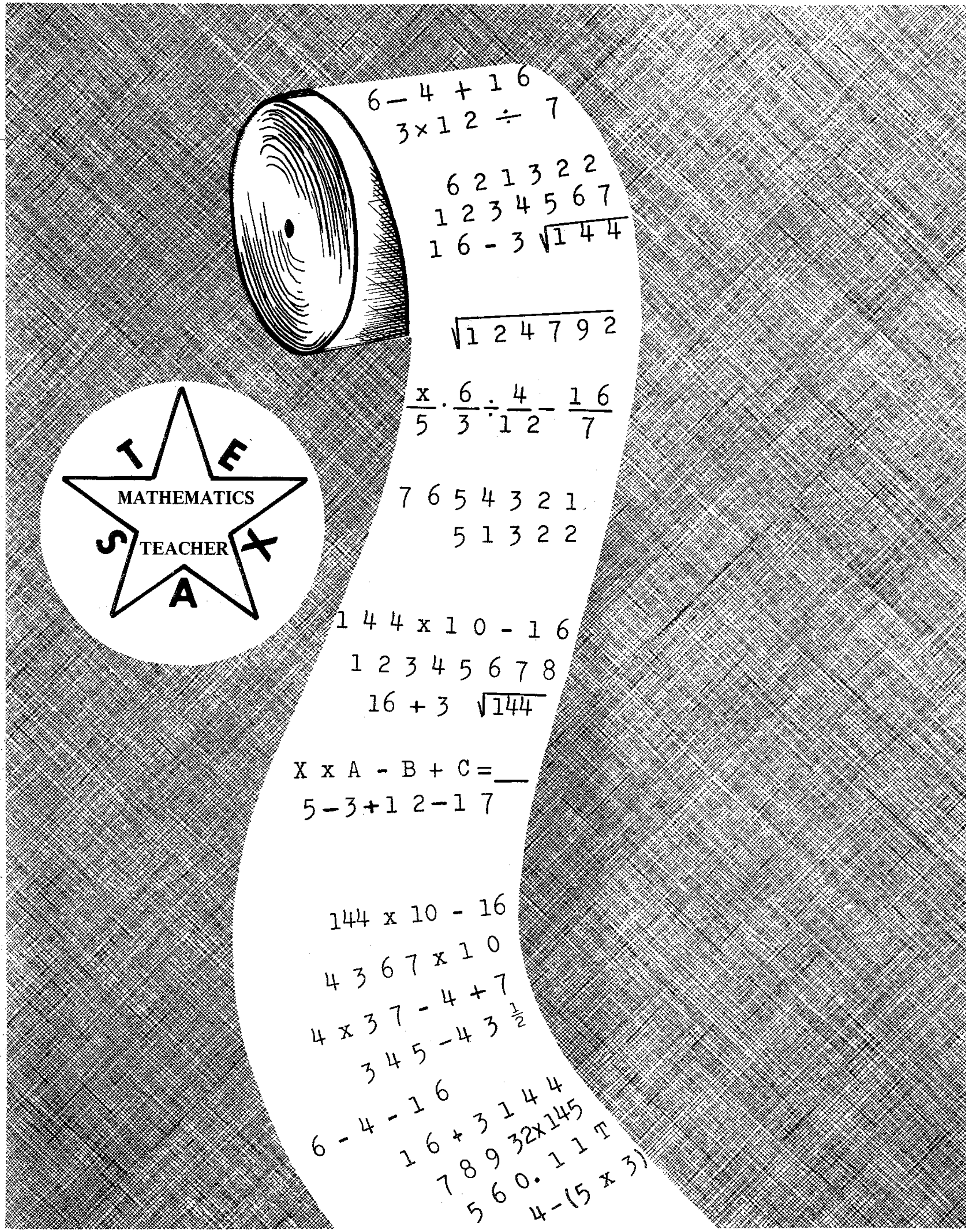

TEXAS MATHEMATICS TEACHER

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Texas Council of Teachers of Mathematics

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PRESIDENT'S MESSAGE

The CAMT conference this fall had the largest attendance ever with over 1500 registered. Janie Schielack, TEA, Barbara Cager, FWISD, Susan Lair, FWISD, Avis Stone, FWISD, Tina Greenlee, FWISD, and Anita Priest, DISD, assisted me with the on site registration. Several others helped by relieving us for breaks and lunch. Thanks to all who attended for their patience and good-naturedness.

House Bill 246 of the 67th Texas Legislature directed the revision of the state approach to curriculum. Each school district that offers kindergarten through twelfth grade will be required to offer a well-balanced curriculum that includes: English language arts, other language, mathematics, science, health, physical education, fine arts, social studies, economics, business education, vocation education, and Texas and United States history. The Legislature has also directed the State Board of Education to designate the essential elements of each of the 12 subjects and to require each school district to provide instruction in those elements at the appropriate grade levels.

The Commissioner of Education will appoint individuals to serve on work committees to gain detailed information and advice from the subject matter specialists. TCTM has submitted four nominations: John Huber, Southeast, Betty Travis, Southwest, Bryon Craig, Northwest, and Patsy Johnston, Northeast.

Diane McGorman, Vice-President TCTM, is now in charge of membership. Please contact her for assistance with joint membership forms or membership problems.

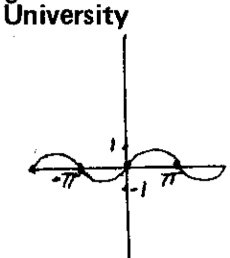
Patsy Johnston

TCTM Journal needs articles for all levels of Mathematics.

