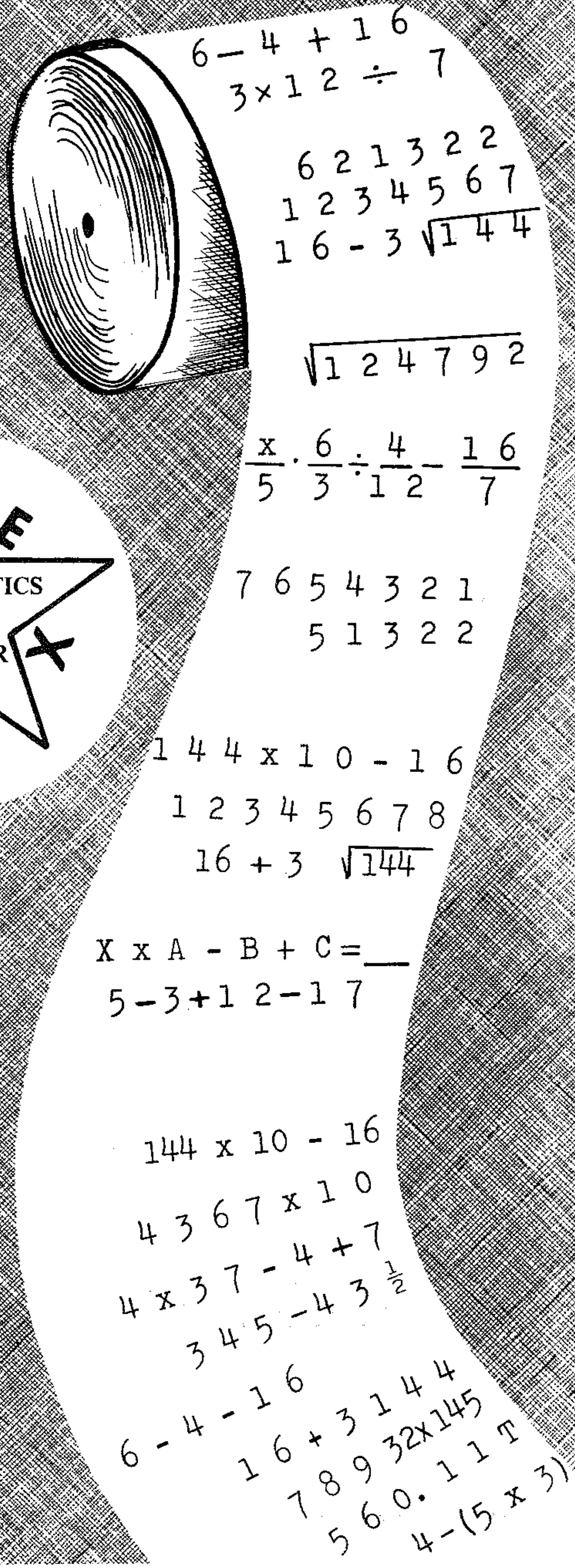
TEXAS MATHEMATICS TEACHER



TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher,* 100 So. Glasgow Drive, Dallas, Texas 75214.

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TEXAS MATHEMATICS TEACHER

Texas Council of Teachers of Mathematics

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PRESIDENT'S MESSAGE

TCTM's part in sponsoring CAMT this year, November 5–7, 1981, will be on site registration. I am requesting all members who are attending to give us one or more hours in assisting with registration. We especially need help early Friday morning.

The 59th Annual Meeting of NCTM in St. Louis, Missouri was informative and enjoyable. A Delegate Assembly of the Council's affilated groups is held each year in conjunction with the NCTM annual meeting. A chief function of the assembly is to provide an interchange of ideas among groups, It is also a highly respected and influential recommending agency on matters pertinent to mathematics education. This year, we voted on nineteen proposed resolutions to be referred to the Board of Directors.

There will be a Fundamentals of Mathematics Conference in Victoria, Texas, February 5–6, 1982. Some areas to be presented are:

- Problem Solving
- Evaluating and Reporting Student Progress in FOM
- Staff Development for FOM Teachers
- Diagnostic-Corrective Teaching Approach to FOM
- Math Anxiety in FOM

I hope the material in the newsletters in useful in your classroom. If you have any ideas, activities, or teaching strategies that you would like to share, please send them to Gordon W. Nichols.

Patsy Johnston

FOM CONFERENCE FOCUSES ON MOTIVATION AND SKILLS

Dr. Robert K. Gilbert
University of Houston Victoria Campus

Sponsored by TCTM, TASM, TEA, Coastal Bend Council of Teachers of Mathematics, and the University of Houston Victoria Campus, the Confernece for the Teaching of the Fundamentals of Mathematics convened on 23–24 January 1981. The Conference was attended by approximately 286 teachers, supervisors, and other math educators from across the state. Speakers from throughout Texas and from Wyoming, New Mexico, Louisiana, and Arkansas were invited. The sessions ranged from calculator workshops to discussions on state guidelines for math curricula. The Conference provided high school and Junior high school teachers, often overlooked or by-passed in the programs of other conferences, with a chance to attend meetings specifically planned for them. Each session focused on one of the fundamentals of mathematics topics or on the pedogogical concerns of these topics. The focal points of the Conference were developing math skills and motivating both students and teachers.

BACKGROUND FOR CONFERENCE

Over the past several years there has been a growing concern over the declining scores on standarized mathematics tests of high school graduates. A cry of "back-to-the-basics" has arisen in many sectors of the American public. Public reactions to this concern range from ignoring the situation to spending huge sums of money and hours of effort to rectify the situation.

Educators have responded in a variety of ways. The National Council of Supervisors of Mathematics (NCSM), encouraged by the public conern for basic computational skills in mathematics, responded by publishing (1977) a list of basic skills in mathematics. This list defines basic skills as more than simply computational skills. Topics such as problem solving, geometry, interpreting data and graphs, and computer literacy are included.

Another reaction to the call for improved math scores came from the National Council of Teachers of Mathematics (NCTM). This group published a booklet entitled AN AGENDA FOR ACTION: RECOMMENDATIONS FOR SCHOOL MATHEMATICS OF THE 1980's. These recent recommendations encouraged schools to develop mathematics curricula that emphasize problem solving skills while still maintaining computational skills. They further recommend that "mathematics teachers demand of themselves and their colleagues a high level of professionalism" which is to include well-prepared and highly motivated teachers.

