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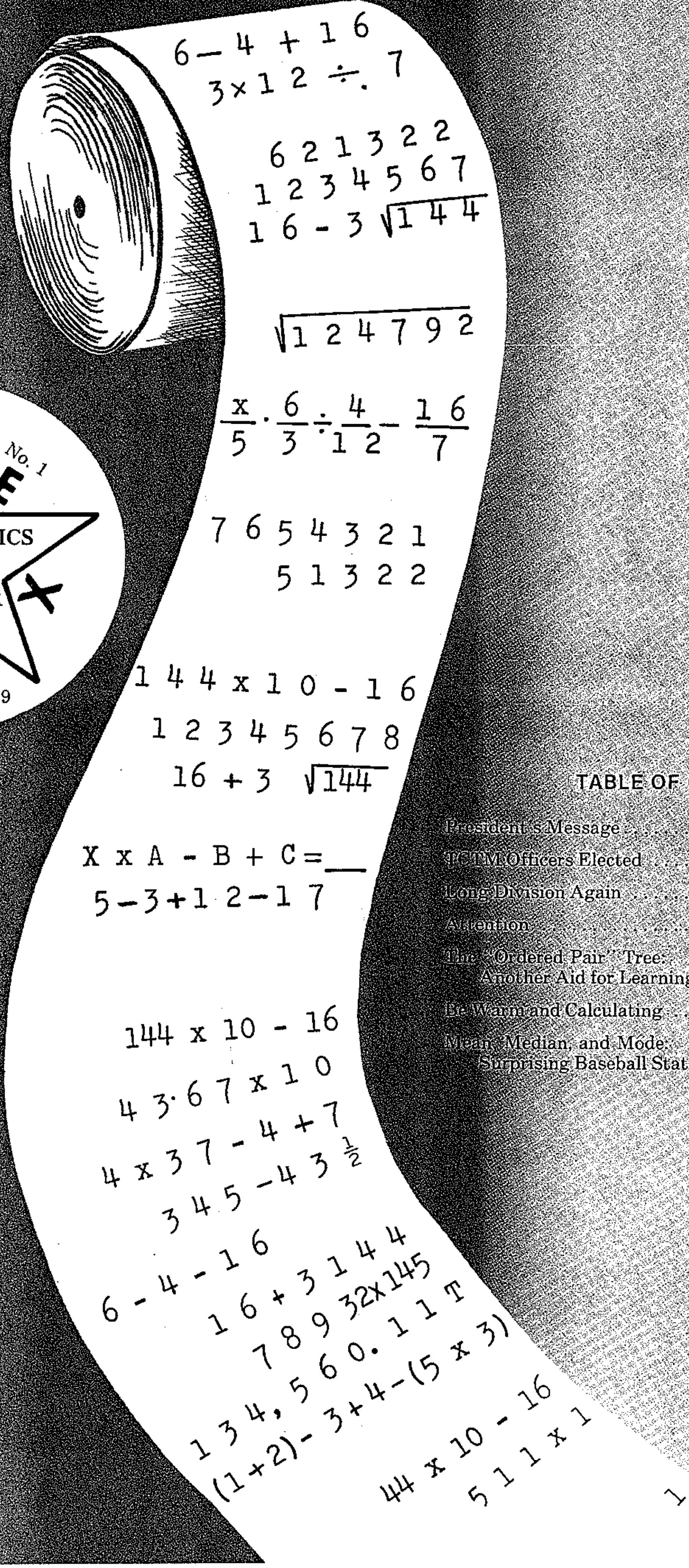

TABLE OF CONTENTS

TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

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PRESIDENT'S MESSAGE

Happy New Year!

As we begin another year, we naturally look back at what was accomplished and look forward to what remains unfinished.

During the term of office of the immediate past-president, Ms. Shirley Cousins, we adopted a new constitution. This constitution provides for four regional directors through which the Executive Board will have geographic representation. One of these directors has already been appointed. Mrs. Joseph Langston will represent the Northeast Region. We are looking for directors from each of the other regions. If you are interested in assuming this responsibility, please let me know as soon as possible.

TCTM programs and services are increasing in quantity, quality, and variety. Our goal is to make TCTM an organization which serves educators and their students at all grade levels. The following is a list of some programs and services which we will be struggling to continue or to bring to reality during the next two years:

1) Drive in Saturday Workshops in each region;
2) At least two issues of the Newsletter each year;
3) Four issues of the *Texas Mathematics Teacher*;
4) Increased enrollment;
5) Continued co-sponsorship of the CAMT meeting in Austin each fall;
6) Assist in organizing additional local councils of teachers of mathematics in Texas;

In order to accomplish these goals we enlist your support and enthusiasm. Are you a member? If not, join now. If yes, get a colleague to join. Let's make 1979 the best year ever.