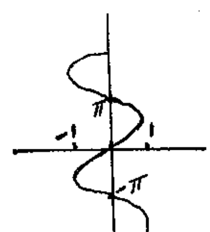
USING THE CALCULATOR ON THE PROBLEM OF THE RECTANGULAR NUMERICAL APPROACH TO INTEGRATION

by Princelett Duhon
Thomas Jefferson High School
Sterling C. Crim
Lamar University

Many of today's students see Calculus and Trigonometry as an impossible task. This is no longer true with the assistance of our modern hand-held calculators. Looking at $\int_0^1 \sin^{-1}x\,dx$ and attempting to determine its value would seem almost impossible to most beginning calculus students. With the use of the calculator we can determine its value quite readily.

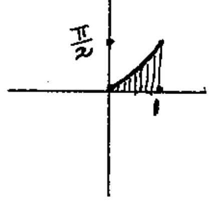


f(x)=sinx



$\sin^{-1}x$

In this program we are to find the area under the curve shown below as $f(x)=\sin^{-1}x$:



$f(x)=\sin^{-1}x$

Using the rectangular numerical approach we take a definite integral to be evaluated $\int_A^B f(x)dx$, divide [A,B] into N subintervals, approximating the integral over each subinterval by finding the area under the curve in each subinterval, and sum the values.

Using calculus we can determine the actual value of $\int_0^1 \sin^{-1}x\,dx$

$$\int_0^1 \sin^{-1}x\,dx = x\sin^{-1}x + \sqrt{1-x^2}\Big]_0^1$$

$$= 1\sin^{-1}1 + \sqrt{1-1}^{\,0} - 0\sin^{-1}0 + \sqrt{1-0}$$

$$= \frac{\pi}{2} - 1$$

$$= 1.5707963 - 1$$

$$= .57079633$$

Using the rectangular rule we can set up the following formula and from it the concluding program.

$$A_N = \frac{1}{N}\left(\sin^{-1}\frac{M_1}{N} + \sin^{-1}\frac{M_2}{N} + \ldots + \sin^{-1}\frac{M_{N+1}}{N}\right)$$

Where N is the number of intervals to be used and M =O to M_{N+1} =N.

This program is for use on the TI-30:

Press 1. [ON/C] [DRG]

2. [N] (where N equals the number of intervals to be considered)

3. [1/X] [STO]

4. [(] [RCL] [X] [M] [)] (where M varies from O to N)

5. [INV] [SIN] [+]

6. Continue steps 4 and 5 until M=N

7. [X] [RCL] [=] Record this value.

8. Repeat program until desired accuracy is reached. (The more intervals used the more accurate the value of the approximate area)

Approximations are found through an iteration process of successively bisecting intervals (N=1,3,5,7,9,11,...) until a predetermined degree of accuracy is reached.

A_1 = 1.5707963

A_3 = .88010976

A_5 = .75089348

A_7 = .69718742

A_9 = .66793166

A_{11}= .64957056

THE ROLE OF FORMAL LOGIC IN MATHEMATICS CLASSES

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As a result of teaching experiences in high school geometry classes and in methods courses for preservice secondary mathematics teachers, the author has become interested in the role of formal logic as a basis for understanding proofs in geometry.

The textbook[1] used by the author in secondary school devotes one chapter to the topic, Induction and Deduction, and includes a presentation of the following principles of logic: conjunction, disjunction, negation of a statement, conditionals, converse, inverse, contrapositive, and patterns of inference. However, in subsequent chapters, the application of the principles of logic was never explicitly shown. It seemed of dubious merit, therefore, to devote time to the study of logic if students did not understand and appreciate the role of this logic in subsequent work in the course.

Experience with college classes of preservice secondary mathematics teachers underscored students' lack of understanding of the applications of logic to mathematical proofs.

The literature reveals differences of opinion among mathematicians and mathematics educators concerning the teaching of formal logic. Some believe that formal logic per se should not be taught. Proofs should be given, and the students will learn intuitively the principles of logic that are used.[2] Hilton says, "I believe that there should be rather little of such explicit, overt appeal to logic and that, in the main, the student should acquire respect and appreciation of sound reasoning through practice rather than through learning explicit rules of logical inference and being trained to apply them."[3] Those who share this belief feel that too much concern for the logical structure of arguments may serve to make some students think that it is the form rather than the substance of an argument that is important.

On the other hand, there are those who believe that there is a good reason for teaching formal logic. They point out that without some rules of inference there is no way to settle differences of opinion over whether an argument is a non sequitur.[4] Exner makes this observation: "If the student has no criteria for making such judgments, the correctness of his proofs is a function of the judgment of his teacher at the moment."[5] If the principles of logic are regarded as means for understanding the arguments presented and are not regarded as ends in themselves, they should enhance the student's intellectual development and offer the potential of transfer to nonmathematical arguments.[6]

Because the author agrees with this second viewpoint, she incorporated a unit of logic into Math 3311, Mathematics for Secondary Teachers, an undergraduate course for preservice teachers. The objective of the unit is to increase the college students' understanding of structure and the role of various strategies of proof in building a structure. With this knowledge and understanding, it is hoped that students will be better prepared to teach proofs in high school geometry in a meaningful way.

The unit on logic designed by the author begins with a definition of "statement" and negation of a statement". It is then postulated that the negation of a true statement is a false statement and the negation of a false statement is a true statement. This postulate is summarized in a truth table. Exercises in writing negations of statements follow. This sequence of definition, examples, postulates, truth tables, and exercises is followed with the terms, "conjunction" and "disjunction" and appropriate theorems concerning their negations. A similar sequence is employed with respect to implications, equivalent statements, converses, inverses, and contrapositives. Then a presentation of the more frequently used inference patterns is made; namely the law of detachment:

$$\begin{array}{l} P \to Q \\ \underline{P} \\ Q \end{array}$$

the law of syllogism:

$$\begin{array}{l} P \to Q \\ \underline{Q \to R} \\ P \to R \end{array}$$

and the law of contrapositive:

$$\begin{array}{l} P \to Q \\ \underline{\;Q\;} \\ P \end{array}$$

The use of these laws is explicitly called to the students' attention when they are employed in a proof as will be demonstrated in the following examples taken from the unit.

