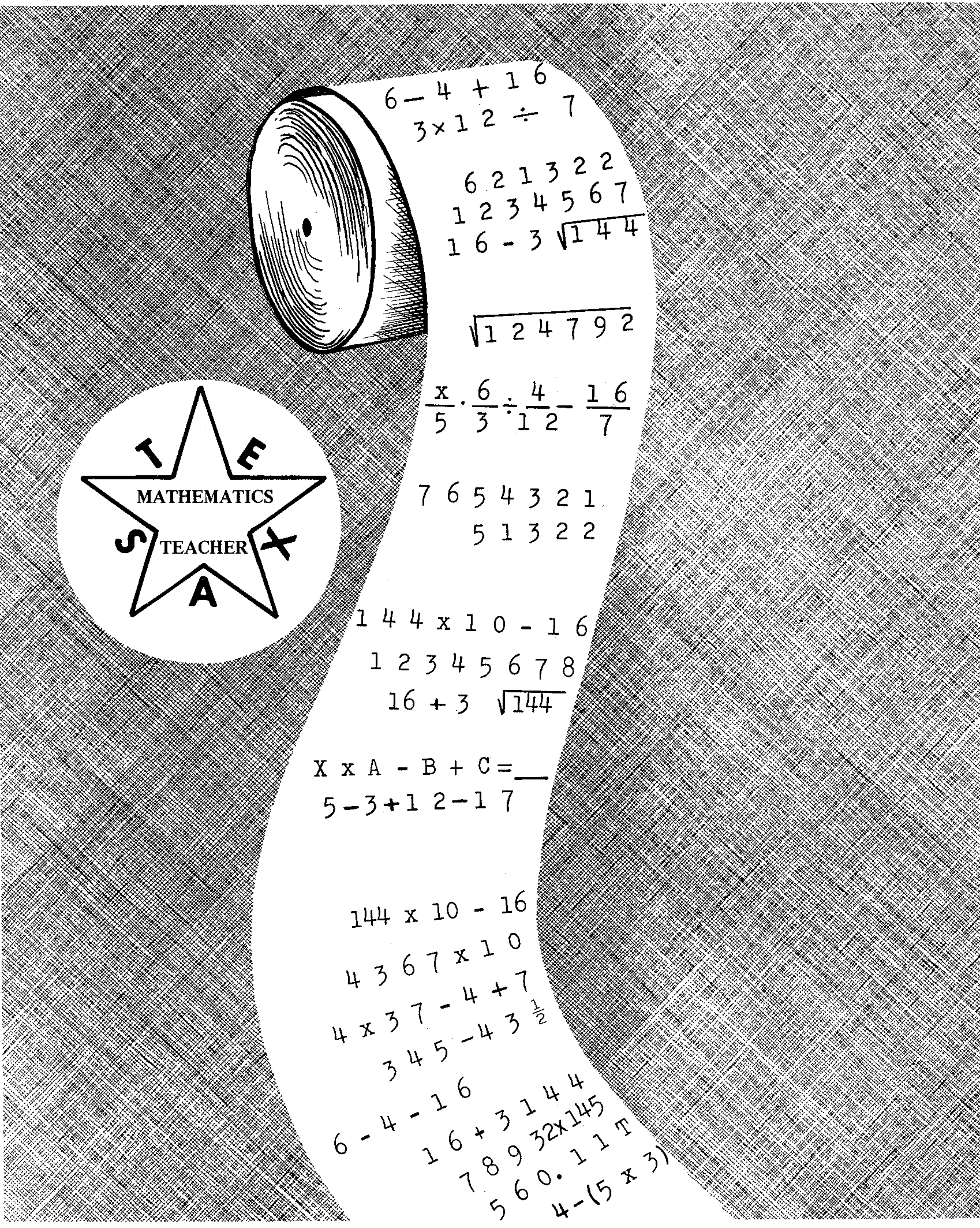
6 – 4 + 1 6
3 × 1 2 ÷ 7
6 2 1 3 2 2
1 2 3 4 5 6 7
1 6 - 3 √1 4 4
√1 2 4 7 9 2
x/5 · 6/3 ÷ 4/12 – 16/7
7 6 5 4 3 2 1
5 1 3 2 2
1 4 4 x 1 0 - 1 6
1 2 3 4 5 6 7 8
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X x A - B + C = __
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144 x 10 - 16
4 3 6 7 x 1 0
4 x 3 7 - 4 + 7
3 4 5 – 4 3 ½
6 - 4 - 1 6
1 6 + 3 1 4 4
7 8 9 32x145
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T E X A S
MATHEMATICS
TEACHER

TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

TEXAS MATHEMATICS TEACHER is published quarterly by the Texas Council of Teachers of Mathematics. Payment of membership fee of $5.00 entitles members to all regular Council Publications.

TEXAS MATHEMATICS TEACHER

Texas Council of Teachers of Mathematics

Volume XXXII | January, 1985 | No. 1

PRESIDENT'S MESSAGE

Dear TCTM Members,

As the new president of TCTM I would like to share my beliefs about the organization. Its responsibilities to you as members, your responsibilities to it, and my responsibilities as its president. One of my colleagues suggested that I use the cliche that the purpose of TCTM is "to meet the needs of the constituents," and we joked about it. However, as I thought about it, that is exactly what I want to say to you. We are in a time of change in education in Texas; each of will be affected by those changes that occur, and TCTM has a role in influencing them where possible and in helping its members as they respond to the changes. As change occurs, needs are likely to change, and one of your responsibilities is to share those needs with us so that TCTM may more effectively assist you in meeting them. Your other responsibilities include participating in TCTM's activities – programs, elections, sales of materials, etc. – and promoting membership among your colleagues. My responsibilities are primarily to make sure that TCTM does, indeed, respond to your needs through its programs and other activities and this journal. Suggestions for any or all aspects of TCTM are welcomed.

Let me remind you of upcoming events: NCTM Annual Meeting, April 17–20, in San Antonio; History of Mathematics special interest group meeting in San Antonio preceding the NCTM meeting April 15–17; NCTM elections – please vote; meetings of local councils (check local listings for time and place) – join and participate!

