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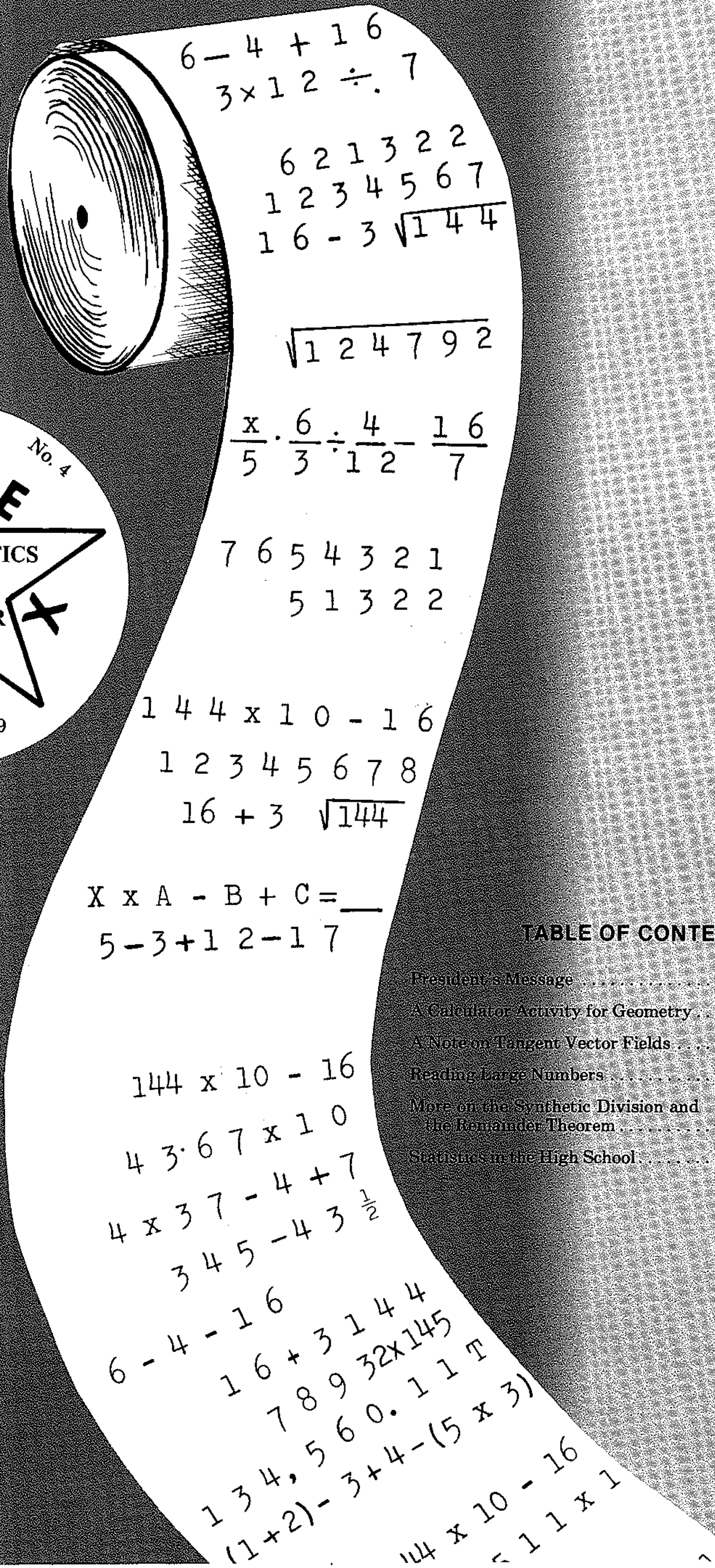


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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.



TEXAS MATHEMATICS TEACHER is published quarterly by the Texas Council of Teachers of Mathematics. Payment of membership fee of $5.00 entitles members to all regular Council Publications.

# PRESIDENT'S MESSAGE

It is that time again. All of us are back in our classrooms and busy as can be, but please take time to send in your 1979-80 dues and to invite your co-workers to join our organization. Use the membership blank in this issue. We need the support of all mathematics teachers in Texas.

Are you looking for new ideas, new ways to help children learn, and new ways to stimulate thought? If so, plan now to attend the annual fall CAMT meeting in Austin, October 25, 26, and 27, where leaders in mathematics education will be prepared to help you in your search. Following the format of past years, the program is designed to attract those interested in mathematics education at all levels—kindergarten through college.

A preliminary notice of this meeting was mailed to you in the May issue of the journal and notices were sent to superintendents and supervisors last spring. TCTM members and others should have received a program booklet and registration form in September.

