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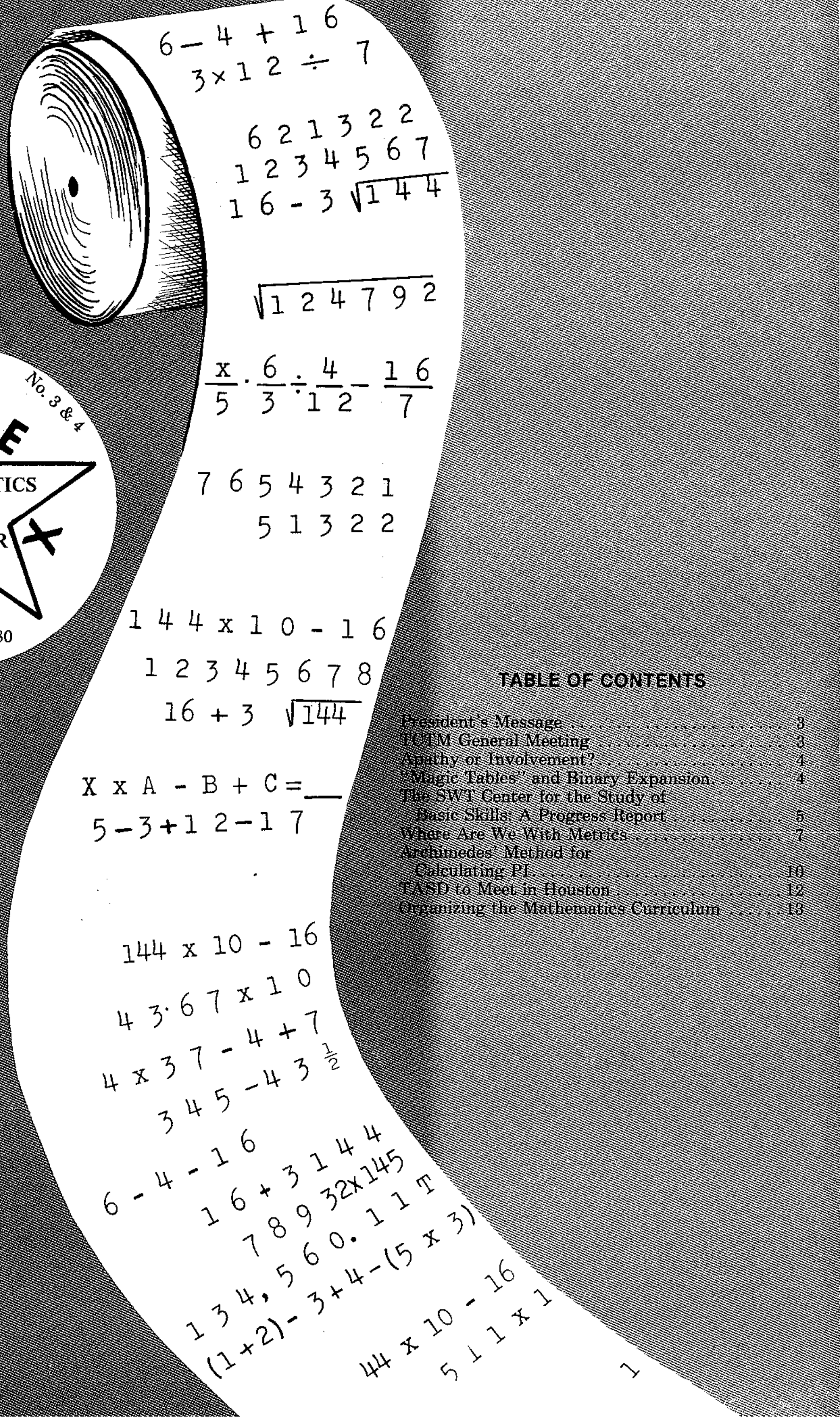
TEXAS MATHEMATICS TEACHER

October 1980



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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher,* 100 So. Glasgow Drive, Dallas, Texas 75214.

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PRESIDENT'S MESSAGE

With mixed emotions, I write my last president's message. It has been a pleasure and a very rewarding experience to serve you. One has to have this privilege before he can fully appreciate what takes place behind the scene during a term of office. The word "cooperation" assumes a new meaning when all the duties begin to present themselves. I wish to thank you for all you have done to help me to try to fulfill my responsibilities.

I am looking forward to the leadership of which I am sure our new president and her executive committee are capable; but, without the assistance of each member, they cannot succeed. When they solicit your help, please be ready to serve. Your support in recruitment of new members is a very special way in which you can help to make our organization grow. Contributing articles to the Newsletter and to the *Journal* is another outstanding way for you to help. Ask your fellow teachers to share their mathematical ideas with other Texas teachers. Also, when it is possible, make plans to attend the state meeting in Austin, as well as Name-of-Site meetings, and the annual meeting of the National Council of Teachers of Mathematics.

