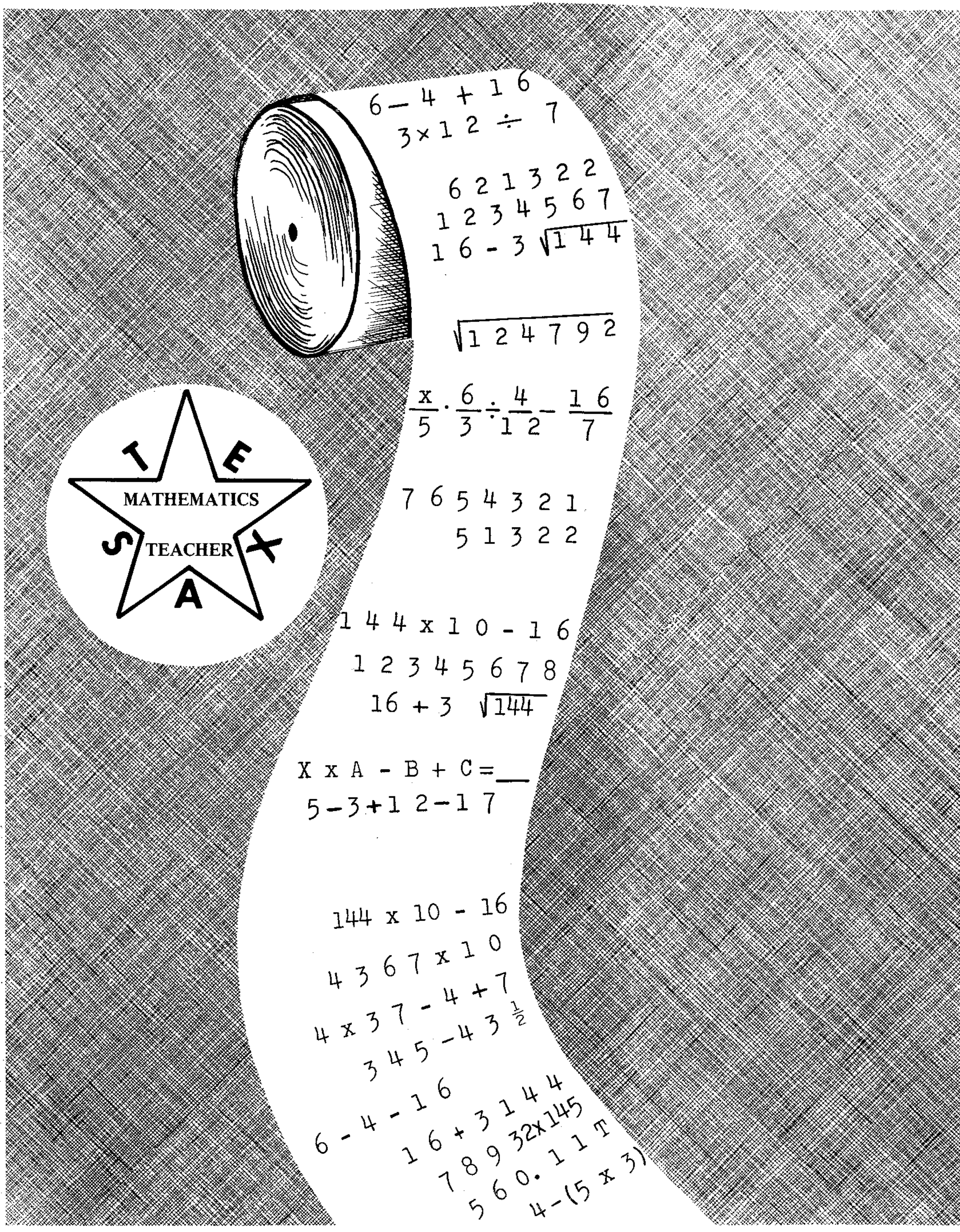
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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher,* 100 So. Glasgow Drive, Dallas, Texas 75214.

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Texas Council of Teachers of Mathematics

Vol. XXIX March, 1982 No. 2

PRESIDENT'S MESSAGE

The object of TCTM is to encourage an active interest in mathematical science, to afford a medium of interchange of views regarding the teaching of mathematics, and to further the cooperative study of problems relating to the teaching of mathematics. Membership in this organization is open to persons who are engaged in the teaching of mathematics in educational institutions both public and private. A concerted effort for the improvement of mathematics education at all levels of instruction is needed throughout the state of Texas, and TCTM hopes to be of assistance to its members.

Today, more than ever before, the study and appreciation of mathemathematics are vital to the intellectual development and to the scientific, industrial, technological, and social progress of society. School districts have an obligation to provide mathematics courses appropriate to the interest and abilities of their students. In order to keep their options open for future careers, students should be encouraged to make maximum use of their talents by studying mathematics. It is time to do more than just emphasize basics. Begin now, in your classroom and district, to encourage excellence in intellectual growth.

— Patsy Johnston

A 1982 LOOK AT MATHEMATICS FOR THE 80's

An Agenda for Action: Recommendations for School Mathematics of the 1980s was published by the National Council of Teachers of Mathematics to present their viewpoint of the directions mathematics programs should be taking in the 1980s. Although the decade has begun, what changes are taking place in Texas classrooms in response to these challenges issued for the 1980s? What has happened to those recommendations? Is problem solving becoming a focus for the mathematics curriculum? Are mathematics programs using computers and calculators to the fullest extent possible? Is the concept of basic skills actually being expanded beyond computational skills? Are curricula being developed to accommodate all ranges of student needs and abilities? Are mathematics educators in Texas preparing for the probability of competency testing? Is the mathematics necessary for the 80s being presented in your classroom? in your school? in your district? In looking for answers to these questions, a few observations about several of the recommendations of the **Agenda** become apparent.

