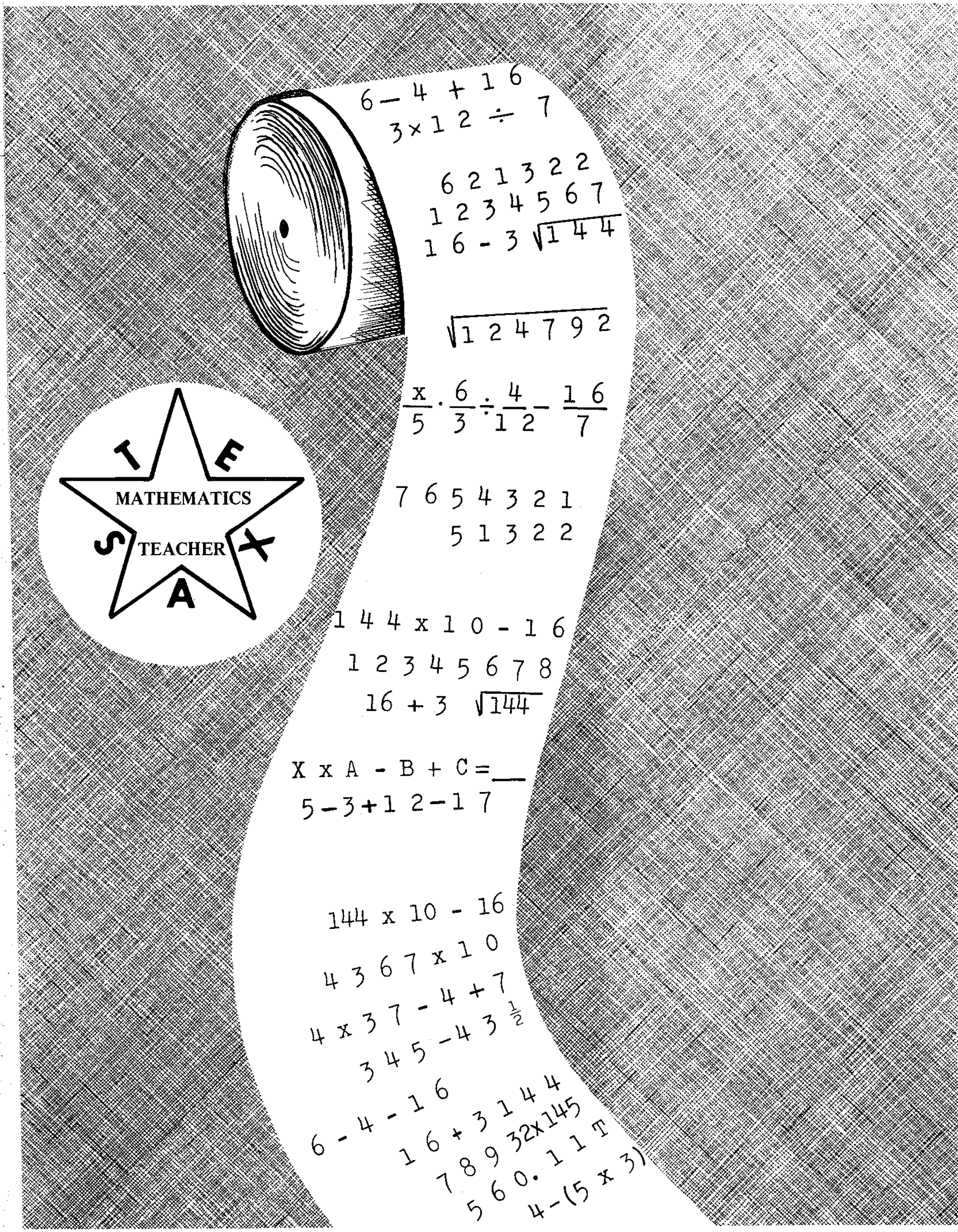
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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher,* 100 So. Glasgow Drive, Dallas, Texas 75214.

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TEXAS MATHEMATICS TEACHER

Texas Council of Teachers of Mathematics

Vol. XXIX May, 1982 No. 3

PRESIDENT'S MESSAGE

The 60th Annual Meeting of NCTM in Toronto, Ontario on April 14, 1982, was informative nad enjoyable. A Delegate Assembly of the coucil's affiliated groups is held each year in conjunction with the NCTM annual meeting. This year we voted on the following twelve resolutions to be referred to the Board of Directors:

1. That NCTIM actively contact state boards of education and the state universities requesting that they seriously consider the development of the Bachelor of Arts in mathematics teaching degree. Consistent with NCTM'S GUIDELINES FOR THE PREPARATION OF TEACHERS IN MATHEMATICS, this BAT degree should cover those college mathematics classes through calculus, and then offer inservice types of courses covering computer science, linear algebra, Euclidean and other geometries, probability and statistics, theory of euqations, and other classes taught in high school. These courses may be taught by staffs from the high school. Such a program should be highly publicized in the high schools to attract students into the program. (Los Angeles City Teachers' Mathematics Association) — Defeated

