

$$6 - 4 + 16$$

$$3 \times 12 \div 7$$

$$621322$$

$$1234567$$

$$16 - 3 \sqrt{144}$$

$$\sqrt{124792}$$

$$\frac{x}{5} \cdot \frac{6}{3} \div \frac{4}{12} - \frac{16}{7}$$

$$7654321$$

$$51322$$

$$144 \times 10 - 16$$

$$12345678$$

$$16 + 3 \sqrt{144}$$

$$X \times A - B + C = \underline{\quad}$$

$$5 - 3 + 12 - 17$$

$$144 \times 10 - 16$$

$$4367 \times 10$$

$$4 \times 37 - 4 + 7$$

$$345 - 43 \frac{1}{2}$$

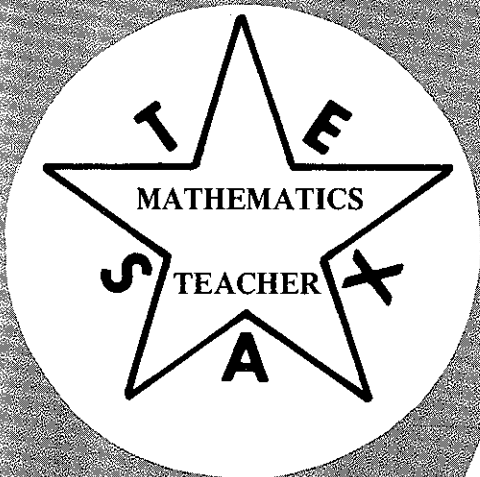
$$6 - 4 - 16$$

$$16 + 3144$$

$$78932 \times 145$$

$$60.117$$

$$(5 \times 3)$$



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PRESIDENT'S MESSAGE

Happy 1981!

With the release of the hostages and the election of a new President, we have hope for a better decade than the last one. However, our country has financial problems with inflation and a large deficit and needs our assistance in correcting them. Unfortunately, these problems are not unique to our government but are faced by TCTM also.

The executive Board of TCTM met during the annual CAMT conference in Austin and decided to make some changes to improve TCTM's financial situation. The first decision was to change the format of the journal, *Texas Mathematics Teacher*. If the financial position of TCTM improves, the old format will be resumed.

The second decision was to help sponsor more mathematics meetings in Texas. The first

meeting was in Victoria, January 23-24, 1981, on the campus of the University of Houston. This meeting involved a *Conference for the Teaching of the Fundamentals of Mathematics*.

Last, but not least, was the decision to co-sponsor a Texas membership drive with the National Council of Mathematics Teachers. Dr. Floyd Vest, vice-president of TCTM, and Mr. Terry Parks, the NCTM southwestern representative for affiliated groups, are working to design a campaign for a joint membership drive this year. Please encourage other mathematics teachers to become members of TCTM.

Your continued cooperation and support is greatly appreciated.

Patsy Johnston

SEE ★
PAGE 7

CRISIS IN MATHEMATICS CLASSROOMS

"The shortage of qualified mathematics teachers in United States classrooms is one of the most pressing problems facing education today," according to Dr. Max A. Sobel, president of the National Council of Teachers of Mathematics. Sobel claims that "if we are not able to supply our students with qualified teachers of mathematics, we will not be able to prepare them for participation in the technological age of the 1980's. Already there are reports that the Soviet Union is far ahead of the United States in providing all of their secondary students with advanced programs in mathematics. We face a serious crisis in this technological decade of steps are not taken to insure an adequate supply of mathematics teachers for our schools."

In a survey just completed by the NCTM in cooperation with the Association of State Supervisors of Mathematics and the National Council of Supervisors of Mathematics, the majority (61%) of mathematics supervisors reported that "certified teachers of mathematics are very difficult to find." In some large cities there was a ratio as low as one applicant for each ten mathematics teaching vacancies. The supervisors felt that over the next two years the situation would

worsen, with 73% predicting it would be very difficult to fill mathematics teaching vacancies with qualified people.

The survey also found that almost 25% of the reported teaching positions in mathematics for the 1980-1981 school year were filled by teachers not permanently certified in mathematics. Faced with classes of students and no certified mathematics teachers, school systems have found that the most popular strategy for dealing with the shortage of qualified applicants is to assign teachers from other fields of preparation to teach mathematics. Competition with industries' salaries and the difficult teaching conditions are the two most frequently cited causes for the shortage.

According the NCTM's AN AGENDA FOR ACTION: RECOMMENDATIONS FOR SCHOOL MATHEMATICS OF THE 1980's, "public support for mathematics instruction must be raised to a level commensurate with the importance of mathematical understanding to individuals and society."

###

Should the Concept of Integers As A Directed Distance Be Taught to Third, Fourth, and Fifth Grade Children

John L. Creswell, University of Houston
Odene Forsythe, Midland College

According to several conference reports including Cambridge Conference on the Correlation of Elementary Science and Mathematics (1967) and the Snowmass Conference on K-12 Mathematics Curriculum (1973), one of the major objectives of mathematics instruction in the elementary school should be an understanding of the real number system. To accomplish this objective, it has been recommended that from the beginning of mathematical instruction the real number line be used to represent the real numbers with the positive and negative integers shown and used on it. It was felt that the operations of signed numbers should be taught in grades three through six.

Contemporary curricula do not generally include the integers until the junior high school level. An examination of nine public school mathematics textbook series for grades 3-6 revealed one series which introduced integers in the third grade, two in the fifth grade, five in the sixth grade and one which did not include integers.

Before the integers are incorporated into the elementary curricula, there are several questions which need answers: 1) Is the study of integers feasible for the elementary school student? 2) At what stage in the child's development would it be best in terms of student understanding to introduce the concept? 3) At what level in the curriculum should the concept be introduced.

Determining the answers to these questions relating to students in grades 3-5 was the primary focus of the study.

Statement of the Problem

The concept of integers is two fold. An integer can be conceptualized as a directed distance or as an additive inverse. The concept of additive inverse is based upon two numbers whose sum is the additive identity zero, and for this an understanding of absolute value is required. Absolute value is based upon distance from zero, therefore an understanding of the concept of directed distance is a prerequisite for understanding additive inverse. Accordingly, this study was confined to the concept of an integer as a directed distance.

Since the directions left and right are used extensively when the concept of integers is presented as a directed distance, this study was also concerned with the possible effects of the level of left-right conceptualization on integers conceptualization.

Using the concept that a task is solved at a given age when at least 75 per cent of the subjects at that age gave the correct response, Piaget (1928) identified three stages in left-right conceptualization. In the first stage, the child relies on his own point of view in distinguishing left and right. In this stage, acquired at approximately five years of age, the ability to designate parts of one's own body in terms of left and right is acquired. In the second stage, the child is able to put himself in another's

point of view and thus recognizes left-right of a person facing him. According to Piaget, this ability is acquired at about age eight. In the third stage, the child is able to consider left and right from the point of view of objects themselves. He can distinguish between the relative term "to the left" and "to the right" with three objects. Piaget reported that this ability is acquired at about age eleven.