Anita Priest

TCTM OFFICERS ELECTED

The recent election of officers for TCTM has added the following personnel to our Executive Board:

Vice-President (Elementary School)

CATHY RAHLFS — Coordinator of Elementary school Mathematics, Amarillo ISD. Before assuming this position Cathy was mathematics consultant for PESO Education Service Center, Region XVI and the Texas Education Agency. Her degrees are as follows: B.S., M.A., University of Texas at Austin. While attending UTA she was a recipient of a Prospective Teacher Fellowship.

Secretary

SUSAN SMITH — High School Mathematics Teacher, Ysleta ISD. Susan has also taught junior high school mathematics and participated in the Engineering Opportunities Program for Teachers at the University of Texas at El Paso. Her degrees are as follows: B.A., M.Ed., University of Texas at El Paso. Susan also participated in an NSF Summer Institute at the University of Texas at Austin.

Parliamentarian

EVELYN ROBSON — K-12 Mathematics Coordinator, Goose Creek ISD (Baytown). Formerly Evelyn taught junior high school and senior high school mathematics in GCISD and Alsine ISD. Her degrees are as follows: B.A., University of Texas at Austin; M.Ed., University of Houston. Evelyn also participated in two NSF Summer Institutes at UH.

LONG DIVISION AGAIN:

Teaching long division has been a thorn in the flesh to many teachers. Why should this operation cause so much more difficulty than the others that precede it? The first and most obvious answer is simply its position in the heirarchy of concepts and operations.

One doesn't attempt to begin instruction in division until multiplication has been taught. This could be because we agree with Piaget when he says children understand the ascending heirarchy before they understand the descending heirarchy. Or it could be that there are more prerequisite skills necessary for this particular operation than there are for the others. Since there seems to be no unique answer to the problem, it would be well to look at some instructional methods proposed by other educators.

In *The Report of The Cambridge Conference on the Correlation of Science and Mathematics in the Schools* published in 1969, some new ways of considering division and a new short division algorithm were proposed. Some examples follow:

> "The student learns first that division facts are derived from multiplication facts. Thus, 32 ÷ 4 is 8 because 8 × 4 is 32. Next (perhaps three weeks later), he practices division of multiples of powers of 10 by one-digit numbers (280 ÷ 4 = 70; 5400 ÷ 6 = 900), deriving his results from the corresponding multiplication facts. Finally, he is led to discover that division distributes over addition. This discovery can be promoted by a consideration of sets or area, or, with very perceptive children, by means of theoretical considerations. For example, since
>
> (30 ÷ 2) + (4 ÷ 2) × 2 =
> (30 ÷ 2) × 2+ (4 ÷ 2) × 2 =
> 30 + 4, it follows that (30 ÷ 2) + (4 ÷ 2) =
> (30 + 4) ÷ 2.
>
> Now the student is ready for the general problem of division by a one-digit number. The process consists of partitioning the dividend into addends whose quotients by the divisor can be found easily. Thus, to find the quotient of 52 by 4, we might proceed as follows:
>
> 52 ÷ 4 = (44 + 8) ÷ 4 = 11 + 2 = 13,
> or 52 ÷ 4 = (48 + 4) ÷ 4 = 12 + 1 = 13,
> or 52 ÷ 4 = (40 + 12) ÷ 4 = 10 + 3 = 13.
>
> The skill acquired in multiplying multiples of powers of 10 by one-digit numbers (mentioned in Section 3) will now pay off as the student searches for useful partitions of the dividend.
>
> 2972 ÷ 4 = (2800 + 160 + 12) ÷ 4 = 700 + 40 + 3 = 743
> 5128 ÷ 4 = (4000 + 1000 + 120 + 8) ÷ 4 = 1000 + 250 + 30 + 2 ÷ 1282
>
> This procedure should be practiced for a considerable time before the algorithm for short division is introduced. The algorithm can take either of the forms illustrated at the top of the page 105 (where the successive steps taken by the student are indicated by script numerals), and the special partitioning can be pointed out to the student.
>
> $$\frac{5128}{4} \rightarrow \frac{\overset{1_1}{5^1 1\ 2\ 8}}{4} \rightarrow \frac{\overset{1_1 2_3}{5^1 1^3 2\ 8}}{4} \rightarrow$$
>
> $$\frac{\overset{1_1 2_3 8}{5^1 1^3 2\ 8}}{4} \rightarrow \frac{\overset{1_1 2_3 8\ 2}{5^1 1^3 2\ 8}}{4}$$
>
> $$\overline{5128}\rfloor\underline{4} \rightarrow \overset{1}{\overline{5^1 1\ 2\ 8}}\rfloor\underline{4} \rightarrow \overset{1\ 2}{\overline{5^1 1^3 2\ 8}}\rfloor\underline{4} \rightarrow$$
>
> $$\overset{1\ 2\ 8}{\overline{5^1 1^3 2\ 8}}\rfloor\underline{4} \rightarrow \overset{1\ 2\ 8\ 2}{\overline{5^1 1^3 2\ 8}}\rfloor\underline{4}$$
>
> (The second form may be preferable since it results in easier-to-read bookkeeping.) It is probably best to avoid $\overline{}$ notation because this notation forces the student to reverse the order in which division problems are usually stated; for example, the conventional notation for "Divide 5128 by 4" requires the student to write the "4" to the left of the "5128."[1]

