never realize that they do not understand the definition.

Practice in determining the size of the expression prior to teaching an algebraic definition of absolute value will help make the definition more meaningful to students.

Consider the following expression:
What is the value of the expression (x - 6), if x 0?
a) always less than 0?
b) always greater than 0?
c) zero?
d) sometimes less than/sometimes greater than zero?

Answer:
On a number line place "(x - 6)" to the left of zero.

(x-6)
———————— 0 ————

That is, if x 0, then (x - 6) 0. "(x -6') is a negative number for any value of "x" that is negative.

Determine the sizes of the following expressions given information regarding the values of the variables. Place a check in the appropriate box.

Expression	Variable Information	Less than Zero	Equal to Zero	Greater than
1. x^3	x 0			
2. $x^2 + 6$	x 0			
3. $x^2 + 6$	x 0			
4. $x^2 + 6$	x = 0			
5. -3x	x 0			
6. 5y	y 0			
7. -5x + y	x 0 y 0			
8. $(x - 14)^2$	x 0			

Expression	Variable Information	Less than Zero	Equal to Zero	Greater than
9. $(x-14)^2$	x)			
10. $(x-14)^2$	x = 0			
11. -f	f 0			
12. $-(f^2)$	f 0			
13. -f + -g	f, g 0			
14. $f^3 + -2g$	f 0, g 0			
15. $-3x^2$	x 0			
16. $-2(x-1)^2$	x 0			

Given the following information, place the non-zero real numbers, "-f, g, h", on a real number line on the proper side of zero and in the proper order:

(CONTINUED ON PAGE 12)

THE FAST TRACK, ACTION AND LINEAR FUNCTIONS

by Kenneth E. Easterday, Auburn University
Kooi Fong Yee, Kajang Silangor, Malaysia

The mathematics curriculum in the 1950's and early 1960's was based on the development of the curriculum in terms of a "spiral curriculum". An idea or concept was to be introduced at one level and developed at that level. The concept would be reintroduced later at a slightly more advanced level and developed again at this new level. As one moved up the spiral, the concept was refined and learned more thoroughly (Johnson & Rising, 1972). This same spiral approach appears to have merit as one way of dealing with applications of mathematics and with the development of problem solving skills.

Two sequences of problems are proposed here. The mathematical concepts transcend the upper elementary grades to collegiate mathematics. The applications are drawn from the physical sceinces and also transcend several grade levels. The exercises lend themselves to computer simulation or to other forms of laboratory experiences. The problem solving skills to be developed are imbedded within the exercises. Students should be encouraged to perform the experiments where appropriate, to seek alternate solutions and to generate other applications.

Yeshurun (n.d.) asserted that any verbal problem leading to an equation which can be solved by precalculus mathematics exhibits one of a few structures which he called connections. These connections are functions or function systems of several variables. His second premise was that the structure and not the wording, is the core of the problem to be solved. He (1979) identified nine such connections and the simplest of these connections is the triple connection. A triple connection applies if the product of two quantities equals a third one. All triple connections can be solved in the same manner.

Problem 1.

Suppose that one has a mass transit train or trolley for intra-city travel. Once such vehicle after acceleration for 2 seconds travels at a constant speed or rate of 30 meters per second for 8 seconds. Find the distance the vehicle travels and represent the relationship graphically.

Junior high school students know that the distance traveled is the rate or speed times the time (d = rt). If they understand that this is a triple connection and essentially the same problem as finding the area of a rectangle, then it is a simple transition to the representation of this problem graphically. Any triple connection can be so represented.

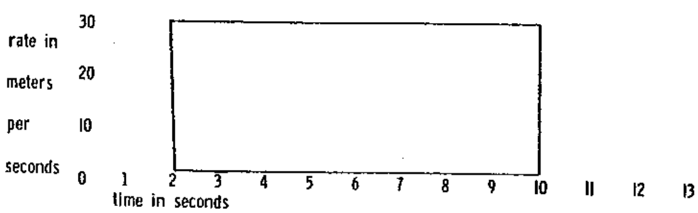


Figure 1. The area of the rectangle equals the distance with constant speed over time

The distance traveled in 8 seconds going 30 meters per second is 8 X 30 or 240 meters. The distance is represented by the area of the rectangle shown above.