Using a sample of 400 children Laurendeau and Pinard (1970) performed a series of five tests dealing with the development of the concept of space. One of the tests was on left-right conceptualization. The subjects used in the study were distributed according to sex, father's occupation, level in school (or number of children in family in cases of preschool children), so as to reproduce exactly the proportions established by the French Canadian population census for the city of Montreal. The ages of the subjects ranged from two to twelve years inclusively, and were classified into six-month intervals up to the age of five years and at twelve-month intervals after that point. Each age interval contained fifty children.

The test used was divided into three sections, each designed to reach one of the three principal phases in the progressive structuring of the concept, which Piaget recognized. The test differs from those used by Piaget (1928) and Elkind (1961) mainly by the number of questions asked in each section and the order of questions in the sections. Piaget and Elkind used only four questions for the sections dealing with parts of the subject's own body and parts of another person's body. Laurendeau and Pinard used six questions for each section; thus, they claim, reducing the danger of success by chance, and alternated the questions not only with left-right dimension but also with parts of the body involved. Again this appeared to reduce the effects of chance by allowing more independence of the child's response.

The results of their study differ very little from Piaget's and Elkind's. Table I gives the results in terms of median age of those successful in each section as well as the age of accession of the sections. It should be noted that Laurendeau and Pinard used a criterion of 50 per cent successes instead of the 75 per cent required by Piaget and Elkind for the age of accession.

Table I
Laurendeau and Pinard Study Results

| | Own Body | Another Person | Three Objects |
|-------------------------------|-------------|-------------------|------------------|
| Median age | 9:0 | 9:9 | 10:5 |
| Age of accession ^a | 5:1 | 7:10 | 9:9 |

^a50 per cent successful

If 75 per cent is required to reply correctly to all questions to determine the age of accession, the age levels change to about seven years, ten years, and twelve years, respectively, for the three levels of left-right conceptualization. Laurendeau and Pinard explained the difference between these ages and those found by Piaget and Elkind by the smaller number of questions used by Piaget and Elkind.

The present study also investigated the effects of sex classification on understanding the concept of integers as a directed distance. While there is no evidence of any difference in overall intelligence due to sex difference (Fox, 1976; Fennema and Sherman, 1976), there is evidence of difference in intellectual areas (Stone and Church, 1957). At all ages, males usually perform better in the realm of quantitative and spatial relationships and females in the verbal sphere (Kagan, 1969; Maccoby and Jacklin, 1974).

In essence the basic problems of this study were to determine, first, if it were feasible in terms of student understanding to place the concept of integers as a directed distance in the elementary school curriculum for students in grades three through five; second, if it were feasible at which level was the concept of an integer as a directed distance appropriate; third, if the level of understanding of left-right conceptualization affected the understanding of the concept of integers; and fourth, if sex classification affected the understanding of the concept of integers.

Need for the Study

If the mathematics curriculum is to be compressed as recommended by the Cambridge Conference Report, (1963), the structure of the real number system should be developed throughout elementary curriculum with as little unnecessary repetition as possible. The present curriculum contains a great deal of repetition by presenting the structure of the natural numbers, the rational numbers, and the integers, separately.

Adler (1972) stated that one of the principal defects in the arithmetic curriculum occurs when the concept of negative integers is not developed early enough. Cochran (1966) feels that signed numbers will be creeping into the teaching of arithmetic more and more. However, Lovell (1971) feels that because of the difficulty young children have with left-right conceptualization, the concept of negative integers might well be left until after the primary grades. He suggests that teaching of negative integers to eight year olds should be examined very carefully.

After having studied the reports on the first and second National Assessment of Educational Progress (NAEP), (1975 & 1980) respectively, one may wonder if indeed eight, nine and ten year olds can learn to do the operations on signed numbers effectively.

Additionally, in a survey of 200 elementary teachers by Creswell (1979), it was found that 162 or 81 per cent expressed a need to have additional instruction before they would feel comfortable in teaching signed numbers. Since no prior research has been reported on the sequencing of the concept of integers, there was clearly a need for this particular study.

Procedures

Pilot Study

The study was designed to involve a treatment period nine days; therefore the investigators constructed nine daily lessons and lesson plans, along with two achievement tests, one written and one oral. This necessitated a pilot study, which was used to determine the appropriateness of: 1) the materials developed for teaching the concept of integers as a directed distance, 2) the grade level range for which the study and materials were designed, 3) the sequence of the materials, and 4) the achievement testing instruments. The pilot study was also used as a means for determining the need for refinement of the materials and achievement testing instruments.

Using the results of the pilot study, it was determined that the grade level range was appropriate for the proposed study. According to scores on the achievement tests, the range appeared to include subjects unable to understand the concept. Using the subjects' reactions, it was also determined that the materials and their sequencing were appropriate for the concept and grade level with very little refinement being necessary.

However, the results of the pilot study indicated a revision of the achievement testing instruments was needed. The written achievement instrument was consequently redesigned from short answer completion to multiple choice. The wording of questions on the oral achievement test was clarified and the instrument was shortened.

Population

The subjects consisting of 49 third grade, 56 fourth grade, and 52 fifth grade students, were selected from students of an elementary school located in a suburb of a large Southeast Texas city. The community from which the school obtains its student population of 1,000 consists of families from the low-middle to middle-middle socioeconomic range.

Treatment and Data Collection

The nine day treatment was conducted by regular classroom teachers, so that a week prior to the beginning of the study, the investigators met with the teachers for two sessions lasting approximately one and a half hour each. During these sessions the following topics were discussed: 1) the inductive teaching method and how it was to be applied in the study, 2) the concept of integers, especially as a directed distance, 3) the terminology of the concept, 4) the sequence of the concept and materials, 5) the method of using and presenting the material, and 6) the application of the lesson plans. The teachers were enthusiastic and cooperative, and seemed eager to be involved in the study.

Two days prior to the treatment period, each subject was tested to determine the level of left-right conceptualization. The test was administered by the investigators and a team of four doctoral students who had been trained in the use of the test.

The instructional period was organized around nine lessons using the inductive teaching strategy which were designed and developed by the investigators. Each lesson plan included: 1) a statement of the objective(s)

of the lesson, 2) a list of materials to be used in the lesson, and 3) an abstract of the lesson. The lesson abstract include the sequencing of the content to be taught, suggested questions to be asked, activities to be included, and procedure for using the materials.

Following the instructional period, using a written and an oral achievement test designed by the investigators, two measures of achievement were collected on each subject.

Data Analysis

With the four independent variables of grade level, age level, levels of left-right conceptualization, and sex, and the dependent variables being scores on the oral and written integer achievement tests, the data were analyzed using two different experimental arrangements:

1. An arrangement containing the independent variables grade, left-right conceptualization and sex, in which the vector of means for integer achievement test scores were tested for significant difference over the levels of the independent variables and the interaction of the independent variables.
2. An arrangement containing the independent variables age, left-right conceptualization and sex in which the vector of means for the integer achievement test scores were tested for significant difference over the levels of age and the interaction of age with the other independent variables in the arrangement.