Speaking of meeting, thank you for helping to send me to the NCTM meeting in Seattle. At the delegate assembly on Wednesday, concerns of various affiliated groups were discussed and voted on. These resolutions are then presented to the board of directors for their consideration. It was at this convention that the "Agenda for Action-Recommendations for School Mathematics of the 1980's" was first released.

Thank you again for your support and cooperation during my term of office. Let's make 1980-81 our best year ever.

Anita Priest

Texas Council of Teachers of Mathematics General Meeting

On Friday, October 31, 1980, during the annual CAMT meeting in Austin, there will be a general meeting of the Texas Council of Teachers of Mathematics. The Executive Committee urges each and every TCTM member, as well as all prospective members, to attend this meeting. Come and meet the newly elected officers along with all members of the Executive Committee and representatives from the National Council of Teachers of Mathematics.

Your ideas and suggestion for building a better TCTM are needed and requested. TCTM can only be as strong as the membership, involvement, and support of mathematics teachers make it.

See you there!

THANKS

TCTM wishes to thank the Fort Worth Council of Teachers of Mathematics for their contribution of $200 to our treasury.

CAMT: Austin — Oct. 30 to Nov. 1

Apathy or Involvement?

"The reward of a thing well done is to have done it." — **Ralph Waldo Emerson**

As teachers of mathematics in the State of Texas, now is the time for each and every one of us to take time to make an important decision. Will it be APATHY toward.... or INVOLVEMENT in our professional organization?

In the last few years many have shown a great deal of apathy. Memberships have declined dramatically. This means that fewer involved people have worked much harder to keep the organizations at the professional level of which all mathematics teachers can be proud.

Apathy has been evident among mathematics teachers in general: among those who have assumed responsibilities as officers of organizations; among those who have accepted assignments on projects, committees and have had responsibilities during conventions; and among the general memberships of organizations by not voting.

Let us name a few incidents ....

Quite a number of teachers have a built-in question when asked to support TCTM by becoming a member. "What do I get for my $5.00?" Or they make a statement such as "I've never seen how I can get any good out of it." These teachers could be real assets to TCTM as members who should get involved and help bring about the good changes they desire.

Even when self-addressed stamped envelopes were enclosed for their convenience, some local council officers have completely ignored requests from members of the TCTM Executive Committee to furnish information which would insure that TCTM could better serve them. To get full benefits from the professional organizations, all groups must participate and cooperate. This should be an inherent responsibility of all who agree to serve as leaders.

There have been quite a few instances of persons accepting certain responsibilities during CAMT; then without even the courtesy of letting the person or committee depending on them know that they could not or would not be able to fulfill their obligations — they fail to show.

It is a privilege to be able to vote on slates of officers, constitutions (or their revisions and amendments), and other items that need approval of general memberships. When votes on many of these things come up, as few as 2.2% of the membership have voted. This means that organizations are trying to serve 97.8% of their members without even having an idea of what their feelings are.

Statewide, TCTM membership is about 750. Memberships in some local councils have declined. This needs to be corrected. Each present member, in any of these organizations, should make a special effort to invite the many mathematics teachers who have neglected to join to become involved. Once enrolled, these teachers should become and remain active in all of the activities. What better way is there for mathematics teachers to have access to peer as well as to professional expertise in the "Return-to-the-Basics" teaching and in the added responsibilities and accountability demanded by the general public through the school administrators than by membership in their professional organizations?

For organizations to function effectively there must be cooperation and unity among the officers and the members. This takes much hard work on everyone's part. So let's stop being a "Let-George-do-It" person and be recipients of the reward that comes to us through the knowledge that we have given ourselves for the betterment of mathematics teaching.

"Magic Tables" and Binary Expansion

M. G. Monzingo
S.M.U.

Consider the rectangular array in Table 1, and pick a positive integer ≤ 31. Given only the columns in which the number appears, I am able to "divine" the number. For example, suppose that the selected number appears in columns 1, 3, and 5. Then, the number is 21. Do you think that I either peeked, or memorized the table? Neither, I simply added the first numbers in each of the columns in which the number appeared, namely, $16 + 4 + 1 = 21$.

Table I

16	8	4	2	1
17	9	5	3	3
18	10	6	6	5
19	11	7	7	7
20	12	12	10	9
21	13	13	11	11
22	14	14	14	13
23	15	15	15	15
24	24	20	18	17
25	25	21	19	19
26	26	22	22	21
27	27	23	23	23
28	28	28	26	25
29	29	29	27	27
30	30	30	30	29
31	31	31	31	31

The purpose of this note is: (1) to illustrate why this table and others like it work (without magic), (2) to show how to construct these tables, and (3) to show the relationship between such tables and the binary expansion of numbers.

