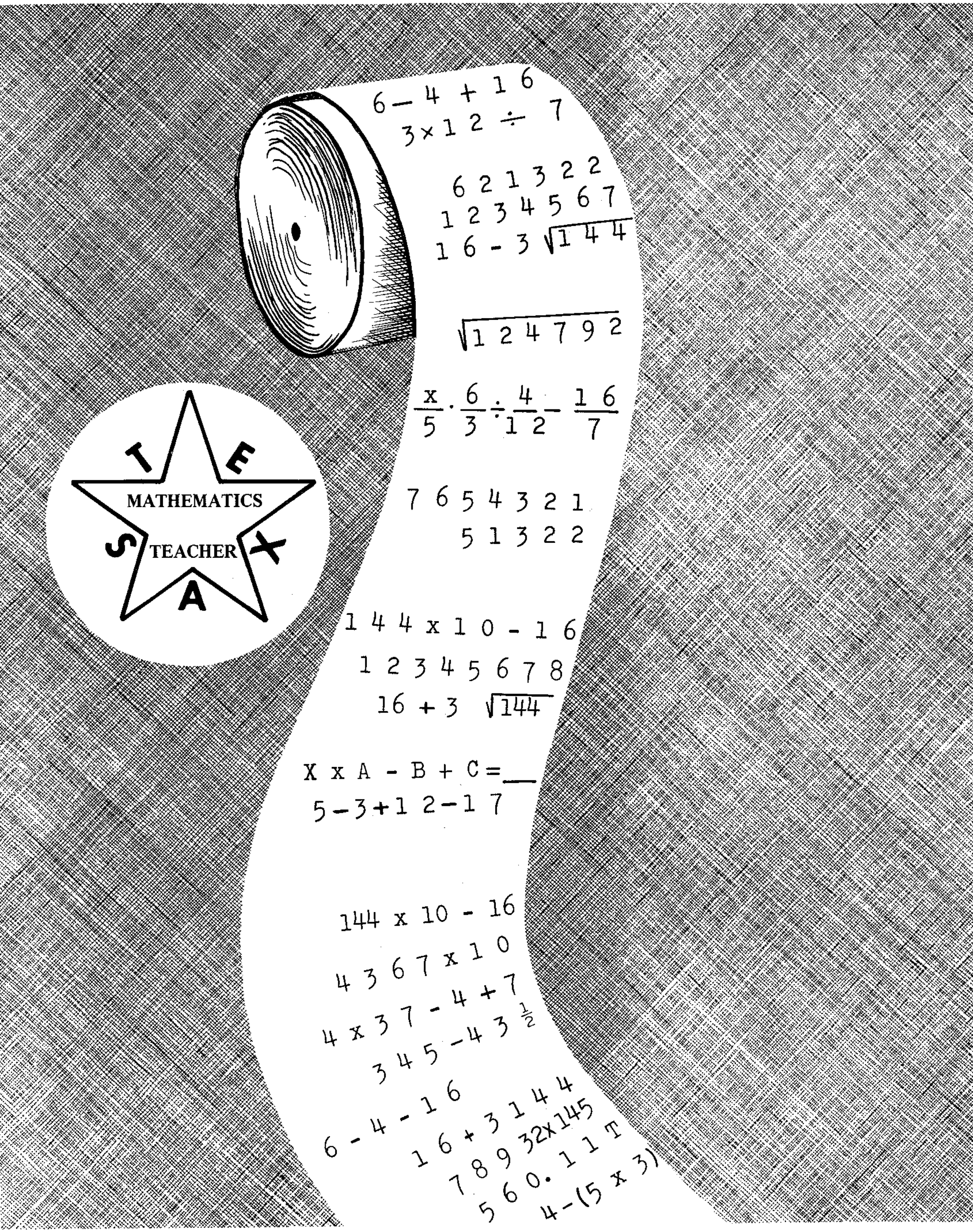
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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

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TEXAS MATHEMATICS TEACHER

Texas Council of Teachers of Mathematics

Vol. XX January, 1983 No. 1

PRESIDENT'S MESSAGE

I hope all of you had a safe and joyous holiday season. January is the time of year for resolutions and revewals – a time of bright hope and promises for the year ahead. I hope all of you have included in your resolutions a renewed dedication to our profession and a willingness to support the projects of TCTM. Your executive committee has several plans in the making and you will be receiving information on these soon. I extend to each of you a personal invitation to become actively involved in TCTM. It is only through the united efforts of all our members that changes can be made.