> 'Recommendation 1: Problem solving must be the focus of school mathematics in the 1980s.'

New texts on the topic are emerging; university courses concerned with the nature and art of problem solving are being offered. Articles in the **Mathematics Teacher,** the **Arithmetic Teacher** and other publications deal with problem solving. Along with these many endeavors, perhaps some words of caution are necessary. By its very nature, problem solving requires creativity, time and innovation. The process cannot be based on rigid routines nor be easily categorized or structured. Problem solving should not be a function of time; 15 minutes every other Friday will not develop the spirit of problem solving.

Every problem in a problem solving situation need not and perhaps should not have an answer. If problem solving is to relate to the world, to allow students to develop skills necessary to deal with the variety of personal, professional and daily experiences of life, then the process should be consistent with real experiences - for which there are often no right answers or for which there are several equally valid options. The problem solving experiences of our students should reflect these situations.

Students should not be programmed to categorize and solve every problem according to some master plan. Problem solving involves many methods and strategies, dependent on the nature and perception of the problem solver. Students should be provided with a foundation from which to work. Strategies need to be explored and discussed, imagination encouraged and a variety of problem solving techniques developed.

Teachers cannot be expected to become proficient at teaching problem solving without some encouragement and direction from their administration and from educational leaders in the mathematics community. Many facets of the topics have been explored in several excellent presentations. More research needs to be done, more programs need to be developed. Seek out those involved and share your experiences. Teachers and students will all benefit.

> 'Recommendation 3: Mathematics programs must take advantage of the power of calculators and computers at all grade levels.'

What place do computers have in our schools today? Is one sitting unused in your library? Who uses computers and for what purpose? In some schools the students cannot touch the machine until they know how to use it. In others, computers are used only to play games. The profusion of computers in every phase of society has raised many questions for educators, questions which must be carefully analyzed to provide sound direction for computer usage in the classrooms.

Schools, parents and educators associate computers with mathematics. All mathematics educators, however, are not necessarily equipped to handle the issue. The average teacher in this country has more than ten years of classroom experience and has had little or no training in the area of computers and calculators. James M. Rubillo in 'A Sign of Strength: Admitting a Weakness' in the October, 1981, issue of the **Arithmetic Teacher** recommends that every school district in the country create an inservice computer awareness program for all its teachers. The goal of such a program would be to 'provide teachers with the background to design a curriculum for the student and then to select the equipment necessary...' When the equipment is chosen before the curriculum is planned, there is often an unused micro sitting on a library table.

We in Texas are fortunate to have many qualified computer personnel who teach university computer programs, present sessions at meetings and provide leadership for sponsored computer workshops. Yet, the audience is limited and the contact, in most cases, brief. Texas school districts need to heed Rubillo's advice and sponsor computer inservice for all its teachers. If schools are to follow through on the NCTM recommendations to take an active part in preparing students to live in a world of computers, the educators in those schools must be given inservice programs to fulfill the challenge.

Although computers have become an adjunct to the mathematics curriculum, they should not be confined solely to mathematics classes nor even to business courses. Mathematics educators must cooperate with educators in other disciplines for the purpose of developing software applicable to those areas and encouraging other teachers to make use of the computer as an educational tool. Computers should become a part of the total school environment, and to accomplish this, inservice computer education is necessary. Get your district involved. Ask for some direction and help from those who work with computers and computer technology.

> 'Recommendation 6: More mathematics study must be required for all students and a flexible curriculum with a greater range of options be designed to accommodate the diverse needs of the student population.'

In the September, 1980, **Mathematics Teacher,** Zalman Usiskin's article "What Should not be in the Algebra and Geometry Curricula of Average College-Bound Students?" suggested some changes in the content of algebra and geometry courses. While controversial, these recommendations could provide a needed impetus for evaluating the traditional mathematics curriculum. There are topics in algebra and in geometry that are not necessary for the mathematical proficiency of every student. How important are rational polynomials for all of your students? Where can you fit computer literacy? Does your curriculum meet the diverse needs of your students?

According to the **Agenda** 'Mathematical competence is vital to every individual's meaningful and productive life.' The same mathematics, however, is not vital to every student's future. Programs must be developed to ensure that everyone will have the mathematics necessary for a variety of potential careers and the mathematics necessary to function as a productive citizen. An often overlooked area that should be considered as basic is probability and statistics. In today's complex world an every increasing amount of data is being collected, analyzed and thrust at the public. A background in probability and statistics will enable an individual to deal competently with this information. Areas of study which at one time required little mathematics now depend on complex statistical techniques.

Statistics, in fact mathematics as a whole, is becoming more visible in its applications to science, technology and nature. Mathematics courses designed to involve these applications can deepen the understanding of a mathematical concept as well as reinforce an emphasis on problem solving. Cooperation with other disciplines will increase an awareness of these applications and provide for the integration of mathematics in other areas. To realistically cope with the varied situations in the world, students need to recognize that mathematics is not an isolated topic.