Based on the method for determining the number 21, can you determine (without peeking) the number which appears in columns 4, 3, and 2? Can you determine in which columns the number 23 appears?

Before going further, it is instructive to study Table 1 to gain more insight. Returning to Table 1, we observe that;

(1) The columns of Table 1 are headed by $2^4, 2^3, 2^2, 2^1, 2^0$.

(2) The information given about the number in question, 21, is only membership (1) or non-membership (0) in each column,

(3) $21 = 1\cdot2^4 + 0\cdot2^3 + 1\cdot2^2 + 0\cdot2^1 + 1\cdot2^0$
$= 10101$ (binary system, or base two)

The third observation certainly suggests that the table is based on the binary number system. Now, can you answer the questions raised earlier?

Despite the fact that the following example does not yield a table nearly as large as Table 1, it will, however, lead to a better understanding of Table 1 and will demonstrate procedures which could be used to construct much larger tables.

The integers 1, 2, and 3 can be expressed in binary as follows:

$1 = 0\cdot2^1 + 1\cdot2^0$ 01
$2 = 1\cdot2^1 + 0\cdot2^0$ 10
$3 = 1\cdot2^1 + 1\cdot2^0$ 11

Next, replace the 1's in the k-th row by the integer k, which yields;

01
20
33

Finally, delete the superfluous 0's, yielding

Table 2

21
33

Though considerably restricted in range, Table 2 has the same "magical" property as does Table 1. This process can be repeated with n = 3, the number of columns, yielding Table 3.

Table 3

4	2	1
5	3	3
6	6	5
7	7	7

Now, the process used to construct Tables 2 and 3 could be used to construct a table larger than Table 1. But, there is yet an easier way! By examining Tables 1, 2, and 3, we can observe that Table 2 can be expanded to Table 3, by adding 4 to the entries of Table 2, placing this block of numbers below Table 2, and adding the column to the left 4, 5, 6, 7. Table 3, in turn, can be expanded by adding 8 to each entry of Table 3, placing this block of numbers below Table 3, and adding the column to the left 8, 9, 10, 11, 12, 13, 14, 15. Another application of this process, adding 16 to each of the entries of the newly formed table, etc., will lead us to Table 1.

Can you now find the "magic" table which will work for all positive integers ≤ 63? ≤ 127?

As a final note, the fact that these tables have their "magical" property can be proven using mathematical induction.

The SWT Center for the Study of Basic Skills: A Progress Report

John J. Edgell, Jr.

Southwest Texas State University

Southwest Texas State University, SWT, began as a normal institution, evolved to a university, has educated a considerable portion of the state's educators and is a continuing, progressive, educational influence. During 1978 SWT instituted a Steeples of Excellence Program designed to fund proposals which have the potential to offer regional, or national leadership.

The Mathematics Department of the School of Science was well aware of developing Mathematics/Education problems, regionally and nationally. The department perceived that a near crisis situation was on the horizon with respect to the preparation of mathematics teachers and in the mathematics teaching/learning process. Within the Steeples of Excellence Program the department could increase efforts in providing leadership in helping to resolve these problems. Consequently the Mathematics Department in conjunction with the Education Department of the School of Education applied for and was awarded university funds from the Steeples of Excellence Program.

Since the Education Department is responsible for the preparation of teachers of Reading and Language Arts, the funding included the three basic skill areas of mathematics, reading and language arts. Hence the Center for the Study of Basic Skills, Center, was initially designed to bring together the unique expertise of the two departments to provide leadership in the study of successful teaching/learning practices with respect to the basic skills of mathematics, reading and language arts. The Center, as depicted in Figure 1, has since expanded to include the Department of English of the School of Liberal Arts. So there are three departments from three schools of the university cooperatively involved in the goals of the Center.

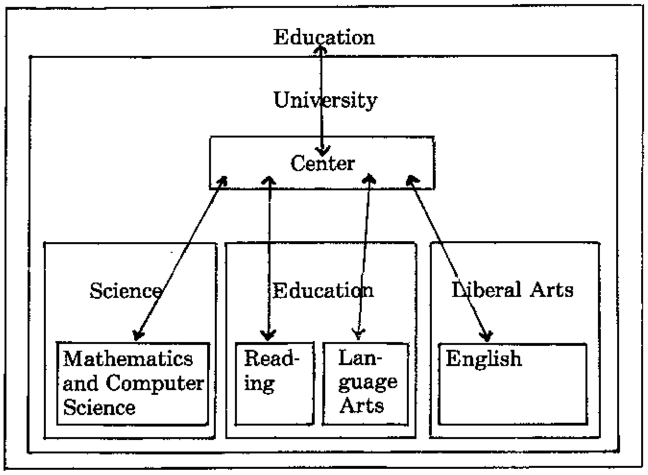

Figure 1. Conceptual Module for the Center.