2. That NCTM actively seek advice from the federal government and other sources on methods of encouraging private industry to support inservice and preservice training programs for persons involved in, or seeking to be involved in, the teaching of school mathematics at all levels and that NCTM make further attempts to alert private industry to the impending crisis imposed on the future stability of the work force if qualified individuals are not available to teach mathematics in elementary and secondary schools. (Association of Mathematics Teachers of New Jersey) — Passed

3. That NCTM take the initiative to outline and encourage the implementation of inservice opportunities for elementary principals such as

   (1) including sessions for elementary principals during NCTM meetings,

   (2) seeking program time at the NA of E. S. P., and

   (3) regularly submitting articles for publication in the National Association of E. S. P. Journals and others — Passed

4. That NCTM change the names of its journals to the ELEMENTARY MATHEMATICS TEACHER and the SECONDARY MATHEMATICS TEACHER. (California Mathematics Council) — Defeated

5. That NCTM collaborate with professional groups in other disciplines (e.g., English, science, social studies) to produce a handbook of microcomputer applications in a variety of disciplines. (Washington State Mathematics Council) — Passed

6. That a serious attempt be made to hold NCTM conventions in locations where costs are reasonable. Also, the Board could explore holding the annual meeting at times when rates are lower. (Ohio Council of Teachers of Mathematics) — Passed

7. That dues for NCTM be approved by the membership. (Ohio Council of Teachers of Mathematics) — Defeated

8. That NCTIM not only publish a Board or presidential candidate's education, affiliations, and publications but require every candidate to submit a statement of personal philosophy and goals he or she would seek of elected. (Los Angeles City Teachers' Mathematics Association) — Passed

9. That NCTM publish materials on problem solving for the primary (K—12) grades. (Greater Toledo Council of Teachers of Mathematics) — Passed

10. That the Board of Directors of NCTM approve a convention registration fee schedule that will approximately balance the convention expenses. (Ohio Council of Teachers of Mathematics) — Defeated

11. That NCTM reduce its nonmember registration fee at regional meetings. (Greater El Paso Council of Teachers of Mathematics) — Defeated

12. That NCTM reinstate offering special school and district rates for regional conferences. (Minnesota Council of Teachers Mathematics) — Passed

The National Council of Teachers of Mathematics through funding by National Science Foundation, is conducting a series of four regional conferences on promoting equity in mathematics education in elementary and secondary schools. The conferences are designed to help educators develop skills and strategies to meet the needs of such underrepresented groups as girls, blacks, language minority students, and Native Americans.

Regional Conference Schedule

Orlando, Florida	October 22–23, 1982
Albuquerque, New Mexico	November 19–20, 1982
Baltimore, Maryland	January 28–29, 1983
Minneapolis, Minnesota	February 18–19, 1983

There will be no registration fee. Participants will be furnished lunches and a packet of materials for conducting equity workshops. If you have any questions, contact:

Equity Project
National Council of Teachers of Mathematics
1906 Association Drive
Reston, Virginia 22091
(703) 620-9840

Application deadlines: Orlando and Albuquerque — June 1, 1982
Baltimore and Minneapolis — October 1, 1982

"I work a five-day week, without pay, plus homework, and this is the thanks I get!"

The American Computer Science League is a nonprofit organization devoted to computer science education. ACSL administers monthly computer science contests for junior and senior high school students, publishes a monthly newsletter containing the results of each contest and items of interest, and awards prizes to outstanding students and schools at local and regional levels. Last year the top-scoring students among the 1500 participants were awarded microcomputers at the year's end All–Star Contest festivities. For more information, write:

American Computer Science League
Box 2417A
Providence, RI 02906

Our new CAG representative to NCTM is Chris Boldt, Eastfield Community College, Dallas County Community College District, Mesquite, Texas 75150. He is replacing Terry Parks who is becoming the coordinator for all the CAG representatives to NCTM.

The TCTM Newsletter needs material for all levels of mathematics. The material should be for immediate classroom use and easy to reproduce. Send to Wayne Miller, P. O. Box 818, Baytown, Texas 77521.

Patsy Johnston

"But if I go every day, won't I wear out my welcome?"

MANUSCRIPTS NEEDED!!!!! Send them to 100 S. Glasgow, Dallas, Texas, 75214.

THE MATHEMATICS CURRICULUM AND THE LEARNER

Dr Marlow Ediger
Northeast Missouri State University
Kirkville, Missouri

Teachers, principals, and supervisors seemingly emphasize the rather heavy utilization of a single or multiple series textbook to provide major sequential learnings for pupils in the mathematics curriculum. Reasons given for a textbook centered mathematics curriculum include the following:

1. The teacher cannot develop his/her own creative units of study in each busy day of teaching. A repurable textbook with a relevant accompanying manual may do a good job of assisting teachers to select vital objectives, learning activities, and appraisal techniques for pupils within the framework of teaching-learning situations.