During the last few months I have noticed an increased surge of professionalism among our mathematics teachers. Our membership is up and continuing to rise. The attendance at CAMT was outstanding, with over 2200 teachers attending that meeting. Everywhere I turn I see eager and motivated teachers – all willing to learn more and share with others. Our Fundamentals of Mathematics Conference is fast approaching and I am receiving 4–5 daily requests on information about that meeting. This is just another example of this renewed interest and dedication. The cheers of Max Sobel, Past-President of NCTM, and 2,000 mathematics teachers still ring in my ears – "Tow, four, six, eight, teaching math is really great." We all need this belief and moral support, and it is through our professional meetings and journals that we are able to support and uplift one another, and provide the caring and sharing of teaching mathematics.

I wish for each of you a safe and prosperous New Year.

Sincerely,

Betty Travis

CAMT

PALMER AUDITORIUM

AUSTIN, TEXAS

OCTOBER 6–8,1983

PROBLEM SOLVING ACTIVITIES TO TEACH THE AREA OF A RECTANGLE

Charles P. Geer, PhD
Texas Tech University, Lubbock, Texas

Finding the area of a rectangle provides the student with an infinite number of problem solving activities that develop and reinforce computational skills. These activities require the student to use simple computational skills to discover the concept of area. They then use these concepts to practice more difficult computational skills.

Teachers who use the array or grid models to help students visualize multiplication facts are really showing them how to find the area of a rectangle. The great stress in today's mathematics on manipulatives, visual learning, and applications makes a class unit on area a natural extension of the basic skills. A unit on area allows students to develop computational skills while discovering problem solving strategies, shows the student the relationship between arithmetic and geometry, and provides an interesting change for students.

The problems presented in this unit are sequential and designed to introduce the concept of area. Samples of many types of problems are given. Each problem requires the student to determine a strategy for finding the answer and then use computational skills to complete his work. Students will use different procedures to solve the same problem. It is often valuable to discuss the strategy a student used to solve the problem as well as the correct answer. Students in this way increase their problem solving strategies and improve their ability to solve problems.

Problem 1:

Find the area of a math book or desk top by tiling (covering without leaving any spaces) with square or triangular tiles, cards, and pieces of paper. Determine the number of square units required to cover each object.

Problem 2:

Use a geoboard to find the area of rectangles. Define one square unit on the geoboard. Make a rectangle that is 3 units long and 2 units wide. How many squares are found in this rectangle? Make other rectangles on your geoboard. Record the length, width, and area of each rectangle on a piece of paper. How many different rectangles can you make?

Problem 3:

Find the area of the three rectangles shown below. In Figure 1 all squares are shown and students need only to determine the total number of squares. In Figure 2 students must understand the concept of area and then compute it. Students in Figure 3 must measure the rectangle before determining its area.

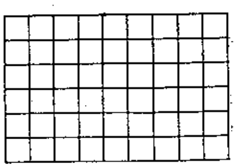


Figure 1

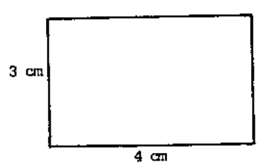


Figure 2

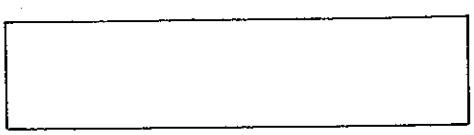


Figure 3

Problem 4:

Complete the table by computing the area of each rectangle.

Length	Width	Area
5 cm	7 cm	35 sq. cm
8 cm	9 cm	
23 mm	17 mm	
145 mm	29 mm	

Problem 5:

Draw and then determine the area of other dimensions of these rectangles.

a. Draw a rectangle with a length of 8 cm and a width of 6 cm. Find its area.
b. Draw a rectangle with a length of 6 cm and an area of 30 square centimeters. Find the width of this rectangle.
c. Draw 3 non-congruent rectangles that have an area of 16 square centimeters.
d. Draw as many non-congruent rectangles as you can with an area of 24 square centimeters.

Problem 6:

Use the area and perimeter of these rectangles to find their length and width. (The perimeter of a rectangle is the sum of the length of all the sides.)

a. Draw a rectangle that has an area of 28 square centimeters and a perimeter of 22 centimeters.
b. Draw a rectangle where the number of square units in the area is equal to the number of units in the perimeter.

Problem 7:

Find the area of these rectangles. What is their total area?

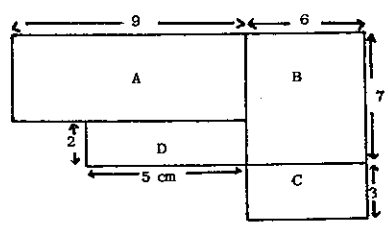


Figure 4

Activities of this type provide an interesting way to develop problem solving skills. They also provide students a chance to review computational skills. Teachers will find these activities enthusiastically attacked by students who will not even realize they are practicing the computational skills they have just learned. After solving these problems many students will create their own problems regarding area and perimeter of rectangles.

