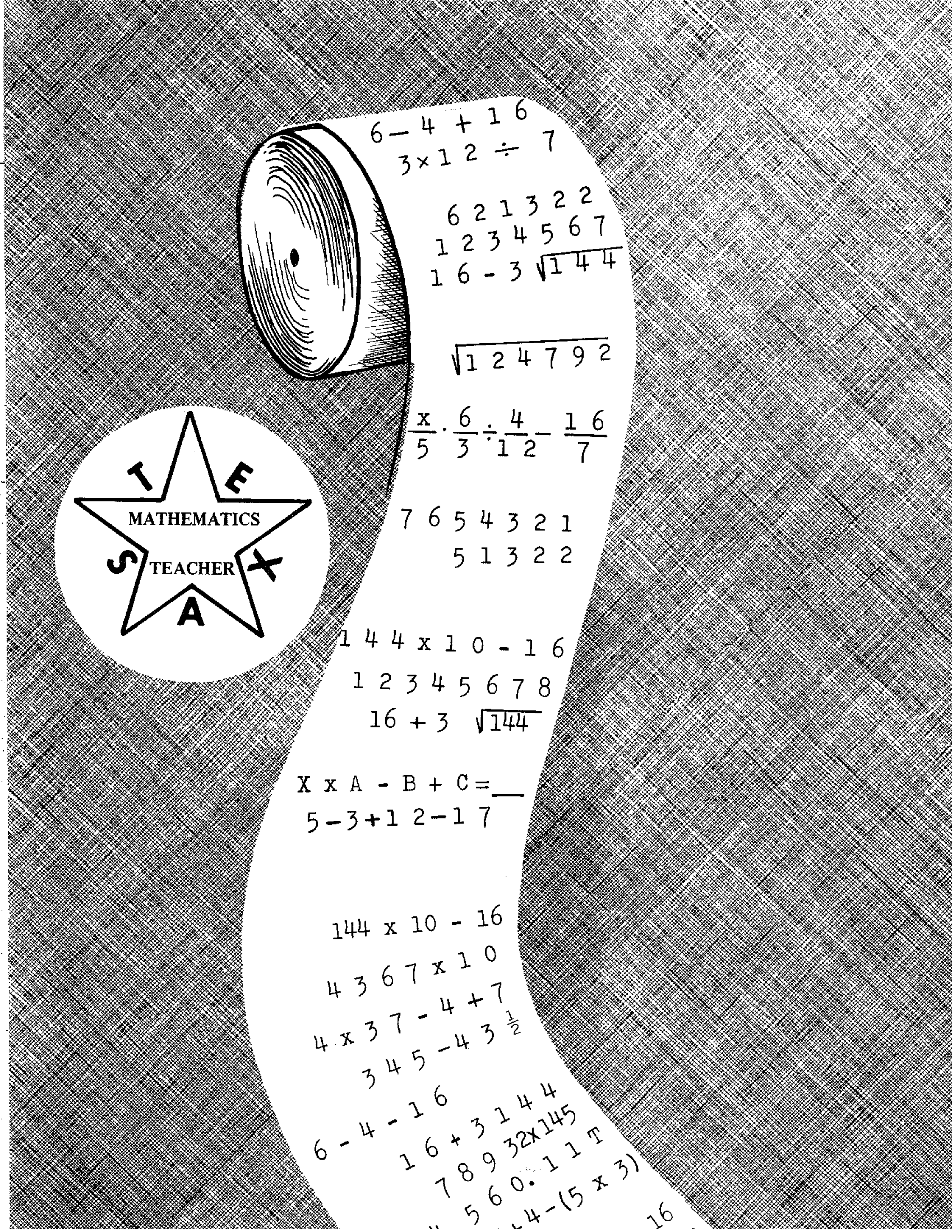
TEXAS
MATHEMATICS
TEACHER
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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

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TEXAS MATHEMATICS TEACHER

Texas Council of Teachers of Mathematics

Volume XXXII | MARCH, 1985 | No. 2

PRESIDENT'S MESSAGE

I have before me two proclamations, one issued by F. Joe Crosswhite, the president of NCTM, and the other by Mark White, the governor of Texas. The former designates the week of April 14–20, 1985, as Mathematics Education Week; it is, in a sense, recognition by our own profession of the importance of what we and our colleagues teach. The latter designates the week of April 17–20, 1985, as Mathematics Education Week **in the state of Texas;** it is recognition by others – the people of the state, as represented by their chief elected political representative – of the importance of what we teach. *(See elsewhere in this journal)*

Both the documents are referring to the fact that the Annual Meeting of NCTM is to be held in San Antonio during the designated time periods. The governor's proclamation specifically refers to TCTM and our sponsorship of the NCTM meeting. That makes me feel good, as does the general recognition of the importance of what we do expressed in both documents.

My primary disagreement with both proclamations is not with what they say, but with what they do not say. They both speak strongly of the importance of mathematics, but neither really comes out and mentions the importance of mathematics, but neither really comes out and mentions the importance of the mathematics teacher. Therefore, my message to you is that I take this opportunity to proclaim the week of April 17–20, 1985, to be Mathematics **Teacher** Week in the state of Texas. Each one of you may feel free to add your own "Whereas . . . . . " statement to my proclamation? you know the ones that apply to you and your colleagues. And I hope to see as many of you as are able to come in San Antonio during our week!

Ralph W. Cain
President, TCTM

REFEREES WANTED

Manuscripts published in the TEXAS MATHEMATICS TEACHER are reviewed by at least three mathematics educators. Members of TCTM who are regular readers of the journal – classroom teachers, supervisors, and teacher educators – and who would like to review manuscripts should write to the TCTM indicating their willingness to serve and the level of interest (Elementary, Secondary, or both). Send to George H. Willson, Box 13857, North Texas State University, Denton, TX 76203. The Editorial Panel will review the responses and make the final selection.

