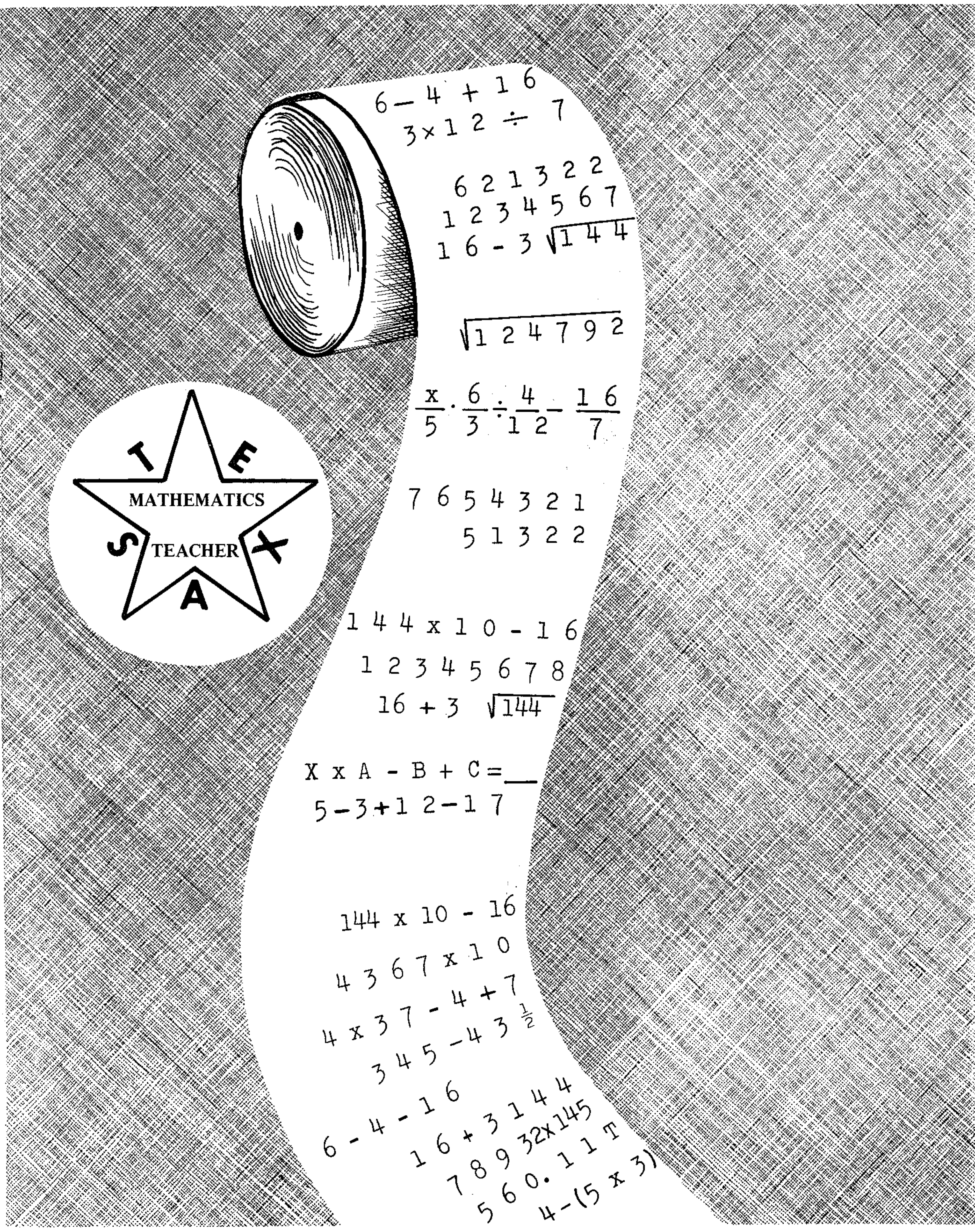
TEXAS MATHEMATICS TEACHER



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PRESIDENT'S MESSAGE

Dear TCTM Members,

Another school year is rapidly drawing to a close and with this closing comes summer vacations, travels, and/or summer studies. My hope is that all of you have a safe, rewarding and stimulating summer and return to us next fall eagerly anticipating another full school year. I hope also that you renew your commitment to TCTM and help us in our efforts to make Mathematics Education in the state of Texas the very best it can be.

This past year in the Texas Council has been both enjoyable and rewarding, yet (as in all activities of life) not without some disappointment. Positive actions include the tremendous CAMT meeting in Austin, the publication of our TCTM journals, the FOM Conference in San Antonio, the selection of a committee to produce a monograph from the Council on Teaching Basic Math Skills and the association of the Texas Society of Professional Engineers with our Council to participate in their National Task Force program of Math Counts. We are excited about working closely with the engineering society in this endeavor.

All of these actions require work and effort; and we encourage each of you to become involved and lend us your support. I truly believe that the future of the state of Texas is one of tremendous possibilities for growth and advancement – especially in the areas of Math/Science Education. We need to work hard to let our expertise and years of experience of teaching mathematics be a factor in this growth effort and provide a leadership role to those making decisions concerning the future.

TCTM encourages your participation. Sign up now to be a speaker at CAMT or the FOM Conference, write an article for our journal or the monograph series, or offer to help in the Math Counts program on the state level. Get involved in TCTM. The effort is well worth the price.

Sincerely,

Betty Travis, Ph.D.

CAMT

PALMER AUDITORIUM
AUSTIN, TEXAS
OCTOBER 6–8, 1983

THE THIRD "R" IN THE BIG RED SCHOOL HOUSE: THE SOVIET CHALLENGE IN MATHEMATICS EDUCATION

Judy Reinhartz
University of Texas at Arlington

In recent years mathematics education in the United States has come under considerable attack. As the average quantitative score in mathematics on the Scholastic Aptitude Test continues to decline (a drop from 502 in 1963 to 466 in 1980) and more and more states legislatively endorse competency testing for promotion and/or graduation, the criticisms against the mathematics curriculum continue to mount. Perhaps of all the disciplines in a curriculum, mathematics receives the most international attention. Many countries of the world to accomodate technological advance are experiencing a "revolution," not in politics alone, but in their mathematics curriculum. A common reaction from American educators and citizens when confronted with the discrepancy between the mathematics curriculum in the United States and other countries, particularly the Soviet Union, is one of defensiveness.

Wirszup in numerous articles sounds the alert and hopes to raise the public awareness concerning the importance of mathematics learning in the United States. He says our survival as a nation is at stake. Some educators discount him as a fanatic, but he points out that the Soviet Union has launched a massive drive to upgrade the tecnological general education program with mathematics at the heart of this reform movement.

Comparing two very diverse countries such as the Soviet Union and the United States is a difficult task. If initial differences in culture, politics and values are not enough, accurate communication is hampered because of language. In some cases, words like **vospitaniyeh** do not have just one English Equivalent. A translation of this term might be "moral education" or inculcating good work habits and good citizenship. Precise meanings of concepts, assumptions, and expectations are often impossible to achieve.