I look forward to working with you in TCTM. If you are willing to take an active role in TCTM activities, please contact me. If you have ideas for the journal, contact me or the editor. Remember that we work for you, but we need some of you to work with us. Let us hear from you

RALPH W. CAIN

The late Dr. E. Glenadene Gibb will be remembered with a memorial lecture presented at the Eleventh Annual Conference of the Research Council for Diagnostic and Prescriptive Mathematics. The conference is being held in Austin, Texas, Saturday, Sunday, and Monday, April 13-15, 1985, at the Wyndham Southpark Hotel. The lecture will be given by Dr. Linda Jenson Sheffield of Northern Kentucky University, who is a former student of Dr. Gibb.

Dr. Gibb was Catherine Mae Parker Centennial Professor of Education at The University of Texas at Austin at the time of her death. She had been a faculty member in Mathematics Education and Mathematics since 1965. She was President of the National Council of Teachers of Mathematics from 1974 to 1976, a founding member of the Research Council for Diagnostic and Prescriptive Mathematics, a member of the Mathematics Association of America, and a member of Delta Kappa Gamma Society International. She was the author of many articles and books including the widely used elementary mathematics series of textbooks published by Scott-Foresman.

Information about the conference may be obtained from the Conference Chair:

Dr. Charles E. Lamb
Curriculum and Instruction, EDB 406
The University of Texas at Austin
Austin, Texas 78712-1294

or the Program Chair:

Dr. Cherry C. Mauk
St. Edward's University
3001 South Congress Avenue
Austin, Texas 78704

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Manuscripts published in the TEXAS MATHEMATICS TEACHER are reviewed by at least three mathematics educators. Members of TCTM who are regular readers of the journal - classroom teachers, supervisors, and teacher educators - and who would like to review manuscripts should write to the TCTM indicating their willingness to serve and the level of interest (Elementary, Secondary, or both). Send to George H. Willson, Box 13857, North Texas State University, Denton, TX 76203. The Editorial Panel will review the responses and make the final selection.

A GRAPHICAL STUDY OF THE QUADRATIC EQUATION

by John Huber, Sam Houston State University
and Joseph Wiener, Pan American University

Several methods of deriving the quadratic formula have been discussed by various authors (Dobbs, 1982; Huber and Wiener, 1983; and Wallace and Wiener; to appear). The purpose of this paper is to examine the nature of the roots of the quadratic equation based on the equivalence of the quadratic equation and a system of equations involving the straight line and hyperbola. An analysis of the nature of the roots of the quadratic equation will be based on the mutual location of certain lines and hyperbolas. This will be followed by a geometrical derivation of Viete's sum and product formulas for the roots of a quadratic equation.

Let $ax^2 + bx + c$ where $a = 0$ be a quadratic equation.

Dividing by a we have $x^2 + px + q = 0$ where $p = \frac{b}{a}$ and

$q = \frac{c}{a}$. If $q = 0$ then we have the trivial quadratic equation

$x^2 + px = 0$, with $x = 0$ and $x = -p$ as solutions. Assume $q = 0$.

Then $x = 0$ so $x^2 + px + q = 0$ is equivalent to

$$x + p + \frac{q}{x} = 0.$$

Then $x + p = \frac{-q}{x}$. Letting f and g be functions with

$f(x) = x + p$ and $g(x) = \frac{-q}{x}$, the solution of the original quadratic equation is given by the x coordinate of the point(s) of intersection of the graphs of f and g. The graph of

$g(x) = \frac{-q}{x}$ is a hyperbola. (See Figure 1).

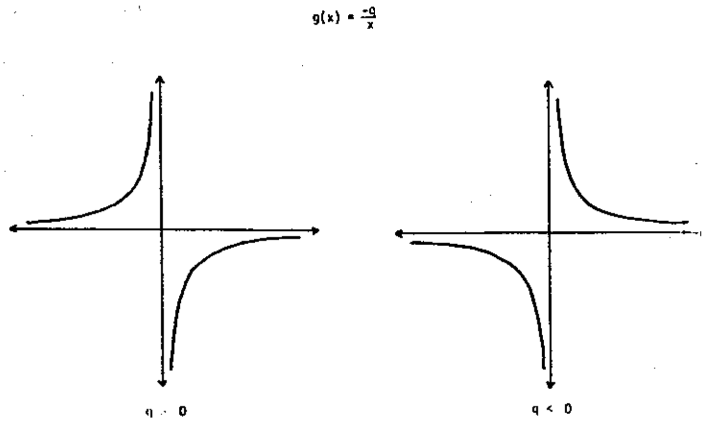

Figure 1

The graph of $f(x) = x + p$ is the straight line with slope 1 and y-intercept

(0,p). (See Figure 2). When does $x^2 + px + q = 0$ $(q = 0)$

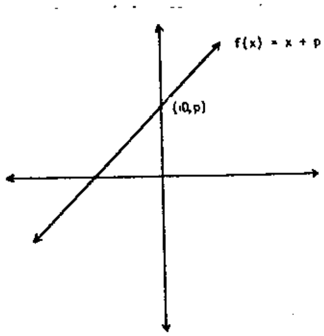

Figure 2

have a real solution? Examining the graphs of f and g with $q < 0$ we see that the graphs of f and g always intersect at two points,

$x^2 + px + q = 0$ always has two real solutions. (See Fiqure 3).

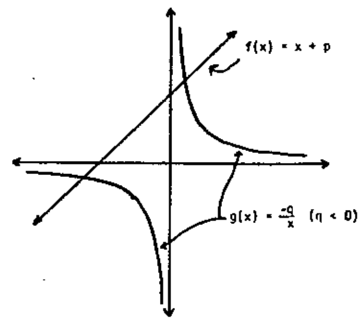

Figure 3

If $q > 0$ then we can have one, two or no real roots. (See Fiqure 4).