2. Developers and writers of reputable mathematics textbooks have spent much time and money in their completed product. Sequential learnings may then be provided when content is presented to learners in the order contained in the adopted mathematics textbook.

3. The utlization of reputable mathematics textbooks has stood the test of time. These texts have been utilized for decades and with needed revisions may be relied upon to provide significant learnings for pupils.

4. With today.s emphasis on the basics, content within a reputable mathematics text might well provide essential learnings for pupils.

THE PSYCHOLOGY OF TEACHING AND LEARNING

Teachers, principals, and supervisors need to test their hypotheses pertaining to methods of teaching and learning against recommended criteria from the psychology of education. Which criteria in the educational psychology arena, based upon research findings, if followed by educators in the school setting, might assist pupils to achieve optimally?

1. Pupils need to experience interesting learning activities to achieve desired ends. Effort put forth by pupils in the mathematics curriculum may then be sustained due to perceived interest.

2. Purpose needs to be developed in each unit of study. When purpose is involved in learning, learners accept reasons for participating in each activity and experience.

3. Meaning must be attached to that which is being learned. If pupils perceive meaning pertaining to facts, concepts, and generalizations being emphasized in each unit of study, they will better understand what is being learned. Learnings acquired meaningfully will also be retained for a longer period of time compared to content gained in a nonmeaningful manner.

4. Adequate provision must be made for pupils of diverse capacity and achievement levels. Each pupil then is guided to achieve optimal development pertaining to understandings, skills, and attitudinal objectives in the mathematics curriculum.

THEORIES OF LEARNING AND THE MATHEMATICS CURRICULUM

Teachers, principals, and supervisors need to analyze and ultimately implement that which is desirable from diverse theories of learning in the educational psychology arena. Implementing a theory or several theories may well assist in providing interesting, purposeful, and meaningful learnings to provide for individual differences in the mathematics curriculum.

Programmed learning emphasizes that pupils progress in very small sequential steps. Thus, for example, a pupil in a programmed textbook may view an illustration, read a sentence or more, and respond to a completion item or question. Immediately thereafter, a pupil may check the correctness of the response given. If the response is correct, reinforcement is then in evidence. If the response was incorrect, the learner may notice the correct answer and still be ready for the next linear item in learning. This procedure may be followed again and again in programmed methods of teaching.

Assumptions inherent in programmed learning to guide optimal achievement of each pupil include the following:

1. Each pupil will be achieving at his/her optimal rate in learning regardless of the rate of progress of others in the class setting.

2. A pupil knows the right answer to a previous programmed item before approaching the next sequential step of learning.

3. The scope of content covered in each step of learning needs to be small. Otherwise, a learner may become lost in attempting to cope with an excessive amount of content at a given time in learning.

4. Specific items of content covered within the framework of programmed learning generally are ordered in a positive manner so that each pupil may make few or no errors in achieving sequential learnings.

5. A pupil should know if his/her response is correct to any item in programmed learning before progressing sequentially to the next linear item. Otherwise, incorrect responses may be practiced.

6. Independently, each pupil may read, view a related illustration, respond, and check his/her own response before attending to the next sequential item. The teacher, of course, gives assistance as it is necessary to do so.

Teachers, principals, and supervisors in analyzing and appraising programmed learning may ultimately wish to implement some or all of the above listed.

Measurably stated objectives in the mathematics durriculum have become quite popular both inside as well as outside the framework of state and district wide approaches to accountability. Objectivies in ongoing units of study in mathematics, such as the following sound familiar:

1. The pupil will solve correctly nine of ten word problems on page seventy in the adopted mathematics textbook.

2. The pupil will add correctly fourteen out of fifteen problems, each containing three two digit addends.

3. The pupil will correctly multiply five of six problems, each factor having three digits.

The teacher generally determines which specific ends pupils are to achieve in each unit of study in mathematics when measurably stated objectives are utilized in the instructional sequence. The teacher also chooses learning activities for learners to attain the desired ends. Ultimately, each pupil's progress is measured in terms of having attained the predetermined ordered measurable objective.

Assumptions inherent in utilizing measurably stated objectivies in the mathematics curriculum include the following:

1. The teacher generally is in the best position, academically and educationally, to select objectives, learning activities, and appraisal techniques for pupils.

2. That which is vital to learn by pupils can be measured.

3. Essential or basic learnings for pupils can be identified with considerable certainty in the mathematics curriculum.

4. Teachers need to be accountable in terms of vital learnings acquired by pupils in the mathematics arena; success by pupils in attaining precise ends also reveals teacher proficiency in teaching.