Recognizing these differences, a comparison study was undertaken. The present study attempted to compare the compulsory education system of the ten year school and its emphasis on mathematics in the Soviet Union with the mathematics curriculum in the public educational system grades 1 — 12 in the United States. This study is based on an examination of text materials, teaching methodology, curriculum materials, articles for teachers, students and the general public, statistics for entrance and graduation requirements for institutions of higher learning, and observations in mathematics classrooms in the Soviet Union and the United States. The recent effort by the Soviets to mobilize their educational system in general, and particularly their mathematics curriculum, poses a formidable challenge to educators and mathematicians in the United States and this challenge warrants close scrutiny.

After decades of educational expansion in the Soviet Union, 98 percent of the school-aged population complete a secondary school or an equivalent education, compared with 80 percent in the United States. [2] The general education schools in the Soviet Union are tuition free, public, and coeducational.

A team of scholars in the 1960's from the U.S.S.R. Academy of Sciences and the U.S.S.R. Academy of Pedogogical Sciences headed by A.N. Kolmogorov, a distinguished mathematician, has been responsible for the major reforms in mathematics education. The Soviets also have achieved excellence in advanced mathematical research. Models of excellence can be seen in the research conducted by P.S. Novikov in mathematical logic/"word problems," Gelfand in functional analysis, and A. Malcea in the relations between logic and algebra.

Academicians who advocate a return to traditional methods, criticize Kolmogorov's approaches to mathematics instruction as too modern. The reform movement in mathematics education which began in 1964 for the "new school mathematics" established the goals for the new program and is responsbile for the curriculum design which was implemented in all schools by the middle of the 1970's. The mathematics curriculum as described includes modern content which emphasizes the theoretical foundations, as well as application of concepts and innovative methodology. According to Kuznetsova and Leant'eva (1977), "the new mathematics curriculum is considerably richer on the conceptual plane." [3]

The mathematics content taught and the methods employed reflect the research being conducted in psychology and learning theory. For Soviet educators and psychologists, unlike those in the United States, learning in school is inexorably linked to cognitive development. Soviets are generally negative toward Piaget and his interpretation of cognitive development. [4]

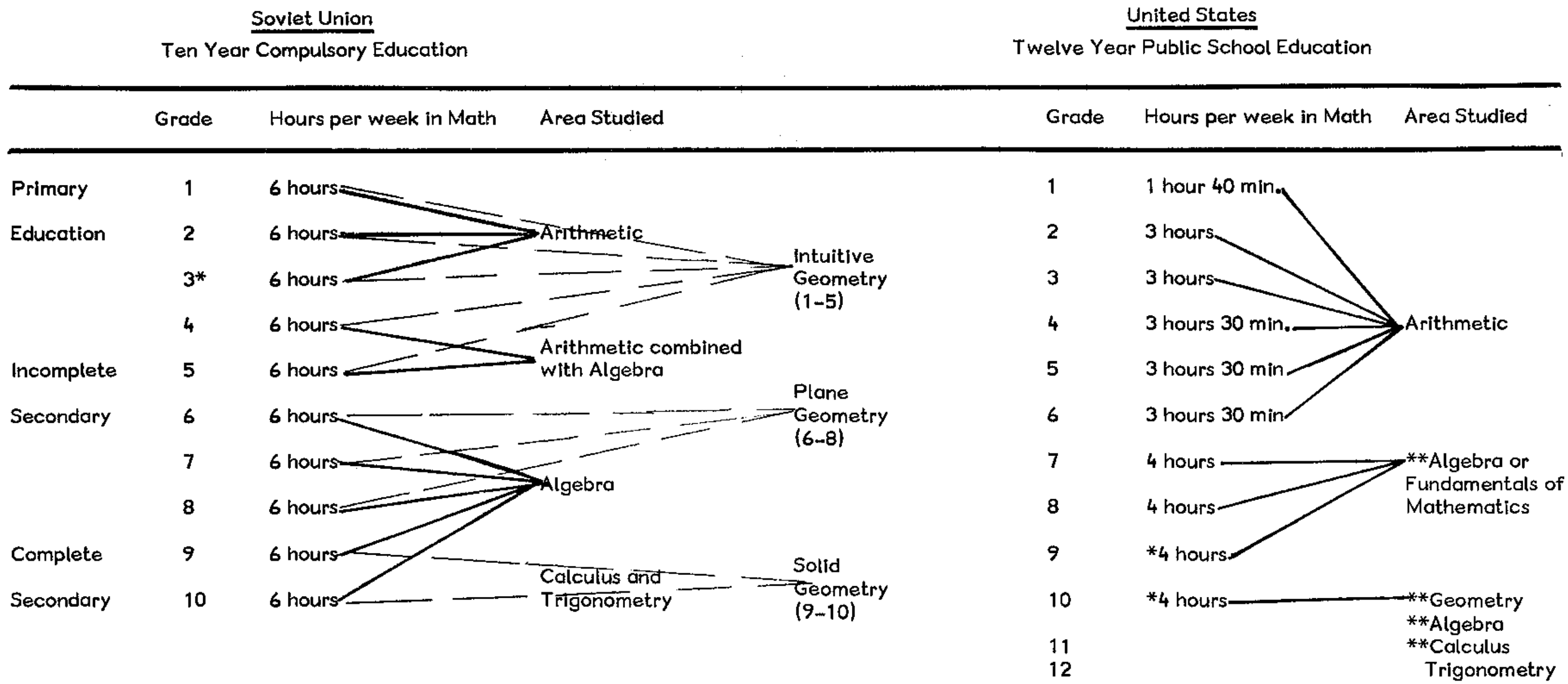
> "In the final analysis, a pupil's mental development is determined by the content of what he is learning. Existing intellectual capabilities must therefore be studied primarily by making certain changes in what children learn at school." [5]

This is in contrast to Piagetian psychology. For the western psychologist, concepts develop independently from the school curriculum. [6] This represents a fundamental difference about how cognition develops.

It would take a child in the United States 13 years of arithmetic, algebra, and calculus to achieve the equivalent mathematics background of a student attending the 10 year compulsory school. [7, 8] Unlike over 5,000,000 graduates of Soviet secondary schools who had studied calculus for two years only 105,000 high school graduates in the United States studied calculus in 1976 (up from 55,000 in 1972—1973) for just one year. In college, the situation did not improve. [9] In the fall 1975, the number of college students studying calculus was only 397,000. In institutions of higher learning in the Soviet Union this number was quadrupled.