In essence, mathematics supervisors and departments must evaluate their curriculum in terms of the students in their district. Mathematics educators must design programs preserving the integrity of the traditional mathematics courses yet meeting the many new challenges. In their search for better programs, for new material, for new and better ways to communicate this material, the Teachers can play a vital role. Your duty as a teacher is to fulfill the recommendations for the 80s. Become involved. Help us help you keep Texas mathematics in touch with the times.

(Adapted from an article in Fall, 1981, issue of **Wisconsin Teacher of Mathematics***)*

TCTM Journal needs articles for all levels of Mathematics.

WHAT ARE SOME FUTURE TEACHERS SAYING ABOUT ELEMENTARY SCHOOL MATHEMATICS?

Anthony Maffei
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Elementary school majors can recall quite well their own elementary school experiences when learning mathematics. For some it was a time of anxiety as evidenced by the imprint it has left on their attitudes toward the subject. Is it possible, then, that some future teachers could unknowingly create an environment for turning students off to learning math? Professional educators at the undergraduate level should seize the opportunity to address this problem in their math methods and/or math content courses.

EVALUATION

From the input of approximately 250 elementary majors from a nearby teacher's college, the writer devised a final questionnaire consisting of 40 distinct statements each expressing a specific trait that could lead to the development of negative mathematics attitudes in the elementary grades. The questionnaire was then administered to a new group of 145 elementary majors who were enrolled in either a math methods or contents course. From their elementary school experiences, they were asked to check only those reasons why they believed elementary school students can develop negative attitudes toward math.

Due to space limitations, only some of those statements receiving a frequency rating of more than 60% will be discussed.* From the experiences of this writer as a math consultant for several school districts, it is also evident that many of these statements can be applied in varying degrees to the current elementary classroom scene.

**The 40 item questionnaire with frequency ratings is available upon request at the above address. Please enclose a self-addressed, stamped envelope when making your request.*

RESULTS

1. Math is taught on a memorization level with few "whys" given for understanding a problem (74%).

Perhaps the best way to explain the "whys" of mathematics on the elementary level is through the use of manipulatives, diagrams, pictures – in short, usually a technique that does not exclusively use symbols to explain a mathematical idea. Unfortunately, many teachers use the textbook as their only curriculum source. Materials which involve hands-on experiences, of the teacher-made or commercially tested type, especially provide a sound foundation for the eventual transition to the symbolic nature of mathematics.

Elementary majors should be thoroughly familiar with these important ways of teaching concepts. They should be given first-hand experiences in designing manipulatives, pictures, models, etc. and also be instructed to use them more than just at the beginning of a new topic but when the need arises.

2. Teachers do not make math interesting, lively, and fun with games, puzzles, etc. (74%).

Some elementary teachers are not aware of the motivational impact that enrichment materials, such as easy to follow games, puzzles, etc., have upon students. However, these are usually readily available from commercial sources, in-services, and periodicals.

On the undergraduate level, students must be shown through experience how a simple math game in the format of a bingo or tic-tac-toe will do more to motivate students to learn their facts and skills than will constant drill work on rows and columns of the same type of problem.

3. Teachers tend to cover topics too quickly before students have a chance to understand them (73%).

More than ever before, principals and teachers are making some progress in addressing the fact that different students learn at different rates. Many instructional models used in the classroom today stress the importance of assessing a student's strengths and weaknesses and of planning an appropriate program of study. There is still much work to be done in this field as educators search for systems that optimize student progress while at the same time minimize paper work.

Future teachers would benefit more from the first hand experiences of observing a classroom which is meeting the learning needs of its students than just by discussing the problem in class.

4. Teachers don't vary their way of teaching math; that is, from seatwork to small groups, to class discussions, to blackboard, etc. (72%).

For the most part, students learn and do their math in isolation whether it be of the paperwork type or just listening to the teacher explain a problem. Some believe that this emphasis is due in large part to the "right answer syndrome" where working in isolation prevents students from cheating (Tobias, 1981). Unfortunately, this method will produce anxiety in students for it fails to take advantage of the interchange of ideas that can come about from utilizing peer interaction, small group discussions, project groups, etc.

Future teachers should be exposed to math that is taught in an atmosphere of open-mindedness where an exchange of ideas between students and teachers is crucial to the learning process and where errors by students, and even by teachers, are not laughed at but are accepted as part of this process. Such an atmosphere can probably be best achieved in a varied instructional setting.

5. Math is taught as either right or wrong with no "in between" (72%).

Since standardized and teacher-made tests place credit on getting the right answer, it is not uncommon for teachers and students to look at math as a subject dealing with only "getting the right answer" rather than with strategies and methods for getting that correct response. Unfortunately this thinking is obviously creating psychological damage for some students.

Teacher educators can help alleviate this problem by incorporating into their course requirements provisions that will train teachers to be able to diagnose a student's problem and to prescribe appropriate remedies. The ultimate ramification of such a requirement should make a teacher aware of a student's thinking processes as he goes about doing a problem. Some credit for work can then be awarded on the basis of how much of the problem employs a correct method even if the final answer is incorrect. Commercial test designers can also help by devising some items asking students to choose from a list of methods, a correct one for solving a specific problem, rather than only asking them to do the problem and find the answer.

6. Students give up when they cannot get the answers quickly (67%).

Students erroneously assume, probably through teacher conditioning, that speed and quickness in getting answers are the signs of a successful mathematics student. As Tobias (1981) points out, one source of anxiety toward math is the time pressure type test. Through the use of flash cards, timed tests, and competition to place first students are being taught that math is a subject that in order to learn well it must be learned quickly. Obviously, there are many students who, because of their learning styles, cannot operate well in this environment.