Humanism as a psychology of teaching and learning has also had strong supporters in the field of education. Humanists believe that pupils with teacher guidance need to have ample opportunities to choose objectives as well as learning activities. Learning centers in the school-class setting may well assist in fulfilling these needs. Each pupil may choose which specific task to work on at any of several learning centers. The pupil sequentially then chooses activities and experiences in the mathematics curriculum. If a pupil is not actively involved in choosing and prusuing, the teacher is there as a stimulator and guidance person to assist the involved learner. There are diverse choices which may be made by pupils in the mathematics curriculum. These choices may include the use of:

1. Programmed learning and measurably stated objectives.

2. Reputable mathematics textbooks to acquire sequential learnings.

3. Laboratory means of learning in the mathematics curriculum.

4. Activity centered methods stressing projects and learning by doing approaches.

Assumptions involved in humanism as methods to aid pupils to achieve optimally in mathematics units of study include the following:

1. Within a flexible framework, the pupil is in the best position to determine what is vital to learn, as well as the means of learning. The psychological rather than a logical curriculum is then in evidence.

The learning environment becomes increasingly humane as pupils are actively involved in determining the ends and means of learning. A teacher or programmer determining the curriculum in selecting measurably stated objectives, activities to attain these ends, as well as appraisal techniques would be frowned upon by humanists.

3. Trust and confidence needs to be in evidence in the school-class environment between educators and pupils. Thus, pupils may be choosers of their own experiences and activities in an atmosphere that emphasizes trust in that learners do make positive decisions.

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THE EFFECT OF PIAGETIAN LEVEL, ATTITUDES, SEX AND TEACHING FORMAT ON ACHIEVEMENT, TRANSFER, AND RETENTION IN ELEMENTARY GEOMETRY

John L Creswell, University of Houston – Judith Hempel, Houston Independent School District

The results of the Second National Assessment of Educational Progress (NAEP) indicate that elementary school students lack understanding of basic geometry concepts such as area and volume along with inability to work with plane figures (Carpenter, et al., 1980). Creswell (1970) found that many elementary school teachers avoided teaching geometry concepts until the "end of the year," and for many the "end of the year" never materialized. Indeed, many elementary school teachers complain about having to teach geometry, which has led some authorities to feel that geometry is the "stepchild" of mathematics. Many felt that elementary school children were not cognitively ready to learn intuitive geometry concepts. This negative attitude of the teacher often affects student achievement (Aiken, 1976)

Moulton (1974) found that many teachers spend a great deal of time having children learn by rote to compute area and volume without much attention to concept development. This teaching format prevails in most teachers according to Lydia Muller-Willis (1970) who found that most teachers felt that their task in the dassroom is to transmit knowledge by stuffing students' memories. This format is called deductive/"telling." Others have found that guided discovery using demonstrations and hands-on experiences is a more effective teaching format (Bruner, 1961). Britt (1980) found that discovery learning enhanced retention and transfer by elementary school students in geometry.

Some researchers have found no significant differences in mathematics achievement through the seventh grade (Fennema, 1974; Creswell, 1982; and Creswell and Johnson, 1981). Others contend that even in elementary school, boys out-perform girls in spatial ability and mathematics achievement (Kail and Siegel, 1977; Guay and McDaniel, 1977; and Luchins and Luchins, 1979).

The purpose of this study was to attempt to determine the extent to which teaching format, e.g., deductive/"telling" or inductive/guided discovery, attitudes, sex, and Piagetian level affected achievement, transfer and retention in geometry.

METHOD

The Sample

Subjects consisted of sixth-grade students selected from an elementary school whose clientele were primarily in the lower to lower-middle socio-economic range. The 112 subjects were from four self-contained classrooms and consisted of 50 males and 62 females.

Treatment

The treatment consisted of three weeks of instruction in geometry involved in teaching teaching basic concepts of area and volume. The four classes were divided into two groups, one of which was taught the deductive/"telling" format, and the other was taught by the inductive/"discovery" format. One teacher taught both groups, thus eliminating the teacher variable.

Since it was not possible to randomly assign subjects to each of the two instructional formats, a pre-test was administered. Results indicated that there was no significant difference in the mean scores of the two groups.

The main features of the deductive/"telling" format were: (1) verbalization of the concept by the teacher was the initial instructional step, (2) the concept was explained verbally and symbolically, (3) students worked examples only after verbal presentations.

The inductive/"discovery" approach was featured by: (1) verbalization of concept was delayed, (2) student was presented with an ordered, structured series of examples of the concept, (3) manipulatives and transparencies were used and students were encouraged to use their own ideas to develop the concept; and (4) students were allowed to discover and verbalize the concept.

Each group received approximately 60 minutes of instruction per day during the 21 day treatment period.

Instrumentation

Piagetian level of the subjects was assessed through the use of Piagetian Developmental Level Task Instrument (St. Martin, 1974) which has been used successfully with middle school students. This instrument along with the attitude scale, was administered prior to the treatment period.

The Revised Mathematics Attitude Scale by Aiken and Dreger was used to assess subjects' attitudes toward mathematics. This instrument has been used with success by the authors for several years.

Since the authors could find no published standardized test with which to assess geometry achievement, such an instrument was constructed. In order to do this, 50 concepts and skills involved in the treatment materials were listed. Three test items for each concept and skill were generated and then made into three alternate forms of instruments designed to assess achievement.