Example 1: Statement: If $\angle$ A and $\angle$ B are vertical angles, then $\angle A \cong \angle B$. $\quad P \to Q$

$\angle$ A and $\angle$ B are vertical angles; therefore, $\quad \underline{P}$

$\angle A \cong \angle B$. $\quad \therefore Q$

This is an example of the use of the law of detachment.

In the unit care is taken to give examples of invalid arguments, such as

$$\begin{array}{l} P \to Q \\ \underline{\sim P} \\ \therefore \sim Q \end{array}$$

and to show by example and by truth tables that they are invalid.

Example 2: Prove: The diagonals of a square bisect each other.

List of known facts:

A. A parallelogram is a quadrilateral with two pairs of parallel sides.

B. A rectangle is a parallelogram with at least one right angle.

C. A square is a rectangle with at least two congruent adjacent sides.

$A \to D$ If a quadrilateral is a parallelogram, then the diagonals bisect each other.

Proof:

If $(C \to B)$ is true and $(B \to A)$ is true, then $C \to A$.

$$\begin{array}{l} C \to B \\ \underline{B \to A} \\ \therefore C \to A \end{array}$$

If $(C \to A)$ and $(A \to D)$, then $C \to D$

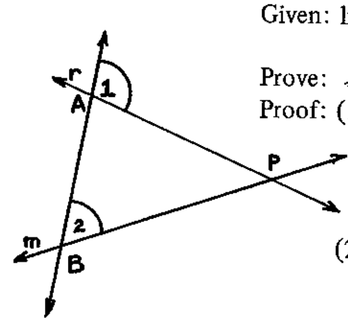
$$\begin{array}{l} C \to A \\ \underline{A \to D} \\ \therefore C \to D \end{array}$$

This is an example of the law of syllogism.

Example 3: Prove: If the corresponding angles formed by the transversal of two lines are congruent, then the lines are parallel. $\quad P \to Q$

Proof:

Let us write the contrapositive of the theorem to be proved: If two lines are not parallel, then the corresponding angles formed by the transversal of these lines are not congruent. $\quad \sim Q \to \sim P$



Given: lines r $\not\parallel$ m and corresponding angles, $\angle 1$ and $\angle 2$.

Prove: $\angle 1 \not\cong \angle 2$.

Proof: (1) $\angle 1$ is an exterior angle of triangle APB; therefore, by the exterior angle theorem, $\angle 1 > \angle 2$.

(2) $\angle 1$ $\angle 2$ and the trichotomy of angles theorem imply $\angle 1 \not\cong \angle 2$.

This is an example of the law of contrapositive. The contrapositive is true; therefore, the theorem is true.

Before presenting examples of indirect proof, the students review the negation of the conditional $P \to Q$ which is $P \wedge \sim Q$. This negation is used in the following example.

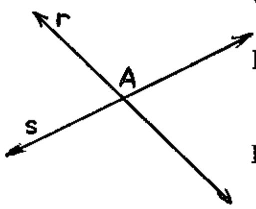
Example 4: Prove: If two distinct lines intersect, then they intersect in only one point.

List of known postulates:

(1) There are at least three points which do not lie on one line.

(2) For any two different points there is exactly one line containing these points.

(3) Every line contains at least two points.

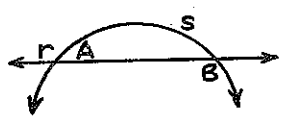


Given: Lines r and s intersecting at point A.

Prove: Lines r and s cannot intersect in more than one point.

Proof:

1. Negate the theorem by writing $P \wedge \sim Q$. If two distinct lines intersect, then they intersect in at least two points A and B.
2. Argue that $P \wedge \sim Q \rightarrow T$. Then for the pair of points A and B we have two distinct lines (Statement T).
3. But it has already been shown that $\sim T$ is true. Postulate 2 states that there can be exactly one line containing these two points.
4. Use an inference pattern to get



$P \wedge \sim Q \rightarrow T$ law of contrapositive

$\sim T$

$\therefore \sim(P \wedge \sim Q)$ which is an equivalent form of the theorem.

This four-step strategy of indirect proof can be applied to any indirect proofs in secondary geometry.[7]

The author has found that the response of preservice mathematics teachers to the study of logic is most encouraging. Hopefully, their knowledge and understanding of methods of proving theorems will enable them to give more explicit attention to strategies of proof as they teach high school geometry courses. Then they will not resort to the miscellaneous methods for proving theorems as cited on Rome Press 1979 Mathematics Calendar:

> . . . reduction ad nauseam, proof by handwaving, proof by intimidation, proof by referral to nonexistent authorities, the method of least astonishment, the method of deferral until later in the course, proof by reduction to a sequence of unrelated lemmas (sometimes called the method of convergent irrelevancies) and finally, that old standby, proof by assignment.[8]

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THE ARBELOS REVISITED

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In his *Book of Lemmas (Heath, 1912)* Archimedes states an elegant result which has come to be known as the arbelos problem. This name comes from the figural relationship existing between configuration of semicircles involved in the problem and the ancient Greek shoemaker's knife, the arbelos (αρβηλο). Archimedes' result can be stated as follows:

> If $\overline{AB}$ is the diameter of a semicircle and C is any point between A and B and if $\overline{AC}$ and $\overline{CB}$ are the diameters of semicircles on the same side of $\overline{AB}$ as the original semicircle, the region bounded by the three semicircles is the arbelos and it has an area equal to that of a circle having as diameter $\overline{CD}$, where $\overline{CD}$ is the perpendicular to $\overline{AB}$ at C and meets the original semicircle at D. *(See Figure 1.)*

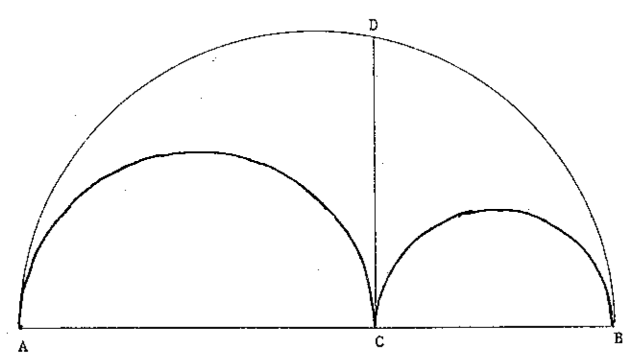


FIGURE 1.

This result can be easily established in the following fashion: Let AC=2R and CB=2r. Then the area of the arbelos is equal to the area of the large semicircle minus the sum of the areas of the inscribed semicircles.

$$\text{Area of arbelos} = \pi\left[\frac{(R+r)^2}{2} - \frac{R^2}{2} - \frac{r^2}{2}\right] = \pi rR.$$

> Now $\triangle$ABD is a right triangle and $\overline{CD}$ is the mean proportional between $\overline{AC}$ and $\overline{CB}$. Thus, the area of the circle with $\overline{CD}$ as diameter is π rR, and the result holds.

It is at this point that one can begin to experiment a little with the basic figure that Archimedes has presented for us to examine.

If one adds segments $\overline{AD}$ and $\overline{BD}$ to the configuration shown in Figure 1 and labels their intersections with the two inscribed semicircles E and F respectively, the diagram shown in Figure 2 results. If one next connects C with E and F, the quadrilateral CFDE appears. Inspection indicates that it possibly is a rectangle. Further inspection indicates that it is further possible that line $\overline{EF}$ is the common external tangent to the two inscribed semicircles.

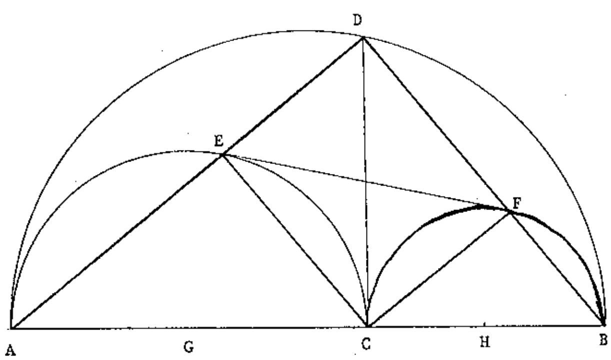


FIGURE 2.

To establish the above conjectures, it suffices to show that CFDE is a rectangle and that the radii of the inscribed semicircles are perpendicular to $\overline{EF}$ at E and F respectively. The argument goes as follows:

> Since $\angle$AEC, $\angle$ADB, and $\angle$CFB are inscribed in semicircles, they are all right angles (See Figure 3.). Furthermore, $\angle$DEC and $\angle$DFC are right angles as they are supplements of right angles. Thus, quadrilateral CFDE has three, and hence, four, right angles and is therefore a rectangle. Using complementary angles, the fact that $\triangle$AGE, $\triangle$EGC, $\triangle$CHF, and $\triangle$FHB are isosceles triangles, and that $\triangle DFC \cong \triangle CED \cong \triangle ECF \cong \triangle FDE$, we have the angle measures as shown in Figure 3. It follows directly that $m(\angle GEF)^o = m(\angle HFE)^o = 90$. Thus, $\overline{EF}$ is the common external tangent to the two inscribed semicircles.

As $\overline{DC} \cong \overline{EF}$, as diagonals of a rectangle, we can restate Archimedes' result as follows:

> The area of the arbelos is equal to the area of a circle having as its diameter the common external tangent segment for the two inscribed semicircles.

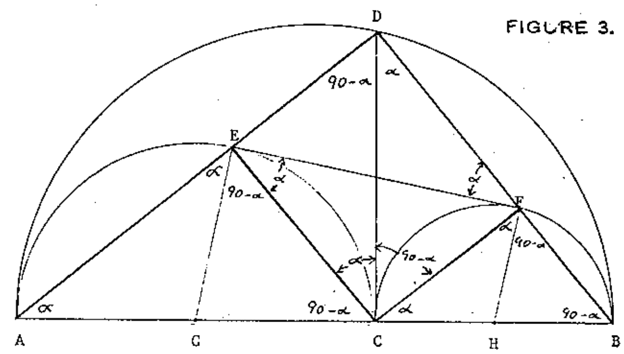


FIGURE 3.

The above theorem makes a good exercise for students who have just completed their study of congruent triangles, angle measures, and tangency. The following exercises are appropriate for secondary students with a knowledge of elementary trigonometric functions and the accompanying basic trigonometric identities.

The first of these additional arbelos' exercises concerns an investigation of the relation of the area of rectangle CFDE to the area of the arbelos. A glance at Figure 4 shows that the area of the rectangle is less than that of the arbelos, since the area of the arbelos is equal to that of the circle which circumscribes the rectangle CFDE. If we let the radius of the semicircle on $\overline{AC}$ be R and the radius of the semicircle on $\overline{BC}$ be r, then CF = 2r cos α, CE = 2R sin α, and the area of rectangle CFDE is 4Rr sin α cos α. This latter expression for the area can be simplified to 2Rr sin 2α.