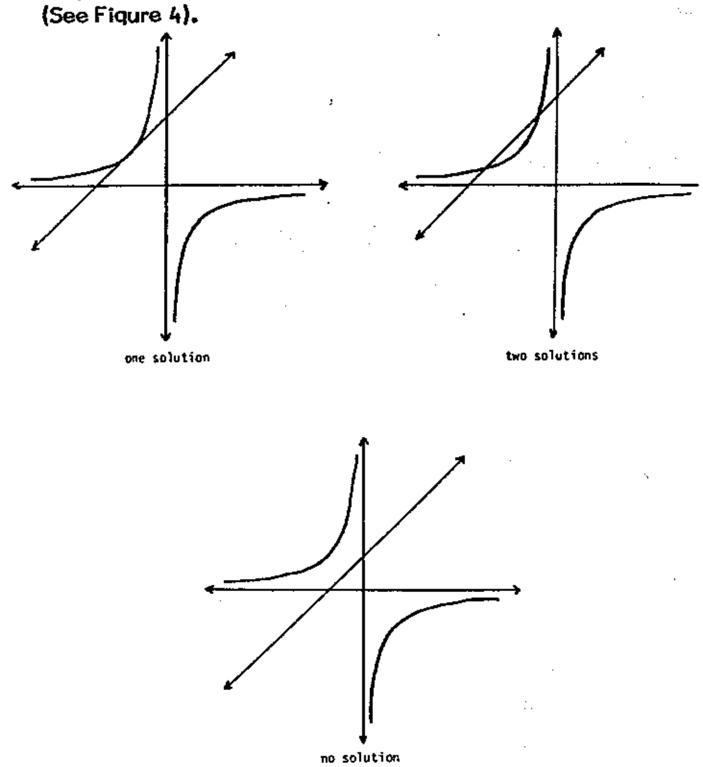

Figure 4

If $x^2 + px + q = 0$ has exactly one solution then f and g must intersect in exactly one point. For f and g to intersect in exactly one point the point of intersection must lie on the axis of symmetry, $y = -x$, of the hyperbola

$y = \frac{-q}{x}$. If not, by symmetry, there would be two points of intersection of $y = x + p$ and

$y = \frac{-q}{x}$. Thus the point of intersection must lie on the line $y = -x$ and the hyperbola

$y = \frac{-q}{x}$. (See Figure 5).

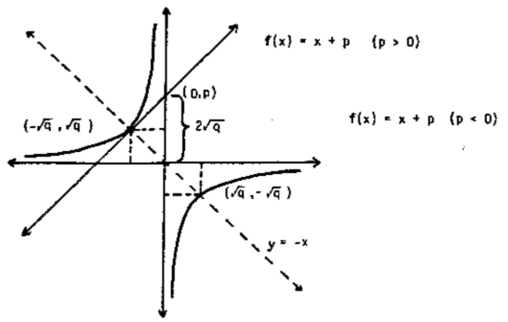

Figure 5

Then $\frac{-q}{x} = -x$ and $x^2 = q$ or $x = \pm\sqrt{q}$ so the points of intersection are

$(\sqrt{q}, -\sqrt{q})$ and $(-\sqrt{q}, \sqrt{q})$ where $q > 0$.

Examing Fiqure 5 we see that this will occur only if

$|p|_2 = 2\ q$. But $|p| = 2\sqrt{q}$ if and only if $p^2 = 4q$ or $p^2 - 4q = 0$. Thus $x^2 + px + q = 0$ with $q > 0$ has exactly one real solution if and only if the discriminant

$p^2 - 4q = 0$. If $|p| > 2\sqrt{q}$ (See Figure 6) and $q > 0$ then

$x^2 + px + q = 0$ has exactly two real solutions. Similarly, if $|p| < 2\sqrt{q}$ (See Figure 6)

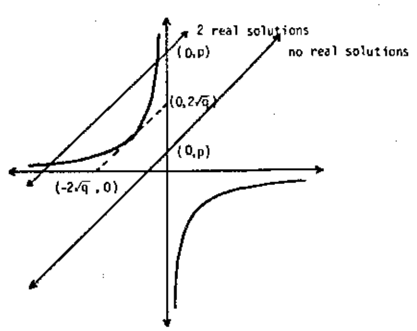

Figure 6

and $q > 0$ then $x^2 + px + q = 0$ has no real soultions. Thus we can conclude

(1) If $q < 0$ then $x^2 + px + q = 0$ has two real solutions;

(2) If $q > 0$ and the discriminant $p^2 - 4q = 0$, then $x^2 + px + q = 0$ has exactly one real solution;

(3) If $q > 0$ and the discriminant $p^2 - 4q > 0$, then $x^2 + px + q = 0$ has two real solutions; and

(4) If $q > 0$ and the discriminant $p^2 - 4q < 0$, then $x^2 + px\ q = 0$ has no real solution.

Suppose $x^2 + px + q = 0$ has exactly one solution. Then the point of intersection of $y = \frac{-q}{x}$ and $y = x + p$ is $(-\sqrt{q}, \sqrt{q})$ if $p > 0$ and $(\sqrt{q}, -\sqrt{q})$ if $p < 0$. (See Figure 7).

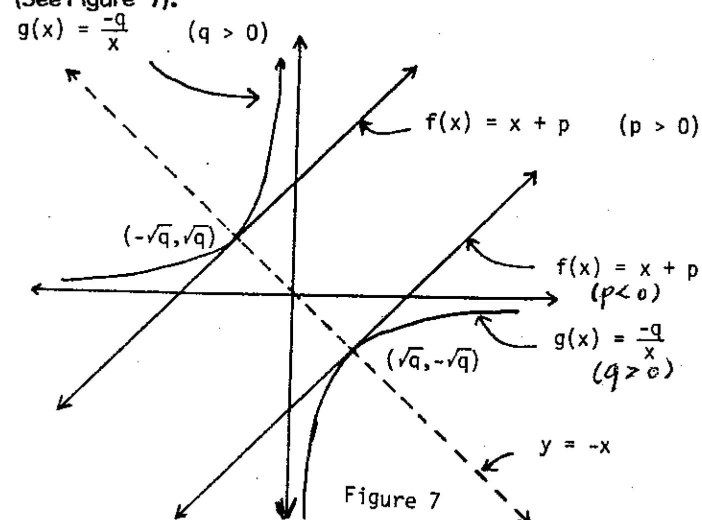

Figure 7

But $p^2 = 2q$ so $q = \frac{p}{2}$ if $p > 0$ and $\sqrt{q} = \frac{-p}{2}$ if $p < 0$. Then the point of intersection of $y = x + p$ and $y = \frac{-q}{x}$ is $(\frac{-p}{2}, \frac{p}{2})$ whether p is positive or negative. Thus, if $x^2 + px + q = 0$ has exactly one real solution the solution is $x = \frac{-p}{2}$.

Suppose $x^2 + px + q = 0$ has exactly two real solutions x_1 and x_2. Consider the case $p > 0$. (See Figure 8).