CAMT

The 32nd Annual Conference for the Advancement of Mathematics Teaching (CAMT) will be held October 24–26, 1985, at the Hyatt-Regency Hotel and the Palmer Municipal Auditorium in Austin, Texas. The theme of this year's conference is "Mathematics: Teaching the Problem Solvers of Tomorrow."

Official Memorandum

By

MARK WHITE

Governor of Texas

GREETINGS: **AUSTIN, TEXAS**

Various programs and activities aimed at encouraging students, teachers, parents, and the general public to become knowledgeable in the field of mathematics are being sponsored by the Texas Council of Teachers of Mathematics during the week of April 17-20, 1985, including the Annual Meeting of the National Council of Teachers of Mathematics in San Antonio.

Mathematics is the language of science and engineering and it paves the way for advances in modern technology. To maintain our state's forward momentum, it is necessary to depend upon advances made in science and engineering.

To remain economically competitive as a state, public and private business in Texas will need a clear understanding of mathematics and the proper use of mathematical models in their economic endeavors.

Mathematics is a valuable intellectual endeavor and can provide a firm foundation for the future success of the state. Our children must become familiar with the language and application of mathematics if they are to cope with the pressures of our technologically based economy.

THEREFORE, I, as Governor of Texas, do hereby designate the week of April 17-20, 1985, as

MATHEMATICS EDUCATION WEEK

in Texas.

In official recognition whereof, I hereby affix my signature this 11th day of January, 1985.

Mark White

Governor of Texas

TEXAS PROGRESS IN PROBLEM SOLVING

C. J. Dockweiler and R. L. Erion
Educational Curriculum and Instruction, Texas A&M University, August 1984

In 1981, the National Council of Teachers of Mathematics published Priorities In School Mathematics. This document was a report of the project of the same name which became known as the PRISM Project. The main focus of the project was to survey the preferences of teachers, teacher educators, and lay people on various aspects of the mathematics curriculum and the teaching of mathematics. In particular, items pertaining to goals, resources, etc., as they relate to problem solving, were included.

During the 1983-84 school year, a survey form was prepared and distributed to the membership of the Texas Association of Supervisors of Mathematics (TASM). The survey included all the PRISM items on problem solving which had received strong support (over 80%) from the PRISM respondents. The items were phrased to elicit supervisor's responses which would reflect the level of activity in their Districts with regard to the teaching of problem solving. Approximately 50% (representing a little over 50 supervisors) of the TASM membership returned the survey. The 54 respondents included 32 District Supervisors and 5 Service Center personnel who reported serving a total of 313 districts. The following table provides an indication of the level of effort expended by the Districts served by the participating supervisors. Keep in mind that each of the categories received strong support (over 80%) in the PRISM Project.

Supervisors' Report of District's Level of Effort Expended Toward Improving Problem Solving

	Significant/Moderate level of effort	No Effort	Not Related to Problem Solving
Problem Solving Goals			
Goal 1: To develop methods of thinking and logical reasoning	91*	9*	
Goal 2: To acquire skills necessary for living	98	2	
Goal 3: To acquire Prob. Solv. techniques vital to well-rounded education	91	7	2*
Goal 4: To develop creative thought process	83	15	2
Goal 5: To apply recently taught math ideas	91	7	2
Problem Solving Techniques			
Tech 1: Construct a table-search for patterns	81	19	
Tech 2: Translate problem into number sentence or equation	98	2	
Tech 3: Solve a simpler problem-extend to original problem	89	9	2
Types of Resources			
Type 1: Resource guide to real-life problems	43	47	8
Type 2: In-service training on problem solving methods	77	23	
Type 3: Materials for modeling problems	76	21	2
Type 4: Supplementary materials with more problems like texts	48	21	11
Teaching Strategies			
Start 1: Problems to challenge students to think	89	11	
Start 2: projects of real-life situations for individuals or teams	70	28	2
Start 3: Problems to introduce math topics	81	9	8

*Percent of Respondents (N=54)

With few exceptions the responses suggest that the districts involved are attempting to provide classroom teachers with programming, training, and materials to improve the teaching of problem solving. These practices and programs are supportive of the results reported in the PRISM survey of a few years ago. A few specific observations may be made to highlight the data as presented in the Table.

1. Only a few supervisors (four to six) report their districts have not expended any effort to achieve goals concerning
   -- developing methods of thinking and logical reasoning;
   -- the acquisition of problem-solving techniques that are vital to having a well-rounded education;
   -- the development of creative thought process; and
   -- the application of recently taught mathematical ideas.
   The importance of these goals to the development of problem solving skills would seem to suggest their inclusion in the public school curriculum at some point.
2. Several supervisors (about nine) report that no effort was directed at constructing tables and searching for patterns.
3. About half of the respondents indicate no effort directed to resource guides for real-life problems. This result may be due to the lack of adequate materials as much as the lack of use of existing materials. The requirement of the present adoption of elementary mathematics texts to include supplementary problem solving resources may alleviate any difficulties associated with this result.
4. Over 20% of the respondents indicated no effort expended to provide
   -- in-service training for problem solving methods for all teachers of mathematics;
   -- materials for modeling problems;
   -- supplementary materials with text-like problems.
5. Assignment of projects involving real-life situations to individuals or groups of individuals received no attention in 28% of the reporting Districts.