To assess validity of the instruments a panel of experts was convened consisting of doctoral students, university professors and sixth-grade teachers. The panel was given a list of behavioral objectives,the content materials, and the assessment instruments to study. The panel was unanimous in its evaluation that the instruments were content valid.

The three instruments were then administered to 150 sixth-grade students (50 for each instrument) from a different school with a similar clientele. Using Kuder-Richardson Formula 20, reliability estimates of .80, .83, and .85 for Forms I, II, and III, respectively, were determined. After conducting an item-analysis, two equivalent forms of the instrument consisting of 50 items each were constructed. These two forms were administered to another 100 sixth-grade students from another school. Again using Kuder-Richardson Formula 20, reliability estimates of .80 and .83 for Forms I and II, respectively, were determined.

Form I was used to assess geometry achievement at the end of the treatment period, while Form II was used to assess retention. Form I was administered immediately following the 21-day treatment. Form II was administered two weeks after administration of Form I.

In a similar manner, a test for transfer was constructed, and of course was used to gether data regarding transfer.

Procedures

A multiple linear regression model was used to predict geometry achievement, retention and transfer from the independent variables of attitude, sex and Piagetian level and teaching format. This technique was used to answer the following: To what extent does sex, Piagetian level, attitude and instructional format effect (1) geometry achievement, (2) retention, and (3) transfer? From these, research hypotheses were generated and tested at $p < .01$ level of significance.

In order to accomplish this, separate regression equations were used with achievement, retention and transfer each acting as dependent variables.

RESULTS

Achievement

Results indicate that as a group the variables sex, teaching format, Piagetian level, and attitude significantly ($p < .01$) contributed to achievement. However, when the effects of teaching format were removed, the others as a group were not significant contributors. By itself, teaching format contributed 24% of the variance of a total variance of 31%.

Retention

As a group, the predictor variables accounted for 13% of the variance in retention. However, by themselves only instructional format ($p < .01$) and Piagetian level ($p < .05$) contributed significantly.

Transfer

The total equation for transfer accounted for 91% of the variance, significant at $P < .01$. The interaction between instructional format and sex, Piagetian level, and attitude accounted for 2% of the variance, significant at $p < .01$. Sex classification accounted for 88% of the variance in transfer, while Piagetian level accounted for only .3%.

Post Hoc Probe

The results of the multiple regression analysis reported above seemed to indicate that additional analysis was in order. Therefore, a discriminant analysis was used to anwer the following question: Are Piagetian levels distinguishable in terms of achievement, attitude, sex of student, retention and transfer? It was found that these variables predicted 49 of 66 cases or 75.2% of concrete level membership. Indeed, the total percentage of correctly "grouped" cases was 73.4, a significant correctly grouped percentage.

DISCUSSION

The results of this study indicate that sex classification was not a significant contributor to the variance in achievement. This supports the findings of most other studies of subjects of this age level (Fennema, 1974; Bruner, 1978; Luchins, 1979; and Creswell, 1982). This variable also did not account for a significant amount of variance in retention. Abreco (1966), Bush (1977), and Dwyer (1974) reported similar results.

Interestingly, instructional format was a significant contributor to math achievement and retention, but not to transfer. This is interpreted to mean that students who were taught by the inductive/"discovery" format achieved and retained more than did students who were taught by the deductive/"telling" format.

Piagetian level was a significant contributor to retention, but not to achievement or transfer. This is somewhat surprising, since most studies have reported a decided achievement advantage for students operating at the formal cognitive level.

One of the most surprising findings was that sex classification contributed 88% of the variance in transfer. This is another unique finding in that no previous studies could be found which reported anything regarding the relationship between sex classification and transfer or knowledge.

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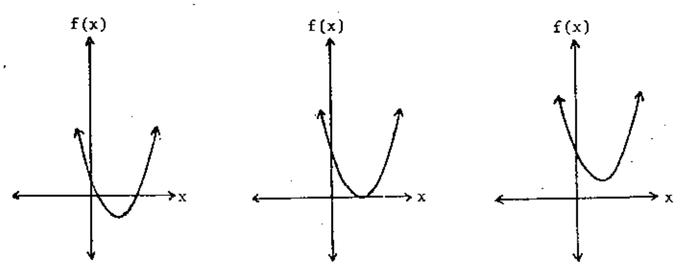
GRAPHING THE COMPLEX ROOTS OF A QUADRATIC

JOHN HUBER
PAN AMERICAN UNIVERSITY

In solving the quadratic equation $Ax^2 + Bx + C = 0$, where A, B, and C are real numbers ($A \neq 0$), it is convenient to determine the roots by graphing the corresponding quadratic function $f(x) = Ax^2 + \beta x + C$. The quadratic equation $Ax^2 + Bx + C = 0$ then has two, one, or no real roots (both complex) depending on whether the quadratic function $f(x) = Ax^2 + Bx + C$ intersects the x-axis in two, one, or no points, respectively. (See Fig. 1.) The purpose of this paper is to provide a graphical interpretation of the later case where the roots are complex.