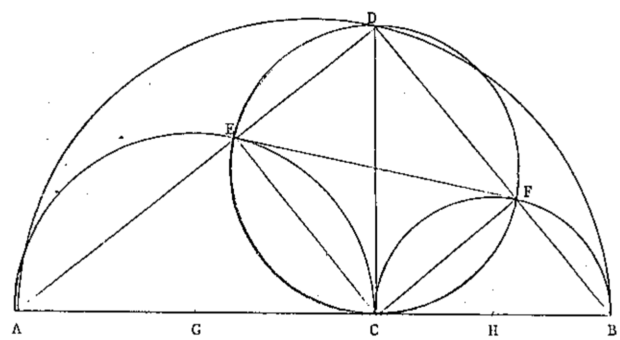


FIGURE 4.

The ratio of the area of the rectangle to the area of the arbelos is then given by the expression (2Rr sin 2α)/Rrπ or (2 sin 2α)/π. This ratio ranges in value from 0 to 2/π as α(the measure of $\angle$BAD) ranges from 0 to π/4. When α varies from π/4 to π/2, the value of the ratio varies from 2/π back to 0, as the resulting geometric configuration for α=0 is the mirror image of the configuration for $\alpha = \pi/2 - \theta$. Thus, the ratio of the area of the rectangle to the area of the arbelos ranges from 0 to approximately 0.63661978.

Another interesting investigation is finding the location of the point C on $\overline{AB}$ which will result in rectangle CFDE being a Golden Rectangle. This would happen when CF:EC::EC:(EC + CF) or when CE:CF::CF:(CE + CF) *(Runion, 1972)*. The solution to this investigation involves a relationship between the measure of $\angle$BAD and the length of $\overline{AC}$. Letting the ratio of AC to CB be x to 1 and m($\angle$BAD) = α, we now solve for the value of x which will determine the length $\overline{AC}$ such that a Golden Rectangle results.

Elementary right triangle trigonometry gives us the following relationships: CE = x sin α and CF = cos α. Using these values in the ratios which would make CFDE a Golden Rectangle, we have the following:

$$\frac{x \sin \alpha}{\cos \alpha} = \frac{\cos \alpha}{x \sin \alpha + \cos \alpha} \quad \text{or} \quad \frac{\cos \alpha}{x \sin \alpha} = \frac{x \sin \alpha}{x \sin \alpha + \cos \alpha} \qquad (1)$$

Since $0 < \alpha < \pi/2$, we have both $0 < \sin \alpha < 1$ and $0 < \cos \alpha < 1$. Solving for the positive value of x in the left hand equation, we get:

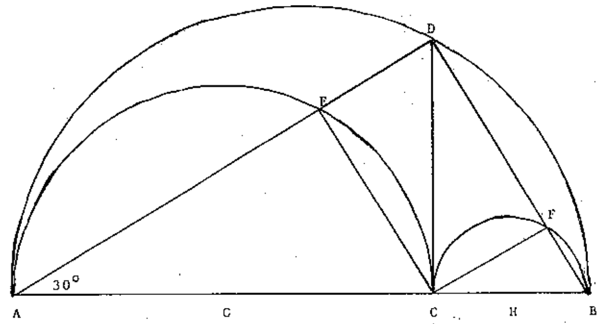
$$(\sin^2\alpha)x^2 - (\sin \alpha \cos \alpha)x - \cos^2\alpha = 0,$$

$$\text{or } x = \frac{+\sin \alpha \cos \alpha + \sqrt{\sin^2\alpha \cos^2\alpha + 4 \sin^2\alpha \cos^2\alpha}}{2 \sin^2\alpha},$$

$$\text{or } x = (\cot \alpha)\left(\frac{1 + \sqrt{5}}{2}\right).$$

The second factor in the last expression is quickly recognized as the Golden ratio itself, r.

Thus the location of the point C along $\overline{AB}$ which will make CFDE a Golden Rectangle when $\angle$BAD has angle measure of α is determined by the values of R and r for which the ratio R/r is equal to (cot α)(r). [If one takes the positive solution to the second proportion in (1) above, the value for the length of $\overline{AC}$ is $x = (\cot \alpha)\left(\frac{\sqrt{5} - 1}{2}\right) = (\cot \alpha)(1/r)$. This results in the other value of (cot α) (1/r) for the point C along $\overline{AB}$ which will result in a Golden Rectangle.]



m($\angle$BAD)° = 30 and CB = 1.5 gives x ≐ 2.8025 and AC ≐ 4.204.

FIGURE 5.

These results, along with hundreds of others already known about the arbelos, should provide students with excellent opportunities to apply their geometric and trigonometric knowledge in a simple setting. The literature on the arbelos is rich and varied *(Raphael, 1973; Ogilvy, 1969; Boyer, 1968; and Gaba, 1940)*. The opportunities for relationships are so numerous that the present way of looking at the Golden Section in the arbelos is only one of many possible *(Bankoff, 1955)*. In fact, the arbelos problem is such an intriguing and deep one that Victor Thebault (1949) referred to the configuration as one of the few simple, yet complex figures, capable of "arousing the curiosity of pupils, developing their imagination, and making them love this attractive branch of mathematics (geometry)."

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"I got 5 out of 10 right on my math test today. Ninety percent isn't bad, huh?"

VIETE'S METHOD FOR APPROXIMATING THE ROOTS OF A POLYNOMIAL

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Pan American University

In *De numerosa,* Francois Viete gives a recursive algorithm, which was in use until about 1680, for successively approximating a root of a polynomial. The algorithm becomes so laborious for equations of high degree that one seventeenth-century mathematician described it as "work unfit for a Christian." However, the algorithm is well suited for today's programmable calculator.