In addition to the survey items which were parallel to the PRISM Project item, several items were included to obtain some information about the specific increase in emphasis or programmatic changes that may have occurred during the last few years. Over 75% of the supervisors reported that their districts had either dramatically or moderately increased the emphasis at the district, building, grade level, and classroom level. An interesting addition to the general increase in emphasis is the perception by 68% of the respondents that only "moderate" increase in problem solving emphasis has been expended by the district to train teachers and provide materials, but the expected growth in the classroom may not be occurring.

In conclusion, it appears that a great deal of problem solving emphasis has been generated in the Districts of the participating supervisors. Obvious cautions should be raised as generalizations are attempted;

-- only supervisors were surveyed;
--only some of the supervisors returned information;
-- and Texas Districts not having representation in TASM were not approched.

These results, therefore, are very tentative and partial, at best. They do indicate that many activities regarding problem solving are taking place and the districts in which these activities are occurring are to be commended and encouraged to continue and expand those efforts.

REFERENCE

National Council of Teachers of mathematics, Priorities In School Mathematics; Executive Summary of the Prism Project, NCTM, 1981.

PROCLAMATION

National Council of Teachers of Mathematics

WHEREAS, *mathematical literacy is essential for citizens to function effectively in society; and,*
WHEREAS, *mathematics is used every day – in the home, on the job, and for recreational activities; and,*
WHEREAS, *the language and processes of mathematics are basic to other areas of the school curriculum; and,*
WHEREAS, *expanding technology demands increased mathematical competence; and,*
WHEREAS, *relevant community and school activities can generate interest in mathematics;*
NOW, THEREFORE, *I, F. Joe Crosswhite, president of the National Council of Teachers of Mathematics, do hereby proclaim the week of 14 - 20 April 1985 as*

MATHEMATICS EDUCATION WEEK

to be observed in schools and communities in recognition of the importance of mathematics.
IN WITNESS WHEREOF, *I have hereunto set my hand and caused the Corporate Seal of the National Council of Teachers of Mathematics to be affixed on this 1st day of November 1984.*

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS (INCORPORATED) CORPORATE SEAL ILLINOIS

President

Executive Director

NCTM

STUFF Strategic Tactics Ultimately For Fun

Dear T.C.T.M. Members:

KEEP UP THE GOOD WORK!!!

The word must be getting out because we are beginning to get "Stuff".

If you would like to see your activities in print, send them soon. We only have one more issue this year. It would be a shame to have to wait until fall. Send the "Stuff" to Harris County Department of Education, 6208 Irvington Blvd, Houston, Texas 77022.

Sincerely,

Bette and Judy
"Stuff" Staff

PRIMARY

CHILDREN'S SUGGESTION FOR GRAPHING ACTIVITIES

1. What month were you born in?
2. What day of the week were you born on?
3. How long is your first name?
4. How long is your last name?
5. Add your first and last name and graph.
6. How many vowels are in your name?
7. What is your favorite drink?
8. What kind of shoe are you wearing – tie, buckle, slip on?
9. Which is your favorite solid shape?
10. Which hamburgers do you like best? MacDonalds, Burger King, Jack-In-The-Box, Wendy's?
11. What do you call your Mom? Mom, Mother, Mommy, Momma?
12. What color are your eyes? Your hair? Your shoes?
13. What kind of hair do you have? Curly? Wavy? Straightt?
14. Who is your favorite – E.T., PacMan or Smurf?
15. Favorite sport?

PRIMARY

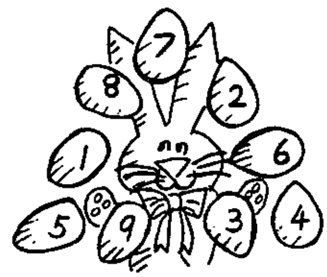
PAD ADD

DIRECTIONS:

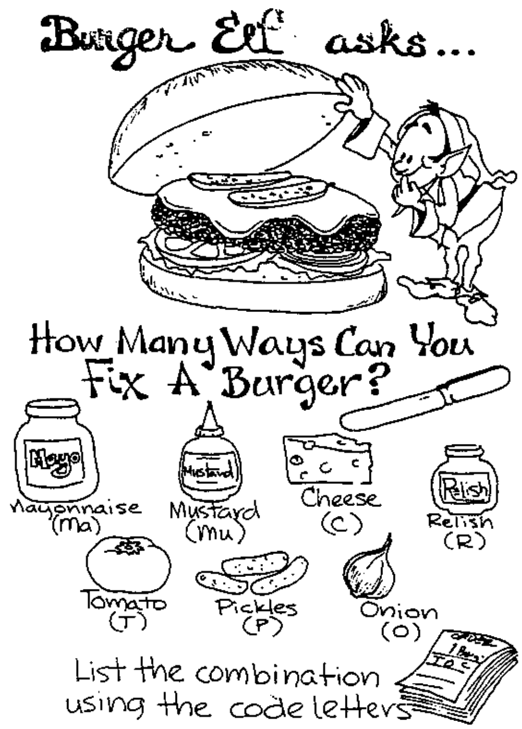
Make a large floor pad using poster board and marking it with a felt tip pen. Divide it into several parts, no particular size or shape. Write a numeral from one to five in each section. The leader tosses two small bean bags onto the pad. The players in the group must immediately add the two numbers on which the bags landed, and the first to say the answer scores a point. The winner is the first player reaching ten points. He/she becomes the new leader and play resumes.

PRIMARY

The Easter Bunny is having an egg roll. Can you help him roll these numbers around so that each side of the triangle adds up to 17? There are several possible ways.