Two Real Roots — One Real Root — No Real Roots

FIGURE 1

Consider the quadratic function $f(x) = x^2 = 6x + 5$. Examining the graph, we find that the vertex V is at (3, - 4) and the graph intersects the x-axis at (1, 0) and (5, 0). [See Fig. 2.]

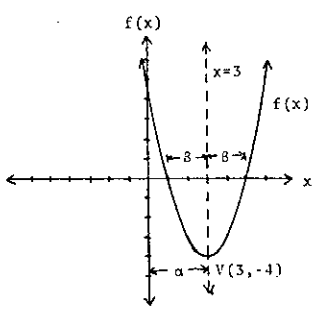


FIGURE 2

Denote the directed distance from the y-axis to the vertex V by a and the distance from the line of symmetry to the points of intersection of the graph with the x-axis as β. [See Fig. 2.] Then the x-coordinates of the points of intersection of the graph and the x-axis are given by $\alpha \pm \beta$. Thus the roots of $x^2 - 6x + 5 = 0$ are $x = \alpha \pm \beta$ where $\alpha = 3$ and $\beta = 2$. [See Fig. 2.]

For the graph of the quadratic function $f(x) = Ax^2 + Bx + C$ to intersect the x-axis intwo points, the discriminant $B^2 - 4AC$ must be greater than zero. Then the vertex is $V = (\frac{-B}{2A}\ \frac{4AC - B^2}{4A})$, the line of symmetry is $X = \frac{-B}{2A}$, and the points of intersection with the x-axis are $(\frac{-B}{2A} + \frac{\sqrt{B^2 - 4AC}}{2A}, 0)$ and $(\frac{-B}{2A} - \frac{\sqrt{B^2 - 4AC}}{2A}, 0)$.

With $a = \frac{-B}{2A}$ and $\beta = \left|\frac{\sqrt{B^2 - 4AC}}{2A}\right|$, the roots are given by $x = \alpha \pm \beta$. [See Fig. 3.]

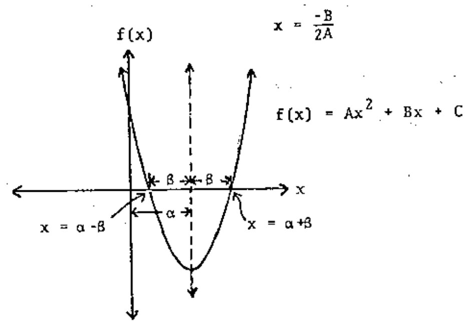


FIGURE 3

Consider the quadratic function $f(x) = x^2 - 6x + 9$. Examining the graph we find that the vertex V is at (3, 0) and this is also the only point at which the graph intersects the x-axis. [See Fig. 4.] Thus the roots of $x^2 - 6x + 9 = 0$ are $x = \alpha \pm \beta$ where $\alpha = 3$ and $\beta = 0$.

For the graph of the quadratic function $f(x) = Ax^2 + Bx + C$ to intersect the x-axis in one point, the discriminant $B^2 - 4AC$ must equal zero. Then the vertex is $V = (\frac{-B}{2A}, 0)$, the line of symmetry is $x = \frac{-B}{2A}$, and the point of intersection with the

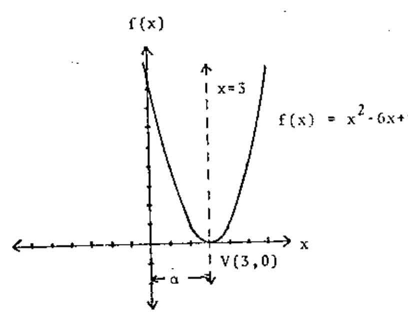


FIGURE 4

x-axis is $(\frac{-B}{2A}, 0)$. With $a = \frac{-B}{2A}$ and $\beta = 0$, the double root is given by $x = \alpha \pm \beta$. [See Fig. 5.]

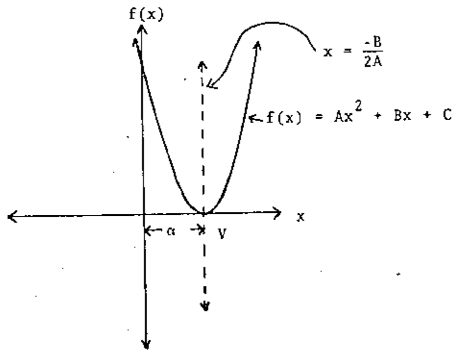


FIGURE 5

Consider the quadratic function $f(x) = x^2 + 4$. Examining the graph we find that the vertex V is at (0, 4) but the graph does not intersect the x-axis at any point. [See Fig 6.]