Consider the quadratic equation

$$x^2 + px + q = 0. \tag{1}$$

Suppose x_1 is our first approximation to a root r of the equation. Then $r = x_1 + \Delta x_1$. Substituting in equation (1) we have

$$(x_1 + \Delta x_1)^2 + p(x_1 + \Delta x_1) + q = 0$$

or

$$x_1^2 + 2x_1\Delta x_1 + (\Delta x_1)^2 + px_1 + p\Delta x_1 + q = 0 .$$

Assuming Δx_1 is so small that $(\Delta x_1)^2$ may be neglected, we have

$$\Delta x_1 = - \frac{x_1^2 + px_1 + q}{2x_1 + p}$$

Then using $x_2 = x_1 + \Delta x_1$ as our next approximation for the root r, we calculate a better approximation for r. Continuing in this manner we have the following recursive algorithm:

$$x_{n+1} = x_n - \frac{x_n^2 + px_n + q}{2x_n + p} \tag{2}$$

As an example of the application of this algorithm, consider the equation

$$x^2 - 4x + 2 = 0 .$$

Using the recursive formula (2) with initial values of $x_1 = 1$ and 3 (See Appendix for Programs), we find the corresponding roots $r_1 \approx 0.5857964376$ and $r_2 \approx 3.414213562$, respectively. (See Tables 1 and 2.)

n	x_n
1	1.0000000000
2	0.5000000000
3	0.5833333333
4	0.5857843137
5	0.5857864376
6	0.5857864376
7	0.5857864376

Table 1

n	x_n
1	3.000000000
2	3.500000000
3	3.416666667
4	3.414215658
5	3.414213562
6	3.414213562
7	3.414213562

Table 2

Viete's method can also be generalized to higher degree polynomials. For example, consider the cubic equation

$$x^3 + px^2 + qx + r = 0 . \tag{3}$$

Assuming Δx is so small that $(\Delta x)^2$ and $(\Delta x)^3$ are negligible, we have

$$x_{n+1} = x_n - \frac{x_n^3 + px_n^2 + qx_n + r}{3x_n^2 + 2px_n + q} . \tag{4}$$

Applying Descarte's Rule of Signs and synthetic division, one can determine that the cubic equation

$$2x^3 - x^2 - 8x - 3 = 0 \tag{5}$$

has one positive root between $x=2$ and $x=3$. Dividing equation (5) by 2 we have

$$x^3 - 0.5x^2 - 4x - 1.5 = 0 .$$

Using the recursive formula (4) with an initial value of $x_1 = 2$ (See Appendix for Programs.), we find the corresponding root $r \approx 2.414213562$. (See Table 3.)

n	x_n
1	2.000000000
2	2.583333333
3	2.429285099
4	2.414350010
5	2.414213562
6.	2.414213562
7	2.414213562
8	2.414213562
9	2.414213562

Table 3

In a similar manner the method can be extended to polynomials of degree higher than three.

For those familiar with the Newton-Raphson method for approximating roots of functions we note that Viete's Method is a special case for polynomial functions. If $f(x) = 0$ has only one root in the interval [a,b], and if neither $f'(x)$ nor $f''(x)$ vanishes in this interval, and if x_1 is chosen as a or b, then the Newton-Raphson algorithm for approximating a root is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} .$$

However, Viete's Method does not require an understanding of derivatives, therefore this method is accessible to second year algebra students in their study of synthetic division, Descarte's Rule of Signs, and roots of polynomials.

APPENDIX

Programs for TI 58 and 59

PROGRAM FOR $x_n+1 = x_n - \dfrac{x_n^2 + px_n + q}{2x_n + p}$

LRN

00	2nd CP	
01	STO	Input x_1
02	01	
03	R/S	
04	STO	Input p
05	02	
06	R/S	
07	STO	Input q
08	03	
09	R/S	
10	1	Initialize n=1
11	STO	
12	00	
13	RCL	Display n
14	00	
15	R/S	
16	RCL	
17	01	
18	R/S	
19	RCL	
20	01	
21	-	
22	(	
23	RCL	
24	01	
25	x^2	
26	+	
27	RCL	
28	02	
29	X	
30	RCL	
31	01	
32	–	
33	RLC	
34	03	
35	)	
36	–	
37	(	
38	2	
39	X	
40	RCL	
41	01	
42	+	
43	RCL	
44	02	
45	)	
46	=	
47	STO	
48	01	
49	1	
50	SUM	
51	00	
52	GTO	
53	13	

LRN

RST

PROGRAM FOR $x_n+1 = x_n - \dfrac{x_n^3 + px_n^2 + qx_n + r}{3x_n^2 + 2px_n + q}$

LRN

00	2nd CP	
01	STO	Input x_1
02	01	
03	R/S	
04	STO	Input p
05	02	
06	R/S	
07	STO	Input q
08	03	
09	R/S	
10	STO	Input r
11	04	
12	R/S	
13	1	Initialize n=1
14	STO	
15	00	
16	RCL	Display n
17	00	
18	R/S	
19	RCL	Display x_n
20	01	
21	R/S	
22	RCL	
23	01	
24	-	
25	(	
26	RCL	
27	RCL	
28	y^x	
29	3	
30	+	
31		
32	02	
33	X	
34	RCL	
35	01	
36	x^2	
37	+	
38	RCL	
39	03	
40	X	
41	RCL	
42	01	
43	+	
44	RCL	
45	04	
46	)	
47	÷	
48	(	
49	3	
50	X	
51	RCL	
52	01	
53	x^2	
54	+	
55	2	
56	X	
57	RCL	
58	02	
59	X	
60	RCL	
61	01	
62	+	
63	RCL	
64	03	
65	)	
66	=	
67	STO	
68	01	
69	1	
70	SUM	
71	00	
72	GTO	
73	16	

SOME IDEAS FOR USING THE CALCULATOR WITH GIFTED AND TALENTED STUDENTS AT THE UPPER ELEMENTARY SCHOOL LEVEL

by Dr. John J. Edgell, Jr.
Southwest Texas State University

This article suggests some ideas and problems related to two issues in public schools. What should we be doing, mathematically speaking, for gifted and talented students and what is the educational role of the calculator. The article has no presumption of offering complete resolutions to either of the issues. It merely suggests a viable alternative to some of the mainstream practices.