Teachers, as well as future ones, should develop in their students an attitude that persistence and not speed is the important key to doing math for "the child who sticks with a problem, who stubbornly turns it around in his mind until it becomes clear to him, may ultimately be a better math student than his 'speedier' classmate." (Tobias, 1981, p. 35).

7. Students never really master the basic skills (66%).

Since many of these respondents were still exposed to the "new math", this response seems appropriate, if we are assuming skills are equivalent with facts since the new math era did stress concepts more than facts. However, skills mean more than facts, a position outlined by the National Council of Supervisors of Mathematics (October 1977, AT).

Consequently, teacher educators should be spending equal time instructing these future teachers on the techniques of imparting such skills as estimation, approximation, geometry, measurement, computer literacy, etc. as well as on computational facts. Some current classroom teachers tend to spend an excessive amount of time on computational facts even when their state's minimum competency tests include the other skills. However, it is difficult to find fault with these teachers when part of the reason is probably due to the fact that these skills were not incorporated in their own pre-service training.

8, **Few practical applications of math to everyday life are given (63%).**

The position papers by the National Council of Supervisors of Mathematics (October 1977, **AT**) and by NCTM's **Agenda For Action Recommendation for School Mathematics of the 1980s** have done much in directing mathematics curriculum toward addressing the needs of everyday situations. Mathematics methods courses at the undergraduate level should also include topics that would help teachers see math as it is used in related careers as well as day-to-day experiences. As a result, fewer students would question the importance of learning the subject as they go from grade to grade.

CONCLUSIONS

Although this article has emphasized certain directions in the preparation of elementary school teachers, it is by no means relegated to that realm. The practicing classroom teacher can also employ these suggestions mainly from an in-service program which addresses these issues. For hopefully, both avenues of attack will improve student attitudes and consequently student achievement in learning mathematics.

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ISSUES IN THE MATHEMATICS CURRICULUM

By Marlow Ediger
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There are bsic, essential learnings which all pupils need to achieve in mathematics, according to selected educators and most parents. No doubt, all pupils need to develop adequate proficiency in addition, subtraction, multiplication, and division. The essentials are relevant for pupils presently, as well as in the adult world. Practical use can then be made of learnings involving the basics in the school curriculum. What has been acquired as essential subject matter in ongoing units of study is used to solve personal and social problems involving mathematics.

ISSUES AND MATHEMATICS

There are teachers emphasizing that pupils achieve proficiency in bases other than base ten. For example, a pupil may achieve understandings of base five numeration. With meaning and interest attached, a pupil might then understand five symbols used in base five. These are 0, 1, 2, 3, and 4. Understanding of place value is very significant. Thus, in 213 base five, there are three ones, one five, and two twenty-fives (3 + 5 + 50 = 58 in base ten). Concrete, semi-concrete, and abstract materials may be utilized as learning activities to assist pupils to understand place value in base five.

To practice application of base five learnings, pupils may utilize toy money to buy selected items in a miniature supermarket in the classroom setting. Empty several boxes, fruit and vegetable containers, and flour sacks, among other items, may have prices stamped in base five values. Base five toy money is then utilized to "buy" chosen items. Adding the prices of a given set of items and determining change are then emphasized in base five learnings.

Advantages given for emphasizing base five learnings include:

1. learners obtain new experiences pertaining to place value and digits used in base five.

2. enrichment experiences are available for gifted and talented pupils, especially on higher grade levels.

3. pupils may expand mathematical thinking in going beyond a single base (base ten) and incorporate experiences involving base five, along with other bases.

Disadvantages given for pupils studying other bases than base ten include the following:

1. learners need to understand and attach meaning to have ten due to its highly functional use in society. Other bases generally do not have a functional value in society.

2. pupils experience frustration in attempting to understand a different base than base ten.

3. time is valuable in the mathematics curriculum. Available time must be given to guide pupils to understand base ten.

A second issue in the mathematics curriculum involves the degree of specificity necessary to state useful objectives. Behaviorists believe that objectives for pupils to achieve should be written in measurable terms. The teacher may then select learning activities for pupils to achieve the chosen ends. After instruction, the teacher may measure if a pupil has or has not attained the measurable objective. A new teaching strategy may need to be chosen to help the unsuccessful learner achieve the objective. Successful learners may work on achieving the next sequential end. Pupils individually may achieve optimally in attaining the specific ends.

Humanists believe in an open-ended amthematics curriculum in which each pupil may choose sequential tasks. The teacher may structure the learning environment. Pupil-teacher planning of objectives, experiences, and appraisal procedures might also be utilized. Pupils, however, decide which tasks to pursue, as well as which to omit. Learners generally choose tasks based on inherent interests, purposes, and personal meaning.

Advantages given for emphasizing behaviorism in the mathematics curriculum include the following:

1. The teacher is certain as to what will be taught in mathematics based on precise objectives selected for pupil attainment.

2. The teacher may measure if a pupil has or has not achieved an objective. A teacher might then be certain if an objective has been attained before a pupil moves on to the next sequential goal. Proper sequence is experiences may then be in evidence for each learner.

Advantages given for humanism, as a psychology of learning, might involve the following:

1. pupils are more interested in learning if choices may be made as to what to learn sequentially.

2. life in society requires that pupils choose and make choices. The school curriculum presently must also emphasize decision making by pupils.

3. a humane curriculum may well be in evidence if input from learners is an end result in curriculum development.