Some teachers have used the distributive property to teach division using the algorithm in this way:

$$4\overline{)2972} \quad = \quad 4\overline{)2000 + 900 + 70 + 2}$$

to solve the problem the pupil still thinks in powers of ten following through as in Figure 1.

Figure 1

$$\begin{array}{l}
4\overline{)2972} = \\
\quad 400 + 300 + 40 + 3 = \qquad 700 + 40 + 3 = 743 \\
4\overline{)2000 + 900 + 70 + 2} \\
\underline{-1600} \\
\quad 400 + 900 = 1300 \\
\qquad\qquad\quad \underline{1200} \\
\qquad\qquad\quad 100 + 70 = 170 \\
\qquad\qquad\qquad\qquad \underline{-160} \\
\qquad\qquad\qquad\qquad 10 + 2 = 12 \\
\qquad\qquad\qquad\qquad\qquad \underline{-12} \\
\qquad\qquad\qquad\qquad\qquad\quad 0
\end{array}$$

So far the only prerequisites mentioned have been knowledge of multiplication, knowledge of the distributive property, and powers of ten. However, it will be quite obvious to math teachers that these algorithms will necessitate approaching division in a less traditional manner.

It should be made clear that the proponents of these algorithms do not advocate teaching pupils any division algorithm until they reach seventh grade level. This would eliminate having pupils experience learning three or four different algorithms as they move from fourth grade through seventh grade.

As an alternative the committee suggested children be subjected to problem solving involving the division concept having them discover their own solutions or estimate to find a reasonable answer. However, they say,

> "it may not be good practice to have students rely completely on estimation methods in obtaining quotients. An occasional challenge to find the exact answer would be in order. In such cases, a student would put to use his understanding of division and whatever ingenuity he could muster. For example, consider the problem '457,452 ÷ 524 = ?' for which the exact answer is required. What is required is the exact answer to the problem '? × 524 = 457,452.' We start by estimating. Clearly, 1000 is too large because 1000 × 524 > 457,452. Perhaps 900 is a better estimate: 900 × 524 = 471,600. Too large. We try 800: 800 × 524 = 419,200. Too small. But now we know that
>
> $$800 < \frac{457,452}{524} < 900$$
>
> By how much is 800 too small? 457,452 − 419,200 = 38,252. So now we must find a number whose product by 524 is 38,252. By experimentation we find that
>
> $$70 < \frac{38,252}{524} < 80$$
>
> So we know that
>
> $$870 < \frac{457,452}{524} < 880.$$
>
> By how much is 870 too small? We continue in this way until we have obtained the answer. This systematic procedure is precisely what is used in the algorithm for long division, but it is superior to the long division algorithm in that the student is aware of what he is doing at each step. The long division algorithm effectively covers up the reasoning because the desideratum is speed.
>
> There is much to be said in favor of the student's going through this kind of 'research' every once in awhile. Not only does it allow him to exercise some good reasoning but it makes him aware of the important fact that there exists a systematic procedure for computing a quotient to any desired degree of accuracy."[2]

These suggestions for instructional change were made in 1969. To date not many of them have been accepted by the classroom teachers. The most negative reaction has probably come from the idea of changing the division sign from $4\overline{)288}$ to $\overline{288}\rceil 4$. This seems odd when you consider the mental gymnastics a child goes through in the usual course of study. First we write $7\overline{)49}$ then 49 ÷ 7. Next we expect them to move to the side division (subtractive algorithm), then by fourth or fifth grade the piling up method and finally our most efficient or traditional algorithm (sometimes called the distributive algorithm).

Since changing the algorithm has not eased the instructional problem, perhaps the way to generate improvement in instruction is to concentrate more on the prerequisites for the operation rather than on the algorithm itself. Then the algorithm can be presented by its relationship to each of the prerequisites.