There are alternatives to either accelerating outstanding students through the standard topical offerings or expanding the depth, more and harder problems, of understanding topics at each level. One alternative is sometimes described as enrichment. An aspect of enrichment may be described as the cultural and historical significance of mathematics. Unfortunately mathematics teachers and poorly trained in this area at all.

The calculator of today is a relatively new calculating device. But calculating devices are not new to mathematics. Man has used calculating devices since he invented the idea of number and pushed beads on a string or whatever to record the idea. In fact, most of the population of the world has always calculated with one device or another. Paper pencil algorithms, as traditionally taught, are a relatively new way of calculating and have been primarily espoused by Western cultures.

There are many good documented educational reasons for using calculators to teach mathematics at the elementary school level. And, there are other factors in considering the calculator as an educational tools. Among which are certain considerations such as calculating power, cost, and availability for determining the relative power of today's instrument with respect to it's predecessors. It is tremendously fast, accurate and capable of an immense number of calculations. The cost has decline to where calculators recommended for elementary school, four basic functions with eight digit read out, are less expensive than the price of a text book. Further, many studies have established that the instruments are available to the vast majority of our students.

Now, what are some of the educational connections between an enriched cultural historical mathematical development and the calculator, other than those stated above? There are several. The following are some suggested problems which are rich in historical significance and the calculator may give insight into discovering solutions. Moreover, these problems seem to be motivational and can provide good opportunities for teaching heuristics.

SIEVE OF ERATOSTHENES
(Practice with the constant function)

Eratosthenes was a Greek mathematician who lived about 200 B.C. He was educated at Alexandria where he was entrusted with the care of the university library. He believed the earth to be round and closely estimated its size. He also determined the Julian calendar (every fourth year has 366 days). One of his mathematical accomplishments has his name, Sieve of Eratosthenes, which involves counting by twos, threes, fives, - - - - -

The idea of a sieve is to separate objects by some means. The objects to be separated are the counting numbers starting with 2, that is:

2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,
16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26,
27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37,
38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48,
49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59,
60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70,
71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81,
82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92,
93, 94, 95, 96, 97, 98, 99, 100, . . .

To count by twos (on the calculator) enter +, 2 and then press = repeatedly. To separate the numbers, cross out every result greater that 2. Circle the first number crossed out.

To count by threes (on the calculator) enter +, 3 and then press = repeatedly. To separate the remaining numbers, cross out every result greater than 3. Circle the first number crossed out (Not previously crossed out).

Continue for fives, sevens, elevens, . . . Notice that these numbers were not crossed out by the preceeding counting process. The numbers not crossed out on the list are classified as ____________________ numbers. The numbers which are circled are called ____________________ ____________________ numbers. The numbers which have been crossed out (i.e. separated form the rest) are called ____________________ numbers.

Did you cross out any numbers on our list when you counted by elevens?

What would be the first counting number to be crossed out when you count by elevens? That number is a __________ ____________________

What about thirteen?, seventeen? . . .

What is the smallest prime?

Is there a largest prime?

Can you predict the next prime?

Can you think of some other extensions to this problem?

KARL FRIEDRICH GAUSS
—Sum of Arithmetic Sequences—

Karl Friedrich Gauss lived durint the late 1700's and early 1800's. His work in mathematics and his mathematical ability earned him the name of Prince of Mathematics. By the age of seven and before his first formal education he had reasoned out the sum of arithmetic sequences such as,

63+67+71+75+79+83+87+91+95+99+103+107+111=

For entertainment he would give the product of a pair of six digit numerals as soon as the last digit of the last factor was given. Such as, 570437 X 832991=
At the age of 22 Karl earned the doctorate in mathematics for proving the Fundamental Theorem of Algebra.

Find the sum of the following arithmetic sequences, discover Karl's technique, if you can.

431+441+451+461+471+481+491+501+511+521=

821+842+863+884+905+926+947=

6213+6313+6413+6513+6613+6713+6813=

What is an arithmetic sequence?

How can you find the sum of an arithmetic sequence easily?
Write some pairs of 6 digit numerals and find the products.
Do you have an expectation for the product before you actually multiply?
How many digits would a product of small 6 digit numerals have?
How many digits would a product of large 6 digit numerals have?
What's a theorem?
What's algebra?
What do you suppose the Fundamental Theorem of Algebra says simply?
Where could you find out more about Karl?

BLAISE PASCAL
—Counting and Addition—

Although Blaise only lived 39 years in the mid-1600's, he made many contributions to the world of mathematics, particularly to the theory of probability. His work in geometry, which was independent of measure, demonstrated the well know observation that mathematics should not be described as the science of quantity. In connection with combinational analysis and probability Blaise used a triangular array of numbers which today carries his name, Pascal's Triangle.

```
                1
              1   1
            1   2   1
          1   3   3   1
        1   4   6   4   1
      1   5  10  10   5   1
    1   6  15  20  15   6   1
```

Do you understand the pattern? Write the next few lines of the triangle.

What is the sum of the first row? The second row? Third row? Fourth row? Is there a general expression for the nth row? in $(1+1)^7 = 1+7+21+35+35+21+7+1$?