Three executive board members have completed their terms and will be leaving the board this year. They are Shirley Cousins, past-president, Betty Hall, vice-president, and Wayne Miller, treasurer. Many thanks to each of you for the great job you have done in serving the membership.

You should have received an election ballot in September, and I hope that you have voted and returned it. The results of this year's election will be announced at the Annual TCTM Luncheon in Austin, October 26.

Thank you for helping to send me to the Annual NCTM Convention which was in Boston, Massachusetts last April. It was a delightful and most rewarding experience.

I hope to see many of you in Austin, October 25-27.

Anita Priest

# A Calculator Activity for Geometry

**John Huber**

*Pan American University*

In tenth grade geometry, the circumference of a circle is defined as the limit of the perimeters of inscribed regular polygons of the circle as the number of sides increases without bound. (Coxford and Usiskin, pg. 537; Jurgensen, et al., p. 323; Nichols, et al., p. 384.) The fact that the ratio of the circumference to the length of a diameter is constant is usually established using similar polygons. This constant is then defined to be $\pi$ and the formulas for the circumference and area of a circle are established. The purpose of this paper is to provide a different approach to determining the circumference and area of a circle using a programmable calculator.

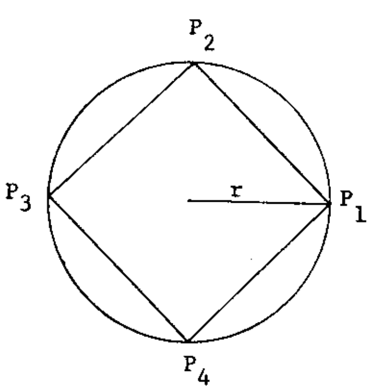


Figure 1

Consider a square inscribed in a circle of radius r. (See Fig. 1.) Constructing the perpendicular bisectors of each side of the square results in an inscribed regular octagon. (See Fig. 2.) Continuing this process a sequence of inscribed regular polygons with $2^k$ ($k = 2,3,\cdots$) sides can be constructed.

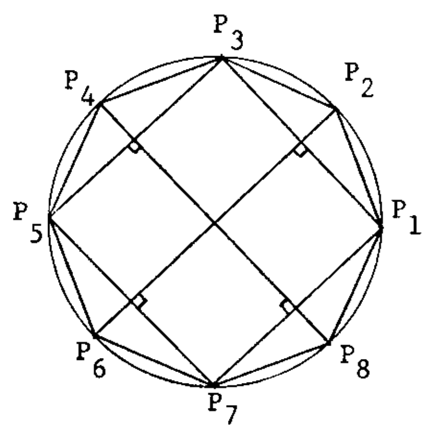


Figure 2

In an inscribed regular polygon with n sides the following results can be established: (See Fig. 3.)

1. The measure of the central angle between consecutive vertices is given by

$$m\angle P_kOP_{k+1} = \frac{360^\circ}{n}.$$

2. The length of each side $s_n$ is given by

$$s_n = 2r \sin\left(\frac{180^\circ}{n}\right).$$

3. The length of the apothem $a_n$ is given by

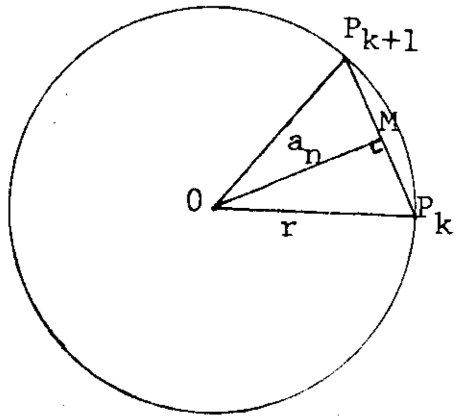
$$a_n = r \cos\left(\frac{180^\circ}{n}\right).$$

4. The perimeter $p_n$ is given by

$$p_n = n\left\{2\left[r \sin\left(\frac{180^\circ}{n}\right)\right]\right\}$$
$$= 2r\left[n \sin\left(\frac{180^\circ}{n}\right)\right]$$