The concept of division can be presented through problem solving at preschool and kindergarten level. The National Council of Teachers of Mathematics thirty-seventh yearbook entitled, *Mathematics Learning in Early Childhood,* has some excellent problem solving suggestions in the chapter on problem solving. The length of this paper will necessitate limiting our study to the concepts directly related to solving the algorithm.

Let us begin with the multiplication facts and the fact that division is the inverse of multiplication. Most texts today insist the teacher begin in the ascending order and teach multiplication facts first. Next the division relationship of the facts such as 6 × 8 = 48 and 48 ÷ 6 = 8 or 48 ÷ 8 = 6 is taught. The concept of the inverse should be continued to include such problems as:

$\begin{array}{r} 24 \\ 8\overline{)192} \end{array}$ inverse   8 × 24 = 192

$\begin{array}{r} 24 \\ 83\overline{)1992} \end{array}$ inverse   83 × 24 = 1992

$\begin{array}{r} 24 \\ 836\overline{)20064} \end{array}$ inverse   836 × 24 = 20064

If one is to solve problems involving more than a one-digit division and dividend then studying multiples of 10, 100, and 1000 is imperative. *Mathematics for the Elementary School* (SMSG) *Grade 5, Part 1* probably has the most comprehensive exercise for this purpose. They suggest having the child make multiplication matrices first for whole numbers, then for multiples of 10, followed by multiples of 100 and 1000. Next the child is expected to fill in the blanks in exercises similar to the following:

"1. Complete with the largest multiple of 10 which makes the sentence true.

a. ____ × 20 < 720   c. ____ × 70 < 3040

b. ____ × 10 ≤ 836   d. ____ × 60 < 5500

2. Complete with the largest multiple of 100 which makes the sentence true.

a. 40 × ____ < 8442   c. 50 × ____ < 36,012

b. 20 × ____ < 5591   d. ____ × 70 < 45,000

3. Find the largest multiple of 100 which makes the sentence true. If there is no multiple of 100, then find the largest multiple of 10.

a. 20 × ____ < 731   c. 40 × ____ < 2449

b. ____ × 46 < 4830   d. 60 × ____ < 45,000"[3]

This author has found that a sufficient amount of the above practice will increase the child's ability to estimate. Since the division algorithm requires making several estimates, each child should be proficient before attempting the next step.

Another means to increase skill in estimating is to teach a unit on rounding off numbers to the nearest ten, hundred, and thousand. The most visual instructional aid for this unit would be the number line. For example, children using it should only experience success in deciding 48 is nearer to 50 than it to either zero or 40.

Balancing an equation sounds out of place in a list of concepts necessary for the division process. This author found children possessing this concept much more competent when trying to solve problems with 2 or 3 digit divisions. Demonstrations with an invicta math balance are quite appropriate for this concept. Also examples that demonstrate how multiples such as $1\overline{)3}^{\,3}$, $10\overline{)30}^{\,3}$, $100\overline{)300}^{\,3}$ elicit the same answer in the same place value so long as both divisor and dividend are multiplied or divided by the same number (property of one).

Children should have practice working with place value in the divisor, dividend and quotient. They practice in telling whether the divisor is a ten, hundred, or thousand. They should learn to read the dividend in the following way $8\overline{)3429}$, 3429 contains 3,429 ones, 342 tens, 34 hundreds, 3 thousands. They also need to be able to give the place value of each digit in the dividend.

Last, the ability to subtract almost goes without saying.

Figure II

```
  93
+  3
  90 <--or--+
4|372   |   |
  360   |  90
   12   |
   12   |   3
    0   |  93
```

Thinking Process

```
4 × ____ < 372 (inverse)
4 × _90_ < 372 (estimation
               using multiple of
               ten)
4 × _90_ = 360 (exact
               multiplication
               using place value,
               place estimation
               and product in
               algorithm)
               (subtraction)
4 × ____ ≤ 12 (inverse)
4 × _3__ = 12 (estimation)
4 × _3__ = 12 (multiplication
              using place value,
              again place in
              algorithm)
              (subtraction)
```

Figure II shows that either the side division or the piling-up algorithm may be used. The right side of the figure shows each concept as it is used in the problem.

First the child restates the division problem in the inverse and it now has the exact characteristics he has just studied in his multiplication unit. Next he should relate to his exercises with multiples of 10, 100, and 1000 to the problem. Teachers should stress that the answers are estimates and sometimes the estimates are not suitable. A discussion on rounding up and rounding down could follow. The exact multiplication should be checked with the dividend to decide if the estimation was correct.

Some correct language to use would be "How many sets of 4 are contained in 372?" or "There are 90 sets of 4 in 372."