This concern was also evident at the state level. Many states initiated testing programs to assess student achievement in the basic skills. Not to be outdone, Texas developed the TEXAS PLAN FOR BASIC SKILLS IMPROVEMENT PROGRAM OF 1978. The major purpose of this program as defined by TEA is to provide leadership for schools attempting to improve instruction in the basic skills areas. An outgrowth of this program was the construction of a statewide testing program. The Texas Assessment of Basic Skills (TABS) was instituted in an effort to assess student achievement in reading, writing, and mathematics. One result of TABS has been a greater concern by both educators and the public in the area of public instruction in mathematics.

The recommendations of NCSM and NCTM and the actions taken by TEA have led to a greater awareness in Texas of the need for better prepared teachers. Professional math groups have long been aware of these needs and their current involvement in alleviating the situation is commendable. More to the point, these professional groups have sponsored the Conference for the Advancement of Mathematics Teaching which meets in Austin each year. Among state conferences held throughout this country, this event must be rated as one of the best. The great variety of presentations of CAMT provides information, materials, and motivation for teachers at all grade levels. With such a wide scope, however, it is obvious that high school teachers of the fundamentsla of mathematics (FOM) are limited in the number of presentations available to them at this meeting.

Most teachers have an annual conference to attend that focuses on their speciality. However, in the case of FOM teachers, there does not appear to be a conference whose primary thrust is in the area of teaching the fundamentals of math. Through the urging of members of the Texas Council of Teachers of Mathematics (TCTW) and the Texas Association of Superivsors of Mathematics (TASM), the FOM Conference in Victoria was planned.

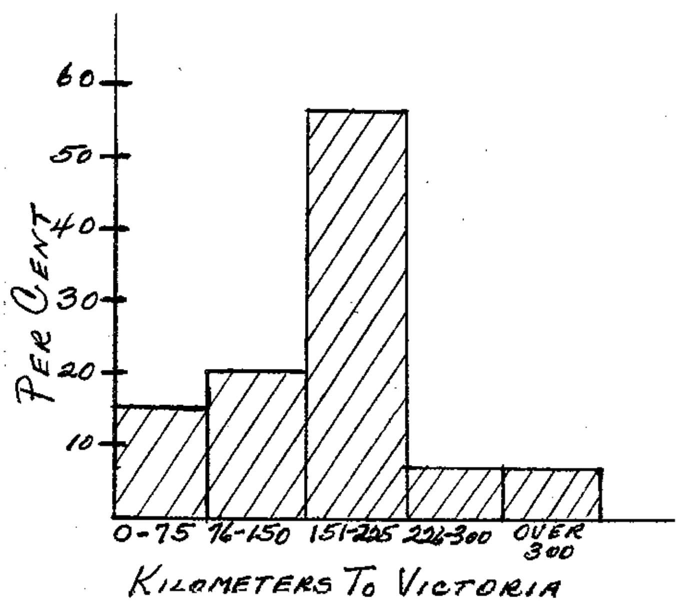
FOM CONFERENCE

In January of 1981 the first FOM Conference was held. Forty-five speakers from four different states volunteered their time to come to Victoria and share their expertise on the teaching of the fundamentals of mathematics. A great many of these speakers are of national renown and have spoken at math conferences throughout the United States. The conference program included 13 workshops, 8 worksessions, 23 regular sessions, and the keynote address. The presentations considered problem solving, graphing, computation, calculators, micro-computers, geometry, consumer math, testing, fractions, and measurement as well as topics concerning bilingual students, state guidelines, and motivation.

Comments from teachers and supervisors attending the conference have been very favorable. The variety of the topics, the enthusiasm of the speakers, and the innovative approaches to teaching math were among the most common compliments received. A large contingent of teachers from Houston ISD expressed their particular pleasure at the organization of the conference. There was an overall belief that participants had an excellent choice of sessions to attend throughout the two-day meeting.

The keynote address entitled, "Why Me, Lord?", was delivered by Dr. Kansky of the University of Wyoming. The data provided by Dr. Kansky strongly supported the contention that schools will need to direct a great deal more attention to teaching the basic skills as described by NCTM. He further encouraged colleges and universities to carefully examine their teacher prepara tion programs in an attempt to improve the mathematical knowledge of future teachers. If the crowd of teachers who came up to ask Dr. Kansky questions after his presentation is a form of evaluating his effectiveness, then we can be assured that he was an overwhelming success.

In examining the geographic distribution of those who attended the FOM Conference the effect of Houston ISD was evident. Of over 225 registrants, approximately 23% of them came from the Houston ISD. The distribution of distances traveled by participants to attend the conference can be seen in the graph below. The largest population centers around Victoria lie with-in the 151–225 kilometer range and, therefore, the above distribution is not surprising. It is encouraging to find that people were willing to travel several hours in order to attend this conference.



The sponsoring groups of TCTM, TASM, TEA, CBCTM, and the University of Houston Victoria Campus wholeheartedly expressed their appreciation to the committee members responsible for organizing and conducting this conference. The response has been so positive that plans for a second "Annual FOM Conference" have been undertaken. The Second Annual FOM Conference is scheduled for 5–6 February 1982 at the University of Houston Victoria Campus. Teachers and supervisors may contact Sally Lewis, Randolph Field High School, Box 2217, Universal City or may contact any TCTM officer for registration materials.

ON THE FORTHCOMING STUDY OF ALGEBRA TEXTS

Stanley J. Hartzler
The University of Texas, Austin

Barth's Distinction says, "There are two types of people: those who divide people into two categories, and those who don't." The author hereby joins the latter of Barth's categories, contending that there are FIVE types of mathematics teachers, classified by their varying responses to nationally published curriculum materials. They are:

(1) Those who assume that text publishers and authors are responsibly attentive to some collection of widely-accepted research findings and theory regarding how students best learn the math that they need personally and professionally, and that texts are steadily improved as the information on learning grows and is refined.

Many such teachers are known by following the book. When the students say, "I don't get it," a category-one teacher responds, "It's in the book. Read it yourself." This writer belonged to category one for the first four months of his teaching career.

(2) Those who suspect that the responsible-publisher assumption avoe is wrong somewhere, but also suspect that they themselves are lousy teachers and lousy users of published material, or that teaching is impossible these days, or that mathematcis is just plain hard to learn for all and much too hard to learn for some students.

(3) Those who know that bad texts are part of today's mathematics learning problems, but are too insecure mathematically, pedagogically, or personally to act, unaware that the problem has solid recognition among professional educators at many levels of all disciplines, and the honor of time as well.

(4) Those who supplement texts with better materials, original or from texts of varied origin and copyright date.

(5) Those (this writer) who are committed to awakening other math teachers and moving them into higher-numbered categories on this scale.