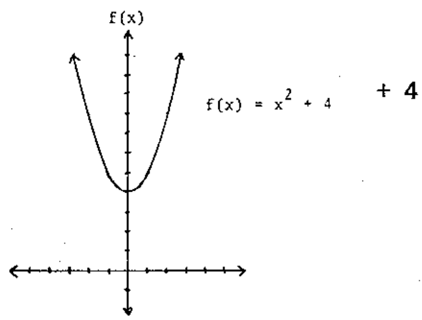


FIGURE 6

Thus $x^2 + 4 = 0$ has no real roots. For the domain of f, denoted D_f, equal to the set of real numbers, f(x) will always be greater than or equal to 4. However, if we extend the domain of f to include the set of all complex numbers such that $x^2 + 4$ is a real number, we can then solve $x^2 + 4 = 0$ over the new domain $D_f' = (x = a + bi \quad a, b \in R \text{ and } x^2 + 4 \in R)$. [See Table I.]

TABLE I

x	f(x)
± 2	8
± 1	5
0	4
± i	3
± 2i	0

In Figure 7a we have the graph of $f(x) = x^2 + 4$, $x \in D_f$. In Figure 7b we have the graph of $f(x) = x^2 + 4$, where $x \in \{bi \mid b \in R \text{ and } x^2 + 4 \in R\}$. In Figure 8 we have the graph of $f(x) = x^2 + 4$ where $x \in D_f' = \{a+bi \mid a, b \in R, x^2+4 \in R\}$. Examining the graphs in 7a and 7b we find that both are parabolas with vertices at (0, 4).

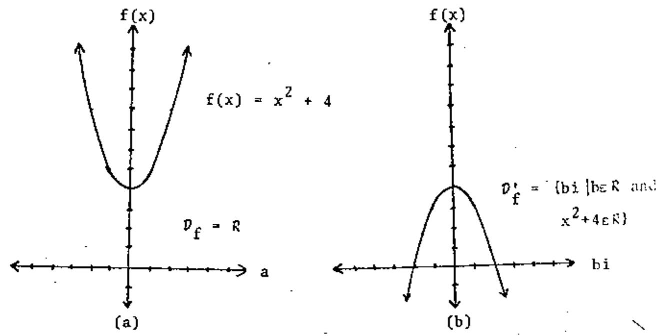


FIGURE 7

In addition, the two parabolas are congruent. Thus, the roots of $x^2 + 4 = 0$ with $x \in D_f'$ are the points where the parabola crosses the complex plane, not just the sx-axis, where $f(x) = 0$.

The image of $f(x) = x^2 + 4$ with $x \in R$ under a reflection in the line $y = 4$ is the parabola $g(x) = -x^2 + 4$ [See Fig. 9a]

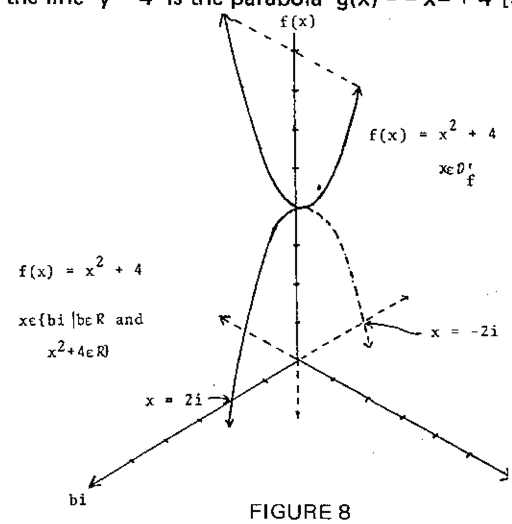


FIGURE 8

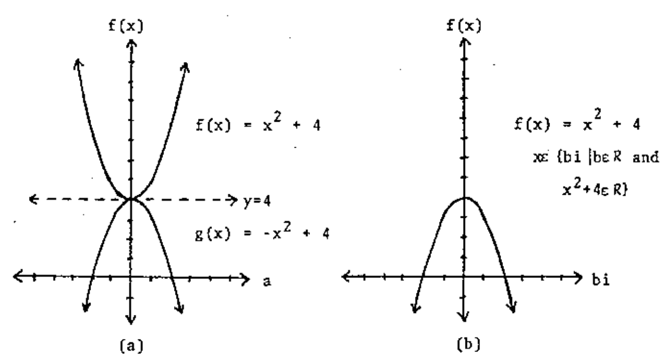


FIGURE 9

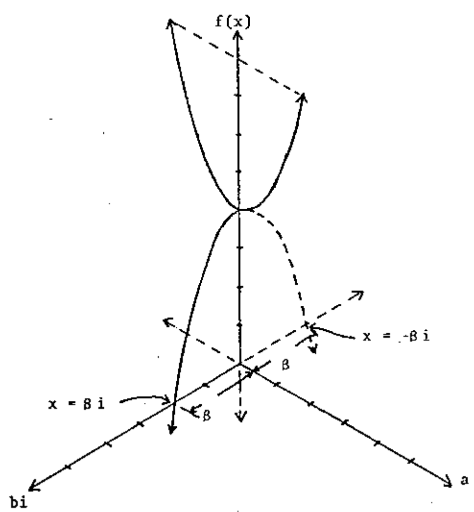


FIGURE 10

For $g(x) = -x^2 + 4$, $\alpha = 0$ and $\beta = 2$. Thus the roots of $x^2 + 4 = 0$ with $x \in D_f'$ are $x = \alpha \pm \beta i$ where $\alpha = 0$ and $\beta = 2$. [See Fig. 9b and Fig. 10]