When the child can correctly think through this process, by following through with the concepts of rounding off, the property of one, and again estimation he can make the transition to a two-digit divisor quite easily. Here again these concepts should be reviewed as they were previously taught and related directly to the problem, as shown in Figure III.

Figure III

```
      76 r 2
    +  6
      70
  49|3726
     3420
      296
      294
        2
```

Thinking Process

```
49 × ____ < 3726 (inverse)
50 × ____ < 3720 (rounding
                 off) (balancing an
                 equation)
 5 × ____ < 372 (property of 1
                or ÷ by 10)
 5 × _70_ < 372 (estimation)
49 × _70_ = 3430 (exact
                 multiplication)
                 (using place value,
                 place estimation
                 and multiplication
                 in algorithm)
                 (subtraction)
49 × ____ < 296 (inverse)
50 × ____ < 296 (rounding off)
 5 × ____ ≤ 30 (balancing
               equation)
               (property of 1 or
               ÷ by 10)
 5 × _6__ ≤ 30 (estimation)
49 × _6__ = 294 (exact
                multiplication)
                (place estimation
                and multiplication
                in algorithm)
                (subtraction)
```

Now the only difference that exists between a two-digit and three-digit divisor problem is the rounding off process. With a two-digit, simply round off both factor and product to the nearest 10. With a three-digit divisor, round off both factor and product to the nearest 100 as shown in Figure IV.

Figure IV

```
       77 r 307
473 | 37268
      34510
       3758
       3451
        307
```

Thinking Process

493 × ___ < 37268 (inverse)
500 × ___ < 37300 (rounding off)
5 × ___ < 373 (property of 1 ÷ by 100)
5 × 70 < 373 (estimation)
493 × 70 = 34510 (exact multiplication) (place estimation and product in correct place value in algorithm) (subtract)
493 × ___ < 3758 (inverse)
500 × ___ < 3800 (rounding off)
5 × ___ < 38 (property of 1 ÷ by 100) (estimation)
493 × 7 = 3451 (exact multiplication) (place estimation and product in correct place value in algorithm) (subtract)

When teaching the transition from the side algorithm to the most efficient or standard algorithm, one can place the estimate in both positions and derive the quotient both ways to show the relationship (as in Figure II). Children respond readily with less frustration when teachers take time to "walk" them through the change.

This author used the methods discussed above and in Figure II, III, and IV to instruct underachievers. By studying the prerequisites and showing the necessary relationships, the children demonstrated the ability to solve a long division problem accurately and with confidence. Teaching long division was no longer the dreaded task of former years.

FOOTNOTES

1. *Goals for the Correlation of Elementary Science and Mathematics,* Houghton Mifflin Co., Boston, MA, 1969, p. 104-105.
2. *Ibid,* p. 105-106.
3. *School Mathematics Study Group* Fifth Grade, Part I, Yale University Press, New Haven, CT, 1963, p. 136.

— Verena Sharkey
University of Delaware
Newark, Delaware

!!ATTENTION!!

HELP IS NEEDED NOW!

To assure more and better communication between the Texas Council of Teachers of Mathematics and the local Councils of Teachers of Mathematics, as well as to have representation from all regions of the state on the TCTM Executive Board, Regional Directors are needed to serve these REGIONS: NORTHWEST, SOUTHEAST, and SOUTHWEST.

The four regions correspond to TSTA Districts and include the counties as indicated in this article.

Please check for your region, and if you can assume one of these positions, do send your name, address, phone number, and region name to

ANITA PRIEST
6647 St. Regis
Dallas, Texas 75217

A listing of the counties in each district, and the districts in each REGION is given below.

NORTHEAST REGION

VI—Austin, Brazos, Burleson, Colorado, Grimes, Houston, Leon, Liberty, Madison, Milam, Montgomery, Polk, Robertson, San Jacinto, Trinity, Walker, Waller, and Washington

VII—Anderson, Angelina, Cherokee, Henderson, Nacogdoches, Panola, Rusk, Sabine, San Augustine, Shelby, Smith, and Van Zandt

VIII—Bowie, Camp, Cass, Delta, Franklin, Gregg, Harrison, Hopkins, Lamar, Marion, Morris, Rains, Red River, Titus, Upshur, and Wood

X—Collin, Dallas, Ellis, Fannin, Grayson, Hunt, Kaufman, and Rockwall

XII—Bell, Bosque, Coryell, Falls, Freestone, Hamilton, Hill, Lampasas, Limestone, McLennan, Mills, and Navarro

NORTHWEST REGION

IX—Archer, Baylor, Clay, Foard, Hardeman, Jack, Knox, Montague, Throckmorton, Wichita, Wilbarger, and Young