Now suppose that the vehicle accelerates at a constant rate from the stopped position to its speed of 30 meters per second and that the vehicle reaches its speed of 30 meters per second in two seconds. Graphically represent its acceleration for these first two seconds and find the distance traveled in these 2 seconds of acceleration.

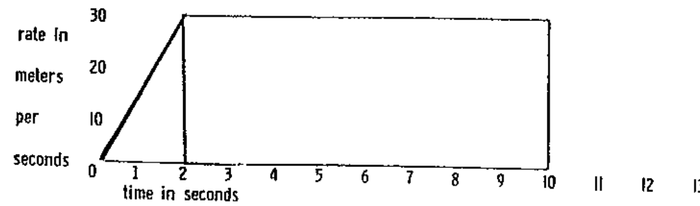


Figure 2. The area of the triangle equals the distance covered during acceleration

The distance traveled in the first two seconds is represented by the area of the right triangle with a base of 2 seconds and an altitude of 30 meters per second. The distance traveled is the area of the triangle (A = 0.5bh) or 0.5 X 2 X 30 or 30 meters. This problem may be classified as a triple connection; however, we need only a right triangle (or 1/2 of the rectangle with height h and length b) to make the graphic representation.

Now is the time to get off of our trolley so we must decelerate to a stop. For comfort we wish to decelerate at a constant rate of 10 meters per second per second. At that rate deceleration, it would take us 3 seconds to stop. Our graphical representation is complete and we can find the distance traveled during deceleration and the total distance traveled on our 13 second trip.

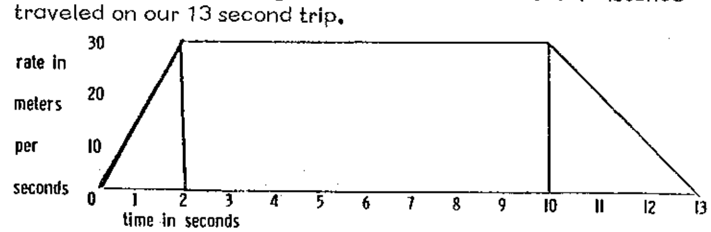


Figure 3. The area of the trapezoid equals the total distance covered

The distance traveled during deceleration is represented by the area of the triangle on the right. This triangle has a base of 3 seconds and an altitude of 30 meters per second. The distance traveled during deceleration is 0.5 X 3 X 30 or 45 meters. Our entire trip of 13 seconds covered 315 meters or the area of the entire trapezoidal region which we have found using component parts.

Incidentally one might ask if we were really on a fast track. At 30 meters per second, how fast were we going in miles per hour? Would we be traveling at a legal speed if we were driving a car?

Problem 2.

Let's stay on the fast track. Suppose a car, A, starts from rest (it was stopped) and travels at a constant acceleration of K miles per minute per minute for the first 10 minutes. Then the car travels at a constant rate or speed of 1.5 miles per minute. Find how many more minutes the car would need to travel to cover the same distance as it covered the first 10 minutes. Then find how long it would take the car to cover three times the distance it covered the first 10 minutes.