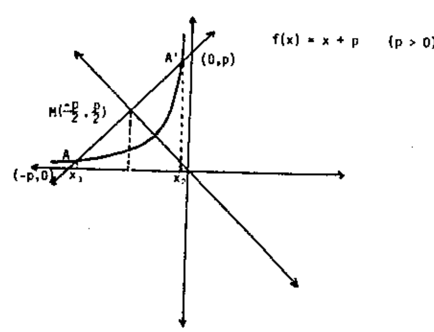

Figure 8

By symmetry M is the midpoint $\overline{AA'}$ and consequently

$$\frac{x_1 + x_2}{2} = \frac{-p}{2} \quad \text{or} \quad x_1 + x_2 = -p\ . \qquad (1)$$

Similarly, (1) is true if $p < 0$. Since x_1 is a solution of $x^2 + px + q = 0$ we have

$$x_1^2 + px_1 + q = 0\ . \qquad (2)$$

Substituting $p = -(x_1 + x_2)$ into (2) we have

$$x_1^2 - (x_1 + x_2)p + q = 0 \text{ or } \quad x_1x_2 = q \qquad (3)$$

Equations (1) and (3) are Viete's formulas for the sum and product of the roots of a quadratic equation. Given Viete's sum and product formulas it is then easy to derive the quadratic formula (Wiener, 1982; and Huber and Wiener; 1983).

REFERENCES

Dobbs, David E. "Discovering the Quadratic Formula." ILLINOIS MATHEMATICS TEACHER, 33 (May, 1982): 27-28.

Huber, John and Wiener, Bella. "Another Derivation of the Quadratic Formula." ILLINOIS MATHEMATICS TEACHER, 34 (May, 1983): 10-11.

Wallace, Edward and Wiener, Joseph. "A New Look at Some Old Formulas." MATHEMATICS TEACHER: (to appear)

Wiener, Bella. "Another Quadratic Formula." MATHEMATICS IN COLLEGE, (Fall, 1982): 21-24.

A GRAPHICAL STUDY OF THE CUBIC EQUATION

by John Huber, Sam Houston State University
and Joseph Wiener, Pan American University

In a number of recent papers (1-4) an interesting pedagogical approach to the study of the quadratic equation has been discussed that is based on the analysis of the mutual location of certain parabolas and lines or hyperbolas and lines. Although this method is not new, it admits of numerous interpretations and enables the students to visualize the derivation of many important properties and facts, including the quadratic formula.

The cubic equation $au^3 + bu^2 + cu + d = 0$ can be reduced to the simplest form

$$x^3 + px + q = 0, \qquad (1)$$

after dividing through by the leading coefficient a and using the substitution

$$u = x - \frac{b}{3a}.$$

Every cubic equation (more general, any algebraic equation of odd power) has at least one real root. Indeed, for large negative values of x the negative term x^3 dominates over the lower degree terms. Therefore, for these values of x the polynomial $x^3 + px + q$ is negative. For large positive values of x the positive term x^3 exceeds $px + q$, which makes the given polynomial positive. Changing from negative values to positive values, this continues function equals zero at some point.

The purpose of this note is to derive geometrically a criteria which allows us to determine immediately when Eq. (1) has only one real root, or when it has three real roots. The approach is based on the study of two graphs:

$$f(x) = x^3 + px, \; g(x) = -q.$$

It is convenient to write $f(x) = x(x^2 + p)$. If $p \geq 0$, then $f(x)$ has only one x-intercept $(x = 0)$. In this case, the graphs of $f(x)$ and $g(x)$ have only one common point. (See Figure 1).

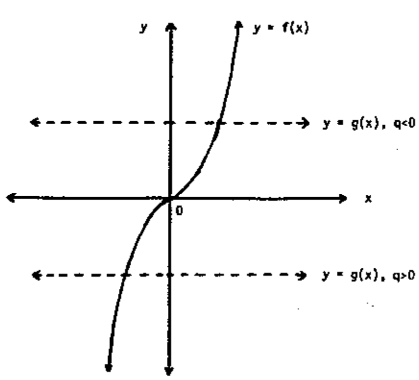

Figure 1

This proves the following particular result. If $p \geq 0$, then Eq. (1) has precisely one real root.

Now, we consider the case $p < 0$. The curve $f(x) = x(x^2 + p)$ has three x-intercepts: $x = -\sqrt{-p}$, $x = 0$, $x = \sqrt{-p}$. This function has one local maximum and one local minimum. To find the coordinates of these two points, we may use the derivative, or solve the problem by elementary means. Namely, take two points on the graph $y = f(x)$ with abscissas x and x_1 and draw a secant line through these points. Its slope is

$$\frac{f(x) - f(x_1)}{x - x_1} = \frac{x^3 + px - x_1^3 - px_1}{x - x_1}$$

$$= \frac{(x^3 - x_1^3) + p(x - x_1)}{x - x_1} = x_1^2 + x_1x + x^2 + p.$$

As x_1 approaches x we find the slope

$$m = 3x^2 + p$$

of the tangent line to the curve at the point with abscissa x. At the critical points the tangent lines are horizontal, hence their abscissas satisfy $3x^2 + p = 0$. From here, we have

$$x_{max} = -(-\tfrac{p}{3})^{\frac{1}{2}}, \quad x_{min} = (-\tfrac{p}{3})^{\frac{1}{2}},$$

and it is easy to see from the graph of $f(x)$ that at x_{max} this function has local maximum and at x_{min} it attains local minimum. Evaluating the function at these points, we find the values of the local maximum and local minimum:

$$f_{max} = 2(-\tfrac{p}{3})^{\frac{3}{2}}, \quad f_{min} = -2(-\tfrac{p}{3})^{\frac{3}{2}}.$$

Consider on the curve $f(x)$ two points

$$M_1(-(-\tfrac{p}{3})^{\frac{1}{2}}, 2(-\tfrac{p}{3})^{\frac{3}{2}}), \; M_2((-\tfrac{p}{3})^{\frac{1}{2}}, -2(-\tfrac{p}{3})^{\frac{3}{2}}).$$

If the horizontal line $y = -q$ crosses the graph of $f(x)$ higher than M_1 or lower than M_2, then Eq. (1) has precisely one real root. (See Figure 2).