(Two notes: people assigned to math teaching jobs but never attempt to teach are not math teachers of any category. Also, many category-three teachers fail to copy from other texts because of copyright law misunderstanding; the truth is that teachers can ditto/photocopy parts of anything published for classroom use, no matter how stern the publisher's warning to the contrary.)

What **evidence** is there that modern American high school mathematics texts, for instance, are not what they should be? For this writer it began with seeing students learning better and coming in with fewer questions (none, in fact) on last night's carefully-written homework which came from such as Milne's HIGH SCHOOL ALGEBRA (Ginn and Company, 1892).

Since this past Christmas, after some sensational national publicity concerning these views, the author has heard from at least a half-dozen category-four mathematics teachers around the USA who supplement modern algebra texts with Milne—1892 SPECIFICALLY.

Secondly, authors themselves say so. One American author of high school algebra spoke freely on the telephone about this, responding to questions about modern publishing policy, I had asked; is it true that current teams of authors often never see or communicate with each other? that they accept opinions and direction from the publisher over their own? that authors are selected on a basis of attractive credentials earned in large states where statewide text adoption policies mean huge profits or financial woe? that homework problems are written not by the text authors but by high school math teachers of uncertain ability living near the publishing house, thereby holding down author royalty? that publishers are forced by teachers to put more effort into what sells books (catalogues, lobbying, text features such as color, charts, nice covers, fashionable ballyhoo) than the educational strengths of the books, because teachers don't know how to look for (and buy) pedagogical quality? that publishers often rewrite what authors submit, to the point that the authors cannot recognize the result?

The author (and others) answered thus: all of the above are common, and any recently-published mathematics text will be the result of at least three of those practices. The author sympathizes with the publishers. So does this writer, recognizing teacher ignorance of what to look for and of what not to be snowed by.

Here's a classic example of teacher ignorance in text selection. Grade school reading teachers evidently operate under the Kodak Myth, "A picture is worth a thousand words". and buy accordingly, forcing publishers to throw content expense and reading quality to the wind in exchange for the best (and most costly) illustrators of general children's literature. Ignored is research showing that when all else (quality of reading content) is held constant, illustrations interfere somehow with the child's learning to read. [1]

So the problem exists, and is complex. There is teacher ignorance and publisher's financial realities; there are myths and fashions about what and how math is to be learned; the label "outdated" is thrown around too freely? there is ignorance about copyright limits and there is poor teacher self-confidence, committment, and training; the appearances of color, author teamwork, and printing layouts falsely implying similar care with content.

Add to these the present non-existence of that large, widely-accepted body of useful emperical research and theory in mathematics education at present. Add also the great and fatuous influence of all the curriculum study groups from the turn of the century through the present, almost none of them having any kind of meaningful philosophical or research foundation from which to operate.

Thus the motivation for a close look at the changes in American school mathematics texts, a hopeful step toward an outline of what issues a teacher could (or should not) be concerned with when choosing a text, with that awareness leading hopefully to a potent demand for better text content.

The study now being undertaken by this writer thus is a mere historical study and not yet a search for quality judgements. It has involved a national search (ongoing) for American high school algebra texts of any date, old or new.

The personal collection now numbers 320 algebra one-algebra two or complete-course samples dating from 1806 to 1981, with University of Texas libraries making available another 150 or so beginning at 1890.

The sample is limited to texts published in the USA and intended for teacher-assisted regular-track high school classes in basic engineering algebra. Research into publisher records indicate that the study sample will thus include from 50% to 60% of the approximately 1000 such texts ever published.

The following scheme will be used to help organize the hundreds of specific objectives. Twelve sections of content (sometimes overlapping) seem for now to cover everything. They are (1) numbers, (2) variables, (3) expressions, (4) functions, (5) algebraic fractions, (6) roots of expressions, (7) powers of expressions, (8) complex numbers, (9) matrices, (10) sets, (11) remainders, and (12) abstract structures.

Each of these twelve are selected because they appear in the sample texts, and not because they should be taught to all students, in the above or any specific order. The listing above is only to ease the recording of data.

Each section will have four subheadings, again with overlap. In each instance the **elements,** their **operations,** resulting **statements,** and **applications** will be examined. This creates a matrix or vector of 48 elements, to which will be added two common items of a supportive nature: computational aids (slide rule, calculator) and logic, again topics for which desireability is open to question.

The 50 categories will be lightly or heavily subdivided as needed. For instance, under the sections of "expressions" and its subheading "statements" lie linear and quadratic equations and inequalities and systems thereof; linear equations alone will get at least 20 other subdivisions, and so forth.

When the objectives are outlined, they will be numbered. Issues of interest to the researcher, based on reading, teaching experience, and preliminary study, will be examined for each lesson appearing in the text. These issues have four classifications, narrow and broad:

(1) **Order of appearance** of a topic in the text will be noted, with items appearing together in a "day's lesson" to be given the same number, and the fact of topic omission noted also. (The term "day's lesson" needs careful defining, as one major change in algebra texts has been the stronger suggesting to teachers of what should be covered as a unit.)

(2) **Approach** will include such as how the information is developed (from arithmetic or elsewhere); development vs. simple recitation of definitions and rules; spiral and sequential cultivation, review of prerequisites; relation to student's previous math or life experience; use of symbols and vocabulary (heavy to insufficient); use of charts and other organizational aids; invitations to student inquiry (or stifling of same); checking methods and their relations to reinforcement of understanding; oral exercises; listings of steps; proof; examples (timely, premature, or irrelevant); illustrations (appropriate or showy); and other issues as they are encountered.

(3) **Homework objectives** (sometimes different from that of the lesson) and how many of each is offered for class use.

(4) **Homework organization** will include use of lesson content, vocabulary, symbols, and examples; timely or premature deviation from lesson examples or extention beyond them; evidence of careful gradation of exercises; evidence of spiraling back to previous exercises in the same or previous lesson; previews of lessons to come; presence of hints; categorization into two or three levels of difficulty; and others.

Such characteristics of each classification will be coded to facilitate recording and processing of data for comparisons.

Each text will also be examined beyond the lessons for three other issues of interest. These are:

(1) Pedagogical aids; reviews for chapters, sections, and the whole book; glossary. index, preface, table of contents, and symbol list; inductive chapter beginnings; and general organizing themes or schemes.

(2) Features of interest: historical notes, biographies, extra topics, challenging problems, references for further study, career information, and opportunities for applying subject matter to daily life.

(3) Features of attraction: color, portraits of mathematicians, pictures for excitement or professional appearance, number of authors and publishers, and more.