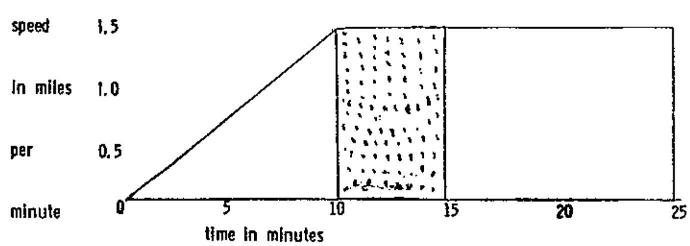


Figure 4. Graphical representation of the distance covered by car A

The distance traveled in the first 10 minutes is represented by the area of the triangle with vertex at the origin. This distance would be 0.5 X 10 X 1.5 or 7.5 miles. To find the number of minutes it takes to cover the distance of 7.5 miles at 1.5 miles per minute is equivalent to finding the length of a rectangle whose area is 7.5 units with a width of 1.5. Such a rectangle would have length of 7.5/1.5 or 5 and is shown above as the dotted region. And to cover 3 times the distance or 22.5 miles at 1.5 miles per minute would take 3 times the time it takes to cover 7.5 miles (3 X 5 = 15). This larger rectangular region is lightly shaded.

Now suppose car B also starts from rest at the same time as car A. Car B has the same acceleration as Car A, but car B accelerates for 25 minutes. Find the acceleration of car B after 25 minutes and the distance car B h as traveled at the end of that time. Then compare the distance covered in 25 minutes by the two cars.

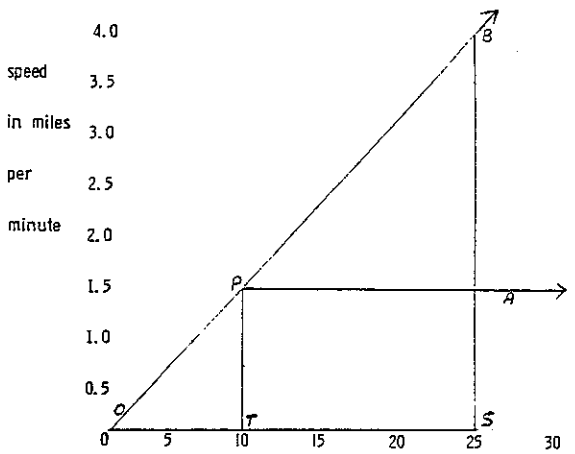


Figure 5. Comparison of the effect of the continued acceleration of car B

Triangles OTP, OSB and PAB are similar. Therefore, BS/PT as OS/OT or BS/1.5 as 25/10. BS = (1.5 X 25)/10 or 3.75. The velocity of car B after 25 minutes of acceleration is 3.75 miles per minute.

After 25 minutes and with a velocity of 3.75 miles per minute, the distance car B has traveled is represented by the area of triangle OSB. The distance car B has traveled is 0.5 X 25 X 3.75 miles or 46.875 miles. During this same time period car A traveled 30 miles -- the area of trapezoid OSAP. One might ask if the graphic appears to be consistent with our findings. From the graphic it is clear that the region of the trapezoid OSAP is included in the region that represents the distance car B traveled.

Are both cars traveling in the fast lane? After 25 minutes car A is traveling at 1.5 miles per minute or 90 miles per hour. Car B is traveling 3.75 miles per minute or 225 miles per hour.

Problem 3.

Consider a new situation. Suppose car A is stopped at a traffic light. The light turns green and the driver in car A accelerates. Just as this is taking place car B, which is traveling at a constant speed, passes car A. Their velocity-time graphics are shown below.