As Figure 1 indicates, the Center communicates across several departments and schools within the university, with related professional groups and with regional educational leaders. The Center draws information from regional educational sources related to basic skill needs and successful basic skill teaching/ learning practices. Further the Center disseminates to the educational region the knowledge gained about successful basic skill teaching/learning practices. Also, the Center brings together the available expertise of the participating departments and influences the preparation of teachers within those departments. This unified effort and the magnitude of the project requires an unusual amount of cooperation between the experts of the departments and the regional educational leaders.

A general purpose of the Center is to identify successful teaching practices at all levels related to the basic skill areas, construct a composite of these successful practices for a regional educational model, evaluate and modify the model as necessary, then disseminate the findings and facilitate regional adoption of the model. Further, a special emphasis will be placed upon the teaching of computer science, an emerging basic skill associated with the mathematics department, as a fundamental tool for teaching other basic skills. A long range goal of the Center is to serve as a catalyst for improved teaching/learning of basic skills at all levels, regionally. Consequently, not only will the Center disseminate successful teaching/ learning practices, but the Center will facilitate the incorporation of these practices as part of the teacher training program. Tentative long range objectives for the Center include: acquiring a library of basic skill related information for the mutual benefit of the Center staff, teacher training programs and regional schools; determining educational management recommendations for implementing successful practices; evaluating a regional basic skills needs profile; accumulating appropriate in-service training strategies; establishing practical regional educational models; revising basic skill area teacher training programs; and reporting important findings to the related professional organizations.

During the past eighteen months the Center has had the following accomplishments. Initially the Center selected a Professional Advisory Board. The Board is composed of a group of nationally recognized leaders in each of the basic skill areas. The general purpose of the Board has been to serve as an evaluative and recommending panel to insure and monitor the quality of the products of the Center. The Board recommends direction for the Center, reviews work accomplished by the Center and facilitates the dissemination of important findings of the Center to professional groups. A "nuts and bolts" accomplishment of the Center is the establishment of a communications network. The communications network includes names, positions and addresses of professional contacts across the state and nationally and also includes similar information on all superintendents, basic skill coordinators, and teacher leaders in each of the one hundred five school districts of Regions XIII and XX of the T.E.A.. The communications network has been computerized and accessibility of this information has greatly enhanced the level of communication between the Center and the related educational leaders. A major accomplishment of the Center has been the development and professional interaction of the Center staff members. The Center has basically served as a forum for those faculty members of the various basic skill area departments who are more than just casually interested in excellence in the training of teachers and in the teaching/ learning process. The Center has sponsored, endorsed or supported faculty members in various ways in participating in professional development activities. The development activities have included seminars by national basic skill educational leaders, workshops by state level educational agencies and informative sessions led by private research organizations. Center staff members have learned about national and international educational assessments, basic tenets of Mastery Learning, skills associated with grant acquirements and the basics of how to disseminate and facilitate programs. Staff members have had strong encouragement and support for educational site visitations. Site visits have occurred with selected educational institutions having a reputation for good basic skill teaching/learning practices or with institutions that have adopted selected Nationally Validated Programs. In the process, the Center staff has developed a working knowledge of National Validated Programs. Finally, there has been increased funds available for Center Staff participation in professional groups. This participation includes attendance at professional meetings and an outlet for publishing findings of the Center. With a special interest in the area of computer science as a fundamental tool for teaching other basic skills, the

Center has acquired several Micro Computer Systems. These systems have been distributed across the basic skill areas of the Center. Additionally, the Center has acquired metric tools to facilitate successful teaching/learning practices relative to teaching/learning the basic skills of measuring. Similarly the Center has acquired a collection of calculators appropriate for training teachers in the great wealth of ideas and products of the massive amount of research relative to calculating via the hand-held calculator. Although the Center has not accomplished the basic goal of constructing a composite model of successful teaching/learning basic skill practices appropriate for the region, the Center has accumulated considerable information relative to this objective. A good beginning in this effort has been a familiarization with the National Validated Programs. This has been followed by site visits to exemplary educational programs. Further accomplishment in this direction has been a recently conducted regional basic skills needs assessment. Although the Center is accumulating information, related to regionally successful educational practices in the basic skills areas, towards constructing a composite regional basic skills educational model, the Center has been able to provide regional leadership by sponsoring three successful conferences. The first conference addressed the teaching/learning of measuring as a basic skill, *Metric: Today's Challenge.* The second conference addressed the teaching/learning of the basic skill of calculating, *Calculators: Tomorrows Challenge.* The third conference served to announce findings of the Center related to the teaching/learning practices in the basic skills, *Important Findings About Teaching Basic Skills.* Each of these conferences brought together national and international leaders with regional leaders and university faculty to share basic skill knowledge with regional educators. In a similar effort to share the acquired expertise of the Center, the Center sponsored the production of a monograph, *Ten Important Findings About Teaching Basic Skills.*