In addition to the 2000 class hours spent studying mathematics in the ten year school, students take elective courses which foster independent work (See Figure 1). Mathematics accounts for one-fifth of the curriculum in the ten year school. To better appreciate the uniqueness of the ten year school, it may be described as a combination of a voacational high school, a technical institute, and a community college in the United States. [10]

Figure 1



Total hours: 2,000 hours

*In grade 3, mathematics is taught as a separate study

665 hours (through grade 9)
(25 hours x 27 weeks (175) days

*(56% of the high schools require 1 or no math courses for graduation. 9% require more than one course of math or science).

**Math course selected depends on high school program

Students are required to attend school from age 7 to 17. Classes begin early in the morning and end in the afternoon. Students go to school on an average of six days a week for almost ten months (September 1 to late May or June depending on the grade level or for approximately 240 days). Mathematics is usually taught in the morning along with science and foreign language. [11] In contrast to the situation in the United States, mathematics specialists teach students beginning with the fourth grade.

Both countries have special schools for nurturing mathematical talent, but in the Soviet Union there is a definite commitment. In the Bratsk school theoretical mathematics (abstract algebra, topology, and non-Euclidena geometry) is emphasized. Contests and quizzes test the student's ability in formualting and solving complicated problems. At the "Kolmogorov School" in Moscow 51 percent of the time is spent learning mathematics and physics. The math/physics curriculum includes algebra and analysis, analytic geometry, linear algebra, discrete mathematics, probability theory, computer programming, and physics. [12]

The curriculum for these schools was extremely successful and served as the prototype for the mathematics curriculum of the ten year school. Mathematically talented students in socialist countries also participate in the well known international Mathematical Olympiad.

In the United States, programs for the mathematically gifted are too few and hapharzardly coordinated to make a significant difference. [13] The Mathematics Education for Gifted Secondary Students program in St. Louis developed by CEMREL includes abstract algebra, linear, algebra, and measure theory. The CERL (Computer-based Education Research Laboratory) computer project at the University of Illinois is probably one of the largest computer systems devoted to student learning.

In the United States, 75 percent of the students study arithmetic for 9 years. During the elementary years, students are taught by elementary teachers who are not trained in mathematics. A cursory review of requirements for elementary certification across the United States reveals that the mathematics requirement ranges from no course to a maximum of two (6 credit hours). In the Soviet Union, teachers with general elementary certification who teach in grades 1—3 receive training in mathematics. This training includes 5 years of algebra and 10 years of geometry and calculus. Teachers assigned to grades 4—10 are especially trained and teach only mathematics. This preparation is almost comparable to a master's degree in a university in the United States. [14, 15]

Secondly, the orientation of the elementary mathematics curriculum in the United States is basic arithmetic with an emphasis on drill. The consequences most often are negative attitudes toward mathematics and feelings of incompetency. [16, 17, 18] However, there are reports that a similar situation is developing in the Soviet Union. Students are failing mathematics in greater numbers and like it less. [19]

In the Soviet Union, geometry is integrated throughout the entire ten year curriculum and solid geometry is emphasized in the last two years (See Figures 1, 2). In contrast, in the United States geometry is offered at the 9th, 10th, or 11th grade level depending on the school district, but only half of the eligible students take it. Those who do complete a course in geometry rarely have a working knowledge of three-dimensional space and are not taught solid geometry.

Lastly, only 4 percent of the graduating seniors take any course in calculus. Soviet students in contrast take differential and integral calculus (See Figure 1).

In the Soviet Union there is a close working relationship among math scholars, educators, psychologists, and methods specialists. They boast of radically different instructional strategies being implemented in the classroom; [20] in actuality teaching styles remain more traditional. Teachers employ a "telling" approach and perceive mathematics as a series of facts. This perception may be a contributing factor to the growing negative attitudes developing toward mathematics. [21] Ideally, the goal of mathematics instruction is to teach the student to solve the problem on his/her own; yet, in practice, the teachers use a deductive approach and provide ready-made solutions. Nevertheless, within a 50 minute lesson, the students are engaged for 37 minutes. [22] This is higher than an average mathematics lesson in the United States, running about 22 minutes. In addition to being on-task longer in the mathematics class in the Soviet Union, the pace of mathematical instruction is more rapid.

While the literature is replete with curricular innovations, most classroom instruction in the United States in mathematics like the Soviet Union remains in the expository mode. Much of the time allotted to mathematics instruction in the United States is lost to interruptions, discipline problems, and procedural matters. [23] In American schools, the textbook used is regarded as the "primary determinant of the mathematics curriculum . . . in terms of content selection and sequencing." [24] Teachers in the United States seem to utilize a "whole group" approach making few accomodations for individual students. Drill is the predominant means of learning.

It is difficult to compare Soviet students in mathematics with their counterparts in the United States. Soviet theorists reject the use of the battery of standardized tests which are administered to students in western schools. Within the Soviet Union, a clinical assessment which involves several personnel (doctor, psychoneurologist, speech therapist, hearing specialist, teacher) and achievement measures which are less controlled. The emphasis is on "daily checking" of student's work. End-of-year examinations are administered, but only at the discretion of local school officials. [25] Students do take academic examinations at the completion of their secondary education (after the eight or tenth year of school). This non-standardized psychometric evaluation which emphasizes achievement relative to the curriculum not to peers may be a feasiable tool for Soviets to use to examine a child's capabilities, but cross-comparisons between countries using percentiles are not possible.

The success of the Soviet mobilization in education is due in part to the high degree of centralization that exists. For those who monitor educational developments in the two countries, centralization seems to be the key difference. A "national curriculum" complete with teacher instructions exists which serves to unify the iffteen diverse republics. [26] Teachers adhere to an instructional system which follows like a script. The lengthy syllabus at each grade level outlines a didactic sequence which essentially includes a lecture first followed by questions. This leaves little autonomy to the teacher for personal preference and individual teaching style.

A second factor which has contributed to the reform efforts in the Soviet Union stems from the politicalization evident in the schools. Politics start at an early age for children in the Soviet Union. Indoctrination procedures begin in the

"Detskii Sad" (nursery/Kindergarten) and continue until death. At age seven, the child becomes a "Young Octoberst", joining the first of a succession of state sponsored youth groups, and enters school. The most important landmark is that of becoming a Young Pioneer at age eleven. Comparing skills, crafts and political ideology are the highlights of this educational experience. And one year of philosophy is required before the student completes "Polynay Srednaya Shkola" (high school). When a Soviet youth reaches the age of 14, they may, if they are found acceptable, join the organization, the Young Communist League (Komsomol). Komsomol members are taught that they must fight for their ideals. They are made aware of the cost of their education and the debt that they must repay.

The purpose of the political indoctrination is to instill a spirit of nationalism and a deep respect for the Motherland, Mother Russia. Politicalization commences at an early age and remains all pervasive throughout the individual's life. Soviet youth grows politically and educationally as decrees are issued from the Central Committee of the Communist party and the U.S.S.R. Council of Ministers. Both provide the centralization necessary to successfully mesh the Soviet educational system with the existing political system.