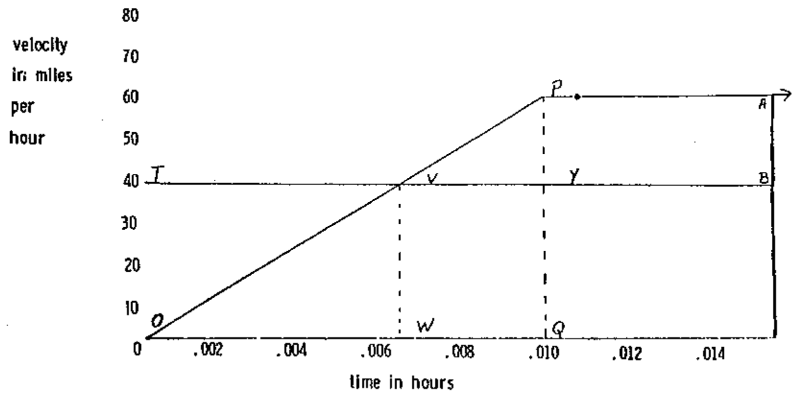


Figure 6. Velocity-time graphics for car A and car B

a. How long does it take car A to be going as fast as car B?
   The abscissa of point V where line segment OP intersects line segment TB identifies the time.
b. At that time how far ahead of car A is car B?
   Car B has traveled twice as far as car A. The area of rectangle OWVT is twice the area of triangle OWV.
c. How long does it take car A to catch car B?
   The distance traveled by car B is 40t. The distance traveled by car A is the area of the triangle OQP plus the area of the rectangle of width 60 and length t-0.010.
   Distance car A = 0.5 X 60 X 0.010 + 60 X (t-.010)
   Distance car B = 40 X t

   40 X t = 0.5 X 60 X 0.010 + 60 X (t-0.010)
   40t = 0.3 = 60t - 0.6
   -20t = -0.3
   t = 0.015 hours
d. How far have the cars traveled when car A catchs car B?
   Using car B's distance statement, the distance is 40 X 0.015 or 0.6 miles. We could also use car A's distance statement as a check.

Problems relating to springs and networking springs provides another important application of triple connections and of physical science based problems which can be introduced in the mathematics classroom. These problems require relatively little equipment to permit comparing actual results with those results obtained with simple calculation.

Problem 4.

Suppose that one has a spring which is 15 centimeters long. Weights are attached to the spring and the length of the spring's stretch is measured.

Mass in grams	5	10	15	20
Extension in cm	2	4		8

a. On a coordinate graph plot the results.
b. What is the relationship, if any, between the extension and the weights that can be inferred from the table or graph?
   There seems to be a linear relationship or linear function. Also one might say that the extension is in direct proportion to the weights.
c. If possible, write an equation which describes your findings in b.
   The extension is numerically equal to 0.4 times the mass of the weights.
d. What extension would one get by attaching a 15 gram weight to the spring?
   6 cm
e. What weight would be required to extend the spring by 18 cm?
   18 = 0.4m or m = 45 g

Problem 5.

The constant, 0.4, in the preceding problem is known as the spring factor. Suppose one has two springs of equal spring factor and length and these springs are arranged in parallel as shown. Each spring independently extends 1 centimeter if a 30 gram weight is attached a single spring.

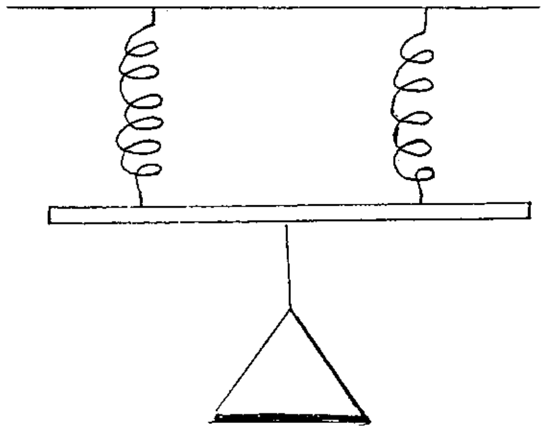


Figure 7. Two springs arranged in parallel

a. What is the extension of the system when a 60 gram weight is attached?

Since the springs are identical and the weight is evenly distributed between the two springs, each spring will give 1 centimeter and consequently the system will extend 1 centimeter.

b. How much weight would be required to extend the system 4 centimeters?

Four times the amount it would take to extend the system 1 centimeter or 240 grams.

Problem 6.

Three springs like the ones in problem 5 are arranged in a series as shown.