Findings and Discussion

Analysis of the data revealed that there is no significant difference in achievement on the concepts as a directed distance due to age, taken at six month intervals, and sex of the subjects. It was also determined that achievement was not affected by the interaction of: 1) left-right conceptualization and grade, 2) left-right conceptualization and sex, 3) left-right conceptualization and age, 4) grade and sex, 5) age and sex, and 6) left-right conceptualization, grade and sex.

Further analysis revealed that there was a significant difference in achievement on the concept of integers as a directed distance among levels of left-right conceptualization and the grades three through five. Using post hoc probes it was found that the significant difference was caused by the written integer achievement test scores.

Using a Scheffe contrast, it was determined that there was a linear trend of the means over the levels of left-right conceptualization of the written integer achievement test scores. The linear trend of the means indicates that moving from one developmental level to the next in the three levels of left-right conceptualization would result in an increase in the performance on the written integer achievement test on the concept of integers as a directed distance.

Additionally, it was determined that there was both a linear and a quadratic trend of the means over the grade levels of the written integer achievement test scores. This phenomenon is possible due to the independence of the tests for the linear and quadratic trends. The linear trend implies that the performance on the written integer achievement test on the concept of integers as a directed distance

increased from one grade level to the next in grades three through five. The quadratic trend implies that the performance on the written integer achievement test on the concept of integers as a directed distance increased from grade three to grade four but decreased from grade four to grade five.

Using the information from both the linear and quadratic trends, it may be inferred that an increase in performance on the written integer achievement test is definitely indicated between grades three and four. The occurrence of an increase or a decrease in performance between grades four and five is questionable. However, by pooling the results of the linear and quadratic trends it would appear that the performance on the written integer achievement test peaks in grade four then trends to level off forming an asymptotic curve.

Table 2
Means of Grade Level Pooled Over Left-Right
Conceptualization and Sex

| Level | n | ICW | ICO |
|-------|----|-------|-------|
| | | Means | Means |
| 1 | 49 | 19.90 | 7.35 |
| 2 | 56 | 27.79 | 10.73 |
| 3 | 52 | 25.60 | 11.37 |

The interpretations of the performance on the written integer achievement test among the grade levels are statistically correct based on the data collected; however, the interpretations should not be based on the data alone. The possible sources which contributed to the existing data structure should be examined. The fifth grade students having more acquired knowledge and maturity than the fourth grade students would be expected to perform better than the fourth grade students; however, an examination of the observed means (Table 2) on the written integer achievement test reveals that there was a decrease in performance between these grades. This decrease was verified statistically by the quadratic trend of the means indicated in the data analysis.

One possible explanation for this occurrence might be the uncontrolled teacher variable. The investigators observed some of the instruction presented by the classroom teachers and due to the personality traits exhibited by the teachers, were of the opinion that the instruction in the fifth grade was not as effective as that in the other two grades. This particular teacher spoke with a low keyed voice, seldom smiled, did not project enthusiasm and in general exhibited some of the tendencies of an introvert. It is the opinion of the investigators that this lack of enthusiastic presentation accounts in part for the decrease in performance indicated by the data between the fourth and fifth grades on the written integer achievement test.

A second potential source accounting for part of the decrease was the type of materials used in the study, i.e., the materials were primarily concrete and semi-concrete in nature. The students in the fifth grade whose intellectual development had reached the abstract stage may

have considered the materials too immature and thus could have been "turned off" by them. This phenomena was noted by Berliner (1976).

Using the first-order residual 'step-down' Scheffe contrast, it was also determined that there was a quadratic trend of the means over the grade levels of the oral integer achievement test scores after statistically removing the effects of the written achievement test. The quadratic trend implies that the performance on that portion of the oral integer achievement test on the concept of integers as a directed distance free from the effects of the written achievement test increased from grade three to grade four and decreased from grade four to grade five.

Conclusions and Implications

Based on the analysis of the data in this study and the potential sources contributing to the data structure it would appear that teaching the concept of integers as a directed distance in the third grade would not be as advisable as in the fourth and fifth grades. Also, if the achievement on the concept is to be evaluated utilizing a written test, those students who have reached level 2 or 3 in the developmental levels of left-right conceptualization would have an advantage over those whose left-right developmental level was lower.

So far as curriculum is concerned, the results seem to indicate that curriculum designers should not include material involving manipulation of integers as a part of third grade textbooks. If integers are included at all, they should be relegated to the appendix as enrichment for talented students.

It would appear that more cognitively mature fourth graders and fifth graders could achieve in doing integer algorithms, although the teachers should be selected very carefully. The teachers who are selected to teach integers to nine and ten year olds, should be enthusiastic and highly motivated. Relating to Creswell's (1979) finding, many elementary teachers would appear to need some in-service training regarding integers before they would feel comfortable in teaching the concept.

Additional research is needed in the following areas: 1) the feasibility of teaching integers using a different approach than as a directed distance, thus avoiding a dependence on left-right conceptualization, and 2) relating achievement to the cognitive level of the student.

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THIS COMBINATION ISSUE

(Vol. XXVIII, Nos. 1, 2, 3,
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has been unduly delayed by serious illness of the typesetter since February 3, 1981. It is our hope to resume the regular October—January—March—May issues during the 1981—1982 school year.

Thanks,

Editor

TOP TEACHING POSITIONS

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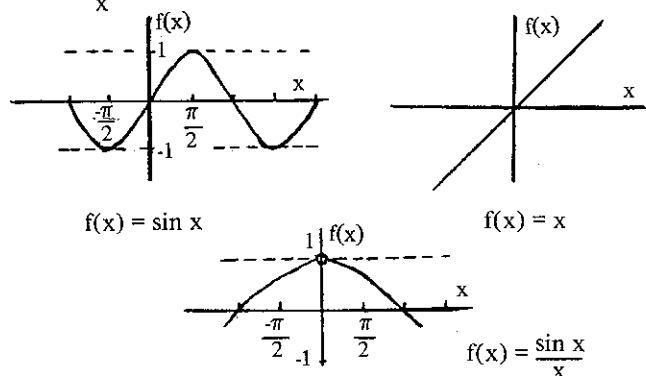
A STUDY OF THE LIMIT OF $\frac{\sin x}{x}$ AS x APPROACHES ZERO

By Rodney V. Horn, Kirbyville High School
Sterling C. Crim, Lamar University

Most beginning Calculus students soon learn that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$. However, the actual process of calculating values of $\frac{\sin x}{x}$ as x approaches 0 involves a

tremendous amount of arithmetic that soon overwhelms the "would-be" eager student. Students of the 80's armed with inexpensive hand-held calculators, can now tackle such problems and spend their time on mathematical ideas rather than on tedious calculations.

The following graphs give us some idea of what happens to $\frac{\sin x}{x}$ as x approaches 0.



We are, of course concerned with $f(x) = \frac{\sin x}{x}$ where x takes on infinitely small values and not what happens when $x = 0$, where our function is undefined. The following algorithm begins at $x = \frac{\pi}{2}$ (radians) and halves that value on each successive iteration. It is designed for use on the TI-30 but could easily be adapted to any calculator with trigonometric functions.