It should not be construed that the adoption of the Soviet mathematics curriculum in the United States is recommended. American schools are different and the philosophy guiding their daily operations is at opposite ends of the educational continuum. This study attempted to analyze the mathematics education in the Soviet Union and in the United States. It is only through such an examination that educators in the Uniteds States will be stimulated enough to consider alternatives.

For the Soviets, mathematics is popularized as being socially useful. The Soviet mathematicians are responsible for translating for the psychologists and the pedogogical scientists the concepts and skills to be taught and methods to be utilized. Within the Soviet educational framework, curriculum development is from the top down and the research paradigm is adhered to seriously. In the United States, perhaps to a fault, new curriculum ideas are relatively easy to introduce and there is much lip-service given to applying research to the classroom. By examining and analyzing different processes, the appreciation of each system unfolds. Comparative studies also help to illuminate common educational concerns.

As innovative as their mathematics curriculum may appear, Soviet pedgogical scholars are voicing concerns. There is evidence that teacher education programs are failing to train teachers to meet the challenges inherent in the mathematics curriculum. [27] Yet, when the requirements for becoming a certified teacher are examined (an entrance exam to enter institutions of higher learning, an exit examination, specialized courses, and a probation period of one year) the reader may be surprised. Unlike their American counterparts, many of the critics of education within the Soviet Union look to reforms in teacher training as the means of reforming the entire educational system.

Soviet scholars are not satisfied with the procedures used to identify and instruct talented individuals. The paradox exists between wanting students to conform and at the same time to be creative. These concerns are not new, nor are they confined to the Soviet Union. They plague educators the world over. If a highly centralized state has been unable to eradicate these problems, educators in the United States can take heart.

It is time to begin to examine the mathematics curriculum in grades 1 – 12 in the United States. If Americans are going to enjoy a high standard of living, to continue to develop technologically, and to maintain a leadership position in the world, we should address the issue of educating its citizens to meet the current and future scientific challenges. Soviets have attempted to do something about reaching this goal; many Americans are not even concerned about such questions as Does the mathematics curriculum in the United States need to be reformed? Should mathematics be taught deductively? or inductively? or both?

After conducting this comparative study, some results and recommendations are offered. One result of this study may be the growing realization that the two countries have many educational concerns in common. Both countries are concerned about the overformalism of the mathematics content at each grade level, the need for a stronger link between mathematical theory and practical application, the cry of back-to-the basics, preserving traditional values, the lack of instruction in computer literacy, and the commitment to a general educational system for all students.

One recommendation is to encourage teachers in the United States to be more conscious of how they manage their time. Time on-task according to research data compiled by Rosenshine and Furst (1971) is an extremely important characteristic of good teaching which relates to achievement gains. [28] As students are engaged for longer periods of time, the emphasis is on both the appreciation of the process involved in solving mathematical problems as well as the product.

Specialized training in mathematics should be required for those planning to teach in the elementary school. With the current shortage of mathematics teachers, [29] teacher education programs should consider adding a mathematics education component to existing requirements. This might become a viable option as teacher candidates become required to demonstrate competency in basic arithmetic on the National Teachers Examination or other standardized tests.

Prospective teachers are coming to me and asking, " . . . how do I teach concepts in mathematics, for example, in geometry when I do not understand these concepts in the first place." One methods course in mathematics for elementary teachers will not solve this problem. By adding more general mathematics courses to a teacher education program, teacher education candidates will gain knowledge and the needed confidence to teach mathematics in the elementary school. This recommendation also might help to improve the self-concepts of these future elementary teachers. Advanced courses such as differential equations, abstract algebra, and multivariable calculus may not be necessary, but certainly mastery of arithmetic skills and basic concepts in geometry and algebra would be a must.

Lastly, schools of education and inservice teachers should continue to cling to the doctrine of individualism. Elementary teachers need to recognize and try to help develop the potential and talents of each individual. Individualism is one of the significant characteristics which separates our educational system from the Soviet's. Nevertheless, this interest in individualized instruction needs to be rekindled and a meaningful diagnostic-prescriptive system should be developed and implemented. A system which employs formative and summative evaluations will yield learning gains and more positive attitudes toward mathematics.

Politics and ideological issues aside, the mathematics curriculum in the United States needs to be evaluated. It would be a refreshing change to have Americans on the offensive in the world educational arena rather than the defensive.

In the final analysis, decisions concerning the mathematics curriculum in grades 1 – 12 in the United States should reflect the very best that educators, psychologists, and mathematicians have to offer young people today and in the future. The entusiasm of the post-Sputnik era which characterized the 1950's and late 1960's needs to be revitalized and nurtured. Our response in 1957 initiated needed curriculum reform. It is time that a second wave of curriculum reform be launched and the mathematics curriculum is a perfect place to begin.

Figure 2

SCOPE AND SEQUENCE OF THE 1975 MATHEMATICS CURRICULUM (Soviet Union)

Adapted from Robert B. Davis, Chap. IV, 1979

Grades 1-3* (ages 7-10)

1. Operations
   - a. addition
   - b. subtraction
   - c. multiplication
   - d. division
2. Laws
   - a. commutative
   - b. associative
   - c. distributive
3. Introductory algebra
4. Measurement
   - a. area
   - b. volume
5. Geometry
   - a. point
   - b. line
   - c. angle
6. Example of a Third Grade Mathematics Curriculum (Pschelko, 1971)
   - 1. skill in reading and writing multi-digit numbers
   - 2. place-value meaning of digits
   - 3. place-value meaning of digits
   - 3. "smaller than" and "greater than" as applied to four, five or seven six-digit numbers
   - 4 expanded notation for place-value numerals (decimal base only)
   - 5. measuring lengths (metric)
   - 6. measuring weight (metric)
   - 7. the study of the 4 arithmetic operations for multi-digit numbers.
   - 8. the solution of "missing addend" or "missing minuend" problems such as 18 + x = 60 or x – 16 = 64
   - 9. experience with expressions such as (932-256) ÷ (68-17)
   - 10. ability to solve word problems such as "1 ton of potatoes was taken from three sections: 450 kilograms from the first, and 3 times less from the second. How many kilograms of potatoes were taken from the third section?"

*There is an important connection between geometry, algebra and arithmetic (Shabanowitz, 1978).