5. The area $A_n$ is given by

$$A_n = \frac{1}{2} a_n p_n$$
$$= \frac{1}{2}\left[r \cos\left(\frac{180^\circ}{n}\right)\right]\left[2r\, n \sin\left(\frac{180^\circ}{n}\right)\right]$$
$$= r^2\left[\cos\left(\frac{180^\circ}{n}\right)\right]\left[n \sin\left(\frac{180^\circ}{n}\right)\right].$$




$$m\angle P_kOP_{k+1} = \frac{360^\circ}{n} \qquad \sin\left(\frac{180^\circ}{n}\right) = \frac{P_kM}{r}$$
$$m\angle P_kOM = \frac{180^\circ}{n} \qquad \cos\left(\frac{180^\circ}{n}\right) = \frac{a_n}{r}$$
$$s_n = P_kP_{k+1} = 2\,P_kM$$
$$p_n = n \bullet s_n$$
$$A_n = \frac{1}{2} a_n p_n$$

Figure 3

Since the only terms in $p_n$ and $A_n$ involving n are $n \sin\left(\frac{180^\circ}{n}\right)$ and $\cos\left(\frac{180^\circ}{n}\right)$, the problem now is to determine what $n \sin\left(\frac{180^\circ}{n}\right)$ and $\cos\left(\frac{180^\circ}{n}\right)$ converge to as n increases without bound. Using a programmable calculator, the following table is generated: (See Appendix for Programs.)

| n | $n \sin\left(\frac{180^\circ}{n}\right)$ | $\cos\left(\frac{180^\circ}{n}\right)$ |
|---|---|---|
| 4 | 2.8284 | 0.7071 |
| 8 | 3.0615 | 0.9239 |
| 16 | 3.1214 | 0.9808 |
| 32 | 3.1365 | 0.9808 |
| 64 | 3.1403 | 0.9988 |
| 128 | 3.1413 | 0.9997 |
| 256 | 3.1415 | 0.9999 |
| 512 | 3.1416 | 1.0000 |
| 1024 | 3.1416 | 1.0000 |

Then the circumference C of a circle with radius r is given by

$$C = \lim_{n\to\infty} p_n$$
$$= \lim_{n\to\infty} 2r\left[n \sin\left(\frac{180}{n}\right)\right]$$
$$= 2\pi r$$

and the area A is given by

$$A = \lim_{n\to\infty} A_n$$
$$= \lim_{n\to\infty} r^2\left[\cos\left(\frac{180^\circ}{n}\right)\right]\left[n \sin\left(\frac{180^\circ}{n}\right)\right]$$
$$= \pi r^2.$$

In conclusion, the programmable calculator can be used to establish the important limits necessary for developing a conceptual understanding of the derivations of the formulas for the circumference and area of a circle.

## REFERENCES


Coxford, Arthur F. and Zalman P. Usiskin. *Geometry: A Transformation Approach.* Laidlaw Brothers, River Forest, Illinois.1971.

Jurgensen, Ray C., Alfred J. Donnelly, John E. Mair, and Gerald R. Rising. *Geometry.* Houghton Mifflin, Boston. 1975.

Nichols, Eugene D., Mervine L. Edwards, E. Henry Garland, Sylvia A. Hoffman, Albert Mamary, and William F. Palmer. *Holt Geometry.* Holt Rinehart and Winston, New York. 1974.

## APPENDIX

PROGRAMS FOR GENERATING n, n sin $(\frac{180°}{n})$, cos $(\frac{180°}{n})$

**TI 58 and 59**

| | LRN |
|---|---|
| 00 | 2nd CP |
| 01 | 2nd Deg |
| 02 | 4 |
| 03 | STO |
| 04 | 00 |
| 05 | 1 |
| 06 | 8 |
| 07 | 0 |
| 08 | ÷ |
| 09 | RCL |
| 10 | 00 |
| 11 | = |
| 12 | STO |
| 13 | 01 |
| 14 | 2nd sin |
| 15 | × |
| 16 | RCL |
| 17 | 00 |
| 18 | = |
| 19 | STO |
| 20 | 02 |
| 21 | RCL |
| 22 | 01 |
| 23 | 2nd cos |
| 24 | STO |
| 25 | 03 |
| 26 | RCL |
| 27 | 00 |
| 28 | R/S |
| 29 | RCL |
| 30 | 02 |
| 31 | R/S |
| 32 | RCL |
| 33 | 03 |
| 34 | R/S |
| 35 | 2 |
| 36 | 2nd Prd |
| 37 | 00 |
| 38 | GTO |
| 39 | 05 |
| | LRN |
| | RST |