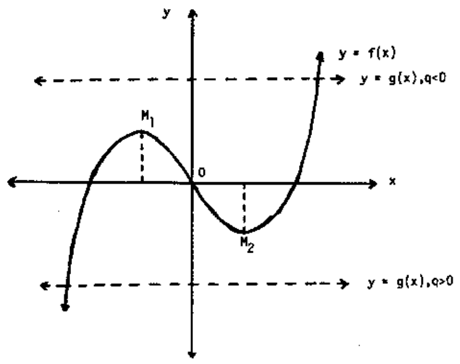

Figure 2

And if the line $y = -q$ passes within the horizontal strip between M_1 and M_2, Eq. (1) has three different roots.

(See Figure 3).

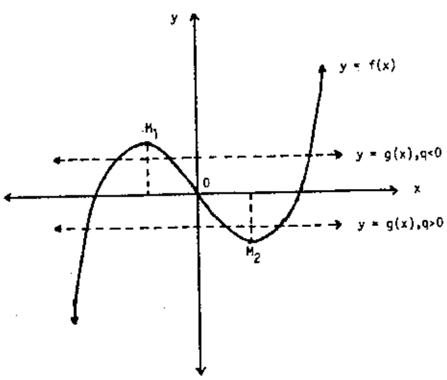

Figure 3

In the first case we have the inequality

$$2\left(-\frac{p}{3}\right)^{\frac{3}{2}} < |q|$$

simplifying of which gives

$$\frac{p^3}{27} + \frac{q^2}{4} > 0 \qquad (2)$$

In the second case we have the opposite inequality

$$\frac{p^3}{27} + \frac{q^2}{4} < 0 . \qquad (3)$$

Observe that (2) holds true, in particular, when $p \geq 0$, $q \neq 0$. Therefore, the condition $p \geq 0$ is only a special case of (2) and may be omitted. We arrive at the following conclusions.

1. If inequality (2) holds true, Eq. (1) has one real and two complex conjugate roots.
2. If inequality (3) takes place, Eq. (1) has three different real roots.
3. If $\frac{p^3}{27} + \frac{q^2}{4} = 0$, then all roots of (1) are real and there is a multiple root. This case occurs when the line $y = -q$ is tangent to the curve $y = x^3 + px$ at the point M_1 or M_2.

Remarkably, the geometric analysis of (1) is simpler than its investigation by Cardano's formulas for the roots of the cubic equation.

REFERENCES

Wiener, Bella. "Deriving the Quadratic Formula without Completing the Square". TEXAS MATHEMATICS TEACHER Jan. 1984, 11.

Wiener, Bella. "Graphing Parabolas without Completing the Square", ILLINOIS MATHEMATICS TEACHER May, 1984, 11-13.

Huber, John and Wiener, Joseph. "A Graphical Study of the Quadratic Equation". THE TEXAS MATHEMATICS TEACHER (to appear).

Wallace, Edward and Wiener, Joseph, "A New Look at Some Old Formulas". THE MATHEMATICS TEACHER (to appear).

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PRIMARY

FIVE LITTLE ROBINS

Five little robbins lived in a tree
Father (thumb)
Mother (little finger)
And babies three (three fingers)
Father caught a bug
Mother caught a worm
And all three babies began to squirm
This one got the bug (indicate finger)
This one got the worm
And this one said
"Now it's my turn".

PRIMARY

DOMINO BINGO

Objective:

Giving sums to 12 for domino patterns.

Directions:

Draw 12 rectangles the size of dominoes on each of 3 cards and insert numbers according to the scheme shown below.

Then, turn a set of dominoes face down and have 3 players take turns drawing dominoes.

If the sum of the domino pattern is equal to a number on the pupil's card, he/she covers it with the domino; otherwise, he/she places it in a discard pile which is to be mixed and reused if nobody wins by covering all squares before the original pile is depleted.

1	6	10
	3	9
7	8	4

3	5	4
9	7	11
6	8	2

2	4	6
5	6	7
10	8	12

INTERMEDIATE SCHOOL

Graphfun with Abe

Solve each problem below. Use these answers to determine your graphing points. Locate and connect these points.

Location of X		Location of Y		X	Y
5 × 8 =	40	37 - 27 =	10	40	10
9 × 4 =		8 + 9 =			
16 + 20 =		10 + 9 =			
2 × 19 =		9 + 10 =			
2 × 20 =		3 × 7 =			
21 + 19 =		15 + 12 =			
11 × 4 =		17 × 2 =			
9 × 5 =		19 + 24 =			
6 × 7 =		5 × 10 =			
3 × 11 =		8 × 7 =			
62 - 48 =		26 + 29 =			
3 × 3 =		2 × 25 =			
4 + 3 =		28 + 16 =			
5 + 6 =		9 × 5 =			
4 × 2 =		13 × 3 =			
27 - 18 =		4 × 9 =			
41 - 33 =		7 × 5 =			
7 + 2 =		16 + 17 =			
53 - 49 =		19 + 5 =			
2 × 2 =		14 + 9 =			
1 + 6 =		11 + 12 =			
84 - 75 =		6 + 18 =			
5 × 2 =		9 + 14 =			
25 - 16 =		62 - 40 =			
6 + 4 =		15 + 6 =			
4 + 5 =		13 + 7 =			
3 + 7 =		2 + 15 =			
25 - 17 =		11 + 3 =			
3 + 5 =		16 - 6 =			
3 × 4 =		24 - 16 =			
39 - 19 =		10 × 1 =			
58 - 44 =		1 × 1 =			
20 × 2 =		5 + 5 =			

ELEMENTARY

ADD-A-TRAIL

Can you draw a special trail to the answer in the bottom box of this puzzle? This trail will be a continuous line connecting some of the numbers in the puzzle. It must end at the number in the bottom box, and the numbers in the trail must add up to the total in the bottom box.

BE CAREFUL! The trail cannot cut corners. It cannot retrace itself. It cannot cross itself. You can use each box only once. There may be more than one way to make the trail.

This is one puzzle that may be easier to do backwards. Try starting your trail with the number immediately above the bottom box.

HINT: First subtract the number in the bottom box from the total of all the other numbers. When you have completed the puzzle, the numbers that are not in the trail you have made will add up to this difference.

6	5	4	5	4
3	6	2	3	2
5	3	1	6	5
4	2	5	4	3
2	1	3	1	6
		79		

ELEMENTARY SCHOOL

EASTER

Easter Sunday is celebrated on the Sunday following the first full moon after the vernal equinox. It must fall between March 22nd and April 25th.