Presently the Center is encouraging staff to continue site visitations, is investigating strategies for helping regional school districts in implementing quality basic skills teaching/learning practices, is investigating available resources for opportunities for extending the research and findings of the Center, is sponsoring the production of a second monograph, and is organizing a fourth conference, *Effective Practices In Teaching Basic Skills.* The Center encourages all interested educators to share their knowledge of the basic skill areas with the Center.

Where Are We With Metrics?

John J. Edgell, Jr.

Southwest Texas State University

The ability to measure and record data is an important, basic societal concern. Consequently, measuring is an important facet of the information to be taught in public schools. Most often the relevant ideas of measuring are embodied in mathematics and therefore, is a primary teaching responsibility of those responsible for teaching mathematics at every level. Changes which affect the way measured data is encoded and perhaps understood are significant basic changes, which have the tendency to affect how and perhaps when certain aspects of teaching measuring should occur. With this basic idea and basic teaching concerns in mind, the Southwest Texas Center for the Study of Basic Skills hosted a measuring conference, *Metrics: Today's Challenge.*[2] This conference was for the benefit of teachers and administrators of schools at all levels within T.E.A. Regions XIII and XX. As evidenced by the large number of educators in attendance and the level of interest and participation, there is considerable concern about the public school role in the metric measuring movement.

As the Mathematics Coordinator for the Center and in preparation for responsibilities associated with the Conference, several activities related to knowing about the teaching of measuring metrically were deemed appropriate. The findings reported in this article are generally the results of activities which include specific preparation for leading conference workshops, determining a regional metric implementation profile[3] for the benfit of workshop leaders and speakers, interaction with speakers and participants and observations subsequent to the conference.

In staying abreast of curricular changes and current classroom practices related to the concern of the Conference, a review of state adopted mathematics textbooks and classroom observations of current teaching practices was conducted. Most of the elementary level texts seem to have a dual approach to understanding measures. Some texts begin with metric units of measure followed by traditional units of measure while others reverse the situation. What occurs in the classrooms varies across the extremes of traditional only to metric only, with an apparent majority ignoring the metrics and teaching the traditional units. At least one first grade teacher was observed struggling with a simultaneous dual development. Almost all of the recent junior high school level mathematics texts have measuring presented metrically. Some have little or no reference to traditional units of measure, while others perhaps include a table of references to traditional measuring units. Classroom observations indicate that junior high school level teachers, when teaching measuring, very often supplement the textbook with materials based upon traditional units of measure. Some completely ignore the metric measures and teach traditional measuring tools and units of measure. High school mathematics texts seem to be oriented towards understanding measuring in terms of traditional units. Teachers generally teach measuring based upon traditional units and then use conversion techniques when teaching metric measuring units, if at all. In reviewing the latest editions of undergraduate university mathematics texts, one generally finds application problems stated in terms of traditional units. A notable exception is a recent trignometry book[4] based upon calculators. An observation which seems clear at this point is that there does not seem to be a uniform strategy for teaching measuring concepts, not even within school districts, K through university levels.

In order to determine the level of implementation of metric measuring education within the two regions, a questionnaire[3] was sent to each of the one hundred five school districts. Sixty-three school districts responded. The responses yield an interesting profile of the regions with respect to the level of metrics education. Seventy-five percent of the districts responding indicated there was on-going metric education occurring within the district. Nineteen percent indicated that there was no adopted plan for the district to teach metric measurement. Contrary to what one would expect after surveying the adopted texts, most of the metric education was reported at the fourth grade and above. This information was also surprising since most of the reported teacher in-service programs were at the elementary level. With respect to the reported metric education programs, there were no programs provided for support staff, only two parent programs and just four administrative programs throughout the two regions. Of the twenty-one responding school districts indicating no in-service metric education programs, nine school districts indicated that there were no plans for sponsoring such programs. With apparently no support staff training, it should be of little surprise that none of the school nurses recored data on children metrically and very little data is recorded metrically by the athletic divisions. Apparently, according to the poll, most of metric education occurs in mathematics classes, followed closely by science classes. The exceptions were a couple of vocational home economics classes which taught or used metrics and a geography class that was taught using metrics. Only six percent of the school districts, indicating ongoing metric education, reported that measuring was taught completely in metric terms. Finally, about sixty percent of the school districts reported using a dual approach and the rest used a traditional based approach with emphasis upon conversions.