**HP 33E**

| | PRGM |
|---|---|
| 00 | f Clear Prgm |
| 01 | g DEG |
| 02 | 4 |
| 03 | ENTER |
| 04 | STO 0 |
| 05 | 1 |
| 06 | 8 |
| 07 | 0 |
| 08 | $x \rightleftarrows y$ |
| 09 | ÷ |
| 10 | STO 2 |
| 11 | f SIN |
| 12 | ENTER |
| 13 | RCL 0 |
| 14 | × |
| 15 | STO 1 |
| 16 | RCL 2 |
| 17 | f COS |
| 18 | STO 2 |
| 19 | RCL 0 |
| 20 | R/S |
| 21 | RCL 1 |
| 22 | R/S |
| 23 | RCL 2 |
| 24 | R/S |
| 25 | 2 |
| 26 | STO×0 |
| 27 | RCL 0 |
| 28 | ENTER |
| 29 | GTO 05 |
| | RUN |
| | g RTN |

| OUTPUT | |
|---|---|
| R/S | n |
| R/S | n sin $(\frac{180°}{n})$ |
| R/S | cos $(\frac{180°}{n})$ |
| • | |
| • | |
| • | |

# A Note on Tangent Vector Fields

**by Richard K. Williams**
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A reasonably well-known theorem in Topology says that any continuous tangent vector field on a sphere must have a singularity, i.e. there is at least one point on the sphere to which the zero vector is assigned. We shall henceforth refer to this theorem as "the singularity theorem." The singularity theorem can be used to prove that each continuous mapping of a sphere into itself must either have a fixed point or map some point to its diametrically opposite partner. (See [2], p. 152.) Let us call this second theorem "the mapping theorem".

It is a simple exercise (see [1], p. 135, problem 4) to derive the mapping theorem without using the singularity theorem. The purpose of this note is to show how the singularity theorem can easily be derived from the mapping theorem. Thus, the instructor who wishes to present these two results to his class has the option as to which theorem he proves first.

Clearly we may assume that the sphere is the unit sphere centered on the origin. Call this sphere $S^2$. If p denotes a point on the sphere, let $-p$ denote its diametrically opposite partner or antipode. Let $\bar{V}(p)$ be the continuous tangent vector field, i.e. $\bar{V}(p)$ is the vector which is tangent to $S^2$ at p. Assume $\bar{V}(p) \neq \bar{0}$ for each $p \epsilon S^2$. Then $\bar{U}(p) \equiv \dfrac{\bar{V}(p)}{|\bar{V}(p)|}$ is a continuous *unit* tangent vector field on $S^2$. Thus if $\bar{U}(p)$ originates at the origin, its terminal point will be a point on $S^2$. Call this terminal point U(p). The correspondence $p \to U(p)$ determines a continuous mapping of $S^2$ into itself. Since $\bar{U}(p)$ is a unit tangent vector at p, U(p) is neither p nor $-p$. This contradicts the mapping theorem, so $\bar{V}(p) = \bar{0}$ for some p.

A well-known illustration of the singularity theorem is obtained by letting the sphere be the surface of the earth and $\bar{V}(p)$ the velocity of the wind at point p. Hence, at all times, there exists a point on the earth where the wind is not blowing. (This point is rarely in Texas.)


**References**

1. B. H. Arnold, *Intuitive Concepts in Elementary Topology*, Prentice-Hall, Englewood Cliffs, New Jersey, 1962.
2. Donald W. Blackett, *Elementary Topology*, Academic Press, New York, 1967.



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# Reading Large Numbers


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It should come as no surprise to teachers and parents that children have trouble reading large numbers. The development of large number notation was itself a very slow process. A brief look at this history can be informative.

There is no evidence that the word "million" was used before the 13th Century. Prior to that time there was little need for or interest in large numbers. When a number representing a million was needed the term a thousand thousand was generally used. The word "billion" appeared in the 14th Century but its acceptance was met with resistance. Instead the term million million was preferred.

In the first Arithmetic Book published in America a billion was used to represent 1,000,000,000 and a trillion was represented by 1,000,000,000,000. The American system of large numbers begins with the ones, thousands, and millions period.

For Example:
249 is read two hundred forty nine ones
249,000 is read two hundred forty nine thousands
249,000,000 is read two hundred forty nine millions.

If we examine the large numbers greater than a millions a clear pattern emerges. The prefix bi means two, hence billions means two sets of three zeros after 1,000. One billion is then 1,000,000,000. The prefix tri means three and trillion means three sets of three zeros after 1,000, thus trillion is written 1,000,000,000,000. The prefix quad means four so we have quadrillion and so on.

When teaching large numbers one must first decide on the largest number the student is expected to read. Can the child stop at quadrillion or is it important for him to recognize a decillion? The following activity may reinforce your student's knowledge and understanding about reading large numbers. An activity for quadrillion involves 12 students. Six of these students will write the same number on their paper, for example 123. Each of the other six students will receive a place value name from ones to quadrillions. One of the children with 123 on their paper will come to the front of the room and the child with the ones paper also comes to the front. Both children hold up their paper and the first child says one hundred twenty three. The second child says ones. This activity continues until one child says 123 and the last child says quadrillions.