A third issue pertains to the degree hand held calculators should be utilized in the mathematics curriculum. The conservative element is with us in advocating that little or no use be made of calculators. Advocates of minimizing the utilization of calculators in the mathematics curriculum believe in the following statements:

1. pupils need to develop proficiency in addition, subtraction, multiplication, and division using paper and pencil in the basic four operations. Audio-visual aids may be utilized to make learnings meaningful to pupils. However, computation skills must be developed using paper and pencil.

2. mastery learning in addition, subtraction, multiplication, and division prepares pupils to become problem solvers in the adult world. Drill and practice are important in committing to memory basic facts in addition, subtraction, multiplication, and division. Using hand held calculators hinders pupils in developing proficiency in arithmetic.

Those advocating rather heavy use of hand held calculators by pupils in arithmetic believe the following:

1. much drudgery can be eliminated in the arithmetic curriculum. For example, once pupils understand a new process in a meaningful manner, learners may then use hand held calculators in problem solving experiences. If pupils understand how to check a long division computation, interest in learning is destroyed if paper and pencil procedures are continually used in checking procedures.

2. pupils may experience drill and practice in responding to basic number pairs using hand held calculators. For example, if a pupil in practicing adding 3 + 5, 5 + 3, 6 + 4, and 4 + 6, among others, he/she may respond orally to 3 + 5 = □ and 5 + 3 = □ before pressing the equals sign on the hand held calculator. In sequence, with more complex learnings involved, learners may again and again meet up with the commutative property of addition A + B = B + A, e.g. (3698 + 4347 = 4347 + 3698).

Advantages given for utilizing hand held calculators rather heavily in the mathematics curriculum include the following:

1. technology is with us and will increasingly influence methods of teaching and means of arriving at specific decisions.

2. pupil motivation for learning may well increase with the utilization of calculators.

3. learners interest in achieving objectives in mathematics might well increase when learning activities are more enjoyable involving the use of technology.

A fourth issue in the mathematics curriculum pertains to the degree geometry should be emphasized in the mathematics curriculum. Lay people in society feel heavy emphasis needs to be placed on the three r's (reading, writing, and arithmetic) in the curriculum. There are selected educators who agree on the rather heavy implementation of a three r's curriculum. What then might be the role of geometry in the mathematics curriculum? During the 1960's "modern" mathematics was emphasized. Mathematicians on the college/university level had much input into the mathematics curriculum for elementary, junior high or middle school, as well as secondary levels of instruction. Numerous mathematicians advocated geometry receiving somewhat equal emphasis as compared to arithmetic in the mathematics curriculum. Reasons given for advocating a strong geometry curriculum include the following:

1. geometry is in eveidence continally in the natural and human made environment. For example, in diverse buildings, there are square, rectangular, and circular windows. Line segments, points, planes, solid figures, and angles are inherent in geometric figures.

2. the concept of **balance** needs to be emphasized in the mathematics curriculum. Thus, geometry, as well as arithmetic, statistics, probability, and algebra need to comprise the total program of instruction in mathematics.

3. there may be no ponflict between metric geometry and arithmetic in the mathematics curriculum. For example, in metric geometry, considerable arithmetic is utilized to determine perimeters and areas of geometric figures.

A fifth issue involves inductive–deductive controversies in teaching. In inductive teaching, the teacher needs to guide pupils to make discoveries. Lecture and lengthly explanations are not desired in inductive methods of teaching. Rather, the teacher uses a variety of materials to set the stage for inductive learning. For example, if pupils are to learn regrouping and renaming in subtraction (borrowing), the teacher might utilize a concrete situation in which the temperature reading dropped from 35° to 19° Fehrenheit. Pupils with teacher guidance may actually read and record the temperature readings. How might the problem by solved as to how many degrees the temperature reading dropped? One pupil may respond with the need to count from 19 to 35 to determine the difference. Another learner may respond with showing 35 sticks and taking 19 away. Thus 35 – 19 = 16. The temperature reading then dropped in degrees. A learner might also show a more sophisticated way of operating to determine the difference between 35 and 19 degrees using a place value chart. Each learner may then understand what is involved when regrouping 35 in terms of 2 tens and 15 ones in a place value chart. Nine ones may then beeremoved from 15 ones, and one ten from two tens. The remainder is 16.

In a deductive method in having pupils understand how many degrees in temperatur readings were involved in dropping from 35 to 19, the teacher may use concrete (manipulative materials), semi-concrete (picture, filmstrips, slides, and study prints), as well as abstract materials. Along with these materials, the teacher in a meaningful way may explain to learners what is involved in regrouping and renaming to determine the difference between 35 and 19. Communicating content moves from the teacher to the pupil. Later pupils with teacher guidance need to apply what has been learned.

ADDITIONAL ISSUES IN THE MATHEMATICS CURRICULUM

There are other relevant issues needing resolving in ongoing units of study in mathematics. These include:

1. an activity centered versus a subject centered mathematics curriculum.

2. subject matter learned by pupils as a means to an end versus subject matter learned as an end in and of itself.

3. an integrated versus a separate subject mathematics curriculum.

4. utilitarian goals versus a basics mathematics curriculum stressing essentials for all learners.

IN CONCLUSION

Teachers, principals, and supervisors need to study vital issues in the mathematics curriculum. After careful analysis of the issues, solutions need to be developed. Ultimately, pupils used to experience relevant objectives, learning activities, and appraisal procedures. Each pupil needs to achieve optimally in mathematics.