At the Conference, George Bright[5] reflected upon metric implementation nationally. Generally, in his opinion, the metric movement seems to have stalled. This is a disappointing situation which seems attributable to several factors, including lack of prepared teaching materials, lack of teacher training, lack of education of the general public and lack of supportive politics. Glyn Wooldridge[6] shared the results of the Canadian metric transition. Basically, he related that the metric transition in Canada was essentially complete and a success. He pointed out; however, that perhaps the basic motivational factors affecting the transition were different. He indicated that big business, industry and an economy based upon international trade worked together with educational institutions for a smooth and successful transition. Also, the fact that a significant part of the Canadian population have French roots aided the metric movement.

Currently, one may notice the following factors influencing metric implementation. Politically supportive actions, particularly at a national level, have been reversed or seemingly ignored. Popular commentaries in local newspapers seem to be negatively oriented. People coping with every day situations seem to be only casually affected, if at all, by metric data.

There are some conclusions which can be drawn from the preceeding information. First, contrary to professional organizations, there is clearly a lack of a unified strategy on the part of educational institutions for teaching measuring metrically! The adopted textbooks seem to be uncoordinated, K-12, and certainly the ongoing teaching lacks direction. In view of the NCTM recommendation of THINK METRIC[7,8,9] with respect to teaching metrics, the majority of the practicing teachers and teaching institutions must have had other authoritative sources for making educational decisions. A personal observation, from the young child's perspective, is that a dual approach to teaching measuring seems to be an unfortunate compromise during a transitional period. Clearly, students are caught in the transition in that they live in a traditional measure based world (locally), but these same children will live in a world where data is determined and recorded in metric terms. Consequently, teachers may make the mistake of opting for a dual approach when in fact young children experience difficulty in understanding measuring on a unilateral basis. Although, there are many opportunities in mathematics for dual ideas to be presented, teachers of mathematics have quite often determined that the duality relationship between ideas should be established later when the student is more "mathematically mature." This may very well be the case with respect to measuring concepts. Secondly, mathematics is often accused of being in its own little world and this seems to be the case with respect to teaching metric measures. It is unfortunate that only teachers directly concerned about teaching measuring experience in-service programs. Similarly it is unfortunate to have metric units only expressed in mathematics and science texts. All courses using measures should complement the efforts of the mathematics teacher, including books for and teachers of such courses. Further, the "real world" of measures for some of these youngsters is related to them by the support staff, including physical education coaches, nurses, secretaries, principles, etc. Hence support staff should be trained on how to complement the metric measuring movement. Also, as in every case when there are major curricular changes, the parents need to share in the educational responsibilities. Thirdly, teaching institutions should not carry the *full* burden of implementing societal changes. Presently, with respect to public support, the situation at best seems to be luke warm. This is somewhat understandable considering the large political boundaries of this country and the vast amount of domestic trade. Without political support, and broad based support of business and industry one can hardly be surprised at a lack of a unified strategy by educators for teaching measuring concepts that involve a change in basic conceptual understandings. Without a real need for data expressed metrically, the public will likely never support metric education.

So, where are we, metrically speaking? It seems that we are experiencing a lull during a transition period. This hurdle requires the cooperation of business, industry, political forces and the public to give momentum to the metric education movement. Teachers of mathematics seem to be presently holding the metric "hot potato." What is occurring during this lull is that a generation of our society is experiencing a poor education with respect to a basic concern to every member of society. The real question should be, can society afford the price of a generation experiencing a poor education in measurement while waiting for business, industry and political forces to combine their efforts with educational institutions in generating educated citizens.

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Archimedes' Method for Calculating PI

John Huber

Pan American University

Using the fact that the circumference of a circle lies between the perimeter of any regular inscribed polygon and that of any regular circumscribed polygon, Archimedes was able to show that π lies between 223/71 and 22/7. The purpose of this paper is to derive the usual recursive formula for the ratio of the perimeter to the diameter of a regular inscribed polygon, show that this formula is unstable for computing devices, and modify the formula to a stable algorithm.