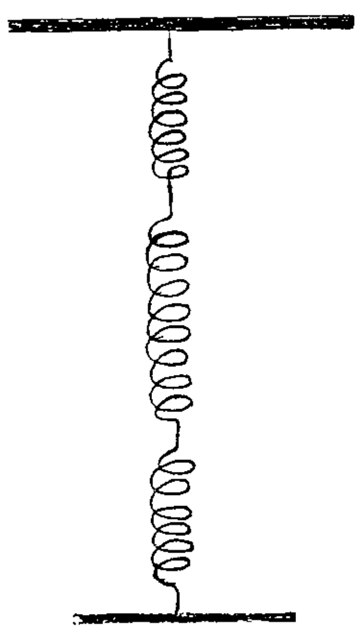


Figure 8. Three springs connected in series

a. Find the extension of this system when a 150 gram weight is attached.

In this case each spring will experience the effect of the 150 gram weight so each spring will extend 150/30 or 5 centimeters. Therefore the total extension of the system is 3 times 5 cm or 15 centimeters.

b. Write an algebraic equation that expresses the relationship between the extension of this system and the amount of weight attached to the system.

$E = 3WF$ where E is the extension of the system, 3 is the number of springs in the system, W is the amount of weight attached to the system and F is the spring factor common to the springs.

c. How much weight must be attached to the system to extend the system 12 centimeters?

Since the spring factor of a single spring is 1 centimeter per 30 grams or the spring factor of the system is 3 centimeters per 30 grams, one could infer that the needed weight would be 12/3 times 30 or 120 grams. An alternate way to compute the needed weight would be to use the equation determined in b.

$$12 = 3W\,(1/30)$$
$$360 = 3W$$
$$120 = W \text{ or } 120 \text{ grams}$$

d. Write a general formula to express the relationship between the extension of a system of a series of springs each with the same spring factor and the weight attached to the system.

$E = NWF$ where N is the number of springs in the system and E, W and F have the same meanings described in b.

Problem 7.

Instead of designing a spring system where weights are attached to system, suppose that we have a single spring that is compressed as a spring in a seat cushion or mattress when weight is applied to the object. A spring of length 18 centimeters is compressed to 16.5 centimeters when a weight of 10 grams is placed upon the spring. Assume that the relationship between the compression of the spring and the weight placed upon the spring is linear.

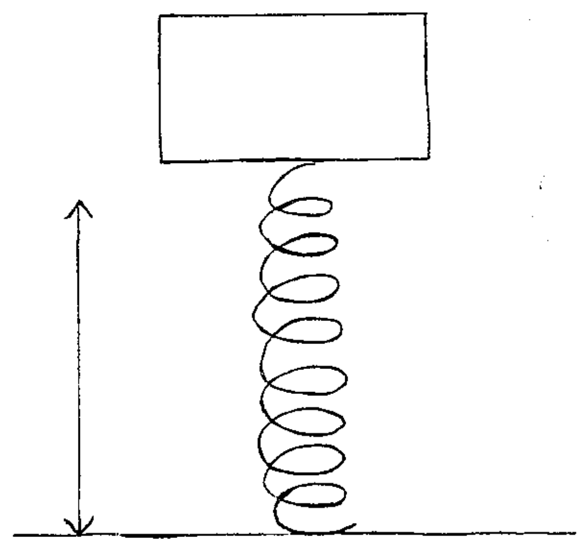


Figure 9. A spring under compression

a. Find the compressed spring factor for this spring.

$$18 - 16.5 = 10F$$
$$1.5 = 10F$$
$$0.15 = F$$ or the compressed spring factor is 0.15 cm per gram

b. What is the compressed length of this spring if a weight of 20 grams is placed on this spring?

Since the relationship is linear and since a weight of 10 grams has the effect of compressing the spring 1.5 centimeters, the 20 grams would compress the spring twice as much or 3.0 centimeters. And the compressed length of the spring is 18 - 3 or 15 centimeters. Also we could use the relationship found in a. If C is the amount of compression or the measure of the length of compression, then C equals 0.15 times 20 or 3.0 centimeter and the compressed length of the spring is still 15 centimeters.

c. What weight would be required to compress the spring to half of its original length?