XI—Cooke, Denton, Hood, Johnson, Palo Pinto, Parker, Somervell, Tarrant, and Wise

XIV—Callahan, Eastland, Fisher, Haskell, Jones, Nolan, Scurry, Shackelford, Stephens, Stonewall, and Taylor

XVI—Armstrong, Briscoe, Carson, Castro, Childress, Collingsworth, Dallam, Deaf Smith, Donley, Gray, Hall, Hansford, Hartley, Hemphill, Hutchinson, Lipscomb, Moore, Ochiltree, Oldham, Parmer, Potter, Randall, Roberts, Sherman, Swisher, and Wheeler.

XVII—Bailey, Cochran, Crosby, Cottle, Dickens, Floyd, Garza, Hale, Hockley, Kent, King, Lamb, Lubbock, Lynn, Motley, Terry, and Yoakum

SOUTHEAST REGION

I—Cameron, Hidalgo, Jim Hogg, Starr, Webb, Willacy, and Zapata

II—Aransas, Brooks, Duval, Jim Wells, Kenedy, Kleberg, Live Oak, Nueces, and San Patricio

III—Bee, Calhoun, Dewitt, Goliad, Jackson, Karnes, Lavaca, Matagordo, Refugio, Victoria, and Wharton

IV—Brazoria, Fort Bend, Galveston, and Harris

V—Chambers, Hardin, Jasper, Jefferson, Newton, Orange, and Tyler

SOUTHWEST REGION

XIII—Bastrop, Blanco, Burnett, Caldwell, Comal, Fayette, Gillespie, Gonzales, Guadalupe, Hays, Llano, Lee, Travis, and Williamson

XV—Brown, Coke, Coleman, Commanche, Concho, Crockett, Erath, Irion, Kimble, Mason, McCullough, Menard, Runnels, San Saba, Schleicher, Sterling, Sutton, and Tom Green

XVIII—Andrews, Borden, Crane, Dawson, Ector, Gaines, Glasscock, Howard, Loving, Martin, Midland, Mitchell, Pecos, Reagan, Terrell, Upton, Ward, and Winkler

XIX—Brewster, Culbertson, El Paso, Hudspeth, Jeff Davis, Presidio, and Reeves

XX—Atascosa, Bandera, Bexar, Dimmit, Edwards, Frio, Kendall, Kerr, Kinney, La Salle, Maverick, McMullen, Medina, Real, Uvalde, Val Verde, Wilson, and Zavala

Each local council that is affiliated with NCTM should have a copy of the TCTM Constitution. If further information is desired, contact the president of a local council.

NORTH-EAST Region	East Texas CTM	MARCIA LYNCH Apartment M-3 2108 Tryon Road Longview, Texas 75601
	Greater Dallas CTM	William H. Onley 11356 Drummond Drive Dallas, Texas 75228
NORTH-EAST Region	North Texas CTM	Reed Jackson 2110 Shawnee Denton, Texas 76201
	Texoma CTM	D. D. McKnight 608 West Walker Denison, Texas 75020
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	San Angelo CTM	James Hodge 2802 Oak Forrest Drive San Angelo, Texas 76901

Math supervisors and consultants may contact

Texas Association of Supervisors of Mathematics
John Veltman, President
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10333 Broadway
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The "Ordered Pair" Tree: Another Aid for Learning Fractions

by Wilson Davis
Delta State University
Cleveland, Mississippi

How do you teach children to determine whether one fraction is greater than, equal to, or less than another fraction? One way, the usual way, is by use of the common denominator method. But there are other ways which may be less formal, and sometimes less certain, but are, nevertheless, good ways to aid in the determination of size order for fractions. If such methods could be used, subtly, to teach other facets of mathematics while teaching about fractions, then, perhaps, the time of the teacher and the pupil would be well spent.

Let's examine a technique for finding other fractions that are equivalent to some particular fraction that has been selected, say 1/2. On a sheet of ordinary rectangular graph paper choose a point near the lower left hand corner of the sheet (refer to Fig. 1) and give it any suitable name such as center, origin, starting point, or whatever seems suitable for your class. Number the horizontal and vertical axes beginning with this point as zero. You may wish to name these axes x and y, respectively, or perhaps P and Q, respectively. Now choose any ordered pair (P,Q) that you like. For example, you may choose (2,1). With a straightedge draw a line through the origin and (2,1). Notice that the line you