Another problem in teaching large numbers is translating from the oral to the numerical representation. In the United States 249 quadrillion, 4 billion, 21 million, 8 thousand, one would be written as 249,000,004,021,008,001. Unfortunately neither the comma nor the period have been standardized. The following techniques have also been used in writing large numbers.

249000004021008001 (with arcs over the groups of three)

: :  : :  ⋮  :  .
249000004021008001

cbacbacbacbacbacba
249000004021008001

And finally we have the metric recommendation that in writing large numbers the commas be replaced with a space, thus 249,000,004,021,008,001 will become 249 000 004 021 008 001.

# More on Synthetic Division and the Remainder Theorem

by Kurt W. Reimann

Division of a polynomial of first or greater degree by another polynomial of first degree only is commonly performed by the process known as synthetic division. In response to student inquiries and interest, as well as my own curiosity, I present herein, first, a generalized method of synthetic division where the divisor polynomial may be of any degree equal to or larger than one and the dividend polynomial accordingly of equal or larger degree than the divisor polynomial. Secondly, a generalization of the familiar Remainder Theorem will be considered.

First, reconsider the common problem of division of a dividend polynomial of degree equal to or larger than one by a linear divisor. Shown below are both the "synthetic division" and "long division" processes.

Example #1 Divide $3x^3 - x + 1$ by $x + 2$

Solution by "Synthetic Division"

$$\begin{array}{r|rrrr} & 3 & 0 & -1 & 1 \\ \underline{-2} & & & & \\ & & -6 & 12 & -22 \\ \hline & 3 & -6 & 11 & -21 \end{array}$$

answer: $3x^2 - 6x + 11 + \dfrac{-21}{x+2}$

Solution by "Long Division"

$$\begin{array}{rllll} & 3x^3 & -6x^2 & +11 & \\ \cline{2-5} x+2 & 3x^3 & +0x^2 & -x & +1 \\ & \underline{3x^3} & \underline{+6x^2} & & \\ & & -6x^2 & -x & +1 \\ & & \underline{-6x^2} & \underline{-12x} & \\ & & & 11x & +1 \\ & & & \underline{11x} & \underline{+22} \\ & & & & -21 \end{array}$$

answer: $3x^2 - 6x + 11 + \dfrac{-21}{x+2}$

As you recall, the leading coefficient of the linear divisor must always be "1" when using synthetic division. [If this is not the case, every term in both divisor and dividend can be divided by the "non-one" coefficient of the linear divisor in order to remedy the situation.] As long as the coefficient of the linear divisor is a "1", the leading coefficient of the quotient polynomial is always the same as the dividend polynomial — in this case, the number "3". Hence, in the synthetic process the number "3" from the top row is simply brought down under the line to be the leading coefficient of the quotient polynomial. The balance of the process hinges on the constant term of the divisor polynomial — in this case, the number "2". The columns that are highlighted in the long division process are clearly the numbers which are mimicked in the synthetic division process. In order to facilitate column addition rather than the more cumbersome column subtraction, the number "$-2$" is used in the "corner" symbol (_|) rather than a "+2". This explains why the signs of the middle numbers in the highlighted column of the long division process are the negatives of their counterparts in the synthetic process.

The reason the "corner box" is in an unorthodox position in the synthetic division will become evident in the discussion that follows.

Now, observe long division where the divisor polynomial is of degree greater than one.

Example #2

Divide $x^4 + 2x^2 + 3x + 7$ by $x^2 + 2x + 1$

$$\begin{array}{rllllll} & x^2 & -2x & +5 & & & \\ \cline{2-7} x^2 + 2x + 1 & 1x^4 & + 0x^3 & + 2x^2 & + 3x & + 7 & \\ & \underline{x^4} & \underline{+ 2x^3} & \underline{+ x^2} & & & \\ & & - 2x^3 & + x^2 & + 3x & + 7 & \\ & & \underline{- 2x^3} & \underline{- 4x^2} & \underline{- 2x} & & \\ & & & 5x^2 & + 5x & + 7 & \\ & & & \underline{5x^2} & \underline{+ 10x} & \underline{+ 5} & \\ & & & & - 5x & + 2 & \end{array}$$

answer: $x^2 - 2x + 5 + \dfrac{-5x + 2}{x^2 + 2x + 1}$

The leading coefficient of the divisor is again "1", therefore the process hinges on the remaining coefficients, "2" and "1", of the divisor. The numbers that must be mimicked are highlighted in the fragmented columns (fragmented because the subtotals are non-essential for synthetic division).