Hoped-for implications of this study include:

(1) the establishment of an inventory of different topic approaches for further study in comparison of relative value; (2) listing of concerns which book buyers should consider or ignore; and (3) establishing more incentive for members of the profession to mobilize themselves away from the blind-faith use of texts and toward genuine instruction of students in mathematics.

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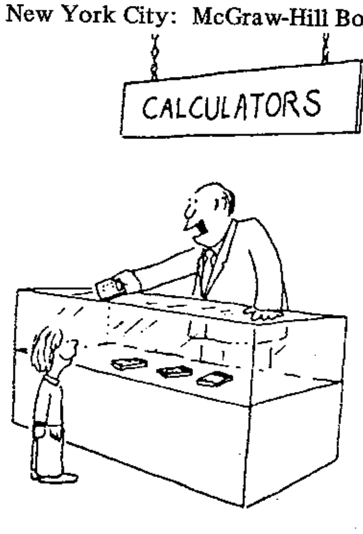

"Here's one you might like. It multiplies, divides, adds, subtracts, and lets you know when the teacher is returning to the room."

SUMMING IT ALL UP

Eleanor S. Pearson
Dallas, Texas

In 1795 at the Ecole Normale in France, Joseph Louis Lagrange said

In 1975 at the Ecole Normale in France, Joseph Louis Lagrange said in his lecture THE OPERATIONS ON ARITHMETIC, "Arithmetic and geometry are the wings of Mathematics. For, whenever, we have reached a result, in order to make use of it, it is requisite that it be translated into numbers or into lines; to translate it into numbers, arithmetic is necessary? to translate it into lines, we must have recourse to geometry." This very eloquent statement was then followed by Lagrange's method for performing the operation of subtraction on whole numbers and multiplication on decimals. Why would such a renowned mathematician in his later years be so interested in algorithms? Did he foresee our mechanical computers to which many algorithms would be applied? Or did he perhaps sense that the differences among students presented many different views of calculations. For whatever reason, Lagrange offered his students a wide range of algorithms to choose from; and following that lead, so will I.

The algorithms I shall attempt to show you are collected from American textbooks copyrighted prior to 1900. I have included some just for fun, some that seem to convey the concept of the operation, and some that are just for special cases. Oh yes, an algorithm is a way to get something done. Algorithms are sometimes referred to as being righthanded or lefthanded. A lefthanded algorithm means to begin with the digit that holds the highest place value, while a righthanded algorithm means to begin with the lowest place value. You may assume that all algorithms are righthanded unless stated otherwise.

ADDITION

As Lagrange said, "As to addition there is not much to say as it is so simple an operation". Really all that can be said is that like units must be added to like units and the result should be expressed in the simplest manner possible.

769	769
+ 487	+ 489
11	16
14	14
16	11
1256	1256

ANALYSIS: Lefthanded and righthanded additions were often used to check the results of addition problems prior to 1900. It is also helpful for beginners, as the partial sums are visible, the place value is stressed, and the final result is written in the simplest manner.

```
  3456
  2247
+ 2325
------
    28
```

ANALYSIS: Double digit addition. Begin with the two rightmost columns. Say 56, add 40 of the 47, say 96, add 7, say 103, continue 123, 128. Write the 28 and carry 1. The rest of the addition I leave to the reader. Just repeat the process, and try, try again.

```
  7  6  9
  8  3  3
+ 2  1  7
---------
```

ANALYSIS: As each place value is independent, you might want to wait and write your simplest form on completion of all the columns.

$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}$

ANALYSIS: Again, we can see that like units must be added and the results expressed in the simplest form. Fractions should be included in this unit as with this example where 7 is the basic unit.

```
  1 foot 4 inches
+ 1 foot 9 inches
-----------------
  2 feet 13 inches
  3 feet 1 inch
```

ANALYSIS: In this example, the sums of the columns are written before writing the simplest form.

Conclusion: Adding is easy. It is the simple form that is the difficult assignment.

SUBTRACTION

Subtraction offers a variety of algorithms. Before we begin, let me say that some people see "8 - 5" as "5 from 8" while others see it as being "5 and 7 is 8". Also a little review of vocabulary:

911	Minuend
– 303	Substrahend
7	Remainder

```
  1 2 3 4
–   3 2 5
---------
    9 0 9
```

ANALYSIS: These are the three most common methods for performing the operation of subtraction. As with addition, the columns are independent of one another. Therefore, when it is necessary to change one of the columns, one may do so by borrowing from the minuend OR reminder, or you may add to the subtrahend. An example of each is given below.

(1) 5 from 14, 9: 2 from 2, 0; 3 from 12, 9.

(2) If you have a student from a foreign country, he may have been taught to add 1 to the subtrahend. If the subtrahend of the next column is 9, then the next two columns are used (9 + 1 = 10). This method was common in the United States prior to 1900. This example would read 5 from 14, 9; 3 from 3, 0; 3 from 12, 9.

(3) This method is known as the "making change" algorithm as it is used in commercial transactions. 5 and 9, 14 2 and 1 and 0; 3 and 9, 12.

I am not too sure that you would call this next example an algorithm, but I think it is a very creative way to obtain a result. It is entitled "two or more subtrahends". Give it a try and you will be glad you own a hand calculator. Oh yes, you have to use the "making change" algorithm from the addition unit.

minuend	1278	ANALYSIS: Since the remainder sought, added to the subtrahends, must be qual to the minuend, we add the columns of the subtrahends, and supply such figures in the remainder as, combined with these sums, will produce the minuend. Thus, 7 and 2 are 9, and 6 are 15 and 3 are 18. Carrying 1 to the next column, 1 and 8 are 9, and 6 etc. (I would hate to finish it and spoil all your fun.)
subtrahends	236 362 387	
remainder	293	

```
  7 4 7 6
- 4 6 1 7
---------
  2 8 5 9
```

ANALYSIS: Lefthanded subtraction is often easier than righthanded, again this one was taught prior to 1900. Beginning at the far left, 4 from 7 is 3 but the next column shows 6 from 4. Therefore, write the remainder 2 for the first column and add 10 to the 4 in the minuend. 6 from 14, write 8. 1 from 7, look at the next column before writing the remainder. Write 5 and add 10 to the 6 of the minuend. 7 from 16 is 9.

```
  30  00  31
- 18  72  29
------------
  11  28  02
```

ANALYSIS: Double the fun with a double column subtraction. (Sometimes this is convenient and sometimes not.) Subtract by double columns. 29 from 31, 02; 72 from 100, 28; 18 from 29, 11. Wasn't that easy! Now I know why Grandaddy was so good at "figuring".