Figure 1

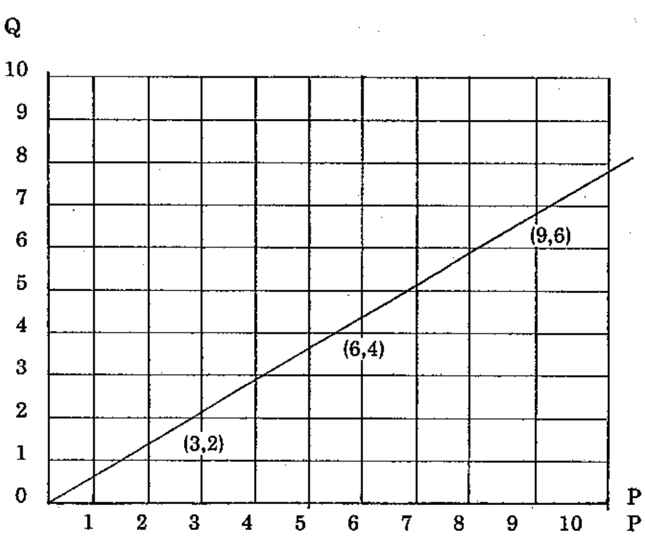

Fig. 1: The "Ordered Pair" Tree showing the equivalence of 2/3, 4/6, and 6/9.

drew also contains the ordered pairs (4,2), (6,3) and (8,4). Any ordered pair on the line when written in the form $\frac{Q}{P}$ will form a fraction equivalent to 1/2. Likewise, select (4,3) and draw a line through this point and the origin. Every point on this line has an ordered pair that, when written in the form $\frac{Q}{P}$, is a fraction equivalent to 3/4.

Some special lines should be noted. The ordered pair (1,1) determines, with the origin, a line representing numbers equivalent to one. The P-axis represents zero, and as the Q-axis is approached moving counter clockwise, the numbers become infinitely large. Thus, each line in the "lower" half of the quadrant represents a number less than one and each line in the "upper" half of the quadrant represents a number greater than one. The arrangement $\frac{Q}{P}$ (rather than $\frac{P}{Q}$) is consistent with the procedure for determining the slope of a line and, in fact, each number on a particular line is precisely the slope of that line. (or the tangent of the angle formed by the line and the P-axis.)

In order to determine whether one fraction is greater than, equivalent to, or less than another the usual procedure is one of finding a common denominator and comparing size of the numerators. Using the "ordered pair" tree to compare fractions we need only to draw the lines representing the fractions and note which line is farthest counterclockwise to decide which is the largest fraction. If the lines are coincident then the fractions are equivalent.

If you have begun to study signed numbers then this teaching device may be extended to other quadrants. For example, lines drawn in the second quadrant represent equivalent negative fractions and rules may be developed to determine size order for the negatives of fractions by the counter clockwise rule. A demonstration model of the "ordered pair" tree for teacher use may be made from pegboard using pegs and string. A piece of pegboard 2'x4' painted with flat green paint and numbered suitably for first and second quadrant ordered pairs will stand in the classroom chalk trough. Wooden pegs and white string work effectively for demonstration.

The "ordered pair" tree gives an opportunity to reinforce number ideas of equivalence of fractions, concepts of "greater than" and "less than" zero, infinite and negative numbers. As an aid in teaching geometry, pupils are introduced to the Cartesian plane, axes, ordered pairs, and plotting points.

Reference:

Howell, Daisy; Davis, Wilson; and Underhill, Leila. *Activities for Teaching Mathematics to Slow Achievers.* Jackson, Miss.; University Press of Mississippi, 1974.

Be ~~Cold~~ Warm and Calculating

Charles E. Lamb

The University of Texas at Austin

There has been much written about the use of hand-held calculators. Comments have ranged from the suggestion that all mathematics be taught using the calculator as a central base, to the other extreme, the suggestion that the calculator not be used as an educational tool at all. Somewhere, in the middle, there is a balance to be reached. At the present time, there has been little research on the long-time effects of calculator use (Troutman and Lichtenberg, 1977). As more and more evidence on the calculator is accumulated, there should be a closing of the gap between the opposing positions. A balance between restructuring the curriculum and using the calculator very little seems reasonable.

While waiting on further evidence, it would be unfortunate if teachers choose not to experiment some on their own. A conservative, but useful approach is suggested by Pollak (1977). He suggests that the calculator be used to attack already existing pedagogical difficulties in the mathematics classroom. This is something every elementary teacher could do without doing strange things to his/her classroom program.

One of the most common problems for the elementary teacher is that of poor student attitude towards mathematics. It might be the case that elementary school children would find the calculator a nice way to make the mathematics classroom more enjoyable. This author has seen positive attitude change on the part of children from the age of five up through the fifth grade. The activities used were strictly of a fun nature. The idea was one of play. The purpose of the work with calculators was to discover how it worked and to look at some "tricks". Children later told teachers that they no longer disliked mathematics class because of the possibilities for fun while studying mathematics.