<u>Grade 4 and 5</u>

6. Fractions
7. Decimals
8. Physics
   - a. simple equations (s=v·t)
9. Geometry
   - a. areas
   - b. volume
   - c. plotting
   - d. tools: use of ruler, protractor and compass to construct geometric figure
   - e. angles
   - f. parallel lines
   - g. theorem of Pythagoras
10. Positive and Negative numbers

<u>Grades 6-7</u>

11. Concept of function
12. Topics:
   - a. inequalities
   - b. graphical representation of functions
   - c. exponents
   - d. polynomial functions
   - e. solving for two unknowns

<u>Grade 7</u>

13. Rational functions
   - a. transformation of expressions

Ex. $$\frac{\frac{1}{x}-\frac{1}{y}}{\frac{1}{x}\ \frac{1}{y}} = \frac{y-x}{y+x}$$

$$\frac{x^2+ax+bx+ab}{x^2-a^2} = \frac{x-b}{x-a}$$

   - b. Binary numerals
   - c. Absolute and relative errors, which includes the formula

$$\Delta\left(\frac{a}{b}\right) = \frac{b\,\Delta a - a\,\Delta b}{b\ (b+\Delta b)}$$

$$\Delta\frac{(a \div b)}{a \div b} = \frac{\Delta a}{a} - \frac{\Delta b}{b}$$

   - d. The function $4 = \sqrt{x}$

14. Geometry
   a. circles
   b. chords
   c. arcs
   d. central angles
   e. intersecting and tangential circles
   f. inscribed and circumscribed circles
   g. translation and rotation

Grade 8

15. Solution of quadratic equations
16. Vieta's coefficeint rules
17. Proofs by mathematical induction
18. Fractional exponents
19. Informal treatment of traditional numbers
20. The logarithmic function

Formulas: $\log_B A = \log_B C \log_C A$

Grade 9

21. Limit of infinite sequences
22. Irrationality of $\sqrt{2}$
23. Limits of functions
24. Continuity
25. Vectors
26. Trigonometry
   a. differentiation of trigonometric functions
27. "Differentiation of exponential and logarithmic functions (application to problems of growth and decay in physics, biology and economics) . . ."
28. Anti-differentiation
29. Definite integrals
30. "Surface areas and volumes of solids of revolution"
31. Harmonic oscillator
32. Probability
33. Geometry
   a. Euclidean geometry
   b. proofs of theorems in

Grades 9 and 10

34. Geometry
   a. measurement
   b. trigonometric functions
   c. law of sines
   d. law of cosines

Footnotes

[1] "College Board Wants to Set Skills Standard," Dallas Morning News, 25 September 1981, sec. A.

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[19] Thomas A. Romberg, Impressions of Soviet Research in Mathematical Education," March 1979. (Mimeographed).

[20] A. N. Kolmogorov, "Modern Mathematics and Mathematics in the Modern School," Soviet Education 16 (1974): 74-78.

[21] Robert B. Davis, Chapter IV in An Analysis of Mathematics Education in the Union of Soviet Socialist Republic.

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[24] L. L. Stewin, "Research Notes: A Note on Pupil Evaluation in the Soviet Union," The Alberta Journal of Educational Research XXVI (December 1980) 276-279.

[26] A. S. Pchelko, "Certain Features of Arithmetic Work in the Third Grade," Nachal'naia Shkola, 1969.

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TEXAS TEACHERS SALARIES, and the SHORTAGE OF TALENTED NEW TEACHERS

by
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In 1954 a starting teacher in Texas with a bachelor's degree earned just $2,800.00 a year on the state minimum salary schedule. With a starting salary of $11,110.00 this year, why is it so difficult to attract high–ability students into the education profession? This article presents one reason why this is true, and reveals the mathematical error in thinking which is made by politicians and administrators who set the salary schedules.

It is informative to see how the purchasing power of $11,110.00 in 1983 compares with that of $2,800.00 in 1954. The following table shows the Consumer Price Index (CPI) for each school year, 1954 through the present, as compiled by the Texas State Teachers Association. Applying the ratio of the CPIs to $2,800.00 shows that it is equivalent to $10,863.00 in 1983 dollars. So the before–tax purchasing power of a beginning teacher now is virtually the same as it was in 1954 (before Sputnik!). Figure 1 shows this fact graphically. After taxes, the purchasing power is considerably lower because the teacher is in a higher tax bracket ("bracket creep").

The situation is the same for beginning teachers with master's degrees. The $3,030 salary in 1954 is equivalent to $11,735 in 1983 dollars. This is about equal to the present $11,880.00 salary on the state minimum schedule.

(The data table goes anywhere after here.)
(Figures 1 and 2 go anywhere after here.)

Figures 1 and 2 also show what has happened in the intervening years. Beginning teachers' purchasing power rose steadily until 1970. Thereafter, it dropped in every year but two, at times by as much as $1,000.00 a year in 1983 dollars. In 1970, starting BA teachers on the state minimum schedule earned the equivalent of $15,556.00 a year, compared to $11,110.00 now. Starting MA teachers earned the equivalent of $17,112.00, compared to $11,800.00 now. It would take an immediate pay raise of $4,446.00 for BA degree and $5,232.00 for MA degree just to bring purchasing power back to what is was in 1970 for beginning teachers.

The underlying cause for the decline in purchasing power is the use of the normal increments in the salary schedule to help compensate for inflation. These increments are designed to reward teachers for experience. By lumping these increments with increases in the salary schedules, the politicians who set the salary schedules have only kept teachers even with inflation. Teachers with 1 to 12 years of experience have about as much purchasing power now as they did when they started teaching.

BEGINNING TEACHERS DO NOT GET A SALARY INCREMENT

An obvious fact, not well recognized by educators and glossed over by politicians, is that beginning teachers do not get a salary increment. This is why the purchasing power of beginning teachers has gone down.

Politicians like to report statistics which put themselves in the most favorable light. For example, former Governor Clements reported that teachers received a 26% pay raise. Not mentioned was the fact that this was 13% for each of two years and that the experience increments had been included. The actual increase in the salary schedules was only 8.5%. Since inflation had been at the 12% to 14% level, beginning teachers lost purchasing power.

The Legislature is now proposing that there be no change in state minimum salary schedules next year. As this article is being written, the evening newscaster reported that teachers would thus be getting only a 4% raise. Let the fact be repeated. **BEGINNING TEACHERS DO NOT GET A SALARY INCREMENT.** If salary schedules are not increased, the very people we are trying to attract into teaching will have even less financial incentive to join the profession next year than they do now.

To attract high ability people into teaching, the Legislature should increase salary schedules by at least $5,000.00 to $6,000.00 a year, thus making starting salaries comparable in purchasing power to those of 1970. Until this is done, we will attract into education only those who are independently wealthy, those who teach merely to augment a spouse's income, or those who are amthematically naive enough to believe that they are really going to get a "26% pay raise."