INTERMEDIATE



INTERMEDIATE SUGGESTED 4 AND UP

DON'T FLIP YOUR LID
CALCULATOR FUN

One of the things people have discovered they can do with calculators for fun is send "messages." Certain numerals look very much like certain letters when the calculator is turned so the display is upside down. Below are some fun problems with hidden messages you can try:

a) Can you name the star of a famous "chiller" movie? To find the answer do:
   1. 43 X 56
   2. Add 300
   3. Multiply by 3
   4. Subtract 46 – Flip!

b) A football player collided with another player because he did the opposite of what he should have done. What did he do?
1. 57 X 36
2. Subtract 198
3. Divide by 3
4. Subtract 6 – Flip!

c) What did the teenager with the strange accent say was his favorite hobby? To find the answer, do:
1. 2,568 + 7,294
2. X 6
3. –2,066 – Flip!

d) If you're on your toes, you can get a job here! To find out, do:
1. 873 X 98
2. + 19016
3. X 10
4. + 8 – Flip!

Brain-stretcher? Make a table of all numerals which, when flipped, resemble letters. Then try to make some message puzzles of your own.

MIDDLE SCHOOL

ROUNDING NUMBERS

OBJECTIVES:

To round numbers
To develop logical thinking

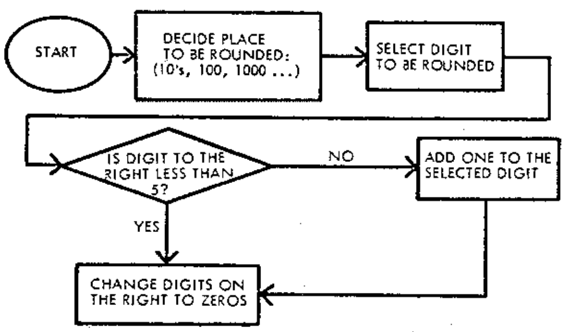
DIRECTION:

Assign students a page of numbers to be rounded.
The steps in this flow chart will teach them how numbers are rounded.

EXAMPLE:

1841 to the nearest hundreds place –
a) 8 is in the hundreds place
b) 4 is to the right
c) 4 is less than 5
d) 4 and 2 and changed to zeros
e) Answer is 1800

Follow the steps in the flowchart to round numbers –



My number is 276 and I want to round it to the nearest 10.

7 is my selected digit.
The digit to the right is not less than 5
7 gets changed to 8
6 gets changed to 0

276 round to 280

Submitted by:
Connie Glasco
North Forest ISD
Houston, Texas

MIDDLE SCHOOL

RATIO-PERCENT DOMINOES

Two, three, or four players may play the game. Each player draws ten dominoes. The first player places one of his dominoes on the table. The next player must add one domino which matches one end of the domino already played or he must play a domino with the equivalent of one on the table. For example, suppose the first player plays a domino with 10% and 80%. In addition to choices of dominoes with 80% and 10% on one end, a player may play one with such equivalents as 1/10, 4/5, 80/100, etc. If a player cannot play any of his dominoes, he must draw from the pile until he gets one that can be played. Dominoes can only be played at the ends so that there is one continuous path. The first player to use all of his dominoes is the winner. In case none of the players can use all of his dominoes, the one with the fewest left wins.

Submitted by:
Alma Mabry
Deer Park I.S.D.
Houston, Texas

MIDDLE SCHOOL

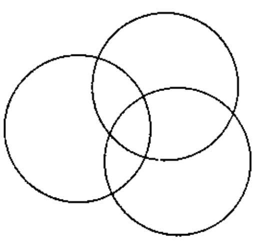
VENN DIAGRAMS

You can test whether a syllogism is valid or invalid by using a Venn Diagram. Named after the man who devised it, John Venn, this diagram uses three overlapping circles. Each circle represents one of the three categories or sets in the syllogism

All flowers are pretty.
All daffodils are flowers
Therefore, all daffodils are pretty

This circle represents things that are pretty (2, 3, 5, 6)

This circle represents flowers (1, 2, 4, 5)

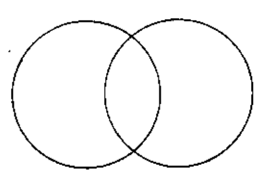


This circle represents daffodils (4, 5, 6, 7)

The first premise all flowers are pretty, means that there are no members of the flower set that are not in the pretty set,

There are no flowers in the shaded area

(Pretty things that are not flowers)



Flowers

Pretty things

Flowers that are pretty

Shade in all of the circle representing flowers that doesn't overlap the circle representing pretty things. (The shading means that there is nothing in that portion of the circle.)

1. All gold is expensive.
Some rings are gold.
Therefore, some rings are expensive. __________

2. All books have pages.
Some books are novels.
Therefore, some novels have pages. __________

3. All taffy is sticky
Some sticky things are yucky.
Therefore, some taffy is yucky. __________

4. All dragons are green
All green things are ugly.
Therefore, all dragons are ugly. __________

5. All children like to play.
All girls are children.
Therefore, all girls like to play. __________

6. All fragile things are breakable.
Some mirrors are fragile.
Therefore, some mirrors are breakable. __________

7. All zebras have stripes.
No zebras are polar bears.
Therefore, no polar bears have stripes. __________

8. No ship is safe.
All ships are large.
Therefore, no large things are safe. __________

9. All men are smart.
All smart things are small.
Therefore, all men are small. __________

10. All butterflies have wings.
All flies have wings.
Therefore, all butterflies are flies. __________

11. All motorcycels have two wheels.
Some motorcycles are fast.
Therefore, some two-wheeled things are fast. __________

12. All clowns are funny.
Some happy people are clowns.
Therefore, some happy people are funny. __________

13. All rocks are heavy.
Some hard things are rocks.
Therefore, some hard things are heavy. __________

SENIOR HIGH SCHOOL

ADVANCED MATHEMATICS BINGO

DIRECTIONS:

1. Write each of the numbers 1-75 on a separate 3" X 5" index card.
2. On each card write a problem that has the same answer as the number written on the card, e.g., number on the card: 14

   Problem: The positive root of $x^2 - 13x = 14$
3. Use regular BINGO cards, regular BINGO rules, and the call cards you made to practice previously taught skills.