1. ON/C DRG
2. π ÷ 2 = STO
3. SIN ÷ RCL =
4. RECORD NUMBER DISPLAYED.
5. RCL ÷ 2 = STO
6. REPEAT STEPS 3-5 UNTIL DESIRED NUMBER OF ITERATIONS IS REACHED.

Results of 6 iterations:

$$\begin{aligned}
 x = \frac{\pi}{2} \quad \frac{\sin x}{x} &= 0.6366197 \\
 x = \frac{\pi}{4} \quad \frac{\sin x}{x} &= 0.90031632 \\
 x = \frac{\pi}{8} \quad \frac{\sin x}{x} &= 0.97449536 \\
 x = \frac{\pi}{16} \quad \frac{\sin x}{x} &= 0.99358685 \\
 x = \frac{\pi}{32} \quad \frac{\sin x}{x} &= 0.99839439 \\
 x = \frac{\pi}{64} \quad \frac{\sin x}{x} &= 0.99959845
 \end{aligned}$$

PHILOSOPHICAL PERSPECTIVES IN THE MATHEMATICS CURRICULUM

Dr. Marlow Ediger
Northeast Missouri State University

Teachers, principals, and supervisors can benefit much from an in depth study of diverse philosophies of education in continually developing the mathematics curriculum. An evaluation of and ultimate implementation of selected educational philosophies may provide guidance and direction in the teaching of mathematics.

Essentialism and the Mathematics Curriculum

Essentialists are rather certain in terms of subject matter learnings for pupils to achieve. The objectives for pupils to attain must be specific and can be identified with general certainty. The teacher with principal or supervisory assistance then is in the best position to choose objectives, learning activities, and appraisal techniques for pupils in each unit of study in the mathematics curriculum. Understanding (facts, concepts, and generalizations) as well as skills objectives (performing diverse operations) are more relevant for learners

to achieve as compared to attitudinal ends. Obedience and discipline are vital concepts for the teacher to emphasize in the teaching of pupils. Reputable mathematics textbooks providing sequential learnings may well provide the majority of activities and experiences for pupils. Meaningful learnings, as well as drill and practice, are vital for pupils to experience. The teacher needs to pretest pupils based on predetermined objectives pertaining to each unit of study. Learning activities must be selected to help pupils attain these precise ends. After instruction, it can be measured if each pupil has attained the selected objective or objectives. Additional learning activities need to be selected by the teacher if specific pupils have not achieved the measurable objectives.

Essentialists basically do not emphasize

- (1) Teacher-pupil planning in determining objectives, learning activities, and evaluation procedures in the mathematics curriculum.

(2) Pupils being involved in choosing what to learn (objectives) as well as the means of learning (activities and experiences to achieve desired ends).

(3) A permissive environment for pupils in pursuing learning activities.

Humanism and the Mathematics Curriculum

Humanism may well be considered a philosophy of education as well as a psychology of learning. Humanists advocate that pupils should have opportunities to select ends (what to learn) and learning activities to achieve desired ends. Thus, the teacher may develop an adequate number of learning stations with, perhaps, five or six tasks or learning experiences at each station. A pupil may then choose the station and the task that he/she wishes to pursue. Teacher-pupil planning may also be involved in selecting additional tasks, than those originally written by the teacher for any one station or center. Thus, the mathematics curriculum can be very open-ended in terms of pupils choosing activities and experiences. Trust is a significant concept to emphasize. Pupils can be trusted to achieve sequentially and be responsible for choices made. If a teacher has feelings of mistrust toward pupils achieving well in the mathematics curriculum, involved learners may then behave as irresponsible beings.

There are numerous kinds of learning activities from which pupils may select in ongoing units of study. These include utilizing content from reputable mathematics textbooks, programmed learning, laboratory approaches, contracts, management systems of learning, and measurably stated objectives or general objectives. The choice to make is the responsibility of the involved learner. Learning is its own reward; tokens and prizes are definitely not needed to motivate learners to achieve well within the framework of mathematics units of study.

Humanists do not advocate

(1) The teacher sequentially selecting measurable objectives, learning activities, and appraisal techniques for each pupil in ongoing units of study.

(2) Obedience and conformity behavior from learners. Pupils must have opportunities to be creative and achieve intrinsic motivation.

(3) Teachers rigidly supervising each sequential step of pupil achievement. Trusting the learner is important in order to develop relevant attitudes within involved pupils, according to humanists.

(4) That which is measurable in pupil achievement is superior to interests, attitudes, and purposes developed by involved learners.

Experimentalism and the Mathematics Curriculum

Experimentalists emphasize change as a key concept in school and in society. Scenes and situations in life are not static nor stable but rather reveal a changing world. Since change is definitely in continual evidence, new problems arise. Thus, individuals, especially within committee and group settings, need to become proficient in identifying problems, gathering related content,

as well as achieve and test a hypothesis (or hypotheses). The original hypothesis may need revising as a result of being tested within the framework of problem solving activities in the mathematics curriculum. School should not be separated from that which is deemed vital, worthwhile, and relevant in society. Thus, pupils with teacher guidance need to identify and solve practical problems which relate school and society. To carpet a classroom, the exact number of square feet or yards can be computed by learners who possess adequate background content. The following additional examples in the utilization of mathematics reflects concepts pertaining to the integration of school and society:

(1) Pupils with teacher assistance measuring ingredients to prepare representative food items related to a culture or nation being studied in social studies.

(2) Learners buying items (using toy or real money) from a miniature supermarket in the classroom setting.

(3) Pupils making wind vanes, hygrometers, barometers, bird houses and feeders, rain gauges, convection boxes, and anemometers in the school-class setting. Measurement and the use of number are relevant concepts to emphasize in these purposeful learning activities.

(4) Pupils working in a mathematics laboratory setting. Thus, learners can engage in activities involving "learning by doing" in weighing, measuring, determining temperature readings, and finding the volume of needed containers within the framework of emphasizing practicality in the mathematics curriculum. Learnings obtained here by pupils may well be useful in school and in the child's personal life in societal settings.

Experimentalists do not advocate

(1) Pupils with teacher guidance completing activities sequentially from a reputable series of mathematics textbook.

(2) Learners working on programmed materials to attain desired ends.

(3) Pupils generally working on activities and projects on an individual basis. Human beings in society work in groups in attempting to solve identified problems.

(4) Teacher predetermining objectives (general or measurable) for pupils to achieve. The overall aim of instruction, according to experimentalists, may well be for pupils to identify and develop solutions pertaining to life-like problems.

(5) Pupils developing learnings based on a logical mathematics curriculum. Rather, a psychological curriculum needs to be evidence; pupils are active individuals in the learning by doing arena.