Let s_n denote the length of a side of a regular polygon of n sides inscribed in a circle of radius r and s_{2n} denote the length of the side of the regular polygon of 2n sides formed by bisecting the arc containing consecutive vertices of the original regular inscribed polygon of n sides. (See Fig. 1.) Using Figure 1 and the Pythagorean Theorem we have

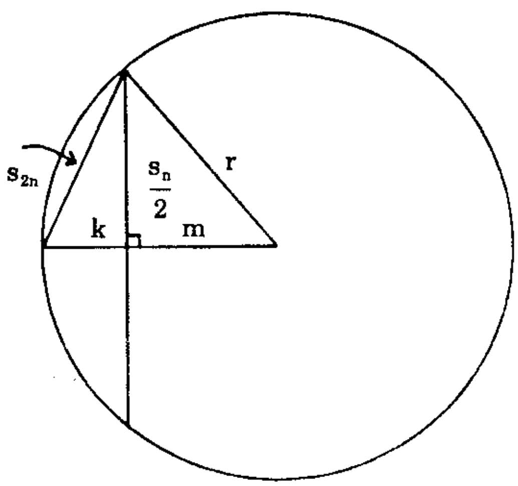

Figure 1

$$r^2 = \left(\frac{s_n}{2}\right)^2 + m^2$$

and

$$m = \sqrt{r^2 - \frac{s_n^2}{4}} \quad .$$

Since $k + m = r$, we have

$$k = r - m$$
$$= r - \sqrt{r^2 - \frac{s_n^2}{4}}$$

so

$$k^2 = r^2 \; - 2r\sqrt{r^2 - \frac{s_n^2}{4}} + r^2 - \frac{s_n^2}{4}$$
$$= 2r^2 - 2r\sqrt{r^2 - \frac{s_n^2}{4}} - \frac{s_n^2}{4} \quad .$$

Again using Figure 1 and the Pythagorean Theorem, we have

$$s_{2n}^2 = k^2 + \left(\frac{s_n}{2}\right)^2$$
$$= 2r^2 - 2r\sqrt{r^2 - \frac{s_n^2}{4}}$$

so

$$s_{2n} = \sqrt{2r^2 - 2r\sqrt{\frac{4r^2 - s_n^2}{4}}}$$
$$= \sqrt{2r^2 - \; r\sqrt{4r^2 - s_n^2}} \quad . \qquad (1)$$

Then considering a regular hexagon inscribed in a circle of radius 1, (1) becomes

$$s_{2n} = \sqrt{2 - \sqrt{4 - s_n^2}} \qquad (2)$$

where $s_6 = 1$ and perimeter/diameter $= \frac{n \cdot s_n}{2}$. Using a programmable calculator, we have the results in Table I. (See Appendix for Programs.) Clearly the ratio does not converge to π.

Knowing that $\lim_{n \to \infty} \frac{n \cdot s_n}{2} = \pi$, why does the algorithm not converge on the calculator? The lack of convergence is caused by the large relative error in the difference $2 - \sqrt{4 - s_n^2}$. Since $\sqrt{4 - s_n^2}$ is close to 2 (See Table II.), the rounding error along with the closeness of 2 and $\sqrt{4 - s_n^2}$ causes a large relative error in $2 - \sqrt{4 - s_n^2}$ resulting in an unstable algorithm. (Conte and de Boor, 1972, pp 13-14.)

To stabilize the algorithm we must remove the difference $2 - \sqrt{4 - s_n^2}$. This can be accomplished by rationalizing the numerator under the radical in (2) , giving us

$$s_{2n} = \sqrt{(2 - \sqrt{4 - s_n^2}) \;\; \frac{(2 + \sqrt{4 - s_n^2})}{(2 + \sqrt{4 - s_n^2})}} \qquad (3)$$

resulting in

$$s_{2n} = \sqrt{\frac{s_n^2}{2 + \sqrt{4 - s_n^2}}} \qquad (4)$$

Eliminating the difference results in a stable algorithm that converges to π . Using a programmable calculator, we have the results in Table III. (See Appendix for Programs.)

Reference

Conte, S.D. and de Boor, Carl. *Elementary Numerical Analysis* 2nd Ed. New York. McGraw-Hill, 1972.

Table I

Number of Sides	Length of Side	Perimeter/Diameter
6	1.0000000000	3.000000000
12	0.5176380902	3.105828541
24	0.2610523844	3.132628613
48	0.1308062585	3.139350203
96	0.0654381656	3.141031951
192	0.0327234633	3.141452473
384	0.0163622792	3.141557615
768	0.0081812081	3.141583911
1536	0.0040906127	3.141590529
3072	0.0020453076	3.141592407
6144	0.0010226544	3.141594284
12288	0.0005113277	3.141597288
24576	0.0002556658	3.141621319
49152	0.0001278358	3.141693413
98304	0.0000639218	3.141885657
196608	0.0000319687	3.142654499
393216	0.0000160000	3.145728000
786432	0.0000080623	3.170208743
1572864	0.0000041232	3.242542203
3145728	0.0000022361	3.517030823
6291456	0.0000014142	4.448731201