TCTM Journal needs articles for all levels of Mathematics.

MARY ELLEN'S DILEMMA: TEACHING PROBLEM SOLVING IN A BACK TO BASICS WORLD

By Harry Bohan
Sam Houston State University
Huntsville, Texas

Mary Ellen is a dynamic sixth grade mathematics teacher, an avid reader of the *Arithmetic Teacher*, and a lifetime member of the NCTM. (On applications with a blank for religious preference she writes NCTM.) With such a background she is naturally committed to the goal of improving the problem solving skills of the students in her classes.

However, Mary Ellen teaches in the Podunk Independent School District whose school board is fanatically committed to the goal of improving scores of the PTBS (Podunk Test of Basic Skills). At the first meeting of the faculty the superintendent of schools acknowledges that beginning this year all teachers will be held accountable for their students making such improvements.

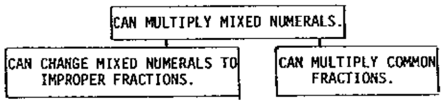
For help in solving her dilemma, Mary Ellen consults Harvey Smedlap, president of the Greater Podunk Council of Teachers of Mathematics who (with great indignation) asks if she has ever read the NCTM *Agenda for Action*. He then informs her that problem solving *is a basic skill*.

However, a quick analysis of the items on the PTBS reveals to our heroine that they are precisely the type of problems the *Agenda for Action* claims problem solving to be *more than*!

Public school teachers today are confronted with just such a situation. The dilemma is caused, in part, by those who present problem solving and computation skills as a dichotomy. Scientists often view mathematics as a subject one must master in order to be able to do science. *How* mathematics is taught is of little or no consequence to them just so students accumulate the skills necessary to function in science. Many mathematics educators seem to have come to view the topic of computational skills *within* mathematics in this same light. They show little concern for *how* computational skills are developed just so they *are* developed so one can use them to solve "real" problems.

Teaching the basic skills and teaching problem solving need not be mutually exclusive events. One adhering to the position that they are mutually exclusive is missing out on many opportunities to teach problem solving skills *while* teaching basic skills.

For example, several years ago a fourth grade class doing a unit on multiplying fractions was about to be introduced to the procedure for multiplying two numbers represented by mixed numerals. In the introduction to the lesson they were told they were about to be taken on what we call in mathematics a "free ride". (In reality it had never been called a free ride until that instant). A free ride was described as a situation in which we were studying a topic that was new, but a topic that could be attained by using other things we had previously learned. As indicated in the "mini-sequence" below,



the ability to multiply mixed numerals stems from being able to change mixed numerals to improper fractions and being able to multiply fractions. The procedure was then taught using a questioning strategy, leading the class to change the mixed numerals to improper fractions and multiplying as shown below.

$$2\ 2/3 \times 3\ 1/2 = 8/3 \times 7/2 = 56/6 = 9\ 1/3$$

The next day the last type of problem to be covered in the unit (multiplying a fraction by a whole number) was being introduced. The instructor had just finished stating that the reason this type of problem had been left until last was because it has been known to be very troublesome to fourth graders. At this point, Clyde, a student in the class, suggested to the instructor that this type of problem was not really hard at all--as a matter of fact, it was a free ride.

Encouraged to explain how this was a free ride he stated, "Look at the problem on the chalkboard."

$$2/3 \times 4$$

"Another name for 4 is 3 3/3 so put 3 3/3 in place of the 4."

$$2/3 \times 4 = 2/3 \times 3\ 3/3$$

"Now I know how to change 3 3/3 to an improper fraction (12/3) and I can use the multiplication rule to finish the problem."

$$2/3 \times 4 = 2/3 \times 3\ 3/3 = 2/3 \times 12/3 = 24/9$$

Clyde had become a problem solver. His solution was one that was new to his instructor--possibly new to mankind. Instead of sitting on the sidelines *watching* mathematics being developed, he had become a *developer of* mathematics. Since that incident, the author has made the free ride strategy an integral part of his teaching. This chance happening has resulted in many elementary students coming to view mathematics in a different and exciting manner.

The free ride is a capability Gagné might call a "cognitive strategy". A cognitive strategy is "a skill of self-management that the learner acquires to govern his own process of attending, learning, and thinking. By acquiring and *refining* such strategies, the learner becomes an increasingly skillful independent learner and independent thinker." (Gagné, 1974)

Experience in teaching the free ride strategy indicate that many students will be able to readily verbalize its meaning after limited exposure. The following quotes are from students who were asked after only one exposure to write out what was meant by a free ride.