As another example, consider the quadratic function $f(x) = x^2 - 11$. Examining the graph we find that the vertex is at (3, 2) but the graph does not intersect the x-axis. [See Fig. 11.] Extending the domain of f to

$$D_f' = (x = a + bi \mid a, b \in R,\ x^2 - 6x + 11 \in R),$$

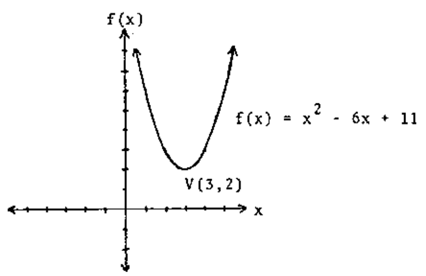


FIGURE 11

the graph intersects the complex plane at two points. [See Fig. 12.] Reflecting $f(x) = x^2 - 6x + 11$ over the line $y = 2$,

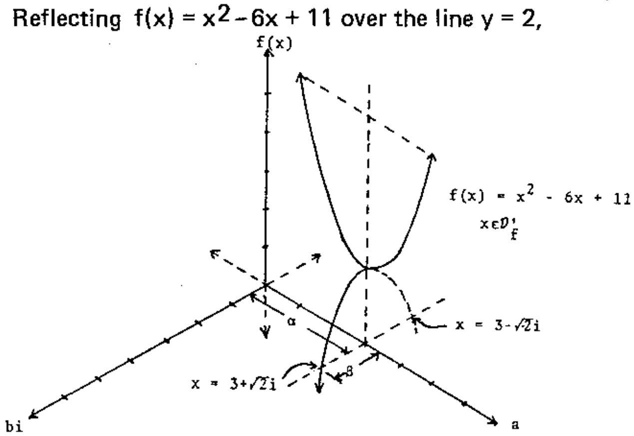


FIGURE 12

$g(x) = -X^2 + 6x - 7$. [See Fig. 13.] Examining the graph of $g(x) = -x^2 + 6x - 7$, we find $\alpha = 3$ and $\beta = \sqrt{2}$. Thus the complex roots of $x^2 - 6x + 11$ are given by $x = \alpha \pm \beta i$ with $\alpha = 3$ and $\beta = \sqrt{2}$. [See Fig. 12.]

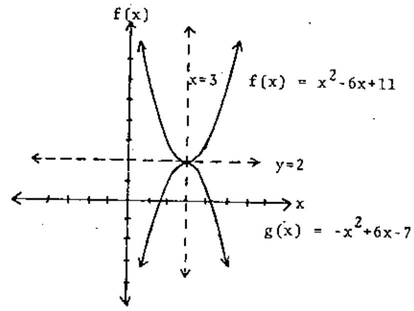


FIGURE 13

For the graph of the quadratic function $f(x) = Ax^2 + Bx + C$ not to intersect the x-axis, the discriminant $B^2 - 4AC$ must be less than zero. Then the vertex is $V = (\frac{-B}{2A}, \frac{4AC - B^2}{4A})$, the line of summetry is $x = \frac{-B}{2A}$, and the points of intersection with the complex plane are $x = \frac{-B}{2A} \pm \frac{\sqrt{4AC - B^2}}{2A}$ i. Thus with $\alpha = \frac{-B}{2A}$ and $\beta = \frac{\sqrt{4AC - B^2}}{2A}$, the roots are given by $x = \alpha \pm \beta$ i.

[See Fig. 14.]

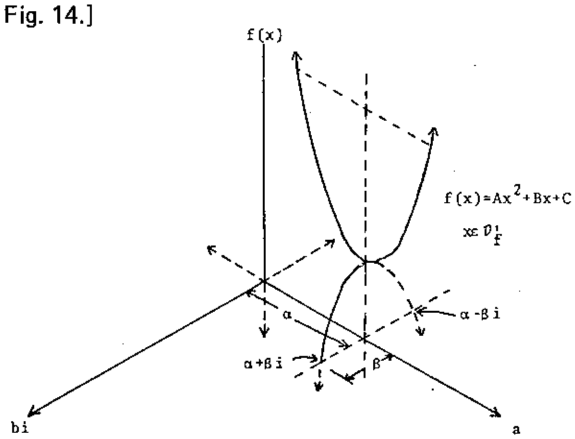


FIGURE 14

In conclusion, we have shown that by extending the domain D of a quadratic function $f(x) = Ax^2 + Bx + C$ to D_f', the set of all complex numbers such that f(x) is real, the roots of the quadratic equation $Ax^2 + Bx + C = 0$ are the x-coordinates where f(x) intersects the complex plane.

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