NOTE: The cards can be made to cover most of the skills taught in one semester and used as a review session.

CONSECUTIVE INTEGRAL LEGS OF A RIGHT TRIANGLE

Donald Skow & Joseph Wiener, Pan American University

Most geometry books (Jurgensen, Brown & King, 1983), (Forester, Cummins & Yunker, 1984), (Ulrich, 1984) that introduce the Pythagorean Theorem of a right triangle,

$a^2 + b^2 = c^2$, where a and b are legs of the right triangle and c is the hypotenuse, use the 3, 4, 5 right triangle as an example or in their exercises. Other examples used are the 5, 12, 13 and 7, 24, 25 right triangles. The 3, 4, 5 triangle is the smallest right triangle that has integral sides. Indeed, let x $(x > 0)$ and $x + 1$ be the legs of a right triangle and $x + 2$ the hypotenuse. Then

$$x^2 + (x+1)^2 = (x+2)^2, \; x^2 - 2x - 3 = 0, \; x = 3, \; x = -1.$$

The value $x = -1$ is rejected since $x > 0$ and the only value of x is $x = 3$ which implies the 3, 4, 5 right triangle. The characterization of all primitive Pythagorean triples is fairly straightforward (Skow, 1981): these numbers are given by

$m^2 - n^2$, $2mn$, $m^2 + n^2$ where m and n are arbitrary positive integers that satisfy the following conditions:

(i) The greatest common divisor of m and n is 1.
(ii) One of the numbers m and n is even and the other is odd, and $m > n$.

Nonetheless, the interest in the study of new ways of generating Pythagorean triples and quadruples continues (Wiener, 1984). The purpose of this article is two-fold:

(1) to find the set of all right triangles with integral sides in which the difference between the hypotenuse and a leg is 1;
(2) to discuss and solve a difficult and interesting problem of how to get all of the right triangles with consecutive integral legs.

Let a and x be the legs of the right triangle and $x + 1$ the hypotenuse. Then

$$a^2 + x^2 = (x+1)^2, \; a^2 + x^2 = x^2 + 2x + 1, \; a^2 = 2x + 1.$$

Since x is integral, $2x + 1$ is odd. Thus the integer a is odd, and letting $a = 1, 3, 5, 7, 9, \ldots$, we get from $a^2 = 2x + 1$, right triangles where the difference between the hypotenuse and a leg is one in the following table:

TABLE 1

k	Leg a	Leg x	implies the right triangle is:
0	1	0	no right triangle
1	3	4	3, 4, 5
2	5	12	5, 12, 13
3	7	24	7, 24, 25
4	9	40	9, 40, 41
....	....	....	....
k	a_k	x_k	

In Table 1, to find right triangles with a side and the hypotenuse greater than the ones shown, let

$a_k = 2k + 1$, $x_k = 2k(k + 1)$, where $k = 0, 1, 2, \ldots$.

To find right triangles with consecutive integral legs, let m and n take on arbitrary values satisfying the above conditions (i), (ii) and get the following table:

Note that Table 2 is only a partial list of the possible Pythagorean triples, which form an infinite set. Notice that Table 2 contains only three right triangles with consecutive integral legs; (1) 3, 4, 5 where n = 1; (2) 20, 21, 29 where n = 2; (3) 119, 120, 169 where n = 5. As the values of n and m get larger the next right triangle with integral legs does not appear until n = 12 and m = 29. Table 3 shows values of n and m that will make only right triangles with consecutive legs.

TABLE 2

m	n	Leg $m^2 - n^2$	Leg $2mn$	Hypotenuse $m^2 + n^2$
2	1	3	4	5
4	1	15	8	17
6	1	35	12	37
3	2	5	12	13
5	2	21	20	29
7	2	45	28	53
4	3	7	24	25
8	3	55	48	73
10	3	91	60	109
5	4	9	40	41
7	4	33	56	65
9	4	65	72	97
8	5	39	80	89
12	5	119	120	169

TABLE 3

k	m	n	Leg $m^2 - n^2$	Leg $2mn$	Hypotenuse $m^2 + n^2$
0	2	1	3	4	5
1	5	2	21	20	29
2	12	5	119	120	169
3	29	12	697	696	985
4	70	29	4,059	4,060	5,741
5	169	70	23,661	23,660	33,461
6	408	169	137,903	137,904	195,025
7	985	408	803,761	803,760	1,136,689
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
k	m_k	n_k	m_k^2	$2m_kn_k$	$m_k^2 + n_k^2$

Notice that in Table 3 the consecutive legs of a right triangle get large rather fast, and that this subset of the Pythagorean triples is also an infinite set.

Now we find a general equation for n_k to show that the set of Pythagorean triples of the type in Table 3 is infinite. Notice in Table 3 that $m_k = 2n_k + n_{k-1}$ and $n_k = 2n_{k-1} + n_{k-2}$.