"Use what you know, to make a problem easier." (A Second Grader)

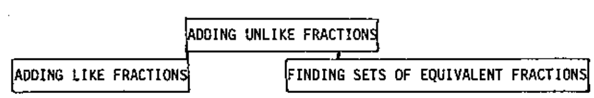
"I think that a free ride is when you're learning a new problem, but then you find out it's not really new because you already know the stuff you need to know to work the problem." (A Fourth Grader)

"When you are introducing something new, you can learn it easier because the two or three things you need to know to work the problem, you already know." (A Fifth Grader)

"I don't know, but I hope I will after today." (A Fifth Grader)

The goal of the teacher is not just to get students to learn the meaning of a free ride, but to get them to use this strategy independently to solve problems in the future. However, just knowing the meaning can help students see mathematics in a different light and get a feeling for the nature of mathematics.

After an initial lesson introducing the free ride strategy to a fourth grade class using addition of unlike fractions, the mini-sequence for this topic was drawn on the chalkboard as shown below.



One of the students asked, "What would you have done if we hadn't known how to add fractions with the same denominators?"

"I would have used some other things you knew to teach you to add like fractions," was the reply.

"What if we didn't know the things in those boxes?" asked another student.

"I would have used other things you knew to teach you those boxes."

At this point a third student, Michelle, commented, "Hey, I think I understand what mathematics is all about. There are a million boxes, all hooked together and all you have to do is work your way up to the top!"

What great insight! What a nice definition of mathematics! What a beautiful moment for her teacher! This was a critical moment in her mathematical experience. Mathematics could now be seen in a new light. Mathematics was no longer a spectator sport! She was ready to become a discoverer of mathematics . . . a producer rather than a consumer.

Another student then asked, "How do you decide which of the things you know to use to work a new problem?" We then discussed how you could brainstorm the problem . . . ask yourself questions such as how the problem differs from similar problems you can work; what; you know about types of numbers in the problem and how you could organize what you do know in some manner and eventually come up with a free ride.

The free ride strategy can help students become independent learners and thinkers. A fifth grade teacher, Jerri Brock* who made this strategy an integral part of her teaching commented,

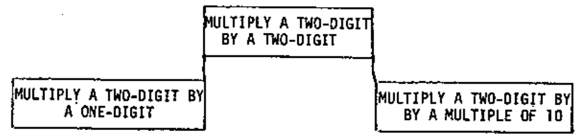
> "I have employed the free ride strategy to get my gifted math group started on an assignment while I work with another. Upon returning to the free ride group, I often find myself unwanted. The students have figured out how to work the problems and would prefer I find something else to do so they can complete their work without interruption. If they haven't figured it out they insist on only a minor clue to put them on the right track. (Some even cover their ears while I give the clue.) The free ride strategy has made my students better tutors. I no longer hear one just giving answers. Instead students give clues related to the skills they know are needed to work a problem."

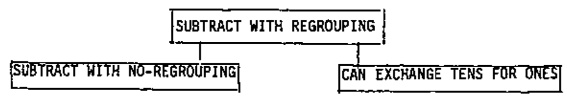
Obviously, the number of exposures to using the "free ride" strategy necessary for students to learn to use it on their own will differ from student to student. Some may never learn it. However, even with the latter students our efforts certainly will not have been wasted as our emphasis has been not only on learning the strategy, but also on learning significant mathematical capabilities. Instantaneous insights as exhibited by Michelle and Clyde are the exception rather than the rule, even with gifted students. Teaching students to use this strategy will require planning by the teacher so that the students have reocurring opportunities to both see the strategy applied and to attempt to apply the strategy themselves. The ability to apply this strategy, as the ability to acquire any meanings and understandings in mathematics,

> "are rarely if ever 'all or none' insights in either the sense of being achieved instantaneously or in the sense of embracing the whole of a concept and its implications at any time. The sudden perception or flash of insight, which is one of the joys of mathematics learning and teaching, comes only to those who have, with thought, struggled to extend or apply concepts which have been partially understood earlier. Further, meanings and understandings themselves change continuously as they are extended, broadened, and applied in different situations." (Jones, 1959)

*Jerri Brock is a teacher in the Aldine Independent School District, Houston, Texas.

Once the student has been introduced to the meaning of a free ride one begins to search for other competencies that can be attacked using the strategy. This search should prove very fruitful as almost every topic we teach in the elementary school fits into a neat "mini-sequence" as shown below. Start by doing a guided discovery lesson which is largely dominated by the teacher.





When closure for the lesson has been accomplished, draw the mini-sequence on the chalkboard and fill in the boxes. Go back and show a few examples of competencies they have developed in the past that were also free rides. Then ask the students if they can think of other things they have learned in the past that were free rides.