The final algorithm for subtraction is beginning a new career after having been swept under the rug for nearly one hundred years. May I present the "Lagrange Algorithm" as he might have presented it to his students in 1795. It is entitle THE COMPLEMENT OF NINE. Abstractly speaking, Lagrange's idea was something like this: If $a - b = c$, then $(10^x + a) - b = 10^x + c$; where x is any positive integer. Since 10^x can be converted to a number such as $9\,9\,9\,{}^{1}0$, the following algorithm will contain two minuends which are NOT written as a sum. Since the subtrachend maybe subtracted from either of the minuends, Lagrange chose to find the complement of 9 (or 10 as in the first column) then add the other minuend. When subtraction is done in this manner, the algorithm contains "carries" instead of "borrowing". Subtract 3851 from 5436.

```
  9 9 9 ¹0   Double
  5 4 3 6    Minuend

- 3 8 5 1    Subtrahend
---------
  1 5 8 5    Remainder
```

ANALYSIS: 1 Complement 10; 9 and 6, 15. Write 5, carry 1. 5 complement 9, 4 and 3 and 4, write 5. 3 complement 9, 6 and 5, write 1. Since the problem was presented as $(10^x + a) - b = 10^x + c$, it would follow that $a - b = c$. Therefore one always subtracts 10^x from the remainder, and that is the reason 1 was written in the thousand's column instead of 11.

Conclusion: Subtraction itself is an easy thing to do; it is the changing of the minuend into a more simple form that causes all the problems. Maybe we should do it like the old timers, "bye nines!"

MULTIPLICATION

"When it becomes necessary" is a quote one of those old math books I read recently. Doesn't look like a profound statement does it? And yet, when you think about the growth of mankind, that statement seems to precede every change. Our world has come a long way since the beginning of multiplication with its repeated additions. We have been sept up into a world of multiplication facts, factors, products, etc. How interesting it would be to trace the path that multiplication took. Think of the many journeys it took like the distributive property, prime factors, the Euclidian algorithm, etc; but I best leave that story for another time. Let's just investigate some of the algorithms before 1900 that were taught in high school.

```
   423
X   23
------
  4230
  4230
   423
   423
+  423
------
  9727
```

ANALYSIS: Adding 423 ten times is 4230. 2 tens and 3 ones of anything your adding gives 23 of those things. This algorithm is the most fundamental principles and makes the "carries" more visible. I recommend it strongly for beginners.

Pulling the ole multiplication fact sheet out of my hip pocket, I shall continue with the most common form of multiplication. Please note that I have done a great deal of mental additions that are not visible on paper. Also note that the zeros were omitted in the partial sums. This is a preference of the author, rather than a part of the basis algorithm.

```
Righthanded                              Lefthanded
      549                     549
X     345                   X 345
---------                   -----
     2745    549 x 5         1647        54900 x 3
    2196    5490 x 4          2196        5490 x 4
+ 1637     54900 X 3    +     2745         549 x 5
---------                   ------
   189405                   189405
```

Here are a few special cases that you might have some fun with.

```
Multiply 327 by 35       327
                        x  7
                        ----
                        2289
                        x  5
                        ----
                       11445
```

ANALYSIS: The factors of 35 are 7 and 5. Multiply by 7, and the result by 5.

```
Multiply 472 by 242      472
                       x 242
                       -----
                         944
                        944
                    +  944
                    --------
                      114224
```

ANALYSIS: Multiply by two, then double this amount. Multiply by two.

```
Multiply 4739 by 357    4739
                       X 357
                      ------
                       33173
                     165865
                     -------
                     1691823
```

ANALYSIS: Multiply by 7. Multiply this result by 5 then write the units digit of this result in the tens place. (7 x 5 x 10 = 350)

Conclusion: Multiplication is much like addition; if you can count, there really isn't much to it. The concept of factors is extremely difficult, and should be introduced only after a thorough understanding of the meaning of multiplication and its algorithms have been acquired.

DIVISION

Poor little ole division. It has been so abused. Ever noticed that it is the only operation that is done almost entirely as a LEFTHANDED algorithm? How many times have you seen it done as a righthanded operation? As, an adult, we probably don't think about that too much, but just imagine what a third grader might think. He is just beginning to read and write. These skills are left to right. Generally, he begins by learning to add, then subtract, then to multiply; and all of these skills are from left to right. Then out of the blue, we reverse the process. Is it any wonder that he might say, "which way do I go?" Therefore, the first algorithm of division will be an attempt to perform a right to left algorithm. A beginner will find that a table will be a convenience whether he has a divisor that is a one digit, or two digit, or three digit number. This table-look-up system was employed by our early electronic computors.

0 x 21 = 0	3 x 21 = 63	6 x 21 = 125	9 x 21 = 189
1 x 21 = 21	4 x 21 = 84	7 x 21 = 147	10 x 21 = 210
2 x 21 = 42	5 x 21 = 105	8 x 21 = 168	

The products can be found by addition. The third product is found by adding the first and second, the fourth is found by adding the first and third, and so on. Ten times the number acts as a check on the addition. Keep uppermost in your mind that division is a series of repeated subtractions as is multiplication a series of repeated additions.

Divide 44164 by 21

21)44164
− 42000   2000
2164
− 2100   100
64
− 63   + 3
1   2103

ANALYSIS: Look at the table and annex as many zeros as necessary. When you have completed subtracting all the 21's possible, the sum of the partial quotients is the entire quotient. (On a typewriter that wasn't so hard, but then I'm pretty good at subtracting lefthanded.)

2 1 0 3   1/21
21)4 4 1 6 4
− 0
4 4
− 4 2
2 1
− 2 1
0 6
− 0
6 4
− 6 3
1

ANALYSIS: Please notice the difference between the algorithm above and this one. Righthanded division does **NOT** use the idea of subtracting column by column, this one does. This one is a three step process that is lefthanded: Step 1. Choose a number from the table. Step 2. Subtract. Step 3. Go to the next column.

21)44164   2103   1/21
21
06
64
1

ANALYSIS: This method uses a table-look-up system and uses the "making change" algorithm found in subtraction. As you probably have noticed the subtrahends have been omitted. This method is taught in many foreigh countries especially in Mexico. For the first subtraction say 42 and 2 make 44. Bring down the 1, 21 and 0 make 21. Bring down 6, 0 and 6 make 6 bring down 4, 63 and 1 make 64.

Now that we have learned the hard way, let's do the same problem again an easier way. I shall assume that the concept of factoring has been introduced to you. I will leave the analysis to you.