TEXAS STATE MINIMUM SALARIES FOR TEACHERS
AND BEFORE-TAX PURCHASING POWER
PREPARED BY PAUL A. FOERSTER FROM TSTA DATA

SCHOOL YEAR	C.P. INDEX	BEGINNING BA ACTUAL	BEGINNING BA 1982$	BEGINNING MA ACTUAL	BEGINNING MA 1982$	--- TOP BA --- ACTUAL	--- TOP BA --- 1982$	--- TOP MA --- ACTUAL	--- TOP MA --- 1982$
54-55	80.2	2805	10863	3030	11735	3453	13373	4434	17172
55-56	80.8	2805	10783	3030	11648	3453	13274	4434	17045
56-57	83.4	2805	10446	3030	11284	3453	12860	4434	16513
57-58	86.0	3204	11572	3429	12384	3852	13912	4833	17455
58-59	86.9	3204	11452	3429	12256	3852	13768	4833	17274
59-60	88.2	3204	11283	3429	12075	3852	13565	4833	17020
60-61	89.4	3204	11132	3429	11913	3852	13383	4833	16791
61-62	90.2	4014	13822	4239	14597	5094	17541	6075	20919
62-63	91.4	4014	13641	4239	14405	5094	17311	6075	20644
63-64	92.6	4014	13464	4239	14219	5094	17086	6075	20377
64-65	93.9	4014	13277	4239	14022	5094	16850	6075	20095
65-66	96.2	4104	13251	4410	14239	5508	17784	6516	21038
66-67	99.1	4104	12863	4410	13822	5508	17263	6516	20422
67-68	102.6	4734	14331	5040	15258	5904	17873	6912	20925
68-69	107.8	4734	13640	5040	14522	5904	17011	6912	19915
69-70	114.2	5337	14516	5877	15984	6507	17698	7731	21027
70-71	119.8	6000	15556	6600	17112	7670	19886	8880	23023
71-72	123.9	6000	15041	6600	16545	8050	20180	9310	23339
72-73	129.6	6000	14380	6600	15818	8450	20251	9780	23439
73-74	142.3	6000	13096	6600	14406	8870	19361	10270	22416
74-75	157.5	6600	13016	7200	14199	9910	19543	11380	22442
75-76	167.7	8000	14817	8600	15928	11780	21818	12780	23670
76-77	177.7	8000	13983	8600	15032	11780	20590	12780	22338
77-78	189.9	8460	13837	9020	14753	13250	21672	14380	23520
78-79	209.0	8540	12692	9110	13539	13380	19884	15090	22426
79-80	237.3	8970	11741	9600	12565	14660	19188	16170	21165
80-81	263.9	9430	11099	10090	11876	15710	18490	17290	20350
81-82	290.3	10230	10945	10950	11716	17400	18617	19140	20478
82-83	310.6	11110	11110	11880	11880	19260	19260	21100	21100

1981-1982 CPI ASSUMES 10% INFLATION.
1982-1983 CPI ASSUMES 7% INFLATION.

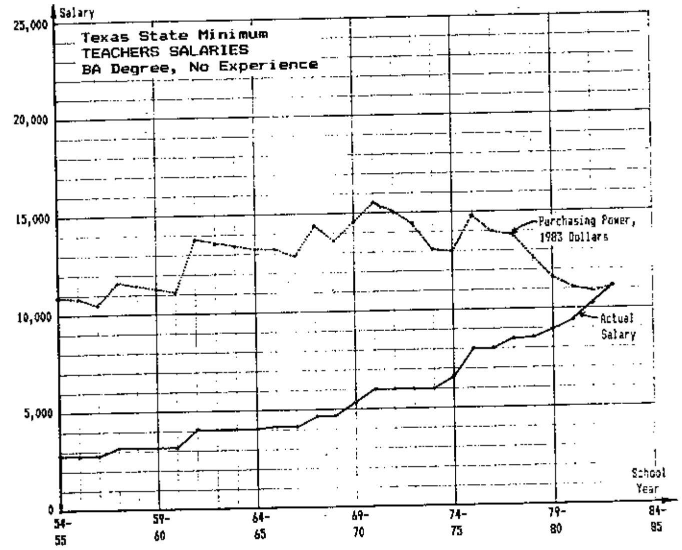


FIGURE 1

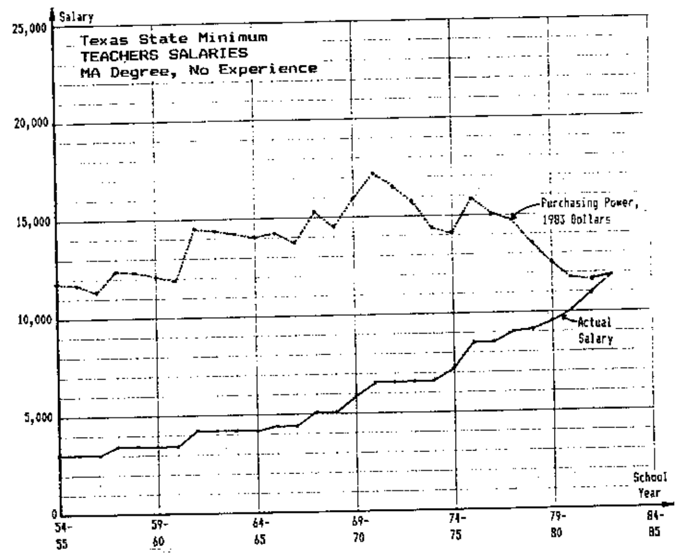
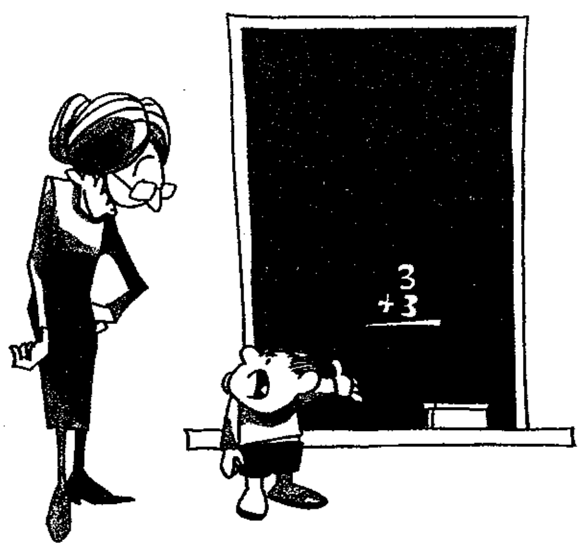


FIGURE 2



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HARMONIC MEANS, GEOMETRIC MEANS AND PI

John Huber
Pan American University

The history of π is long and fascinating (Beckmann, 1977). Around 300 B. C. Euclid proved that the ratio of the area of a circle to the square of its diameter is constant for all circles. Archimedes was then able to show that the area of a circle is equal to the area of a right triangle whose sides have length equal to the radius and circumference of the circle. Thus the problem of finding the area of a circle became the problem of calculating the circumference of a circle. By approximating the circumference of a circle with regular inscribed and circumscribed regular polygons with 6, 12, 24, 48, and 96 sides, Archimedes was able to show that π lies between

$$3\frac{10}{71} \text{ and } 3\frac{1}{7} \quad .$$

The purpose of this paper is to derive the recursive relations

$$1/P_{2n} = (1/P_n + 1/p_n)/2 \qquad (1)$$

and

$$p_{2n} = \sqrt{P_{2n}p_n} \qquad (2)$$

where p_n and P_n denote the semiperimeters of the inscribed and circumscribed regular n-gons of the unit circle. Observe that (1) and (2) are harmonic and geometric means, respectively.