In subsequent lessons, the involvement of the teacher should diminish until, like the teacher mentioned earlier, you feel "unwanted". That feeling of being unwanted should be accompanied by feelings of exhilaration. As Trump and Miller suggest, "the basic goal of every teacher, insofar as students are concerned, should be to become dispensable as rapidly and as completely as possible." (Trump, 1979)

The goal of making oneself unwanted by our students is repulsive to many teachers. It does seem to take from us many of the "warm" moments that occur when closure for a lesson is accomplished. Indeed, it is just such "warm" moments that keep many of us in the teaching profession. In reality, the reverse is true.

Michelle gives a beautiful explanation of the nature of mathematics after a free ride lesson. To her it is an off-the-cuff comment . . . her teacher rates it as one of the highlights of his teaching career.

Given a chance to exhibit his creativity, Clyde discovers a unique free ride for the problem 2/3 X 4. He takes his discovery in stride--his teacher gets goose-bumps.

Try using the free ride strategy. It can go a long way toward helping create an environment in which students attack the entire mathematics curriculum as a turned-on problem solver. Rather than lose "warm moments", use of the free ride strategy can make us the benificiary of a share of the thrill of the many creative discoveries and flashes of insight of the students we teach. Just possibly, you may have an experience similar to the two mentioned above. If you do, you will be able to say to yourself "Now I remember why I'm a math teacher!"

REFERENCES

Gagne, Robert. The Conditions of Learning, Holt, Rinehart, and Winston, Inc., New York, 1965.

Gagne, Robert. "Educational Technology and the Learning Process." Educational Researcher, Vol. 3, No. 1, pp. 3-8. January, 1974.

Jones, Phillip S. "The Growth and Development of Mathematical Ideas in Children." The Growth of Mathematical Ideas Grades K-12. Twenty-fourth Yearbook of the National Council of Teachers of Mathematics, Washington, D.C., pp. 1-6, 1956.

NCTM, Agenda for Action. NCTM, Washington, D.C., 1980.

Trump, Lloyd and Delmas F. Muller, Secondary School Curriculum Improvement. Allyn and Bacon, Inc., Boston, 1976.

THE IMPROVEMENT OF TEACHING AND VOCATIONAL SATISFACTION

By Howard L. Penn and Clifford A. Hardy
Mountain View Community College — North Texas State University

INTRODUCTION

At the end of the last decade, the Mathematical Association of America and the National Council of Teachers of Mathematics produced the document "Recommendations for the Preparation of High School Students for College Mathematics Courses." In an attempt to determine to what extent these recommendations for the improvement of the mathematics program were accepted by teachers of mathematics, a study has been conducted utilizing three groups of mathematics teachers. An additional aspect of the study was to determine to what extent a relationship exists between a positive attitude toward the recommendations and vocational satisfaction in mathematics teaching.

PROCEDURE

In order to measure attitudes toward the recommendations and toward vocational satisfaction the **Survey of Certain Practices and Attitudes of Pre-Calculus Mathematics Instructors** and the **Purdue Scale to Measure Attitude To Toward Any Vocation** instrument was sent to a random sample of high school, community/junior college, and senior college/university instructors who taught pre-calculus college preparatory type courses. Of the 151 instructors contacted, a response was received from 42 high school, 56 community college, and 53 senior college instructors.

The survey instrument employed was the result of a questionnaire designed to measure the attitudes of the subjects toward the recommendations. Content validity was established by jury techniques and the test-re-test reliability coefficient was found to be .69. The validity and reliability of the Purdue Scale used to measure vocation satisfaction has been extensively reported in the literature (Remmers 1960). Pearson r correlation coefficients were calculated for the two attitude measures for the three groups utilized in the study.

RESULTS

The results of the correlation calculations are presented in the table below. As can be seen there is a significant relationship between attitudes toward the MAA—NCTM recommendations and vocational satisfaction for the community college and senior college instructors. That is, the more favorably these two groups viewed the recommendations, the more likely they are to express a higher degree of satisfaction with their assigned task of teaching college preparatory mathematics. No significant relationship between the variables was found, however, for the high school instructors involved in the study.

CORRELATION BETWEEN ATTITUDE VARIABLES

Group	N	Recommendation Scale	Vocational Satisfaction Scale	Pearson r
High School	42	1.785	8.166	0.0338
Community College	56	1.740	8.154	0.3058*
Senior College	53	1.658	8.091	0.3634*

* significant at the .05 level

In addition to the significant relationships reported above, it is interesting to note that the senior college teachers reported the strongest favorable attitude toward the MAA—NCTM recommendations while the high school teachers reported the lowest. On the other hand, the reverse order appears to be true for the vacational satisfaction scores. This effect can probably be explained by the fact the the total sample was asked to respond to the vacational satisfaction scale on the basis of teaching college preparatory classes only, thus controlling the occupational prestige factor somewhat. Similar results have been reported by Weaver (1977) who compared various occupations in terms of job satisfaction with the effect of occupational prestige factored out. Hopefully, the results of this survey will be of interest to those concerned both with improving standards as suggested by the recommendations and to those concerned with morale factors such as job satisfaction.