$\frac{44164}{21}$ 7)44164

3)6309

2103   1/21

Let's do that same algorithm again, and I will give you some help.

$\frac{5855}{168}$ 3)5855   2 x 1 = 2

7)1951   5 x 3 x 1 = 15

8)278   6 x 7 x 3 x 1 = 126

34   143/168

This one is fun!

$\frac{432789}{25} = \frac{432789 \times 4}{25 \times 4} = \frac{1731156}{100} = 17311 \quad 56/100$

Conclusion: Division should be considered as repeated subtractions until the beginner has a thorough understanding of the algorithm for division. Then he may proceed to factors. Please remember that factoring is one of the highest of all mathematical skills and is quite difficult to learn. My proof? I ask you to factor 18154. How quickly did you do it? Did you use an algorithm? The most important personal remark that I would like to make is the following: A problem written in the form 5)5555555 implies that specific algorithm be employed to achieve a result? whereas a problem written a/b implies that any algorithm may be used. I think that it is interesting that people respond to a/b with the concept of factoring instead of the traditional algorithm for division. Wouldn't fractions be easier if we wrote all problems of division as a/b???

What is an algorithm? It is just a way to get a job done.

PHYSIOLOGICAL STRESS AS INDICATED BY INCREASED HEARTRATE EXPERIENCED BY ELEMENTARY SCHOOL PUPILS IN SOLVING VERBAL PROBLEMS

By Jim Bezdek, June Buhler, George H. Willson
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Teaching students to solve verbal problems is one of the major concerns of educators today as it has been in the past. As early as 1927, the National Council of Teachers of Mathematics examined the students' inabilities to solve verbal problems and again in 1970 the National Council called for more research in this area. Currently some researchers are looking at the difficulties encountered in this area which include how children are taught as well as the physiological conditions which influence learning. For example, some of the recent investigations have been directed toward the components involved in the process of problem solving as well as the types of problems to be solved, but very little work has dealt with the influence of stress on learning. Some investigations examining the relationship of stress on learning. Some investigations examining the relationship of stress and learning have been reported for other disciplines, but essentially in the area of mathematics very little has been done. Due to this situation, the researchers associated with this study set out to examine verbal problem solving and stress.

A faculty research grant was obtained supporting a three year study. Development of the technology needed to collect the data, development of software, obtaining the necessary hardware (physiograph) and field testing was done the first year. A clinical study involving children being instructed using two methods of instruction, as well as monitoring stress levels, was conducted during the second year. Findings of the study are reported in whcih stress and achievement were recorded in a regular classroom setting for both methods of instruction during the third year.

The problem of the study (conducted in the classroom) was to measure, to compare, and to anaylze the amount of stress, as indicated by heartbeat rate, experienced by students during teaching learning activities in sixth-grade mathematics.

The purposes of the study were:

1. to measure, to analyze, and to evaluate the arousal responses of sixth-grade students while learning to solve verbal problems in mathematics in a regular classroom?
2. to determine if two different teaching-learning approaches for solving verbal problems in sixth-grade mathematics in a regular classroom produce different levels of arousal responses;
3. to determine the amount of learning produced by two different teaching-learning approaches for solving verbal problems in sixth-grade mathematics in a regular classroom; and
4. to extend the body of knowledge for teaching mathematics.

The pretest-posttest control group design was used to accomplish the study. Due to high costs, limited resources, and other restrictions, only one physiograph was available. With only one physiograph, only two students could be monitored at the same time. This restriction forced the use of a small sample; thus, a matched-pairs sample was used.

The population for the study was composed of 463 sixth-grade students enrolled in one school of a large urban school district. From the population, a matched-pairs sample consisting of twelve pairs was used. Pairing was made using percentile scores from the Iowa Test of Basic Skills and assigning to four strata or Quartiles. Using random numbers five pairs from each strata were selected. Final assignment to either the control or experimental group was complete by drawing names from a hat.

During the instruction of the two groups, the teacher used the same method of teaching. The daily lesson of both groups was divided into three major part. These parts included activities with the teacher making an explanation of solving verbal problems; students working independently solving verbal problems. Students taught by the standard approach worked with verbal problems with only the exact information needed. Students in the realistic approach used three types of problems. These were problems with the exact information needed for the solution, problems with information not needed and problems with missing information. While being taught, both groups were monitored for heartbeat rate, using the physiograph, from the beginning of the study to its conclusion. Pre-post tests were administered to measure changes in achievement.

Four hypotheses were used to accomplish the purposes of the study. These hypotheses were tested using t- tests for differences between the means.

1. Hypothesis One was that the amount of stress as measured by heartbeat rate during the teaching-learning activities would be significantly higher than during a non-active or nonexperimental period.

Data involving both groups during the teaching-learning period and during the nonactive period revealed the difference between the means to be signigicant at the .05 level.

2. Hypothesis Two was that no significant difference between the amount of stress as measured by heartbeat rate sould occur between the standard group and the realistic group during the teaching-learning activities. A t- test for independent samples produced a t- value that was not significant to the .05 level.

3. Hypothesis Three stated that no significant difference would occur between the pretest-posttest gain scores on the verbal-problem test which included only verbal problems with the exact information needed for the solution given in the proper order for the realistic group and the standard group. The t- value from a t- test for two related samples was not significant at the .05 level.

4. Hypothesis Four was that the pretest-posttest gain scores on the verbal-problem test which contained verbal problems with the exact information needed for the solution as well as problems which had both additional and missing information for the realistic group would be significantly higher than the pretest-posttest gain scores on

the same verbal-problem test of the standard group. The $\underline{t}$ - value resulting from a t - test for two related samples was significant at the .05 level.

The following conclusions were formulated from an analysis of the findingso f the study.

1. Since verbal problem-solving activities do cause significantly increased heartbeat rate in sixth-grade students, then verbal problem solving does cause stress.

2. Since students taught by both the standard approach and the realistic approach feel significantly increased stress during teaching-learning activities involving verbal problems, then neither approach is more effective than the other approach in minimizing stress in students while solving verbal problems.

3. Since students taught by the realistic approach made greater gains in solving verbal problems which have additional information or missing information than students who have not been taught by this method, then the realistic approach helps students to succeed in problem-solving activities.

4. Since students taught by the realistic approach made greater gains on verbal-problems tests and feel no more stress than students who are taught by the standard approach, then students can be taught by the realistic approach to problem solving with more positive results.