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GROUP PROPERTIES

By Patsy J. Johnston
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Group Properties

1. Closure
2. Associativity
3. Existence of Identity
4. Existence of Inverses

$$\begin{array}{ll} (a + b) + b = a + (b + b) & (b + a) + a = b + (a + a) \\ b + b = a + a & b + a = b + a \\ a = a & b = b \end{array}$$

Abelian Group Properties

1. Closure
2. Associativity
3. Existence of Identity
4. Existence of Inverses
5. Commutativity

$$\begin{array}{ll} (a + a) + b = a + (a + b) & (b + b) + a = b + (b + a) \\ a + b = b + a & a + a = b + b \\ b = b & a = a \end{array}$$

$$\begin{array}{ll} (a + b) + a = a + (b + a) & (b + a) + b = b + (a + b) \\ b + a = a + b & b + b = b + b \\ b = b & a = a \end{array}$$

$$\begin{array}{l} (b + b) + b = b + (b + b) \\ a + b = b + a \\ b = b \end{array}$$

Using the following example, the group properties will be discussed.

Example: Let $s = \{a, b\}$

| | | |
|---|---|---|
| + | a | b |
| a | a | b |
| b | b | a |

| | | |
|---|---|---|
| x | a | b |
| a | a | a |
| b | a | b |

Closure for addition of real numbers states that for all real numbers a and b , $a + b$ is a real number. Examine closure for the set s for addition. If any two elements of s are added, is the sum always an element of s ? If yes, then s has closure. If no, then s does not have closure. If there is a chart such as in the example, then check the answer part of chart. If there are no foreign elements (no elements which do not belong to s), then there is closure.

Associative property for addition of all real numbers states that for all real numbers a , b , and c , $(a + b) + c = a + (b + c)$.

To check for associativity of the s for addition, it is required to take all possible combinations of the elements of s and check them.

$$\begin{array}{l} (a + a) + a = a + (a + a) \\ a + a = a + a \\ a = a \end{array}$$

There are three positions for elements in the associative example $(a + b) + c = a + (b + c)$

and two elements in the set s . The eight possibilities have all been tested and found valid. Therefore, s is associative with respect to addition.

To know what $(b + b)$ equals, read the chart. Go across from b and down from b and where the lines cross is the answer a .

Since the information about the sets to be examined is unknown, the commutative property cannot be assumed. Therefore, you must read the chart for $(a + b)$ by starting with a on the left and move right. Then find b at the top and move down. When these two lines cross, read the answer b .

In this exercise, three examples of the associative property will be accepted as proof that the set is or is not associative. Realize that this is only an assumption for this exercise because as the number of elements in the set increase, the number of possibilities to check increase in a geometric progression. For a set with four elements, there would be sixty-four possibilities to check for associativity. The proof would take too much time in this exercise. Therefore, always show your three examples or your answer will not be accepted. The only exception will be if the first or second example is false. It only takes one example which does not work to prove that a set is not associative.

There must be an identity element for the set under a specified operation. The number 0 is the additive identity element for all real numbers.

$$a + 0 = a \quad \text{and} \quad 0 + a = a$$

Notice that this property has to be commutative. Therefore, there is no identity element for subtraction for the set of real numbers.

$$3 + 0 = 3 \quad \text{true}$$

$$0 - 3 = 3 \quad \text{false}$$

This means that the set of real numbers for subtraction does not form a group, an Abelian group, or a field.

Does s have an identity element for addition? Yes, it is a .

$$a + a = a$$

$$a + a = a$$

$$b + a = b$$

$$a + b = b$$

If there is a chart showing all elements under an operation, check to see if there is a row and column with the elements listed in the same order as the elements in the row and column headed by the $+$ sign. The second column is the same as the first row and is headed by a . The second row is the same as the first row and is headed by a . Each set has only one identity element or none. Therefore, if the column and row headings are not the same, there is no identity element. In this case $a = a$; therefore, a is the identity element.

There must be an inverse element for each element in the given set for the given operation. For the set of real numbers for the given operation addition, each real number a has an additive inverse, $-a$, such that $a + (-a) = 0$. The element plus its additive inverse must equal the identity element for the set under addition. If there is no identity element, there can be no inverses.

Using the $+$ chart to find the inverses for addition, look across the row for element a until you find the identity element, a , and then look up. The inverse element is the element in the column heading, in this case a . The inverse of element b is b . Find the element b in left-hand column, look across that row to find the identity element, a and look up to column heading, b , which is the inverse.

Examine s under addition to see if it meets the requirements of a group.

First, s has closure because there are no foreign elements in the chart.

Second, s is associative as proved in the discussion.

Third, s has an identity element a .

Fourth, s has inverses.

Therefore, s does form a group for addition.

Examine s under multiplication to see if it meets the requirements for a group.

First, s has closure because there are no foreign elements in the chart.

Second, s is (appears to be) associative from the following three examples.

$$(a \times a) \times a = a \times (a \times a)$$

$$a \times a = a \times a$$

$$a = a$$

$$(a \times b) \times a = a \times (b \times a)$$

$$a \times a = a \times a$$

$$a = a$$

$$(a \times b) \times b = a \times (b \times b)$$

$$a \times b = a \times b$$

$$a = a$$

Third, s has an identity element, b , for multiplication.

Fourth, s does not have inverses because a does not have an inverse.

Therefore, s does not form a group for multiplication.

The Commutative Property for the set of real numbers under addition states that for all real numbers a and b , $a + b = b + a$.

To test a set for the Commutative Property if there is a chart, draw or think a diagonal line from the operation sign to the bottom of the chart. If the chart is symmetrical across the diagonal, then the set is commutative.

Look at the chart for s under addition.

| | | |
|-----|-----|-----|
| $+$ | a | b |
| a | a | b |
| b | b | a |

The chart is symmetrical across the diagonal. The element b is opposite b across the line which means

$$b + a = b$$

$$a + b = b$$

$$b + a = a + b$$

Therefore, s meets the requirements for an Abelian group. An Abelian group has all the properties of a group plus the Commutative Property.

Is s commutative under multiplication? Yes

| | | |
|----------|-----|-----|
| \times | a | b |
| a | a | a |
| b | b | b |

It is symmetrical across the diagonal which shows:

$$\begin{aligned} b \times a &= a \\ a \times b &= a \\ \therefore b \times a &= a \times b \end{aligned}$$

However, s is *not* an Abelian group because it is not a group. Now proceed to the four hand-out sheets. On Sheet 1, the given set $\{x, y, z\}$ and the operations Δ and Ω are introduced.

line one

$$\begin{aligned} (x \Delta y) \Delta z &= \\ y \Delta z &= \\ x &= \end{aligned}$$

line two

$$\begin{aligned} x \Delta (y \Delta z) &= \\ x \Delta x &= \\ x &= \end{aligned}$$

line three

$$\begin{aligned} (x \Delta y) \Delta z &= x \Delta (y \Delta z) \\ x &= x \end{aligned}$$

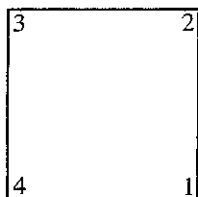
Yes Δ appears to be associative from the one example. Remember one example does not prove associativity.

Sheet 2 follows the same pattern of questions as Sheet 1.

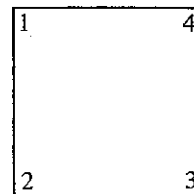
Sheet 3 demonstrates a practical application for the operation \dagger as defined in terms of rotations of a square. Take a square sheet of paper and number the corners 1, 2, 3, and 4. The number of degrees in a circle is 360° . Therefore, a complete rotation or no rotation would be A .