REFERENCES

Remmers, H. H. **Manual for the Purdue Master Attitudes Scales.** West Lafayett, Indiana: Purdue Research Foundation 1960.

Weaver, C. N. "Occupational Prestige as a Factor In the Net Relationship Between Occupation and Job Satisfaction." **Personnel Psychology, 1977,** 30, p. 607-612.

DICE POKER PROBABILITIES

Bonnie H. Litwiller and David R. Duncan
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University of Northern Iowa
Cedar Falls, Iowa

Most students and teachers are familiar with the card game called poker. An interesting and related game of chance is dice poker. This game uses much of the same terminology as card poker. The rules of dice poker follow.

Two or more players participate. For a given game each player has one "turn". Suppose it is Earl's turn. Earl does the following:

A. He rolls the five dice.
B. He may pick up any or all of the five dice and roll them again. This is optional as he may decide to terminate his turn with the results of the first roll.
C. Once again Earl may pick up any or all of the five dice and roll them. This third roll is again optional as Earl may wish to terminate his turn with the results of the second roll.

After Earl completes his turn he then records the five dice that are showing. Call this the "dice poker hand". The dice are then passed to Lynn who takes his turn; this continues until everyone has had one turn.

It then must be determined who won the game. The winner is the player with the highest ranking hand. The ranks are:

1. Five of a kind (highest rank).
2. Four of a kind (4 dice show the same number while the fifth die is different).
3. Full house (3 dice show one number; 2 dice show another number).
4. Straight. There are three admissible straights (1, 2, 3, 4, 5; 2, 3, 4, 5, 6; and 3, 4, 5, 6, 1). As in card poker, the ace can be considered as either high or low.
5. Three of a kind (3 dice show the same number; the other two dice show two additional numbers).
6. Two pairs (2 different number pairs while a fifth die is different).
7. One pair (2 dice the same; the other three show three different additional numbers).
8. No pairs (none of the above).

When Earl rolls the five dice, what is the probability of his achieving each of the eight defined outcomes on his first roll? To compute the probabilities, repeated use of the Fundamental Principle of Counting will be made; that is, if task 1 can be done in n_1 ways and if, following task 1, task 2 can be done in n_2 ways, the total number of ways in which tasks 1 and 2 may be consecutively performed is $n_1 \cdot n_2$. Also, recall that

$\binom{m}{n} = \frac{m!}{n!(m-n)!}$ and represents the number of ways that a subset of n elements may be selected from a set of m elements.

To determine the total number of outcomes when five dice are rolled, visualize the five dice in positions such as:

Die Position

1 2 3 4 5

Since each die may display any of six faces, the total number of possible outcomes is $6 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 6^5 = 7776$. Each of the six faces is equally likely to appear on each of the five dice; thus, it may be concluded that all 7776 "5-dice-results" are equally likely.

To determine the probability of each of the eight defined outcomes, we will compute the number of the 7776 results which will produce that desired outcome. This number will then be divided by 7776 to compute the probability.

Outcome 1: Five of a kind

Since all 5 dice show the same number, the only choice concerns which number will show. Since this choice may be made in 6 ways, there are six results yielding the 5 of a kind outcome. The probability is thus

$$\frac{6}{7776} = .001.$$

Outcome 2: Four of a kind

Perform the next three tasks consecutively.

A. Identify the number which will occur 4 times. This task may be performed 6 ways.
B. Select the 4 dice which are to display the repeated number. This may be done in $\binom{5}{4} = 5$ ways.
C. Select the number which is to be displayed on the fifth and "different" die. This can be done in 5 ways.

These 3 tasks may be consecutively performed in $6 \cdot 5 \cdot 5 = 150$ ways.
The probability is then $\frac{150}{7776} = .019.$

Here and henceforth, following each task, the number of ways that the task can be performed will appear in parentheses.

Outcome 3: Full House

A. Identify the number which will occur 3 times. (6)
B. Select the 3 dice that are to display the number chosen in A.
$$\left(\binom{5}{3} = 10\right)$$
C. Choose the number which is to appear on the remaining 2 dice. (5) The probability is then

$$\frac{6 \cdot 10 \cdot 5}{7776} = \frac{300}{7776} = .039.$$

Outcome 4: Straight

A. Select which of the 3 admissible straights is to be used. (3)
B. Determine the location of the second highest valued die. (4)
D. through F. Determine the locations of the third, fourth, and fifth highest valued die. (3, 2, 1)

The probability of a straight is thus

$$\frac{3 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{7776} = \frac{360}{7776} = .046$$