Table II

Number of Sides	$\sqrt{4 - s_n^2}$	$2 - \sqrt{4 - s_n^2}$
6	1.732050808	0.2679491924
12	1.931851653	0.0681483474
24	1.982889723	0.0171102772
48	1.995717846	0.0042821535
96	1.998929175	0.0010708250
192	1.999732276	0.0002677243
384	1.999933068	0.0000669322
768	1.999983267	0.0000167331

Table III

Number of Sides	Length of Side	Perimeter/Diameter
6	1.0000000000	3.000000000
12	0.5176380902	3.105828541
24	0.2610523844	3.132628613
48	0.1308062585	3.139350203
96	0.0654381656	3.141031951
192	0.0327234633	3.141452472
384	0.0163622792	3.141557608
768	0.0081812081	3.141583892
1536	0.0040906126	3.141590463
3072	0.0020453074	3.141592106
6144	0.0010226538	3.141592517
12288	0.0005113269	3.141592619
24576	0.0002556635	3.141592645
49152	0.0001278317	3.141592651
98304	0.0000639159	3.141592653
196608	0.0000319579	3.141592653
393216	0.0000159790	3.141592654
786432	0.0000079895	3.141592654
1572864	0.0000039947	3.141592654
3145728	0.0000019974	3.141592654
6291456	0.0000009987	3.141592654

APPENDIX

PROGRAMS FOR GENERATING n, s_n, and $\frac{n \cdot s_n}{2}$ using $s_{2n} = \sqrt{2 - \sqrt{4 - s_n^2}}$.

TI 58 and 59	
	LRN
00	2nd CP
01	STO
02	01
03	R/S
04	STO
05	02
06	R/S
07	x^2
08	RCL
09	02
10	+/−
11	+
12	4
13	=
14	$\sqrt{x}$
15	+/−
16	+
17	2
18	=
19	STO
20	02
21	2
22	2nd
23	Prd
24	01
25	RCL
26	01
27	R/S
28	RCL
29	02
30	$\sqrt{x}$
31	R/S
32	×
33	RCL
34	01
35	÷
36	2
37	=
38	R/S
39	GTO 08
	LRN
	RST

HP 33E	
	PRGM
00	f Clear Prgm
	f FIX 9
01	STO 1
02	R/S
03	STO 2
04	R/S
05	g x^2
06	RCL 2
07	CHS
08	ENTER
09	4
10	+
11	f $\sqrt{x}$
12	ENTER
13	CHS
14	2
15	+
16	STO 2
17	2
18	STO×1
19	RCL 1
20	R/S
21	RCL 2
22	f $\sqrt{x}$
23	R/S
24	ENTER
25	RCL 1
26	×
27	ENTER
28	2
29	÷
30	R/S
31	GTO 06
	RUN
	g RTN

INPUT	
6	R/S
1	R/S

OUTPUT	
R/S	n
R/S	s_n
R/S	Perimeter/Diameter
•	
•	

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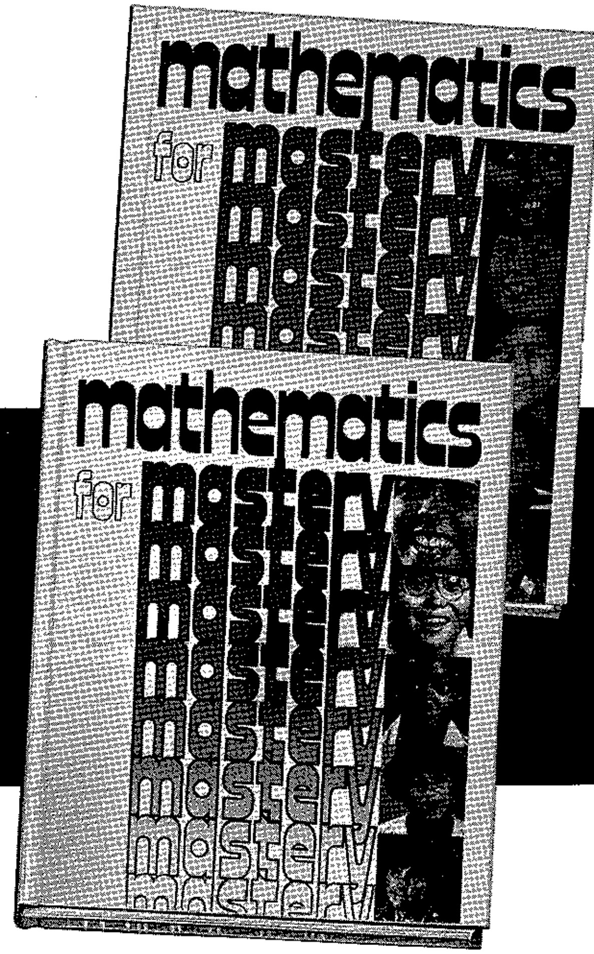

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