The same division is performed below-synthetically. Note that the numbers "$-2$" and "$-1$" are used in the corners in order to again facilitate addition rather than subtraction. Also note that the corner numbers are listed vertically in descending order according to the terms to which they correspond, top to bottom.

$$\begin{array}{r|rrrrr} & 1 & 0 & 2 & 3 & 7 \\ \underline{-2} & & -2 & 4 & -10 & \\ \underline{-1} & & & -1 & 2 & -5 \\ \hline & 1 & -2 & 5 & -5 & 2 \end{array}$$

Since the leading coefficient of the divisor polynomial is "1", the leading coefficient of the quotient polynomial is the same as the dividend polynomial. Accordingly, the "1" from the top row is brought down as the leading coefficient for the quotient polynomial. [I note here, that as was the case with synthetic division by a linear divisor, division by a divisor of degree greater than one still requires that the divisor's leading coefficient be a "1".] At this point the process must be modified from that used in example #1. In order to mimic the process of long division here, the number "1" brought down below the line is multiplied first by the "$-2$" in the uppermost corner with the resultant product recorded in the next column in the same row as the "$-2$". Next, the same number "1" is multiplied by the "$-1$" in the lowermost corner with the resultant product recorded two columns over in the same row as the "$-1$". Now the next column is added yielding the sum of $-2$ and recorded below the line. The same process is now applied to this "$-2$" as was applied to the "1" previously. (The reader should compare the two processes throughout to verify the equivalency.)

Since the quotient polynomial must be 2nd degree, only the first three numbers below the line (an nth degree polynomial has n+1 terms) are subject to the process described in the previous paragraph. The remaining columns are simply added and their sums recorded below the line. The first three digits below the line are the coefficients, in descending order, of the quotient polynomial $(x^2-2x+5)$, and the last 2 digits represent the coefficients, in descending order, of the remainder polynomial $(-5x+2)$.

Consider the example below.

Example #3

Divide $x^7-x^4+2x^2-x+7$ by $x^3+2x^2-1$

Solution by Synthetic Division:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 1 | 0 | 0 | −1 | 0 | 2 | −1 | 7 |
| −2 | | −2 | 4 | −8 | 16 | −28 | | |
| 0 | | | 0 | 0 | 0 | 0 | 0 | |
| 1 | | | | 1 | −2 | 4 | −8 | 14 |
| | 1 | −2 | 4 | −8 | 14 | −22 | −9 | 21 |

Note that the quotient polynomial must be of 4th degree, therefore only the first five numbers below the line are subject to multiplication by the corner numbers "−2", "0", and "1". The quotient polynomial is $x^4-2x^3+4x^2-8x+14$ and the remainder polynomial is $-22x^2-9x+21$.

Here's an example where the suggested process could save time.

Example #4

Given that −1 is a root of multiplicity three of the polynomial $x^5+4x^4+7x^3+7x^2+4x+1$, find the remaining two roots.

*Solution:*

Since −1 is a root of multiplicity three, then x+1 is a factor of multiplicity three. Hence, divide the given polynomial by $(x+1)^3$ or $x^3+3x^2+3x+1$.

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 4 | 7 | 7 | 4 | 1 |
| −3 | | −3 | −3 | −3 | | |
| −3 | | | −3 | −3 | −3 | |
| −1 | | | | −1 | −1 | −1 |
| | 1 | 1 | 1 | 0 | 0 | 0 |

Note, as expected, the remainder polynomial is zero. Applying the quadratic formula to $x^2+x+1$ shows the remaining roots to be $\frac{-1\pm\sqrt{3}\,i}{2}$ or $\frac{-1\pm i\sqrt{3}}{2}$.

We know from the Remainder Theorem that when a polynomial of degree equal to or larger than one is divided by x-a, the constant remainder, R, equals P(a). Now consider a polynomial divisor p(x) of degree $n \geq 1$ dividing another polynomial P(x) of degree $m \geq n$. Naming the quotient polynomial Q(x) and the remainder polynomial R(x), we have

$$P(x) = p(x)\,Q(x) + R(x)$$

which is true for all x. Suppose $r_i$ is a root of p(x), i.e., $p(r_i) = 0$. Then the equation above becomes

$$P(r_i) = p(r_i)\,Q(r_i) + R(r_i)$$

or $$P(r_i) = 0 \cdot Q(r_i) + R(r_i)$$

or $$P(r_i) = R(r_i)\,.$$

Clearly then, the value of P(x) at $x = r_i$ can be found by evaluating $R(r_i)$ — a generalized Remainder Theorem.