To find out the date of Easter this year:

1. Divide the number of the year by 19.
   Let the remainder = A
2. Divide the number of the year by 4.
   Let the remainder = B.
3. Divide the number of the year by 7.
   Let the remainder = C.
4. Multiply 19 times A and add 24. Divide this by 30.
   Let the remainder = D.
5. Take (2 X B) + (4 X C) + (6 X D) + 5 and divide by 7.
   Let the remainder = E.
6. Easter day will be on 22 + D + E.
   If your number is larger than 31, then it goes over into April.

MIDDLE SCHOOL

AGE PROBLEM

A little girl said: "I am five years old, grandpa. How old are you?" The old gentleman replied: "Your father is eleven times as old as you, and I am as many years old as he will be when you are one-third of my age." How old is grandpa? Note: Although more sophisticated means can be used, trial and error techniques can be used to arrive at grandpa's age of 75.

MIDDLE SCHOOL

PROBABILITY USING A CALENDAR

Have a calendar showing the month for use with the activity.

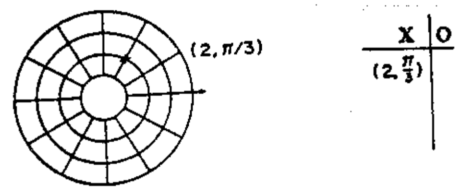
1. Make a set of cards for each date of the month. EXAMPLE: (1-30, 1-28, or 1-31).
2. The cards are put in a box and drawn one at a time.
3. Ask questions about the p robability of drawing a date that is:
   a. on a Saturday
   b. on a Monday
   c. an even number
   d. an odd number
   e. not a Saturday
   f. the 32nd
   g. a day in the month, e.g. a day in February
   h. a multiple of 5
   i. not on a Wednesday
   j. a multiple of 3

SENIOR HIGH SCHOOL

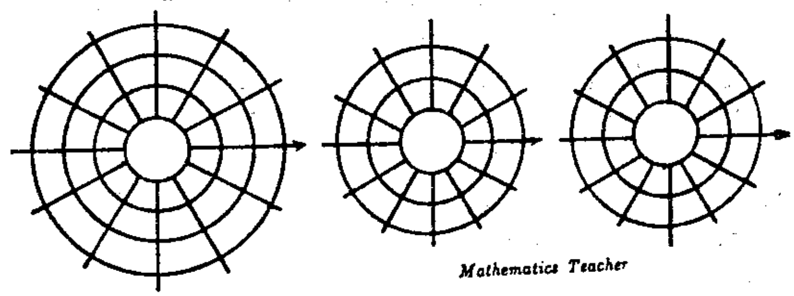
TIC-TAC-TOE IN POLAR COORDINATES

Directions:

1. Use a polar-coordinate system with circles of radius 1, 2, 3, and 4 and radial ines every /6 radians or 30°.
2. The two players alternate putting an "X" or an "O" at a point.
   the point is designated by giving the polar coordinates.



3. The first player to place 4 of his/her marks in a row along a radial ine, a circle, a diagonal, or spiral is the WINNER.



Mathematics Teacher

February 1974

MULTI-LEVEL ACTIVITY

SKILLS DRILL BINGO

Directions:

1. Have a set of regular Bingo cards or let each student make his own. The numbers on the cards are from 1 to 75. There are 5 rows and 5 columns with the center square designated "FREE".
2. Problems are read and the students cover the solution. The level of the game is determined by the difficulty level of the problems.

PROBLEM	COVER
$2 + 3$	5
or $\sqrt{25}$	5
or 10×2^{-1}	5

3. The winner is the first student to cover one row, one column or one diagnol.

NOTE: You will need 75 problems--one for each number 1 - 75.

TCTM Journal needs articles for all levels of Mathematics.

Problems and Contests

The following solutions were presented to the problems in the October, 1984 issue:

Problem: 1 From Dr. H. Don Allen, The Mathematical Log, The Martian Fragment Number Challenge.

The Fragment illustrated derives from an early, little known exploration of the Red Planet. Martians go about addition, subtraction, multiplication, division, and exponentiation exactly as we do. They even use identical symbols for these operations. They use base ten and use the same digits we do but with a difference. Each symbol has a unique value but that value differs from what it would be on Earth. Decipher the fragment and answer the last equation.

$$8 \times 7 = 8$$
$$4 \times 9 = 39$$
$$51 - 2 = 2$$
$$7^4 = 6$$
$$3 + (4 \times 5) = 34$$
$$40 + 04 =$$

---

1) 8X7=8 2) 7^4=6 3) 4X9=39 4) 51-2=2

By 1) 7 is our 1 or 8 is our 0. By 2) 7 is not our 1, Thus 8 is our 0.
By 2) 7 is our 2 or 3 and 4 is the other number. If 7 is our 2, 4 is our 3 and 6 is our 8.
If 7 is our 3, 4 is our 2 and 6 is our 9.

Earth's Values	0	1	2	3	4	5	6	7	8	9
Martians Values	8	3	7	4		9			6	
	8		4	7						6

By 3) if 4 is our 3, then 3 X <u>5</u> = <u>15</u>, thus 9 is our 5 and 3 is our 1.
By 4) 5 must be our 1 but their 3 is our 1. NO SOLUTION
By 3) if 4 is our 2, then 2 X__ = ____, , NO SOLUTION.

Therefore, Problem #1 has no solution or Problem #1 is a misprint.
Submitted by: Donald P. Skow, Pan American University, Edinburg, TX

Jay Snyder, Crockett High School, Austin, TX, also determined that the problem had no solution as it was printed. He also surmised that there may have been a misprint--a correct assumption--the third line should have read 51 ÷ 2. The corrected problem leads one to the conclusion that the Martian 40 + 04 corresponds to our 36 + 63 = 99. Thus, the last equation is 40 + 04 = 11.

Problem: 2 From the New York State Mathematics Teachers Journal.

A man had an 8 gallon keg of wine and a jug. One day he drew off a jugful of wine and filled up the keg with water. Later on, when the wine and water had been thoroughly mixed, he drew off another jugful, and again filled up the keg with water. The keg then contained equal quantities of wine and water. What was the capacity of the jug?