Outcome 5: Three of a kind

A. Choose the number which is to appear 3 times. (6)
B. Select the 3 dice that are to display the number chosen in A.
$$\left(\binom{5}{3} = 10\right)$$
C. Identify the 2 numbers which are to appear on the remaining 2 dice.
$$\left(\binom{5}{2} = 10\right)$$

The probability is $\frac{6 \cdot 10 \cdot 10}{7776} = \frac{600}{7776} = .077$

Outcome 6: Two pairs

A. Select the 2 different numbers that are to be used in the 2 pairs.

$$\left(\binom{6}{2} = 15\right)$$

B. Select the 4 dice on which the 2 pairs are to occur.

$$\left(\binom{5}{4} = 5\right)$$

C. Select which, of the 4 dice chosen in B, each of the 2 pairs will occupy.

$$\left(\binom{4}{2, 2} = \frac{4!}{2!2!} = 6\right)$$

D. Choose the number to appear in the fifth and different die. (4) The probability is thus

$$\frac{15 \cdot 5 \cdot 6 \cdot 4}{7776} = \frac{1800}{7776} = .231.$$

Outcome 7: One Pair

A. Select the number to be used in the pair. (6)

B. Select the 2 dice on which the number chosen in A is to appear.

$$\left(\binom{5}{2} = 10\right)$$

C. Choose the 3 different additional numbers that are to appear on the dice.

$$\left(\binom{5}{3} = 10\right)$$

D. Identify which of the 3 remaining dice each of these 3 numbers will occupy.

$$\left(\binom{3}{1, 1, 1} = 6\right)$$

Thus the probability is

$$\frac{6 \cdot 10 \cdot 10 \cdot 6}{7776} = \frac{3600}{7776} = .463$$

Outcome 8: No pairs

Outcomes 1 through 7 have accounted for 6816 of the 7776 possible 5-dice results. Consequently, only 960 results remain for this outcome. The probability is then

$$\frac{960}{7776} = .123$$

The readers who are card poker players will recall that the hands in card poker are ordered in reverse order of their likelihoods; that is, the least likely poker hand (straight flush) has the smallest probability. The second least likely (4 of a kind) has the next higher value, etc. The most likely hand (no pairs) has the least value.

In designing the common rules for dice poker, essentially the same ranking for the dice hands was used as in card poker. (Note there are no flushes in dice poker although 5 of a kind is possible.)

This leads to a strange anomaly. "No pairs" in dice poker is fairly unlikely (.123). This makes no pairs less likely than either 2 pairs (.231) or 1 pair (.463) and should lead to no pairs outranking these two outcomes. If this were done, the strategy of dice poker might change considerably. Players would want to get rid of single or double pairs in favor of no pairs.

Challenges for the reader

1. For the various outcomes resulting from the first roll on a turn, compute the probability that the hand could be improved by subsequent rolls of some of the dice. In practice, the number of dice that a player rerolls may be influenced by the turns of previous players.
2. If six dice were to be used, name and compute the probability of the different hands that might result in the first roll of the dice. Compute their probabilities.
3. Using the other regular polyhedral dice and the rules described at the beginning of this article, compute the probabilities of the different outcomes on the first roll of the dice.

Bibliography

The Way to Play: The Illustrated Encyclopedia of the Games of the World, sPaddington Press Ltd., New York City, (1975).

SPATIAL IMAGERY: A TOOL FOR TEACHERS OF MATHEMATICS

By Raymond Brie
Texas Christian University

If you believe that teaching mathematics with the aid of models is a fruitless and unproductive endeavor, then it could be time for spatial imagery to enter your mathematics classroom. Spatial imagery is a cognitive ability that all learners have at their disposal (Smith, 1964; Piaget and Inhelder, 1971). It is a mental construction of a real or nonexistent object, process, or concept.

Mathematics content can be mentally coded (processed) either verbally, spatially or in combination of the two (Paivio, 1971). Classroom teachers usually emphasize verbal processing when students are learning mathematics. Traditional mathematics education is only marginally concerned with helping learners develop the ability to mentally manipulate either concrete or pictorial models. However, spatial imagery can be a tool for learning addition, subtraction, multiplication, division and place value concepts.

Mathematics educators have historically supported the use of concrete and pictorial models during instruction (Brownell and Henrickson, 1950; Dienes, 1960; Payne, 1975; Scott and Neufeld, 1976). However, teachers in general have not incorporated the use of models as part of their teaching methodology. Many teachers fail to see the connection between using models and the resulting intellectual growth of the learner. Unfortunately, researchers have not fully clarified this relationship (Moses, 1982; Wacksmuth, 1981). If models can be shown to foster conceptual development, teachers may be encouraged to use them in their mathematics programs.