Example #5

Given $P(x) = x^6+2x-1$, find P(2) and P(−1) using the generalized Remainder Theorem just considered.

Solution: Construct the polynomial whose roots are "2" and "−1". $p(x) = (x-2)(x+1) = x^2-x-2$. Now divide synthetically in order to find the remainder polynomial R(x).

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| | 1 | 0 | 0 | 0 | 0 | 2 | −1 |
| 1 | | 1 | 1 | 3 | 5 | 11 | |
| 2 | | | 2 | 2 | 6 | 10 | 22 |
| | 1 | 1 | 3 | 5 | 11 | 23 | 21 |

Hence $R(x) = 23x + 21$

$P(2) = R(2) = 23(2) + 21 = 67$

and $P(-1) = R(-1) = 23(-1) + 21 = -2$

Example #6

Given $P(x) = x^5-2x+1$, find P(x) at x=1, −1, and 2 using the generalized Remainder Theorem.

Solution: as in previous problem, construct $p(x) = (x-1)(x+1)(x-2)$ or $p(x) = x^3-2x^2-x+2$, then divide:

| | | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 0 | 0 | 0 | −2 | 1 |
| 2 | | 2 | 4 | 10 | | |
| 1 | | | 1 | 2 | 5 | |
| −2 | | | | −2 | −4 | −10 |
| | 1 | 2 | 5 | 10 | −1 | −9 |

Therefore $R(x) = 10x^2-x-9$. Accordingly

$P(1) = R(1) = 10(1)^2-(1)-9 = 0$

$P(-1) = R(-1) = 10(-1)^2-(-1)-9 = 2$

$P(2) = R(2) = 10(2)^2-(2)-9 = 29$

—BORROWED

# Statistics in the High School

by

Dennis G. Haack*

The high school mathematics curriculum is continually changing. One of the more recent changes has been the inclusion of a course in statistics, see Pieters (1976). As to the specific makeup of a high school statistics course, there is not likely to be agreement. As to the primary objective of such a course, there should be agreement. The purpose of this paper will be to look at the objective of a high school statistics course.

The key to the development of any course in statistics is deciding what statistics is. Statistics has, since the publication of R. A. Fisher's *Statistical Methods for the Research Worker* in 1925, been thought of as a set of research tools. In this regard statistics is the investigation of a population. The population of interest may exist or may be created by the researcher.

The study of an existing population is by a sample survey. A part of, or sample from a population is selected and studied. Examples of sample surveys include opinion polls, marketing research surveys, TV-viewing and radio-listening surveys, and preelection polls. A 100% sample is referred to as a census. Of interest in the study of sample survey techniques is how a survey is designed as well as how to analyze and interpret survey data.

On the other hand, a researcher may wish to study a population which he creates. For example, an agricultural researcher might test a fertilizer on a crop which he has planted on a test plot. The researcher is simulating the use of the fertilizer by farmers, that is, he tries to create a population which would exist if farmers used the fertilizer on their crops.

Another example of the investigation of a created population involves research on the effects of a drug. A population is created in the laboratory which would simulate use of the drug if the drug were put on the market.

As with the study of an existing population, the study of a created population involves a researcher with the design of his experiment as well as with the analysis and interpretation of experimental data. So we see that statistics is the study of a population which exists or is created. Statistics provides a set of tools which are required by an investigator for the design of a population study and the analysis and interpretation of the data generated by the study.

Traditionally, statistics courses at all levels have been an attempt to teach statistics as the study of a population. Distinction between experimental and sample survey investigations may or may not be made.

*The author is the section leader of the Biostatistics and Epidemiology Section of the Tobacco and Health Research Institute at the University of Kentucky. As a member of the National Council of Teachers of English (NCTE) Committee on Public Doublespeak, Dr. Haack writes and lectures on the misuse of the language of statistics.

But statistics has become more than a research tool. Listen to the news this evening, or read a newspaper or a newsmagazine. Listen to public officials and advertizers. Statistics has become a language in its own right. This language pervades the media making it nearly impossible to understand a newscast without being quite familiar with the language of statistics. What are these words we hear; "estimates", "significance", "projections", "averages", etc.? We are bombarded by numbers. But what do the numbers mean?

This is what statistics is to most Americans: a language which is very often used and too often misused. Statisticians have, for the most part, not taught about the language of statistics. Even students who have completed a traditional course in statistics cannot usually understand this language.

Statistics can be thought of as the tools required for the study of a population or statistics can be thought of as a language. We must decide which types of statistics we are to teach our students.