MONDAY'S CHILD IS FAIR OF FACE, TUESDAY'S CHILD IS . . . . .

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The reader likely has heard of (or, perhaps, even has seen) someone with the ability to do the following: given the year, month, and day of the month, the person in question is able to name the day of the week.

In this note, a formula will be derived which, when the appropriate information is supplied, determines the day of the week. The formula naturally is based on the Gregorian calendar (with a slight modification which will be discussed later). The Gregorian calendar, the one presently used, was instituted by Pope Gregory XIII on February 24, 1582, replacing the Julian calendar. Incidentally, the Gregorian calendar was not adopted in the U.S.A.until 1752. Why was it necessary to replace the Julian calendar? It is unfortunate, perhaps, that the length of time for one revolution of the earth about the sun, one year, is not an integral multiple of the length of time for one revolution of the earth about its axis, one day. In fact, one year is approximately 365 days, 5 hours, 48 minutes, and 46 seconds, or 365.2422 days. The Julian calendar was based on a 365.25 day year. At first thought, it might seem that the use of the Julian calendar, or for that matter, any calendar which did not compensate for the difference between .25 and .2422 would cause no serious difficulties. But, when one realizes the need for accurate predictions for changes in seasons using the calendar, one can see, for example, that after many years, the first day of Spring will not occur on the same day of the year as in the past. In fact, from the year of the institution of the Julian calendar in 46 B.C. until the institution of the Gregorian calendar in 1582, the deviation had amounted to 10 days. As a result, 10 days were dropped from the year 1582; Friday, the fifth of October became Friday, October fifteenth. The net effect was a "realignment" of the calendar with the seasons.

How does the Gregorian calendar better compensate for the estra .2422 of a day? The reader, obviously, knows that the year 1896 was a leap year; but, the reader may not be aware that the year 1900 was not a leap year. The rule for determining whether a year will be a leap year with the Gregorian calendar is as follows: if the year is divisible by 4 but not by 100, the year is a leap year; if the year is divisible by 100 but not 400, the year is not a leap year; and if the year is divisible by 400, the year is a leap year. As a contrast, with the Julian calendar, every year divisible by 4 would be a leap year.

The reasoning behind this rather complicated, and seemingly somewhat contrived, rule is that after 4 years, we have gained .9688 of a day. Hence, if we add a day to our calendar (leap year), we are "behind" .0312. After 100 years (25 occurrences of the above), we are "behind" .7800 of a day; thus, we do not add a day to our calendar (not a leap year). This will mean that after 100 years, we have gained .2200 of a day. After, 400 years (4 occurrences of the above), we have gained .8800 of a day. Therefore, it is time to add a day to our calendar (leap year). Although much more accurate than the Julian our calendar (leap year). Although much more accurate than the Julian calendar, the Gregorian claendar has an "error" of approximately .0003 of a day per year.

Now, to derive our formula it will be necessary to choose a starting point and, as will soon be seen, to keep track of the number of leap years. For convenience, we choose the year 1600 as our starting point, and our first objective will be to find the numbers of leap years between the year 1600 and the year N ($\geq$ 1600). The brackets denote the greatest integer function; for example, $[3.2] = 3$. Let L denote the number of leap years between the year 1600 and the year N; then

$$L = \left[\frac{N-1600}{4}\right] - \left[\frac{N-1600}{100}\right] + \left[\frac{N-1600}{400}\right],$$

where the terms progressively add a 1 for each year divisible by 4, remove the 1 for each year divisible by 100, and replace the 1 previously removed for each year divisible by 400. Using the fact that $[x+n] = [x]+n$ if n is an integer,

$$L = \left[\frac{N}{4}\right] - \left[\frac{N}{100}\right] + \left[\frac{N}{400}\right] - 400+16-4.$$

Next, using the division algorithm, $N=100C+D$, where

$$0 \leq D < 100,$$

$$L = 25C + \left[\frac{D}{4}\right] - C - \left[\frac{D}{100}\right] + \left[\frac{C}{4} + \frac{D}{400}\right] - 388.$$

Since $0 \leq \frac{D}{100} < 1$, $\left[\frac{D}{100}\right] = 0$. Also, since

$$0 \leq \frac{D}{400} < \frac{1}{4}, \left[\frac{C}{4} + \frac{D}{400}\right] = \left[\frac{C}{4}\right].$$

Thus, $$(1)\quad L = 24C + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 388.$$

Now, 365 = 52.7 + 1 and 366 = 52.7 + 2. Hence, if a year is not a leap year, we "move forward" one day of the week the next year; so that, for example, if July the fourth were on a Tuesday of a non-leap year, then July the fourth would be on Wednesday of the following year. Furthermore, if the year were a leap year, we "move forward" two days of the week the next year; so that, for example, if July the fourth were on a Tuesday of a leap year, then July the fourth would be on Thursday of the following year.