A rotation of 90° would be one fourth of 360° and one fourth of a complete rotation. Each corner number would rotate one position counter-clockwise. This rotation would be named B . The rotation named C would be 180° and each number would move two positions counter-clockwise. The rotation named D would be 270° and each number would move three positions counter-clockwise.

Fill in the \dagger chart first. For example, to solve $B \dagger C$, take the numbered square of paper and place it in B position.



Turn it C rotations which would be 180° or two positions counterclockwise.



Refer to the positions defined for A, B, C , and D . It is now D positions. Therefore,

$$B \dagger C = D.$$

To fill in the \dagger chart correctly, locate B in the left-hand column and move right. Locate C in top row and move down. Write D in the space where the B row and C column cross.

| \dagger | A | B | C | D |
|-----------|-----|-----|-----|-----|
| A | | | | |
| B | | | D | |
| C | | | | |
| D | | | | |

After the \dagger chart is completed, answer the rest of the questions, using the information given in the example and the definitions of a group and an Abelian group.

Sheet 4 demonstrates an operation, Δ , described in terms of rotations and reflections. Cut out an equilateral triangle, and place a Ω in one angle on both sides of the paper, a \dagger in another angle on both sides of the paper, and a \circ in the third angle on both sides of the paper. The rotations are defined similarly to the square rotations on Sheet 3. The reflections are defined in the following manner:

D —hold at the top and the middle of the bottom and turn the triangle over.

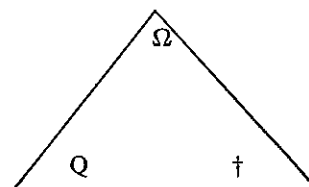
E —hold on the left corner and the middle of the opposite side and turn the triangle over.

F —hold on the right corner and the middle of the side opposite and turn the triangle over.

Now fill in the Δ chart. For example, fill in row D .

$$D \Delta A =$$

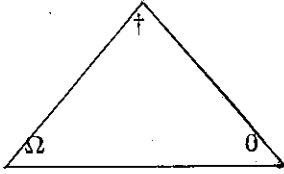
Place the triangle in D position



and rotate A or 360° or 0° which leaves the triangle in D position. $D \Delta A = D$

$$D \Delta B =$$

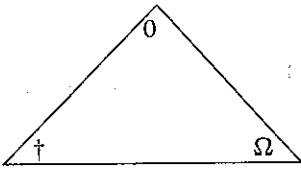
Place the triangle in D position and rotate the triangle B or 120° counterclockwise.



Compare the triangle with the definitions on Sheet 4. The triangle is in F position. $D \Delta B = F$

$$D \Delta C =$$

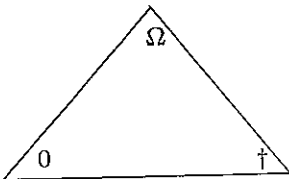
Place the triangle in D position and rotate C or 240° .



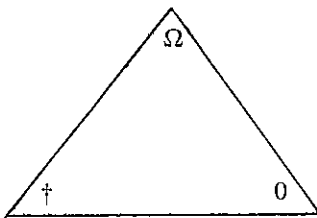
Compare the triangle with the definitions on Sheet 4. The triangle is in E position. $D \Delta C = E$

$$D \Delta D =$$

Place the triangle in D position.



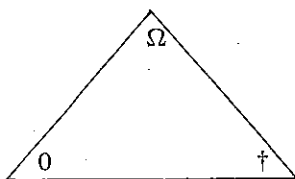
Reflect along the line of symmetry indicated for D (hold top and middle of bottom and turn the triangle over).



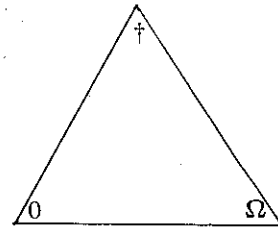
Compare with definitions pictured on Sheet 4. The triangle is in position A . $D \Delta D = A$

$$D \Delta E =$$

Place the triangle in D position



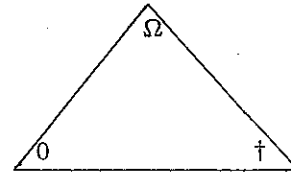
Reflect along the line of symmetry indicated for E (hold left corner and middle of side opposite and turn the triangle over).



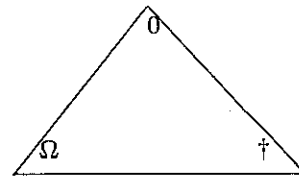
Compare triangle with the definitions on Sheet 4. The triangle is in position C . $D \Delta E = C$

$$D \Delta F =$$

Place triangle in D position.



Reflect along the line of symmetry indicated for F (hold right corner and middle of side opposite and flip).



Compare the triangle with the definitions on Sheet 4. The triangle is in position B . $D \Delta F = B$

After the Δ chart is completed, answer the rest of the questions.

Sheet 1

Given: the set $\{x, y, z\}$
the operations Δ and Ω

Answer the following questions based on the tables.

| | | | |
|----------|-----|-----|-----|
| Δ | x | y | z |
| x | x | y | z |
| y | y | z | x |
| z | z | x | y |

| | | | |
|----------|-----|-----|-----|
| Ω | x | y | z |
| x | z | y | x |
| y | y | x | z |
| z | x | z | y |

What is the value of $(x \Delta y) \Delta z$? _____

What is the value of $x \Delta (y \Delta z)$? _____

Does Δ appear to be associative? _____

What is the value of $(x \Delta y)$? _____

What is the value of $(y \Delta x)$? _____

Does Δ appear to be commutative? _____

What is the value of $x \Omega (y \Omega z)$? _____

What is the value of $(x \Omega y) \Omega z$? _____

Does Ω appear to be associative? _____

What is the value of $(z \Omega y)$? _____
 What is the value of $(y \Omega z)$? _____
 Does Ω appear to be commutative? _____
 Does the set have an identity element for Δ ? If yes, name it. _____
 Does the set have an identity element for Ω ? If yes, name it. _____

Sheet 1

Given: the set $\{x, y, z\}$
 the operations Δ and Ω

Answer the following questions based on the tables.

| Δ | x | y | z |
|----------|-----|-----|-----|
| x | x | y | z |
| y | y | z | x |
| z | z | x | y |

| Ω | x | y | z |
|----------|-----|-----|-----|
| x | z | y | x |
| y | y | x | z |
| z | x | z | y |