A technique that can be successful when using models is to ask the learner to construct or imagine the model in his/her head. Next ask the student to mentally move the spatial image. For example, rotate the image — or cut the image and move the parts. Finally, ask the learner to make the image disappear. In a few seconds, ask the learner to mentally reconstruct the spatial image. If the learner can reconstruct the image at this time, the student probably has the ability to use it as a model for learning mathematics.

Dirkes (1980) has suggested the following guidelines for a teacher interested in systematically using spatial imagery as part of his or her mathematics program:

1) Give initial presentations with pictures and objects. Seldom refer to a concept without drawing a picture. (The teacher should provide concrete and pictorail models when developing new mathematics concepts.)

2) Ask students to draw pictures on graph paper or at least suggest that they draw pictures large enough and with space between items. (The teacher should encourage students to organize their pictures, drawings or illustrations so the models help focus on the concept or the problem. Does the model clarify the mathematics that is presented? If not, the student's thought process needs to be examined.)

3) Encourage students to draw original pictures that express ideas which are meaningful to them. Often they will need to evaluate and to revise their pictures many times to agree with a concept. The revisions are to be expected as part of the problem-solving process. (Teachers can use these drawings as a diagnostic tool. Can the drawings help both the students and teachers examine spatial thinking?)

4) When students have difficulty expressing ideas in words, ask them to draw a picture of what the concept means to them. Examine the drawing diagnostically. (Some students can't express their ideas verbally. Provide these students with and alternative way of communicating. Allow them to be successful learners! The students' learning style may differ from your teaching style.)

5) Compare related concepts through visual representations (p. 11). (Ask students to form spatial images. The teacher should stimulate the students to use the spatial images to compare and contrast mathematicsl ideas. The images could be rotated, separated, combined or superimposed to accomplish the task.)

Processing information through these techniques is fundamental to developing a student's spatial imagery thinking.

Mathematics programs tend to emphasize the verbal coding of mathematics content and relationships. Teachers should provide an alternative coding process for students. The use of spatial imagery as a mathematics tool can enable the learner to think differently and more effectively. Since spatial imagery is an important part of cognitive development (Bishop, 1980; Clements, 1981, 1982), it would seem logical to include spatial imagery development in an effective mathematics program.

REFERENCES

Bishop, A. J. Spatial abilities and mathematics education — A Review. **Educational Studies in Mathematics,** 1980, (3), 257-269.

Brownell, W. A. and Henrickson, G. How children learn information concepts, and generalizations. **Learning and Instruction: 49th Yearbook, Part I,** Chicago: National Society for the Study of Education, 1950.

Clements, N. A. (Ken) Visual imagery and school mathematics (part 1). **For the learning of Mathematics,** 1981, 2 (2) 2-9.

Clements, M. A. (Ken) Visual imagery and school mathematics (part 2). **For the Learning of Mathematics,** 1982, 2 (3), 33-39.

Dienes, Z. P. **Building up mathematics.** London: Hutchinson Educational, Ltd, 1960.

Dirkes, M. A. Say it with pictures. **Arithmetic Teacher,** 1980, 28 (3), 10-13.

Moses, B. Visualization: A different approach to problem solving. **School Science and Mathematics,** 1982, 82 (2), 141-147.

Paivio, A. **Imagery and verbal processes.** New York; Holt, Rinehart and Winston, 1971.

Payne, J. N. (Ed.) **Mathematics learning in early childhood.** (37th Yearbook National Council of Teachers of Mathematics), Weston, Virginia: National Council of Teachers of Mathematics, Inc., 1975.

Piaget, J. and Inhelder, B. **Mental Imagery in the Child.** New York: Basic Books, 1971.

Scott, L. F. and Neufeld, H. Concrete instruction in elementary school mathematics: Pictorial vs. Manipulative. **School Science and Mathematics, 1976,** 76 (1), 68-72.

Smith, I. M. Spatial ability: **Its educational and social significance.** San Diego, California: Knapp. 1964.

Wacksmuth, I. Two modes of thinking — also relevant for the learning of mathematics? **For the Learning of Mathematics,** 1981, 2 (2), 38-45.

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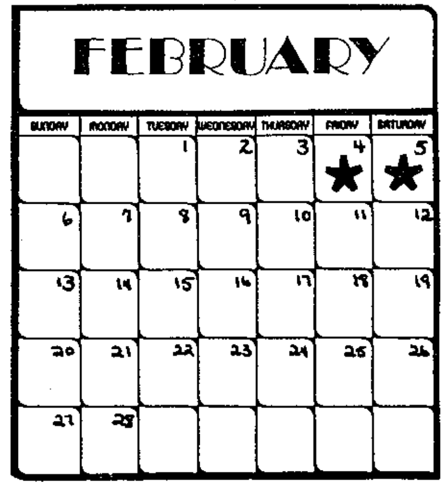

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