From the values of n in Table 3, a sequence of 1, 2, 5, 12, 29, . . . can be formed. Define this sequence recursively as $n_0 = 1$, $n_1 = 2$,

$$n_k = 2n_{k-1} + n_{k-2}$$

where $k \geq 2$. Let $n_k = x^k$, then $n_{k-1} = x^{k-1}$ and $n_{k-2} = x^{k-1}$. Therefore,

$$x^k = 2x^{k-1} + x^{k-2}.$$

Divide the above equation by x^{k-2} and set the equation equal to zero. This yields:

$$x^2 - 2x - 1 = 0.$$

By the quadratic formula, $x = 1 \pm \sqrt{2}$. Let $x_1 = 1 + \sqrt{2}$ and $x_2 = 1 - \sqrt{2}$ then

$$n_k' = (1 + \sqrt{2})^k \text{ and } n_k'' = (1 - \sqrt{2})^k.$$

The variable n_k satisfies a linear difference equation with the boundary conditions n_0 and n_1. Therefore, $n_k = A\,n_k' = B\,n_k''$ where the values of A and B are determined by the boundary conditions of $n_0 = 1$ and $n_1 = 2$. To determine n_k, it is necessary to find the values of A and B. Since

$$n_k = A\,n_k' + B\,n_k'',$$

substituting the values of n_k' and n_k'' yields

$$n_k = A(1 + \sqrt{2})^k + B(1 - \sqrt{2})^k. \quad (1)$$

For k = 0 and k = 1, equation (1) yields:

$$n_0 = 1 = A + B$$

$$n_1 = 2 = A(1 + \sqrt{2}) + B(1 - \sqrt{2}).$$

This system of equations yields:

$$A = \frac{\sqrt{2}}{4}(1 + \sqrt{2}), \quad B = -\frac{\sqrt{2}}{4}(1 - \sqrt{2}).$$

Substituting the values of A and B into equation (1) and simplifying yields:

$$n_k = \frac{\sqrt{2}}{4}\left[(1 + \sqrt{2})^{k+1} - (1 - \sqrt{2})^{k+1}\right]. \quad (2)$$

With the n_k term and the fact the $m_k = 2n_{k-1} + n_{k-2}$, any right triangle with consecutive legs larger than the values given in Table 3 can be generated. The reader might want to write a computer program that will generate Table 3 and additional rows, but that is beyond the scope of this article.

We conclude this article with an example on how equation (2) generates Table 3. The ith triangle in Table 3 is when the value of k is i − 1. Let's find the 4th triangle in Table 3 which implies k = 3:

$$n_3 = \frac{\sqrt{2}}{4}\left[(1 + \sqrt{2})^4 - (1 - \sqrt{2})^4\right] = 12.$$

To find m_k, remember that $m_k = 2n_k + n_{k-1}$ and in this example k=3. Thus $m_3 = 2n_3 + n_2$ and

$$n_2 = \frac{\sqrt{2}}{4}\left[(1 + \sqrt{2})^3 - (1 - \sqrt{2})^3\right] = 5.$$

Therefore $m_3 = 2(12) + 5 = 29$. The legs of the right triangle are $m_k^2 - n_k^2$ and $2n_kn_k$. Hence $m_3^2 = n_3^2 = 29^2 - 12^2 = 697$ and $2m_3n_3 = 2(29)(12) = 696$. The hypotenuse is

$$m_3^2 + n_3^2 = 29^2 + 12^2 = 985.$$

Therefore, the 4th triangle with consecutive integral legs is the 696,697,985 right triangle.

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SOME SUMMATIONS

BY Joe F. Allison

Many claims of equality that require proofs by induction contain summations. It is one thing to inductively establish a claim; it is something else to come up with the "compact" expression equal to the given summation. For example, the most familiar summation and its compact "trade off" are represented by

$$1+2+3+\ldots+n = \frac{n(n+1)}{2}, \text{ or } \quad \Sigma\, k = \frac{n(n+1)}{2} .$$

(For the following, let us assume all summations will take place with the index dummy staring at one and incrementing by unity to n.)

One usually appeals to a result from Gauss' childhood to derive the righthand expression above. Gauss' insight is not as immediately useful with:

$$\Sigma\, k^2 , \quad \Sigma\, k^3 \quad , \ldots , \quad \Sigma\, k^n .$$

Before exhibiting a method that will deliver a desired compact righthand expression, we need to establish some theorems.

Theorem 1: $\Sigma\, ca_k = c \;\; \Sigma\;\, a_k$

Proof: $\Sigma\, ca_k = ca_1 + ca_2 + ca_3 + \ldots + ca_n$

$$= c(\,a_1 + a_2 + a_3 + \ldots .\, a_n\,) = c \;\; \Sigma\, a_k .$$

Theorem 2: $\Sigma\, (\,a_k + b_k\,) = \Sigma\, a_k + \;\Sigma\, b_k .$

Proof:

$$\Sigma\, (a_k + b_k\,) = (a_1 + b_1\,) + (a_2 + b_2\,) + \ldots + (a_n + b_n\,)$$

$$= a_1 + a_2 + \ldots + a_n + b_1 \;\; + b_2 + \ldots + b_n$$

$$= \quad \Sigma\, a_k \quad + \quad \Sigma\, b_k \quad .$$

Theorem 3: $\Sigma\;\, c = nc$.

Proof: $\Sigma\, c = c + c + \ldots + c = c\,(\,1 + 1 + \ldots .+ 1) = nc$.

The first two of the theorems tell us that summations are linear operators. The third is just an arithmetic convention of convenience for compactly writing the sum of the same repeated addend.

Now we get on with our developments.