What is the value of $(x \Delta y) \Delta z$? _____
 What is the value of $x \Delta (y \Delta z)$? _____
 Does Δ appear to be associative? _____
 What is the value of $(x \Delta y)$? _____
 What is the value of $(y \Delta x)$? _____
 Does Δ appear to be commutative? _____
 What is the value of $x \Omega (y \Omega z)$? _____
 What is the value of $(x \Omega y) \Omega z$? _____
 Does Ω appear to be associative? _____
 What is the value of $(z \Omega y)$? _____
 What is the value of $(y \Omega z)$? _____
 Does Ω appear to be commutative? _____
 Does the set have an identity element for Δ ? If yes, name it. _____
 Does the set have an identity element for Ω ? If yes, name it. _____

 x
 x
 Yes
 y
 y
 Yes
 x
 z
 No
 z
 z
 Yes
 x
 No

Sheet 2

Given: the set $\{A, B, C, D\}$
 the operations Δ and Ω

Answer the following questions based on the tables:

| Δ | A | B | C | D |
|----------|-----|-----|-----|-----|
| A | C | D | A | B |
| B | D | A | B | C |
| C | A | B | C | D |
| D | B | C | D | A |

| Ω | A | B | C | D |
|----------|-----|-----|-----|-----|
| A | C | D | A | B |
| B | B | C | D | A |
| C | A | B | C | D |
| D | D | A | B | C |

What is the value of $B \Delta (C \Delta D)$? _____
 What is the value of $(B \Delta C) \Delta D$? _____
 What is the value of $A \Delta (B \Delta C)$? _____
 What is the value of $(A \Delta B) \Delta C$? _____
 Does Δ appear to be associative? _____
 What is the value of $C \Delta D$? _____
 What is the value of $D \Delta C$? _____
 What is the value of $B \Delta C$? _____
 What is the value of $C \Delta B$? _____
 Does Δ appear to be commutative? _____
 What is the value of $A \Omega B$? _____
 What is the value of $B \Omega A$? _____
 What is the value of $B \Omega D$? _____
 What is the value of $D \Omega B$? _____
 Does Ω appear to be commutative? _____
 What is the value of $(A \Omega B) \Omega D$? _____
 What is the value of $A \Omega (B \Omega D)$? _____
 What is the value of $B \Omega (C \Omega D)$? _____
 What is the value of $(B \Omega C) \Omega D$? _____
 Does Ω appear to be associative? _____

Sheet 2

Given: the set $\{A, B, C, D\}$
 the operations Δ and Ω

Answer the following questions based on the tables:

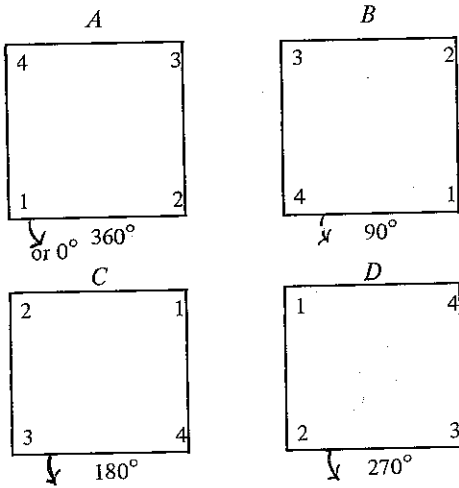
| Δ | A | B | C | D |
|----------|-----|-----|-----|-----|
| A | C | D | A | B |
| B | D | A | B | C |
| C | A | B | C | D |
| D | B | C | D | A |

| Ω | A | B | C | D |
|----------|-----|-----|-----|-----|
| A | C | D | A | B |
| B | B | C | D | A |
| C | A | B | C | D |
| D | D | A | B | C |

What is the value of $B \Delta (C \Delta D)$? _____
 What is the value of $(B \Delta C) \Delta D$? _____
 What is the value of $A \Delta (B \Delta C)$? _____
 What is the value of $(A \Delta B) \Delta C$? _____
 Does Δ appear to be associative? _____
 What is the value of $C \Delta D$? _____
 What is the value of $D \Delta C$? _____
 What is the value of $B \Delta C$? _____
 What is the value of $C \Delta B$? _____
 Does Δ appear to be commutative? _____
 What is the value of $A \Omega B$? _____
 What is the value of $B \Omega A$? _____
 What is the value of $B \Omega D$? _____
 What is the value of $D \Omega B$? _____
 Does Ω appear to be commutative? _____
 What is the value of $(A \Omega B) \Omega D$? _____
 What is the value of $A \Omega (B \Omega D)$? _____
 What is the value of $B \Omega (C \Omega D)$? _____
 What is the value of $(B \Omega C) \Omega D$? _____
 Does Ω appear to be associative? _____

 C
 C
 D
 D
 Yes
 D
 D
 B
 B
 Yes
 D
 B
 A
 A
 No
 C
 C
 A
 C
 No

Rotations



| \dagger | A | B | C | D |
|-----------|---|---|---|---|
| A | | | | |
| B | | | | |
| C | | | | |
| D | | | | |

\dagger is an operation described above in terms of rotations.

Let $S = \{A, B, C, D\}$

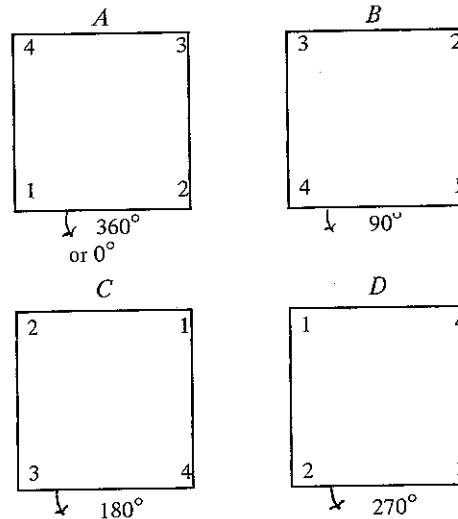
Fill in the chart and answer the following questions.

1. What is the identity element for \dagger ? _____
2. What is the inverse for each element? _____

| | |
|---|--|
| A | |
| B | |
| C | |
| D | |

3. Is S associative for \dagger ? _____
4. Is S commutative for \dagger ? _____
5. Is S a group for \dagger ? _____
6. Is S an Abelian group for \dagger ? _____

Rotations



| \dagger | A | B | C | D |
|-----------|---|---|---|---|
| A | A | B | C | D |
| B | B | C | D | A |
| C | C | D | A | B |
| D | D | A | B | C |

\dagger is an operation described above in terms of rotations.

Let $S = \{A, B, C, D\}$

Fill in the chart and answer the following questions.