In what follows, 0 will denote Sunday, 1 will denote Monday, etc. This should seem rather natural. In addition, March will be the first month, April will be the second month, etc. This may not seem so natural. (This is the slight modification of the Gregorian calendar). The purpose in "naming" March the first month is so that the extra day during a leap year can be added as the last day of the year. The result will be of tremendous help in simplifying our work. The trade off is that March the first will be "New Year's Day," so that we should not change the year number for the months of January or February. That is, for days in the months of January and February of the year N, the year will be considered to be N-1. Therefore, let March 1, 1600 be our starting point and let "a" denote the day of the week for this date. From March 1, 1600 to March 1, N we will have "moved forward" 1 day for every non-leap year and 2 days for every leap year. Thus, we have "moved forward"

$$100C + D + 24C + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 388$$

days. Therefore, if "b" denotes the day of the week of March 1, N,

$$b \equiv a + 100C + D + 24C + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 388 \pmod 7$$

which reduces to

$$(2) \quad b \equiv a + D + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 2C - 3 \pmod 7.$$

Next, let N = 1981. March the first of 1981 was on a Sunday; thus, b = D, C = 19, and D = 81. Substitution into (2) and solving yields a = 6 (mod 7); Saturday! This simplifies (2) to

$$(3) \quad b \equiv 3 + D + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 2C \pmod 7.$$

Since there are 31 days in March and 31 = 4.7+3, April the first is 3 units "greater" (mod 7) than March the first. Rev. Christian Zeller discovered that the increments between the months, like the 3 mentioned above, could be computed, modulo 7, using [2.6m-.21], where 1 denotes March, 2 denotes April, etc.

For m = 1, [2.6-.2] = 2; so,

$$b \equiv 1 + [2.6-.2] + D + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 2C \pmod 7.$$

It follows that if "c" denotes the day of the week of the first day of the m-th month of the N-th year, then

$$c \equiv 1 + [2.6m-.2] + D + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 2C \pmod 7.$$

Finally, if "d" denotes the day of the week of k-th day of the m-th month of the N-th year, then

$$(4) \quad d \equiv k + [2.6m-.2] + D + \left[\frac{D}{4}\right] + \left[\frac{C}{4}\right] - 2C \pmod 7.$$

Now, for a few examples. The answer to our First question should be known to every student of history.

EXAMPLE 1: What day of the week was December 7, 1941?

C = 19, D = 41, m = 10, k = 7

d = 7 + [26-.2] + 41 + [41/4] + [19/4] – 38

= 7 + 25 + 41 + 10 + 4 – 38 = 0 (mod 7)

Sunday!

Recall that for the months of January and February, the year number will be for the "previous year," as in

EXAMPLE 2: What day of the week was January 1, 1981?

C = 19, D = 80, m = 11, k = 1

d = 1 + 28 + 80 + 20 + 4 – 38 = 4 (mod 7)

Thursday!

As a final example, suppose an attempt is made to find the day of the week for February 29 for a year which is not a leap year. As one would guess, the formula should yield the day of the week for March the first of that year. This is precisely the case, although the situation is a bit more complicated than one might think. In going from February 29 of some non-leap year to March 1 in the same year, we not only change the value for k, but also the values for m and N. As it happens, the differences cancel, yielding the correct day of the week for March 1, as in

EXAMPLE 3: What day of the week was February 29, 1981 ?

C = 19, D = 80, m = 12, k = 29

d = 29 + 31 + 80 + 20 + 4 – 38 = 0 (mod 7)

Sunday!

The actual date is March 1, 1981, and the "correct" values are

C = 19, D = 81, m = 1, k = 1,

which yield

$d = 1 + 2 + 81 + 20 + 4 - 38 = 0 \pmod 7$

Sunday!

As for references, any good encyclopedia will have a section on calendars, including discussions on both the Julian and the Gregorian calendars. The two references listed below have discussions on the formula we have derived. The second reference, ELEMENTARY NUMBER THEORY, also considers the problem of determining the month and the day of the month of Easter. In addition, questions of the following type are considered: given a year N and a month m, what days of the month will, for example, Saturday "fall on"?

REFERENCES

1. Archibald, R. G., AN INTRODUCTION TO THE THEORY OF NUMBERS, Columbus: Merrill Publishing Co., 1970.

2. Uspensky, J. V., and M. A. Heaslet, ELEMENTARY NUMBER THEORY, New York: McGraw-Hill Book Co., 1939.

MATHEMATICAL "FOOD FOR THOUGHT"

Problem solving must be the focus of school mathematics. A **problem** exists when a person is faced with a goal to be achieved but encounters obstacles to reaching that goal because no known algorithm or procedure is available to resolve the difficulty. Deliberation is required to determine a procedure for reaching that goal. The process of attacking such a problem – of accepting the challenge; formulating the appropriate questions? clarifying the goal; defining, testing, and executing the plan of action? and evaluating the outcome – is what is meant by problem solving. A problem cannot be called a problem until it is accepted by the problem solver who takes it and becomes personally involved in its solution. A question which has been preceded by a rule or formula of theorem or algorithm that, if applied correctly, guarantees a correct answer, is an exercise, not a problem. Exercises have their usefulness and importance; but, anything that is stated in words – the familiar "story problem" – should not be considered a problem. The thing that differentiates a problem from an exercise is the previous learning of the problem solver. What is one person's problem can be another person's exercise. We need a new teaching style. Problem solving is not a sequence of activities to be performed or an answer to be determined so much as it is an attitude of mind, and to make problem solving the focus of school mathematics means to create a learning environment permeated by the spirit of inquiry.

Teachers **do** too much, **talk** too much, **show** too much, **explain** too much. In short, teachers treat pupils as consumers of information rather than as creators of knowledge. The students still use textbooks, as well as a wide range of other resources and learning aids, but they use them in a variety of different ways. The teacher could pose questions that the pupils do not yet know how to solve, and the class activity focus not on the answer but on figuring out how to get the answer. Some problems contain insufficient data; the students must decide what additional information they need and how to get it: consult references; make observations; take measurements; sample public opinion; or collect, organize, and interpret data. Other problems contain too much information, and students must decide what is necessary and what is extraneous. Once a problem is solved – evaluate both the answer and the procedure; find different methods for solving the same problem; look for other problems that could be solved in a similar manner; create problems; suggest new questions that arise from the present answer; develop formulas or algorithms they can use again; relate their new knowledge to other things they already know about mathematics.