USING STRATEGY GAMES TO TEACH MATHEMATICS IN THE ELEMENTARY SCHOOL

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University of Houston

There is a belief among some people, teachers included, that anything worthwhile has to be difficult to achieve. Nothing of importance can be obtained without hard work. For this group, since mathematics is worthwhile and important, it must be acquired with pain and difficulty. To which you say, "Surely you jest!" But, jest we do not. Why else would these teachers stick slavishly to the textbook, assign pages and pages of home work, use reams of dittoed problem exercises, and give tests with the regularity of an empty gas tank. And, why else would these teachers never use manipulative materials, puzzles and games, and apply mathematics to real life problems. You may or may not know a teacher that fits this description; but, then, perhaps most of us partly fall into this category.

This article has been written for the elementary school teacher who would like to add some fun or more fun to the mathematics program. The emphsis is on the use of games to promote interest in mathematics, improve problem solving and reasoning ability, and increase speed and accuracy in computation. These are games we have used or have observed being used with elementary age students. The games included are representative of types of games that can be successfully used in mathematics programs.

Games can be analyzed in relation to the amount of luck and the amount of strategy that playing them involves. Winning in Number Bingo is a function of luck. In contrast, winning in Tic-Tac-Toe involves a trategy and very little, if any luck. Many games involve both luck and strategy. Dominoes and Triominos, card games like Uno and War, and Backgammon are examples of games requiring a combination of luck and strategy to win.

We have found that the games requiring more strategy have greater potential for developing reasoning and problem solving skills. Gallagher (1980) cautions that there may be little transfer from this type of problem solving to solving real-world problems. We recognize that the problem solving required with games may not have a direct transfer to other types of problem solving. There are however other benefits to be gained from using games. Players of games are required to follow directions and rules, a skill important to all areas of the curriculum, especially mathematics. The game player must pay attention and observe the moves of the other player(s). This skill is also important for being able to learn mathematics.

A strategy game is a game where the player that moves in certain ways will increase his/her chances of winning or decrease his/her chances of losing. The following games involve strategy and can be used effectively with students in grades 1 – 6.

1. Tic-Tac-Toe or X's and O's:

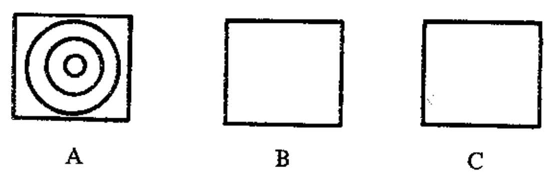
   This games is well enough known that it need not be described.

2. Circle of Pennies

   Arrange ten pennies in a circle. Players (two) are to take turns removing one or two pennies. If two are taken, they must be next to each other. The person who takes the last penny wins.

3. Luca's games or Tower of Hanoi

   A set of objects of decreasing size are used. This coulc be coins, cut paper circles or squares, or cut wooden objects. The objects are placed on a marked area with two other marked areas. i.e.

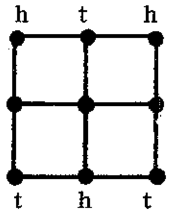


   The game is played by moving the objects, one at a time, from the space marked "A" to the space marked "B". A larger object cannot be placed on top of a smaller object. The objective of the game is to make the least possible moves. (Note: The difficulty of the game can be increased by increasing the number of objects; i.e., moving four objects is more difficult than moving three objects.)

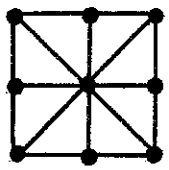
4. Nine Men's Norris

   This game is similar to Tic-Tac-Toe. The game is played on one of the following boards that can be drawn on paper:

A



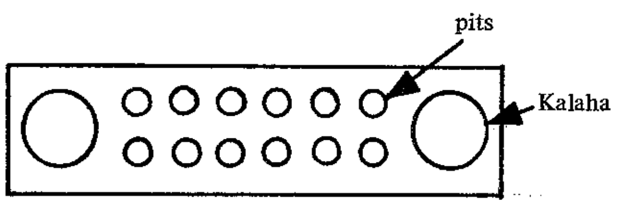
B



Six objects, three of one color or type are used to make the moves; i.e. pennies. The game begins by placing tail-head-tail on one side of the board and head-tail-head on the other side. (see "A" above) One player is heads and moves only tails. The players move one space at a time along the marked lines. A player wins when, on board A, three of the pieces she/he is moving are in a row either horizontally or vertically. The same is true with board B, however, moves and wins are possible on the diagonals as well as the horizontals and verticals.

5. Kalaha

This game is over 7,000 years old. It is played on a board similar to the one below.



(Note: and egg carton with a cup at either end could be used for this game.)

The game is played as follows:

a. The two players sit behind the two ranks of six pits on the board between them. Each pit contains three balls. The purpose of the game is to accumulate as many balls as possible in the larger bin (Kalaha) to each player's right.

b. Each player in turn picks up all the balls in any one of his own six pits and sows them, one in each pit, around the board to the right, including, if there are enough balls, his own Kalaha, and on into his opponent's pits (but not his Kalaha).

c. If the player's last ball lands in his own Kalaha, he gets another turn.

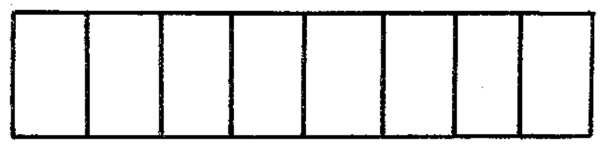
d. If the player's last ball lands in an empty pit on his own side, captures all his opponent's balls in the opposite pit and puts them in his own Kalaha together with the capturing ball.

e. The game is over when all six pits on one side or another are empty.

f. All balls in the pits on the opposite side go into the opponent's Kalaha, and the score, one point per ball, is determined by who has the most balls. The winner is the player who first gets a designated number of points; i.e. 50.

6. The Ladder Game

This game is played on a board constructed like a ladder; i.e.



The numerals 1 to 100 are written on cards that will fit on the spaces in the ladder. A player draws ten cards, numerals down, and takes one to place on the ladder. The numerals are to placed from the smallest at the bottom to the largest at the top. Once a numeral has been placed it cannot be moved. For example, if 96 was drawn and placed on the top space and then 98 was drawn, the 98 could not be placed. The object of the game is to place as many numerals as possible.

(Note: This game can be played with younger children by using only 20 numeral cards with the numerals 1 to 20 on them.)

There are a number of commercial games that can be used profitably with elementary school students. A partial list of those we have used are:

1. Dominoes and Triominos
2. Card games: War, Slap Jack, Battle and Fish
3. Stay Alive
4. Othello
5. Chinese Checkers
6. British Squares
7. Connect Four

Games can add an important dimension to mathematics program in the elementary school. Games can be found for most of the topics that are taught and they do add excitement and interest to learning mathematics. Try them. You and your students will find them educational as well as enjoyable.