We will first do a derivation for $\Sigma\;\, k$ in a different way. This will set up our general procedures for the cases where Gauss' insight does not transfer. To wit:

$$(\;k + 1\,)^2 - k^2 = k^2 + 2k + 1 - k^2 = 2k + 1 .$$

Now we pre-apply summation operators:

$$\Sigma[\,(k + 1\,)^2 - k^2 \;] \quad = \quad \Sigma\,(\,2k + 1\,) .$$

Taking advantage of the linearity feature on the right, we have:

$$\Sigma\,[\,(\,k + 1\,)^2 - k^2\,] \quad = 2 \quad \Sigma\, k + \quad \Sigma\;\, 1$$

or,

$$(1 + 1)^2 - 1^2 + (2 + 1)^2 - 2^2 + \ldots + (n + 1)^2 - n^2 = 2 \;\; \Sigma\;\, k + n$$

Rewriting vertically the lefthand sum of terms,

$$\begin{array}{l} \quad (1 + 1)^2 - 1^2 \\ + \; (2 + 1)^2 - 2^2 \\ + \; (3 + 1)^2 - 3^2 \\ \quad \vdots \qquad \vdots \\ + \; (\,(n-1) + 1)^2 - (n-1)^2 \\ + \; (n + 1\,)^2 - n^2 \qquad = 2 \quad \Sigma\, k \quad + n . \end{array}$$

Observe that the lefthand side telescopes:

$$-1 + (n + 1)^2 \;\; = \;\; 2 \;\;\; \Sigma\;\, k \quad + n , \text{ or}$$

$$2 \;\; \Sigma\, k + n \quad = (\,n + 1\,)^2 - 1 = n^2 + 2n + 1 - 1, \text{ or}$$

$$2 \quad \Sigma \quad k \quad = \quad n^2 + 2n - n , \text{ and}$$

$$\Sigma \quad k = \frac{n^2 + n}{2} \quad = \frac{n\,(\,n + 1)}{2} \quad .$$

This gives us Gauss' childhood observation but gotten at by a more general method. We continue and obtain the compact expression for $\Sigma\;\, k^2$.

$$(\,k + 1\,)^3 - k^3 = k^3 + 3k^2 + 3k + 1 - k^3 = 3k^2 + 3k + 1,$$

$$\Sigma[(\,k + 1\,)^3 - k^3\,] \;\; = \Sigma\,[\,3k^2 + 3k + 1 \;] = 3\,\Sigma\, k^2 + 3\,\Sigma\, k + \; 1,$$

+

or, vertically on the left,

$$\begin{array}{l} \quad (1+1)^3 = 1^3 \\ + \;\; (2+1)^3 = 2^3 \\ + \;\; (3+1)^3 = 3^3 \\ \quad \vdots \qquad \vdots \\ + \;\; (\,(n-1) + 1)^3 - (n-1)^3 \\ + \;\; (n+1)^3 - n^3 \quad = 3 \;\; \Sigma\, k^2 + 3 \;\; \Sigma\;\, k + n . \end{array}$$

Again, the lefthand side telescopes:

$$-1 + (n+1)^3 \;\; = 3 \quad \Sigma\, k^2 + 3 \quad \Sigma\;\, k + n$$

or,

$$3 \;\; \Sigma\, k^2 + 3 \left(\frac{n^2 + n}{2}\right) + n = (n+1)^3 - 1^3 ;$$

$$3 \;\; \Sigma\, k^2 = n^3 + 3n^2 + 3n + 1 - 1 - \frac{3}{2}n^2 - \frac{3}{2}n - n;$$

$$3 \quad \Sigma \;\; k^2 = n^3 + n^3 + \frac{3}{2}n^2 + \frac{1}{2}\; n = \frac{2n^3 + 3n^2 + n}{2} \qquad ;$$

$$\text{so,} \quad \Sigma \;\; k^2 \;\; = \;\; \frac{n\,(n+1)\,(2n + 1)}{6} \quad .$$

Thus a general procedure for obtaining $\Sigma\, k^n$ is suggested.

Notice, too, that here we have opportunities to apply and exploit binomial expansions.

UTSA Summer Institute

MATHEMATICS--A workshop exploring the heristics of solving problems by math modeling as well as the techniques for integrating problem solving into the classroom will be offered to high school mathematics teachers through the University of Texas at San Antonio's 1985 Summer Institute.

In its sixth year, the Institute offers educators and other working professionals the opportunity to earn graduate level credit in intensive summer sessions that last from one to three weeks.

"Problem-Solving Seminar" is one of 18 comprehensive workshops offered by the Summer Institute. It will meet July 8-26.

Teaching the course will be Dr. Maurice Burke, an assistant professor in the Division of Mathematics, Computer Science and Systems Design, at UTSA. Dr. Burke teaches computer literacy, algebra and geometry for elementary school teachers at UTSA.

Summer Institute courses offer from one to three graduate credit hours which can be applied to degree programs and certification requirements by students admitted to the programs.

Persons interested in attending an Institute session who are not UTSA students must file an application for admission to the University. Those who do so before May 1 can register for the courses by mail. Those who file after May 1 must attend the limited registration on May 30 or July 8. UTSA students currently enrolled do not need to apply for admission; instead, they can register for Institute courses during the regular registration for the summer session that is held in April.

COMPUTER LITERACY--Educators representing all disciplines and grade levels can learn to evaluate existing software packages, generate their own classroom software and teach computer literacy during a three-week course being offered by The University of Texas at San Antonio through its 1985 Summer Institute.

In its sixth year, the Institute offers educators and other working professionals the opportunity to earn graduate level credit in intensive summer sessions that last from one to three weeks.

"Computer Literacy for Educators is one of 18 comprehensive workshops offered by the Summer Institute. The course also will see participants learning to program in the BASIC computer language using IBM microcomputers. It will meet July 15 - August 2.

Teaching the course will be Dr. Betty Travis, an assistant professor in the Division of Mathmatics, Computer Sciences and Systems Design at UTSA. Dr. Travis teaches computer literacy at UTSA and has conducted previous Institute courses on this topic. She earned the 1984 UTSA-AMOCO Teaching Award.