Let X be the number of gallons that the jug holds.
One day a man drew off a jugful of wine and replaced the wine with water. Now we have 8-X gallons of wine of which $\frac{8-X}{8}$ parts are wine and X gallons of water of which $\frac{X}{8}$ parts are water. Later, he drew another jugful of the wine and water mixture and refilled the keg with water. Now, the amount of wine is $(8-X) - (\frac{8-X}{8})X$ and the amount of water is $x - (\frac{X}{8})X + X$.

Since the amount of wine and water is now equal, set either equation equal to 4 gallons and solve for X. Both equations will yield a value of X to be $8-4\sqrt{2}$ gallons.

Therefore, the capacity of the jug is $8-4\sqrt{2}$ gallons.

Submitted by: Donald P. Skow, Pan American University, Edinburg, TX. Mr. Skow has also contributed these problems from his book <u>Common Sense in Mathematics, Book I.</u>

ALWAYS ONE

What is the smallest number divisible by 7 which, divided by any of the numbers 2 to 6 inclusive, leaves a remainder of 1?

FLEA MARKET

Joe G. had only two citrus trees left to sell so that he could go home one Saturday afternoon from the local flea market. He hurriedly sold both of them for $9.00 each. He made a 50% profit on one tree and took a 40% loss on the other tree.

How much profit did he make on both trees?

Old mathematics texts are a delightful source of problems. This one comes from Eaton's Elementary Algebra by William F. Bradbury, 1876.

Two sums of money amounting together to $1600 were put at interest, the less sum at 2 percent more than the other. If the interest of the greater sum had been increased 1 percent, and the less diminished by 1 percent, the interest of the whole would have been increased one fifteenth; but if the interest of the greater had been increased 1 percent while the interest of the other remained the same, the interest of the whole would have been increased one tenth. What were the sums and the rates of interest?

Solutions or problems should be sent to Diane McGowan, Route 1, Box 259, Cedar Creek, TX 78612 by February 22, 1985.

To stimulate an interest in mathematics a teacher may involve students in school wide mathematics competition. Anderson High School, Austin, offers a monthly puzzle contest with prizes submitted by local merchants. One contest asked students to write natural numbers using four given numerals such as 3245 and the symbols for addition, subtraction, multiplication, division, the radical sign and exponents. Numbers used for the index of a radical or for exponents must be one of the given digits. For example:

$13 = (4 \times 5) - 2 + 3$ and $4 = \sqrt{4} \div 2 + 5 - 3$

The student who represents the greatest number of consecutive numbers is the contest winner. The contest may be made more difficulty by requiring that the numbers be used in a specific order.

Another puzzle contest could consist of four or five problems of varying point value.

An organization which was begun to promote an interest in mathematics in the high school and junior college is Mu Alpha Theta which is cosponsored by the Mathematical Association of America and the National Council of Teachers of Mathematics. There are over 1300 chapters in 46 states and in Canada, Japan, the Canal Zone, and Switzerland. Texas has more chapters than any other state and is well represented each year at the national convention thanks to sponsors such as Sister Scholastica, Blessed Sacrament, San Antonio; Paul Forrester, Alamo Heights, San Antonio; Lynn Britton, John Jay, San Antonio and Caro McGill, West Orange, Orange. The state convention for Mu Alpha Theta will be held in San Antonio at John Marshall High School on Feb. 16 and 17. For more information of the state convention which involves speaker sessions and competitions write to Kay McCormick, John Marshall High School, 8000 Lobo Lane, San Antonio, TX 78240. Nonmembers may attend. For an application to charter a club at your school write Mu Alpha Theta, 601 Elm Ave., Room 423, Norman, OK, 73019.

THE FOURTH R--THE NEW KID ON THE BLOCK

by Thomas H. Arcy, The University of Houston--University Park, Houston, Texas
and Charles E. Lamb, The University of Texas at Austin, Austin, Texas

One quickly remembers the old cliche about the important topics in the traditional school curriculum. Children were admonished by their parents and teachers to learn the "3 R's". Of course, these were the old standbys of Reading (readin'), Writing (ritin') and Arithmetic (rithmetic).

The importance of these topics has persevered throughout the history of education in the United States. One only has to look at accounts of the history of education in America to confirm this fact. Reading and Writing allow oral and written communication to be conducted, while arithmetic (mathematics) makes it possible for a person to quantify and organize his/her environment. In recent years, a new emphasis has been placed on these fundamentals in education via a movement termed "Back to Basics".

When the term basics is broadly defined and interpreted, no one would deny the importance of a solid foundation for education. However, it is often the case that as society develops and new scientific discoveries are made, it becomes necessary to revise priorities. Such is the case with the "3 R's".

In the past few years, our society has seen a revolution in technology. We are entering the "information age" of computers. Predictions made for the future and the impact of computers on society is mind boggling. Within a short period of time, it is expected that almost no part of our lives will remain unaffected by the evergrowing influence of the computer.

With this in mind, it might be advisable to add a new basic to the traditional "3 R's". In order to make things work out well, it should be called the fourth "R". Computer literacy will be a nice complement to reading, writing, and arithmetic. It will build upon and help to expand each of these fields. Giving the computer a space alongside the traditional "3 R's" is a big step, but a necessary one.

In conclusion, I know you're wondering where does the "R" come from? Well, just by chance I think we'll call it the fourth "R" for Random Access Memory.

Reading
w Riting
a Rithmetic
Random Access Memory

Figure 1: "The Four R's"

GEOMETRIC REPRESENTATIONS OF SOME INTEGER SUMS

Sister M. Geralda Schaefer

The study of the properties of numbers, or number theory, as this subject is now known, originated in the School of Pythagoras around 540 B.C. The Pythagoreans regarded numbers as having characteristic designs which they depicted as pebbles or dots in the sand, and from these beginnings the ancient Greeks and later the Arabs developed geometric algebra which provided a link between geometry and arithmetic or algebra.

It is not unusual today to disassociate geometry from both arithmetic and algebra; however, the study of the geometric representations of generalizations gives students an opportunity to find patterns and relations and to derive new patterns and relations from others. Activities of this foster analytic and creative thinking. they also generate enthusiasm, stimulate interest and give insight into the nature and beauty of mathematics.

As a trivial but important example, the product xy of two positive integers can be depicted as the number of dots in a rectangular array of x rows with y dots in each row. this simple idea can be expanded to establish in geometric fashion some interesting theorems involving integer sums.