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MORE ON THE CONNECTEDNESS OF THE REAL LINE
(Elementary Topology)

by Richard K. Williams
Department of Mathematics
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It is intuitively clear that the connected subsets of the real line are the various kinds of intervals – open, closed, open-closed, closed-open, bounded or unbounded. It is an easy matter to prove that a connected set must be an interval. While the usual proof of the connectedness of any interval is not too difficult, it seems simpler and more natural to first prove that the real line is connected, and then to show that any interval is connected by observing that an interval is a continuous image of the real line and that connectedness is preserved under continuous transformations.

In [1], this author gave a simple and different proof that the real line is connected. The proof used a well-known theorem concerning the structure of open sets of real numbers.

In this paper, a different and even simpler proof of the connectedness of the reals will be given. Nothing more than familiarity with the definitions of open set, closed set, and connectedness is needed.

Theorem: The real line is connected.

Proof: Assume that the reals are not connected, i.e. that $R = G_1 \cup G_2$, where G_1 and G_2 are disjoint, non-empty open sets. Since $G_1' = G_2$ and $G_2' = G_1$ and G_2 are also closed.

Let $X \in G_1$. Since G_1 is open and G_2 is closed, if any numbers in G_2 are less than X, there is a largest such number, i.e. there would be a $y \in G_2$ such that $y < z$ and $(y,z] \subseteq G_1$. But then y would not be an interior point of G_2, contrary to the fact that G_2 is open. Hence $(-\infty, x] \subseteq G_1$.

Similar reasoning shows that $[x, \infty) \subseteq G_1$, so that $R = G_1$, contradicting $G_2 \neq \emptyset$. Therefore R is connected.

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INNOVATIONS VERSUS BASICS IN THE CURRICULUM

Dr. Marlow Ediger
Northeast Missouri University

Much emphasis in American society is being placed upon pupils mastering the basics in the school curriculum. Generally, the three R's (reading, writing, and arithmetic) are perceived to to comprise basic learnings needed by pupils to be effective participators in society. The business world and numerous workers in American society have concluded that selected individuals in the United States have not acquired proficiency in reading, writing, and arithmetic at a desired level.

There are no doubt more innovative concepts, generalizations, and technology available today to improve the school curriculum than ever before. Presently there also appears to be much criticism of objectives, learning activities, and demonstrated achievement of pupils in the school setting. Among others, the following have been recommended innovations in the school curriculum.

1. programmed learning, accountability laws, measurable objectives, and exit objectives.
2. problem solving, critical thinking, and creative thinking.
3. career education, moral education, and environmental studies.
4. team teaching, the nongraded school, open space education, learning centers, schools emphasizing the basics, and education by choice.
5. behavior modification, reinforcement theory, and Teacher Effectiveness Training (TET).
6. individualized reading, teacher-pupil planning, the free school movement, and inquiry methods of learning.

Among the above listed innovations, there also are conflicting inherent goals and philosophies. For example, programmed learning (item 1) is quite different from problem solving methods of teaching and learning (item 2). The programmer in reputable programmed materials determines objectives, each small sequential step of learning for pupils, as well as which is the correct precise response for pupils to make. In contrast, problem solving acrivities emphasize learners identifying problems within a stimulating environment. Pupils with teacher guidance then locate content directly related to the identified problem or problems. Hypotheses are developed and evaluated in terms of offering possible solutions pertaining to the previously chosen problems. In eclectic approaches to teaching and learning, teachers, of course, may and do utilize both programmed learning and problem solving experiences in the school curriculum.

RELEVANCY IN THE CURRICULUM

Regardless of the innovations being evaluated or utilized presently, there are selected criteria applicable to eaching-learning situations presently, as well as in the future. Thus, pupils need to achieve optimally in the following areas,

1. having an inward desire to learn. If pupils and adults have an intrinsic purpose to learn, significant understandings and skills will be achieved in an optimal individual manner. Positive attitudes do give direction in achieving in diverse facets of accomplishment.
2. wishing to identify and attempt to solve problems.. Situations in life require that each person identify and work in the direction of solving vital problems. This is necessary in order to survive as well as to impro improve situations in society.
3. developing skill to think critically. In a democracy, there is considerable freedom to print and write factual content as well as opinions. Thus, learners need to become proficient in analyzing content in terms of being accurate versus inaccurate, biased versus objective, and realistic versus fanciful.
4. thinking creatively. Improvements in society come about due to creative thinking skills of individuals. Novel, unique content is then being presented to change status quo situations. Creative thinking aids in bringing inventions, philosophies, values, and ways of living to the attention of others. Thus, the school curriculum and the curriculum of life must encourage creative thinking abilities within learners.

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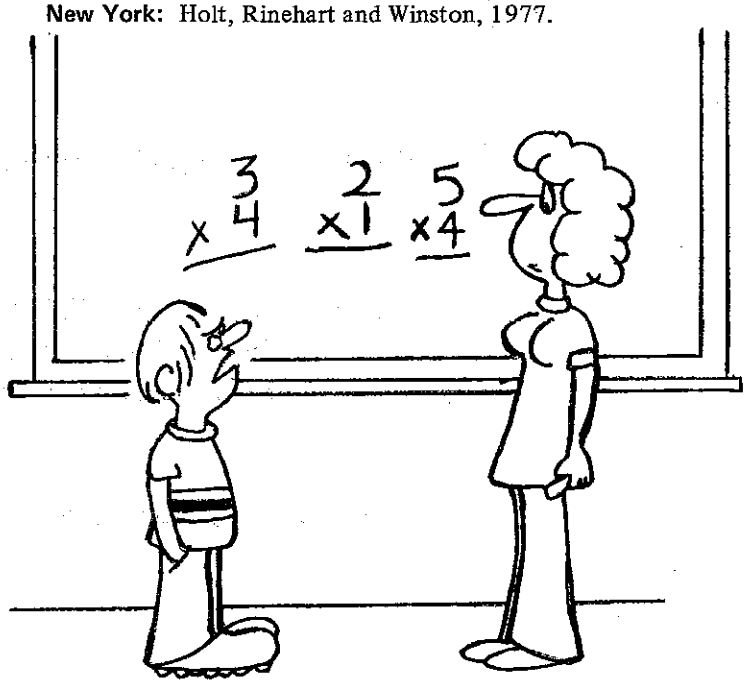

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