Summer Institute courses offer from one to three graduate credit hours which can be applied to degree programs and certification requirements by students admitted to the programs.

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COMPUTER SCIENCE--A course focusing on the use of the computer as a means of illustrating mathematical concepts in the classroom will be offered at The University of Texas at San Antonio during the 1985 Summer Institute.

In its sixth year, the Institute offers educators and other working professionals the opportunity to earn graduate level credit in intensive summer sessions that last from one to three weeks.

"Computers for Mathematics Teachers" is one of 18 comprehensive workshops being offered through the Institute. The course will involve participants in developing a wide variety of mathematical concepts with the use of microcomputers. It will run June 10-June 28.

Teaching the course will be Dr. Maurice Burke, an assistant professor in the Division of Mathmatics, Computer Science and Systems Design. Dr. Burke teaches computer literacy, algebra and geometry for elementary teachers.

Summer Institute courses offer from one to three graduate credit hours which can be applied to degree programs and certification requirements by students admitted to the programs.

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For additional admission or registration information, contact the Office of Admissions and Registrar at (512) 691-4532 or write that office at The University of Texas at San Antonio, San Antonio, Texas, 78285.

For more information on this specific Summer Institute workshop, contact SANDRA JORDAN, the College of Sciences and Engineering, at (512) 691-4450.

PROBLEMS AND CONTESTS

Diane McGowan

Intraschool mathematics competitions are popular throughout Texas. Sponsoring a contest with subject area tests and UIL tests in calculator and number sense can be a monumental undertaking and more than a teacher or math club should try for a first contest. For its first sponsored mathematics competition Anderson High School, Austin, invited the other eight Austin high schools to a mini-contest after school for an hour and a half. Autie Doerr, the Mu Alpha Theta, sponsor, modeled the contest after competitions which occur at the national Mu Alpha Theta competition each summer. The mini-contest consisted of two rounds--one a cyphering round and the other a 25 minute school test. The cyphering round consisted of 10 questions. The students were given the ten questions on half-slips of paper placed upside down in a folder. A number appeared on the front of eachfolder. This was the "Mathlete" number which the student must write on each answer sheet to receive credit for his answer. The student is allowed two minutes to answer each question. He pulls the question, answers it, put his hand with the paper up, runners pick up the answers. If the question is answered correctly in one minute the individual receives two points, in two minutes he receives one point. The student with the most points is the winner. For the school test each school was given 5 copies of the test, all the students present from a school worked on the test and turned in one answer sheet. Each type of test had two divisions-ninth-tenth and eleventh-twelvth. Grading is done on the first contest during the school test and awards are presented after the school test.

Cyphering competition is common in the east and southeast. This type of competition could also be conducted within a school by dividing students into teams of 5 or 6 for the second test. Questions on both tests should be basic mathematics subject area questions. The sponsor should also have some simple tie breaker questions ready.

If you have suggestions for competitions or contest organization, please send them to Diane McGowan, Route 1 Box 259, Cedar Creek,TX, 78612.

Solutions to February Problems

Always One

What is the smallest number divisible by 7 which, divided by any of the numbers 2 to 6 inclusive, leaves a remainder of 1?

The smalles number is 301. The L.C.M. of the numbers 2 to 6 inclusive is 60.

Thus $60k + 1 = 8k + \frac{4k+1}{7}$ which implies that $k = 5$

Hence, $60(5) + 1 = 301$ is the number.

Flea Market

Joe G had only two citrus trees left to sell so that he could go home one Saturday afternoon from the local flea market. He hurriedly sold both of them for $9.00 each. He made a 50% profit on one tree and took a 40% loss on the other tree. How much profit did he make on both trees?

None He lost $3.00. $9.00 is 150% of $6.00 which implies he had a $3.00 profit. $9.00 is 60% of $15.00 which implies he had a $6.00 loss.

These two questions were submitted by Donald Skow from his book Common Sense in Mathematics, Book 1.

Mr. Skow also submitted a solution to the problem from Eaton's Elementary Algebra, 1876:

Two sums of money amounting together to $1600. were put at interest, the less sum at 2 percent more than the other. If the interest of the greater sum had been increased 1 percent, and the less diminished by 1 percent, the interest of the whole would have been increased one fifteenth; but if the interest of the greater had been increased by 1 percent while the interest of the other remained the same, the interest of the whole would have been increased one tenth. What were the sums and the rates of interest?

Let x = less sum \$1600; 1600-x = the greater sum and r= the interest rate of the larger sum. The interest equations are:

1) $(r+2)x + r(1600-x) = 1600r + 2x$

2) $(r+1)(1600-x) + (r+1)x = \frac{16}{15}(1600r+2x)$

3 $(r+1)(1600-x) + (r+2)x = \frac{11}{10}(1600r+2x)$

Solving 2) and 3) gives $x = (400r)/7$ and substituting this value of x into 3) yields that R. = 7. Therefore, the less sum is \$400 invested at 9% and the greater sum is \$1200 invested a 7%.

The following question is taken from a February, 1966 Mathematical Log, the publication of Mu Alpha Theta:

A man dropped a bottle over the side of a boat while rowing upstream. He continued to row upstream with the same steady expenditure of effort for three minutes after dropping the bottle. He then turned the boat around and rowed downstream for a distance of two miles at which point he recovered the bottle (two miles relative to the bank of the river from the point where he turned the boat around to the point at which he recovered the bottle). How fast was the man traveling (relative to the bank of the river) as he was rowing downstream?

Please send a solution to this problem or your suggestions for mathematics enrichment by use of problems and contests to me. Your help is needed to make this column interesting and useful.

PLEASE SOLICIT NEW MEMBERSHIPS!

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