$9 = 0.15W$ where W is the desired weight.
$W = 9/0.15$ or 60 grams.

Consider some of the problem solving skills and generalizations which are contained within these problems. In problem 1 a problem solving skill is to recognize that problems involving $d = rt$ and $A = lw$ are of the same general type, triple connections, and that problems of this type can be expressed as area of regions. Additionally the introduction of problems of this type provide a basis for the later development of finding the area under a curve and the use the derivative to find velocity or speed and acceleration in calculus. The ability to graphically represent problems and to use this representation as either a means of solution or as a way of verifying the reasonableness of results is a skill that students need to develop. Implied in this three part problem is the combining of smaller problems as a means of solving a more complex problem.

Hopefully students will see that this process can and should be reversed; one can often resolve a complex problem by an analysis by parts.

The concepts in problem 1 are continued in problem 2. This problem places greater emphasis on the practical applications of geometry to aid in the solution. Both algebraic and geometric concepts are used to assess the reasonableness of results.

Problems 1 to 3 constitute a sequence of problems that are conceptually interrelated; but, the actual solution of one problem does not affect the results of another problem. In problem 3 the solution to part XaY is geometric. Can it be solved algebraically from the given information? On what basis might one choose to use a geometric or algebraic approach to solve a problem?

The solutions to the sequence of spring problems appear to be more algebraic than geometric. Must that be the case? Are all, some, or none of these problems triple connections? Can generalizations regarding springs be drawn from these problems? Are these problems consistent with students; previous knowledge? What happens if a 10 kilogram weight is attached to the systems in problems 5 and 6? Is there a difference between being able to compute the extension or compression and what would actually happen? Why? What if springs in a system are connected in a series with non-identical springs?

In seeking to provide for transfer of learning in mathematics to its many areas of applications, mathematics educators have an opportunity to develop problem solving skills in addition to assisting in transfer of learning. The interrelationships between mathematics and its applications provide a means for enriching students' knowledge and appreciation of mathematics and the society.

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(CONTINUED FROM PAGE 8)

Assume the Following:

f 0	f g
g)	-h -f
-h 0	h f

Using the information in the box above, decide on which side of zero the following numbers are located:

Number	Left	Right
f^3		
(f - g)		
$-g^2$		
h^2		
h + g		
(-g) + (-f)		

Teachers should insist students read the expression (-x) as the opposite of x. This also demands that we as teachers do the same. The correct reading of this expression should begin in the early grades. The incorrect reading of the symbol is not easily changed. "Negative X" sends messages to the student that complicates and confuses concepts an algebra teacher must teach. It's time to tell it like it is! That is, (-x) is the "opposite of x or the inverse of x", not "negative x".

(WISCONSIN TEACHER OF MATHEMATICS, Winter, 1986)

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Proclamation

Whereas, mathematical literacy is essential for citizens to function effectively in society; and,

Whereas, mathematics is used every day - in the home, on the job, and for recreational activities; and,

Whereas, the language and processes of mathematics are basic to other areas of the school curriculum; and,

Whereas, expanding technology demands increased mathematical competence; and,

Whereas, relevant community and school activities can generate interest in mathematics;

Now, therefore, I, F. Joe Crosswhite, president of the National Council of Teachers of Mathematics, do hereby proclaim the month of April 1986 as

Mathematics Education Month

to be observed in schools and communities in recognition of the importance of mathematics.

In witness whereof, I have hereunto set my hand and caused the Corporate Seal of the National Council of Teachers of Mathematics to be affixed on this 1st day of January 1986.

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- Entries from any local, state, and provincial logo contests may also be submitted in the national contest.
- Each entrant's name, grade, and school address must be typed or printed legibly on the back of the entry.
- Entries must be mailed to NCTM by a classroom teacher or mathematics supervisor whose name, address, telephone number, and signature, plus member number for NCTM members, appear on the back of the entry as verification of originality.
- A $3 submission fee must accompany each entry.
- All entries and fees must be received by 30 April 1986.

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