Mathematics is . . . .

. . . not the same as arithmetic (computation), nor is it algebraic manipulation, nor geometric proofs. It is not a collection of rules and theorems and algorithms, but a system of representing and communicating new ideas. Problems do not have ready solutions, and they make you work and guess and search and experiment and deliberate and evaluate. Figure out how to do things which could not have been done previously. Running into blind alleys and having to retreat and begin anew is mathematics in its living, growing form.

. . . not rigid. There is not one right way to do a problem. Often there is not even one right answer. An important aspect is to look for many ways to solve a problem, to compare those ways and to see which methods are more precise, more reliable, more efficient, more generalizable to other situations, or less difficult to use or to recall.

. . . not magical or capricious. We do not "change a sign" or "move a decimal" or "invert a fraction" because of some hocus-pocus that mysteriously produces the right answer. Rather, mathematics is reasonable and understandable and predictable. Patterns and regulatities help us to understand mathematical ideas and to solve new problems.

. . . is all around us. It most assuredly is not confined to the pages of textbooks or worksheets. Familiar things have mathematical properties and students should develop a habit of looking for mathematical aspects of everyday things. Ask: "Waht if?" or "What if not?"

. . . is creative. It is not "all finished" and tied together in rules and algorithms. Mathematics and problem solving are open-ended, never-ended. Mathematical facility is accessible to everyone. We can ask our own questions and create our own problems and find ways to solve problems we did not know we could solve. We can have fun with mathematics, and we can know the satisfaction and real joy of personal accomplishment that come from creating something uniquely our own – of looking on it and knowing it is good.

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SUMMER INSTITUTE/MATHEMATICS

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Mathematics, statistics, computer literacy and programming language will be covered in five institutes at The University of Texas at San Antonio this summer.

Offered through UTSA's Division of Mathematics, Computer Science, and Systems Design, the courses are part of the 1982 Summer Institute, which offers educators and other professionals an opportunity to earn graduate level credits in a short-term, intensive format at a time when they are able to return to campus.

Institutes and their dates are: Mathematics for In-Service Teachers, May 31 - June 18; Problem-Solving for Secondary Teachers, June 14 - July 2; Computers and Human Relations, July 6 - 26; Programming Lanuage Instruction -- PASCAL, July 21 - August 10; and Methods of Statistics I, July 21 - August 10.

Dr. Ray Carry, professor at The University of Texas at Austin and nationally recognized expert in the field of mathematics education, will teach the institute for in-service teachers of mathematics who wish to improve their knowledge of fundamental math concepts.

→ more →

The institute covering mathematics problem-solving will be conducted by Dr. Jim Wilson, professor and head of the department of mathematics education at the University of Georgia. Dr. Wilson, authority on problem-solving, will include activities from Mathematical Discovery by George Polya.

Experience with computers and discussion of their use in society will be included in the computers and human relations institute, to be taught by Dr. Betty Travis, assistant professor of computer science at UTSA.

The programming institute will be taught by Howard Smith, computer science instructor at UTSA, and will cover programming in PASCAL and editing on MUSIC systems.

Dr. Ram Tripathi, associate professor of mathematics at UTSA, will be teaching the statistics course which covers basic statistical techniques to analyze data and interpret output.

The graduate level classes which comprise the Summer Institute are considered regular summer offerings of the university, and interested persons must be admitted to the university before registering for any of the classes.

Participants should apply for admission and have completed admission files by May 3. Students enrolled at UTSA for the 1982 spring semester may register for institutes without re-applying for admission to UTSA unless they were transient students.

For admission or registration information, phone the UTSA Office of Admissions and Registrar, (512) 691-5432, or write the UTSA Office of Admissions and Registrar, San Antonio, Texas 78285.

Questions about the Institute may be directed to: Summer Institute Coordinator, Division of Continuing Education, The University of Texas at San Antonio, Texas 78285, (512) 227-9147.

MU ALPHA THETA
12th National Convention
St. Louis, Missouri

The Twelfth Annual Mu Alpha Theta National Convention, co-sponsored by the National Council of Teachers of Mathematics and the Mathematical Association of America will be held on the campus of Washington University in St. Louis, Missouri, August 8-11, 1982. Speakers from various fields of mathematics and science will address the participants. There will also be a major emphasis placed on mathematical competitions. In addition to the regular Math Bowl, exciting math relays* on numerous topics as well as chess and backgammon tournaments, and a Rubic's Cubathon will be held to challenge contestants. Trophies, medals, and certificates will be awarded. All interested high school students are invited to attend the convention and participate in the math contests. For further information contact Akehiko Takahashi at Wentzville High School, #1 Campus Drive, Wentzville, Missouri.

*Math Relays (20-minute written tests) cover these topics:

1. functions
2. trigonometry
3. number theory
4. logic
5. conics
6. logs and exponents
7. sequences and series
8. word problems
9. mini-calculator
10. graphing
11. complex numbers
12. triangle properties
13. math history
14. metric
15. elementary calculus
16. matricies
17. equations and inequalities
18. set theory
19. math analysis
20. probability
21. circles
22. polynomials
23. math puzzles and problem solving
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