A first course in statistics should not try to teach statistics as a research tool. There are two main reasons for this. First, the study of statistics, a research tool, requires students to memorize the use of formulas, if not to memorize the formulas themselves. Students become so involved with learning to calculate statistics that they fail to learn what the statistics mean. Retention of the manipulative skills is minimal causing students to have little, if any knowledge of statistics after a course of this type is completed.

A second reason why a first course in statistics should not teach statistics as a research tool is that students, after taking a traditional statistics course, are no better able to understand the statistics they'll encounter in the media than they were before the course started. The better students might be able to run a t-test, but they are not likely to have a feeling for what is involved with a determination that, say, significantly more animals in a treatment group developed cancer than did animals in a control group. Some of the students might be able to calculate the probability of selecting a red ball from an urn, but they may not know how to interpret the statement, "The probability of rain is 20% today." That is, the most we can hope of a student is that he or she will become a manipulative "whiz." A student might become quite good at "plugging and chugging": plugging numbers into a formula and chugging until a number results. Yet our students are not likely to be able to interpret the statistics they might have learned to calculate.

Statistics should be taught as a language rather than as a research tool. Students should first be taught how to interpret statistics. A student will be much better off being able to understand statistics than to just be able to calculate them.

Statistics can be taught as a language. It is being done at the University of Kentucky, see Haack (1976). The idea behind the course is to downplay the calculation of statistics while concentrating on how to interpret statistics. In fact, students do not calculate any statistics in the course. There is, therefore, no need for mathematical formulas. The course is conducted in a strictly verbal, nonsymbolic manner. Examples used in the course come from the media. Ideally, students will be able to apply the principles they learn to statistics they will encounter, or have encountered in other areas.

One of the major drawbacks with a nonsymbolic statistics course has been the lack of a text, requiring a large amount of work of the teacher. Texts are now becoming available (see, for example, Haack (1979)).

One of the more interesting aspects of teaching statistics as a language is that students become genuinely excited about being able to detect misuses of statistics. When I started this experiment in teaching a few years ago, I did not look forward to trying to find examples of the misuse of statistics. Such examples are, of course, very instructive. As I began looking for cases of the misuse of statistics, I became awed by how easy examples were to find. I became more and more convinced that the course of this type was needed. Students also relish catching advertisers and public officials misusing statistics, that is, detecting doublespeak.

Doublespeak is the "involved, inflated, and often deliberately ambiguous use of language" (*Webster's New Collegiate Dictionary*), see Rank (1974) and Dieterich (1976). The misuse of the language of statistics is statistical doublespeak. Statistical doublespeak can be avoided if statistics are properly understood, see Haack (1977). This is the objective of the course I propose.

It is possible to teach statistics as a language. It is a challenging, yet rewarding undertaking. As you contemplate offering a course of this type, you might want to look at some of the books which can be used as reference material. There are a few good, readable books which may help you teach about statistics, the language.

With emphasis on sample surveys there are:

i. Gallup, G. (1972). *The Sophisticated Poll-Watchers Guide.* Princeton Opinion Press.
ii. Roll, C. W., Jr. and Cantril, A. H. (1972). *Polls: Their Use and Misuse in Politics.* Basic Books.
and iii. Wheeler, M. (1976). *Lies, Damn Lies, and Statistics.* Liveright.

These books lack adequate discussion of the science of studying an existing population but do give a good discussion of the "art" of sample surveying.

On the general topic of statistics and statistical doublespeak consider:

i. Bross, I. D. J. (1957). *Scientific Strategies in Human Affairs: To Tell the Truth.* Exposition Press.
ii. Campbell, S. (1974). *Flaws and Fallacies in Statistical Thinking.* Prentice Hall.
iii. Federer, W. T. (1973). *Statistics and Society.* Dekker.
iv. Hauser, P. M. (1975). *Social Statistics in Use.* Russell-Sage Foundation.
v. Huff, D. (1954). *How to Lie with Statistics.* Norton.
vi. Messick, B. M. (1968). *Mathematical Thinking in Behavioral Sciences.* Readings from *Scientific American.* Freeman.
vii. Mosteller, F. (editor) (1973). *Statistics By Example.* Addison-Wesley.
viii. Reichard, R. (1974). *The Figure Finaglers.* McGraw-Hill.
and ix. Taner, J. (editor) (1972). *Statistics: A Guide to the Unknown.* Holden-Day.

You will find these books to be very interesting. Taner's collection of essays is an excellent source for the statistics course I propose. The essays are on the application of statistics in just about any area that students might have an interest.

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