The early Greeks studied figurate numbers and their properties; e.g., triangular numbers. The first four triangular numbers are shown in Figure 1. In studying the pattern of

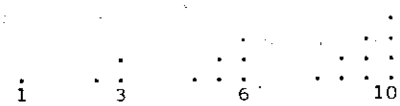


Figure 1

these triangular numbers the Pythagoreans found that 1 + 2 + 3 + 4 + . . . + n, the sum of the first n positive integers could be represented as a triangular number as seen in Figure 2. When

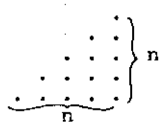


Figure 2

two configurations of the nth triangular number are arranged to form a rectangle, the result is a n(n + 1) rectangle. So 2(1

$$2(1 + 2 + 3 + \ldots + n) = n(n + 1) \text{ and}$$

$$1 + 2 + 3 + \ldots + n = n(n + 1)/2$$ Figure 3

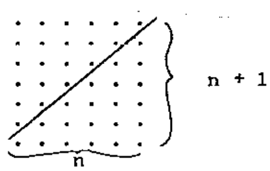


Figure 3

The Pythagoreans also considered other figurate numbers: square numbers, pentagonal numbers, hexagonal numbers, etc. Geometrically, it can be shown that the sum of two consecutive triangular numbers is a square number. Figure 4

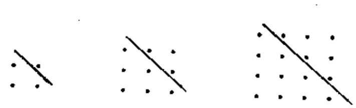


Figure 4

If we consider a square array partitioned as shown in Figure 5, we see that the sum of any number of consecutive odd integers, starting with 1, is a square number.

$$1 + 3 + 5 + \ldots + (2n - 1) = n^2$$

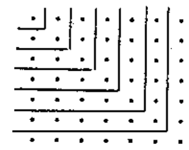


Figure 5

The geometric deduction of this result would probably be more interesting to students than the usual proof by mathematical induction. As a corrollary to this generalization, we can see from an inspection of Figure 5 that the difference of successive squares is an odd integer:

$$n^2 - (n - 1)^2 = 2n - 1$$

which is easily verified algebraically. The L-shaped piece on Figure 5 represents an odd number and is called a gnomon (carpenter's rule) and it is recognized that each odd number is the gnomon of a square number.

We can represent successive square numbers geometrically with the following configuration:

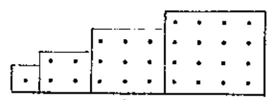


Figure 6

so this figure represents $1^2 + 2^2 + 3^2 + \ldots + n^2$. Now if we reexamine Figure 6 and straighten each gnomon into a column, we have

$$1^2 + 2^2 + \ldots + n^2 = 1 + (1+3) + (1+3+5) + \ldots + (1+3+5+ \ldots (2n-1)$$

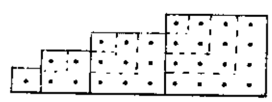


Figure 7

Using Figure 7 and grouping columns of equal height, we can represent the sum $1^2 + 2^2 + \ldots + n^2$ by the pattern below. Figure 8

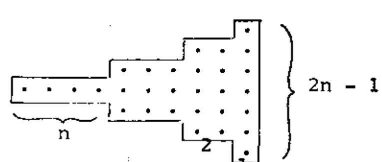


Figure 8

Now if we combine two copies of Figure 6 with Figure 8 to form a rectangular array (Figure 9), we find that

$$3(1^2 + 2^2 + 3^2 + \dots + n^2) = (1 + 2 + 3 + \dots + n)\ (2n + 1)$$

Substituting $n(n + 1)/2$ for $(1 + 2 + 3 + \dots + n)$ gives.

$$3(1^2 + 2^2 + 3^2 + \dots + n^2) = \frac{n(n + 1)}{2}(2n + 1)$$

and $1^2 + 2^2 + 3^2 + \dots + n^2 + \frac{n(n + 1)\ (2n + 1)}{6}$, an identity obtained by Archimedes (287-212 B.C.).

A striking three-dimensional verification of this formula using unit cubes can be demonstrated for $n = 3$. The figure below represents $1^2 + 2^2 + 3^2$. Six such piles can be stacked to form a rectangular prism with sides of length 3, 4, and 7, or $n, n + 1, 2n + 1$ which can be generalized to the identity

$$6(1^2 + 2^2 + \dots + n^2) = n(n + 1)\ (2n + 1)$$

and $$1^2 + 2^2 + \dots + n^2 + \frac{n(n + 1)\ (2n + 1)}{6}$$

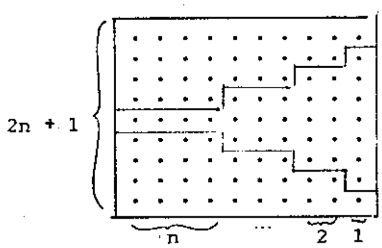


Figure 9

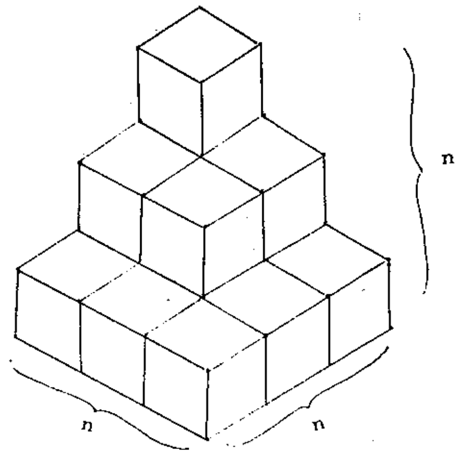


Figure 10

These investigations require only elementary mathematical knowledge yet they provide valuable experience in problem solving and may give greater insight into the development of these identities, the relationship between geometry and algebra, and the unity and beauty of mathematics.

REFERENCES:

Eves, Howard. An Introduction to the History of Mathematics, third Edition, Holt, Rinehart and Winston, New York, 1969, pp. 52-57.

Gardner, M. "'Look-see' Diagrams that Offer Visual Proof of Complex Algebraic Formulas", Scientific American, October 1973, pp. 114-118.

Kung, H. L. "Another Geometric Introduction to Mathematical Generalization", Mathematics Teacher, April, 1972, pp. 375-376.

In Loving Memory

Mrs. J. William Brown, wife of the editor,

passed away January 8, 1985,

after a long illness.

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