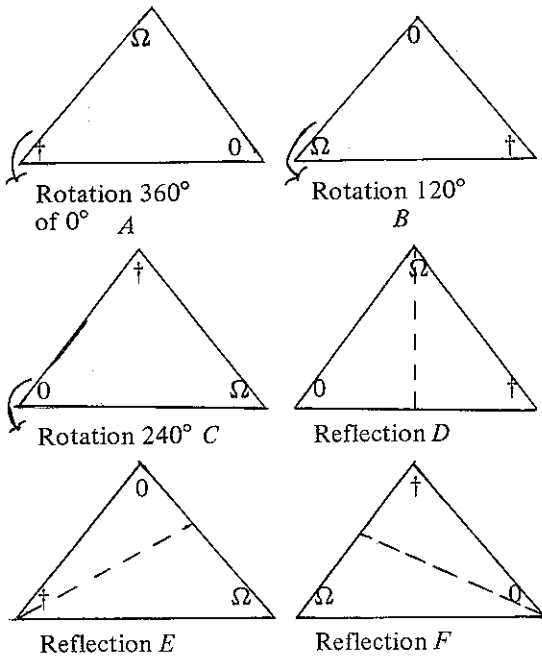
1. What is the identity element for \dagger ? A
2. What is the inverse for each element? _____

| | |
|---|---|
| A | A |
| B | D |
| C | C |
| D | B |

3. Is S associative for \dagger ? _____
4. Is S commutative for \dagger ? _____
5. Is S a group for \dagger ? _____
6. Is S an Abelian group for \dagger ? _____

Yes
Yes
Yes
Yes

Sheet 4
Rotations and Reflections



| Δ | A | B | C | D | E | F |
|----------|---|---|---|---|---|---|
| A | | | | | | |
| B | | | | | | |
| C | | | | | | |
| D | | | | | | |
| E | | | | | | |
| F | | | | | | |

Δ is an operation described above in terms of rotations and reflections. Let $S = \{A, B, C, D, E, F\}$

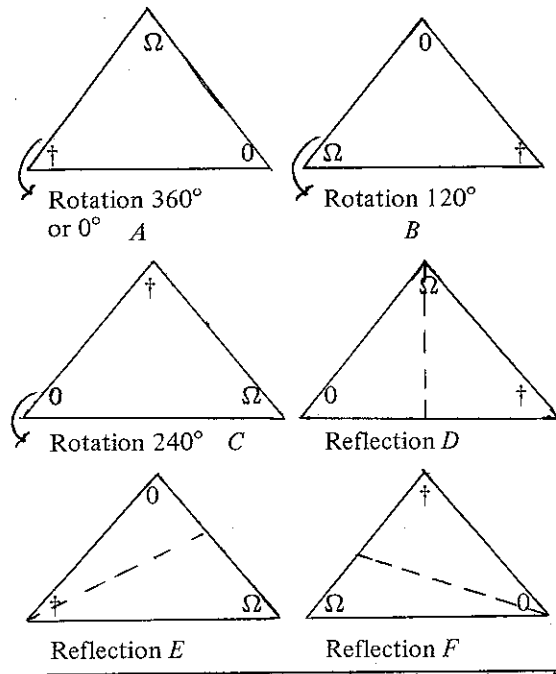
Fill in the chart and answer the following questions.

1. What is the identity element for Δ ? _____
2. What is the inverse for each element if it exists? _____

| | |
|---|--|
| A | |
| B | |
| C | |
| D | |
| E | |
| F | |

3. Is S associative for Δ ? _____
4. Is S commutative for Δ ? _____
5. Does S form a group for Δ ? _____
6. Is S an Abelian group for Δ ? _____

Sheet 4
Rotations and Reflections



| Δ | A | B | C | D | E | F |
|----------|---|---|---|---|---|---|
| A | A | B | C | D | E | F |
| B | B | C | A | E | F | D |
| C | C | A | B | F | D | E |
| D | D | F | E | A | C | B |
| E | E | D | F | B | A | C |
| F | F | E | D | C | B | A |

Δ is an operation described above in terms of rotations and reflections. Let $S = \{A, B, C, D, E, F\}$

Fill in the chart and answer the following questions.

1. What is the identity element for Δ ? _____ A
2. What is the inverse for each element if it exists? _____

| | |
|---|---|
| A | A |
| B | C |
| C | B |
| D | D |
| E | E |
| F | F |

3. Is S associative for Δ ? Yes
4. Is S commutative for Δ ? No
5. Does S form a group for Δ ? Yes
6. Is S an Abelian group for Δ ? No

ATTITUDES TOWARD TEACHING COLLEGE PREPARATORY COURSES IN MATHEMATICS

by Clifford A Hardy
North Texas State University

and Howard L. Penn
Mountain View Community College

INTRODUCTION

Research in the area of vocational satisfaction and teaching has accumulated as concern for faculty morale and its relationship to successful teaching has increased. In a recent study Friedlander (1978) provides evidence supporting the finding that while most faculty members report they are generally satisfied with their jobs they also tend to reveal relatively widespread feelings of discontent with their working conditions.

An often overlooked aspect of job satisfaction, as least as far as the literature is concerned, involves the actual classroom teaching assignment as compared to the broader and more varied vocational role of the faculty member. As a morale factor this can be especially important in specialized areas such as mathematics, where developmental math programs assume more importance and where a faculty member's sole teaching assignment can indeed be completely devoted to college preparatory type courses. In an attempt to measure the degree and direction of attitude in this area a study was recently conducted that tends to supplement findings previously reported in the literature. (Goldstein and Anderson 1977)

PROCEDURE

In order to measure attitudes toward job satisfaction, the Purdue Master Attitudes Scale entitled, "A Scale to Measure Attitude Toward Any Vocation," was sent to a random sample of 56 mathematics instructors in Texas community/junior colleges with over 2000 students. The instructors contacted were asked to respond on the basis of teaching college preparatory type courses only. For the basis of this study, a college preparatory course would refer to any course below calculus, analytic geometry, probability and statistics, finite mathematics and below any higher level mathematics course. Of the instructors contacted, 43 responses were received. The validity and reliability of the instrument employed have been established.

According to Remmers (1960):

Beyond their face validity, these scales have demonstrated validity both against Thurstone's specific scales with which they show typically almost perfect correlations and in differentiating among attitudes known to differ among various groups.

An advantage of the Purdue Master Attitude Scale is that the norms are "built in" in the sense that what is being measured is the affective value of an attitude with the indifference point on the scale being a score of 6.0. That is, scores above 6.0 indicate a favorable

attitude toward the attitude object (Remmers, 1960). Accordingly, t-distribution values were calculated for the attitude dimension under study in order to determine whether or not the mean value of the scaled attitude scores differed significantly from the indifference point of 6.0.

RESULTS

The results of the t-distribution values are presented in the table below. As can be seen, the mean attitude scores for the total group differed significantly from the indifference point of 6.0. That is the general attitude of the community/junior college mathematics teachers toward the teaching of college preparatory courses was not only favorable in direction was found to significantly favorable from the indifference midpoint at the .01 level.

t-distribution Values For
Attitudes Score (N=43)

| Variable | Mean Attitude Direction | Mean Attitude Score | Indifference Midpoint | Standard Deviation | t-value | p |
|--|-------------------------------|---------------------------|--------------------------|-----------------------|---------|------|
| Satisfaction with Teaching College Preparatory Courses | Favorable | 8.154 | 6.000 | 0.899 | 15.49 | .01* |

*Significant at the .01 level

The results obtained generally adhere to findings previously discussed in the literature. For example, in a survey of mathematics faculty, Goldstein and Anderson (1977) reported that faculty members surveyed preferred to allocate the majority of their time to the teaching of students as opposed to what they were actually practicing in their broader vocational role. Since the ultimate success of a mathematics program, as in any other discipline, depends to a certain extent at least, upon the morale and the attitude of the individual instructor involved (Taddeo 1976), the findings of this particular survey should be of interest in that regard.

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