Let s_n and S_n denote the length of the side of a regular polygon of n sides inscribed and circumscribed in a circle of radius r.

Then $p_n = (n \cdot s_n)/2$ and $P_n = (n \cdot S_n)/2$.

Finally, the sequences $\{p_n\}$ and $\{P_n\}$ are respectively monotonic increasing and decreasing sequences converging to π.

Using Figure 1 with Δ AED $\sim \Delta$ ACB we have

$$\frac{AD}{ED} + \frac{AB}{CB} \quad . \qquad (3)$$

By symmetry ED = DB so

$$AD = AB - DB$$

and

$$AD = AB - ED \quad . \qquad (4)$$

Substituting (4) into (3)

$$\frac{AB - ED}{ED} = \frac{AB}{CB} \quad . \qquad (5)$$

Solving (5) for ED,

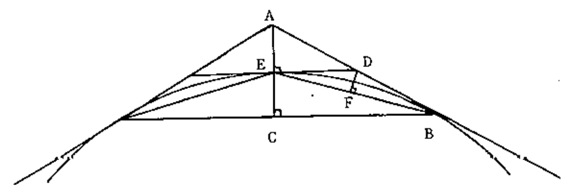
$$ED = \frac{AB \cdot CB}{AB + CB} \quad . \qquad (7)$$

Using Figure 1 with ΔECB $\sim \Delta$ DFB we have

$$\frac{EB}{CB} = \frac{DB}{FB} \quad . \qquad (8)$$

Substituting $FB = \frac{1}{2}EB$ and DB = ED into (8)

$$\frac{EB}{CB} = \frac{ED}{\frac{1}{2}EB} \quad . \qquad (9)$$



$AB = S_n/2$ $\qquad ED = S_{2n}/2$

$CB = s_n/2$ $\qquad EB = s_{2n}$

FIGURE 1

Solving (8) for EB,

$$EB = \sqrt{\frac{ED \cdot CB}{2}} \quad . \qquad (9)$$

Substituting $AB = S_n/2$, $ED = S_{2n}/2$, $CB = s_n/2$ and $EB = s_{2n}$ into (7) and (9), and simplifying

$$S_{2n} = \frac{S_n s_n}{S_n + s_n} \qquad (10)$$

and

$$s_{2n} = \sqrt{S_{2n} s_n} \quad . \qquad (11)$$

Finally, substituting

$$s_n = \frac{2 \cdot p_n}{n}, \quad S_n = \frac{2 \cdot P_n}{n} \quad ,$$

$$s_{2n} = \frac{2 \cdot p_{2n}}{2n}, \text{ and } S_{2n} = \frac{2 \cdot P_{2n}}{2n} \text{ into}$$

(10) and (11), and simplifying

$$P_{2n} = \frac{2P_n p_n}{P_n + p_n}$$

or

$$1/P_{2n} = (1/P_n + 1/p_n)/2 \qquad (12)$$

and

$$P_{2n} = \sqrt{P_{2n} P_n} \quad . \qquad (13)$$

Using a microcomputer with $P_6 = 3$ and $P_6 = 2\sqrt{3}$ in (12) and (13) we have the results in Table I. (See Appendix for Program.)

REFERENCE

Beckmann, Petr. A HISTORY OF PI. The Golem Press, 1977.

TABLE I

6	3	3.4641016
12	3.10582854	3.2153903
24	3.13262861	3.15965994
48	3.1393502	3.14608621
96	3.14103195	3.14271459
192	3.14145247	3.14187305
384	3.1415576	3.14166274
768	3.14158389	3.14161017
1536	3.14159046	3.14159375
3072	3.14159211	3.1459293
6144	3.14159252	3.14159272
12288	3.14159262	3.14159267
24576	3.14159265	3.14159266
49152	3.14159266	3.14159266
98304	3.14159266	3.141 59266
196608	3.14159266	3.14159266
393216	3.14159266	3.14159266
786432	3.14159266	3.14159266
1572864	3.14159266	3.14159266
3145728	3.14159266	3.14159266
6291456	3.14159266	3.14159266

APPENDIX

APPLE BASIC PROGRAM

```
10   HOME
20   PRINT "THIS PROGRAM APPROXIMATES PI USING"
30   PRINT "ARCHIMEDES' HARMONIC-GEOMETRIC
     MEAN METHOD."
40   PRINT:PRINT
50   N=6: I=3: C=2*SQR(3)
60   PRINT TAB(3); "N"; TAB(10; "INSCRIBED"; TAB(24);
     "CIRCUMSCRIBED"
70   PRINT
80   PRINT N; TAB(10); I; TAB(25); C
90   FOR K=1 TO 20
100    C=(1/I + 1/C)/2
110    C=1/C
120    I=SQR(I*C)
130    N=2*N
140    PRINT N; TAB(10); I; TAB(25); C
150  NEXT K
160  END
```

APPLE PASCAL PROGRAM

```
PROGRAM PI;
  USES TRANSCEND;
  VAR   K: INTEGER;
     C,I: REAL;
       N: INTEGER [10 ];
  BEGIN
       N:=6; I: = 3; C:= 2*SQRT(3);
       WRITELN ('THIS PROGRAM APPROXIMATES
              PI USING');
       WRITELN ('ARCHIMEDES HARMONIC-GEOMETRIC
              MEAN METHOD')'
       WRITELN;
       WRITELN;
       WRITELN; ('N': 10, 'INSCRIBED': 16, 'CIRCUM-
              SCRIBED' : 15);
       WRITELN;
       WRITELN; ('N' : 10, I: 10, C: 10);
       FOR K: = 1 TO 20 DO
         BEGIN
              C: = (1/I = 1/C)/2
              C: = 1/C;
              I: = SQRT (C*I);
              N: = 2*N;
              WRITELN (N: 10, I: 10, C: 10)
         END;
  END.
```

NOTICE

The Department of Mathematics, The University of Texas, Austin, has a special offering this summer for high school mathematics teachers. It will offer a three week workshop designed to assist those who wish to prepare themselves to teach an Advanced Placement calculus course. The workshop has been set up as a graduate course in Mathematics. The dates are June 6 – 24.

M396C Topics in Advanced Placement Mathematics

The instructor will be Professor Don Edmondson of the Department of Mathematics. He has had extensive experience working with mathematics teachers in summer institutes and academic year institutes; he has been an invited speaker at numerous NCTM and CAMT meetings.