Do you know of any uses of such a triangle? Describe them. Are there patterns in the diagonals? Describe them.

Can you find the sum of some of the diagonals? Easily? How? Here is an alternative way of writing Pascal's Triangle.

```
1  1  1  1  1 ...
1  2  3  4  5 ...
1  3  6 10 15 ...
1  4 10 20 35 ...
1  5 15 35 70 ...
.  .  .  .  .
.  .  .  .  .
.  .  .  .  .
```

Does this arrangement suggest other patterns for generating the array of numbers? Write a couple more rows.

Why did Pascal live such a short life?

Did you know that Pascal is credited as creating the first calculator?

What happened to his calculator?

MOSCHOPULUS
—Magic Squares—

Moschopulus wrote a treatise on magic squares in 1460.

Some Magic Triangles

```
   2        3        4        5         4
  6 4      6 2      5 1      3 1       8   5
 1 5 3    1 4 5    2 3 6    4 2 6     6     3
                                     1  9  2  7
```

Can you describe what determines a magic triangle?

SOME MAGIC SQUARES

From the above what do you suppose the sum of entrees of a row or column or diagonal must be? Verify your supposition.

2	9	4
7	5	3
6	1	8

33	62	76
100	57	14
38	52	81

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

646	182	264	400
200	464	383	446
300	364	282	546
346	482	564	100

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

9	23	16	30	23	27	30
35	14	28	21	35	28	7
12	40	19	33	26	5	33
38	17	45	24	3	31	10
15	43	22	15	29	8	36
41	20	13	27	20	34	13
18	11	25	18	32	25	39

The last square has an incorrect entree, can you determine where it is, what it should be and describe your process?

Write some of your own!

Is there a pattern?

Will the pattern work on larger sized squares?

For more information on Moschopulus you might refer to, Ball, W. W. Rouse, **A Short Account of the History of Mathematics,** New York, Dover Publications, Inc., 1960.

MAGIC PRODUCT SQUARE

39	91	104	52
6	14	16	8
15	35	40	20
132	308	352	176

Your initial supposition didn't work, did it? Try this, choose a collection of four factors such that none of them are in the same row or column and multiply. Try other factors? Surprised?

Problems for teaching certain aspects of mathematics with select groups of students should be selected with a great deal of care. Probably fewer problems, but problems with quality characteristics are well worth the time and effort. Additionally, open ended problems with bright students may bring them away from an emphasis upon the solution, but rather an emphsis upon extending the problem and creating other problems. Hopefully such problems will stimulate discussions from a viable and growing perspective rather than generate a closed list of rules.

7th and 8th GRADE "BRAIN TEASERS"

Susan Evans
North Pines Junior High, Spokane, WA 99206

1. 9 8 7 6 5 4 3 2 1
Place addition or subtraction signs between some, or all, of the digits to make a problem with an answer of 100. Do not change the order of the digits.

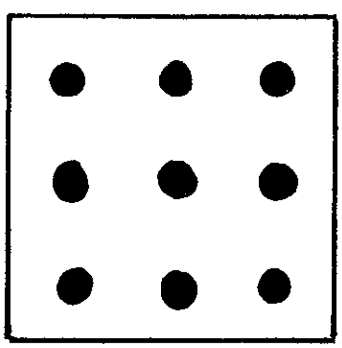
2. Fill in the missing digits for each multiplication problem.

```
    □ □ 7
  x 3 □ □
  -------
  □ □ □ 3
  □ 1 □ □
□ 5 □ □ □
---------
□ 7 □ □ 3
```

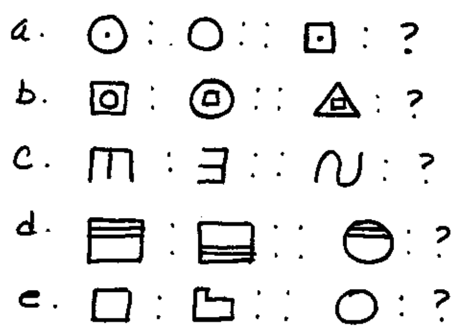
```
    □ 7 6
  x   □ □
  -------
  1 8 □ □
□ □ □ □ □
---------
□ □ 9 2 0
```

```
    2 □ 9
  x   □ □
  -------
    □ 5 □
□ □ □ □ □
---------
□ □ □ 0 6
```

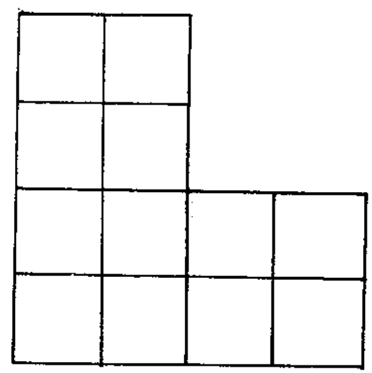
3. Below is a picture of nine dots in a square. Draw two more squares, inside the large square, to divide the large square into nine sections with exactly one dot in each section.



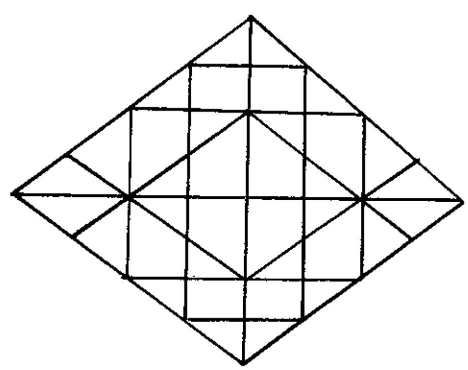
4. Ratio and proportion



5. Divide the figure below into four equal parts. Each part must have the same size and shape.



6. Find the number of (a) triangles and the number of (b) quadrilaterals in the figure below.



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