The class will not assume that participants have a high level of calculus skills. It will examine the fundamental problems of calculus and provide participants the opportunity to develop their skills along with broadening their understanding of the concepts. To accomplish its goals the class will meet from 8:30 – 10:00 and 1:00 – 2:30 with the break period being used for coffee, problem practice, informal exchanges, and lunch.

The cost of the course is $67 for Texas residents, $162 for non-residents. For information or registration forms write or call Don Edmondson. (512/471-7711 or 512/452-4507).

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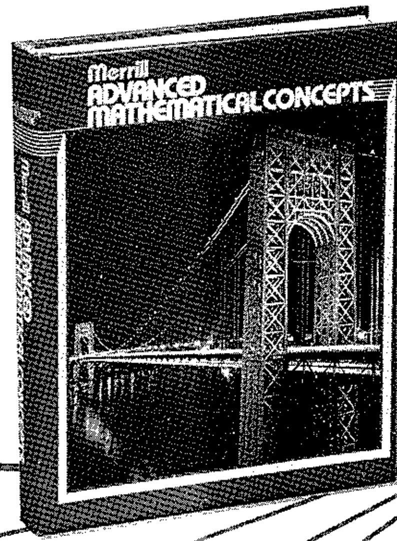

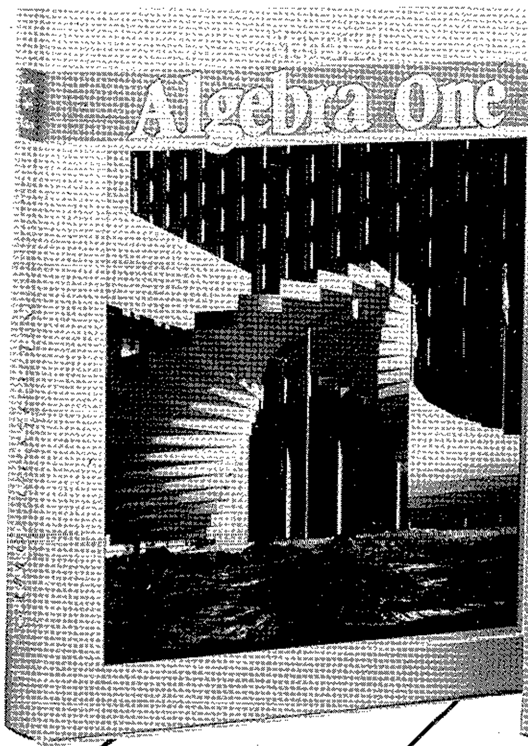

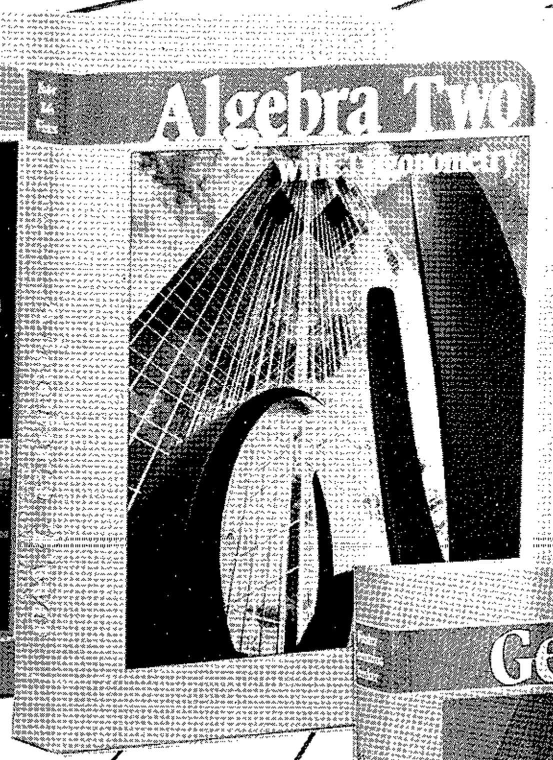

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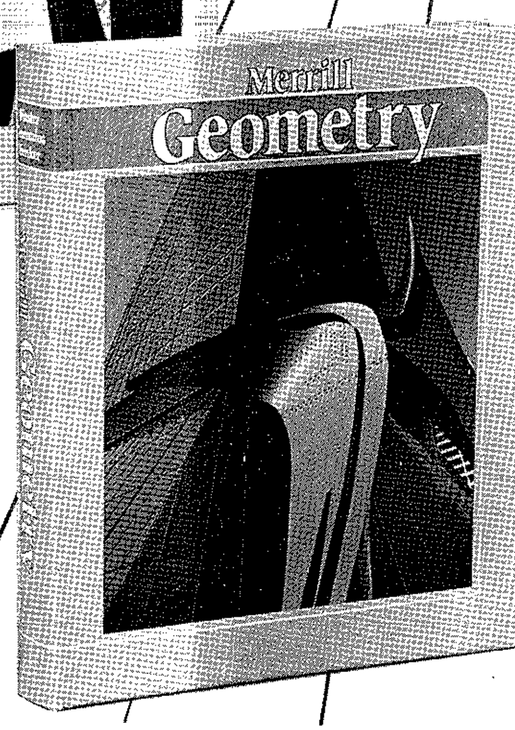

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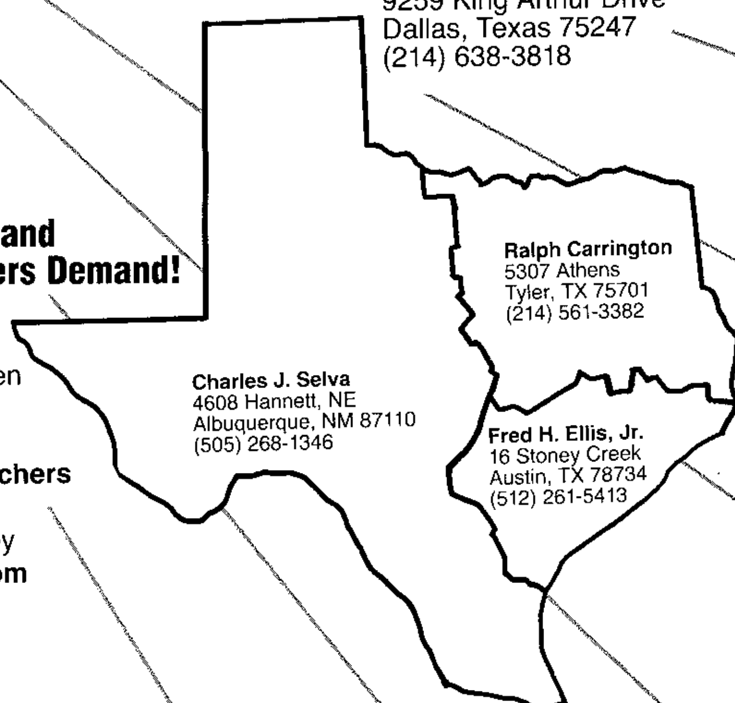

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