Elementary teachers would be wise to investigate activities that would be enjoyed by their students while using the calculators. In many cases, this experience may be a way to transform the mathematics situation from one that is cold and calculating to a situation where the interactions are warm and calculating.

References

Pollak, H. O. et al. What may in the future computers and calculators mean in mathematics education? In H. Athen and H. Kunle (eds.), *Procedings of the Third International Congress on Mathematical Education.* Karlsruhe, Germany: Organising Committee of the 3rd ICME, 1977.

Troutman, A. P. and Lichtenberg, B. K. *Mathematics: A good beginning — Strategies for teaching children.* Monterey, CA: Brooks/Cole Pub. Co., 1977.

Supplemental Readings

Article concerning hand-held calculators in *The Arithmetic Teacher* and *The Mathematics Teacher* from The National Council of Teachers of Mathematics.

Mean, Median, and Mode: Surprising Baseball Statistics

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Teachers of statistics often point out to their students that the three measures of central tendancy (mean, median, and mode) describe data in markedly different ways. To illustrate this point, we considered baseball data recorded in *The Baseball Encyclopedia* (3rd edition, Macmillan Publishers, New York, 1976).

The number of hits made by players who retired before 1975 were studied. Players who were only pitchers were not included. Since they do not have an opportunity to play every day, they do not bat as frequently as other players. Table I reports a summary of "hitting data".

Table I

Number of Hits	Number of Players	Cumulative Number of Players
0	614	614
1	334	948
2	222	1170
3	165	1335
4	127	1462
5	97	1559
6	83	1642
7	91	1733
8	69	1802
9	71	1873
10	61	1934
11	62	1996
12	55	2051
13	39	2090
14	40	2130
15	36	2166
16	44	2210
17	49	2259
18	46	2305
19	34	2339
20	39	2378
21	36	2414
22	24	2438
23	38	2476
24	15	2491
25	32	2523
26	27	2550
27	31	2581
28	20	2601
29	27	2628
30	17	2645
31	22	2667
32	15	2682
33	15	2697
34	16	2713
35	20	2733
36-40	88	2821
41-45	111	2932
46-50	82	3014
51-55	89	3103
56-60	68	3171
61-65	55	3226
66-70	67	3293
71-75	51	3344
76-80	55	3399
81-85	33	3432
86-90	58	3490
91-95	44	3534
96-100	38	3572
101-105	26	3598
106-110	39	3637
111-115	34	3671
116-120	40	3711
121-125	30	3741
126-130	34	3775
131-135	40	3815
136-140	28	3843
141-145	43	3886
146-150	36	3922
151-155	28	3950
156-160	32	3982
161-165	19	4001
166-170	28	4029
171-175	26	2055
176-180	21	4076
181-185	28	4104
186-190	20	4124
191-195	24	4148
196-200	18	4166
201-205	21	4187

Table I (cont.)

Number of Hits	Number of Players	Cumulative Number of Players
206-210	11	4198
211-215	22	4220
216-220	19	4239
221-230	41	4280
231-240	29	4309
241-250	36	4345
251-260	39	4384
261-270	51	4435
271-280	29	4464
281-290	40	4504
291-300	32	4536
301-310	35	4571
311-320	39	4610
321-330	33	4643
331-340	26	4669
341-350	19	4688
351-360	32	4720
361-370	22	4742
371-380	12	4754
381-390	29	4783
391-400	18	4801
401-410	24	4825
411-420	19	4844
421-430	19	4863
431-440	23	4886
441-450	18	4904
451-460	19	4923
461-470	23	4946
471-480	24	4970
481-490	17	4987
491-500	24	5011
501-510	16	5027
511-520	22	5049
521-530	13	5062
531-540	18	5080
541-550	18	5098
551-560	13	5111
561-570	11	5122
571-580	9	5131
581-590	8	5139
591-600	17	5156
601-620	25	5181
621-640	30	5211
641-660	16	5227
661-680	25	5252
681-700	26	5278
701-720	23	5301
721-740	20	5321
741-760	25	5346
761-780	28	5374
781-800	24	5398
801-820	28	5426
821-840	20	5446
841-860	19	5465
861-880	17	5482
881-900	19	5501
901-920	22	5523
921-940	20	5543
941-960	14	5557
961-980	24	5581
981-1000	16	5597
1001-1020	14	5611
1021-1040	13	5624
1041-1060	11	5635
1061-1080	17	5652
1081-1100	12	5664
1101-1120	22	5686
1121-1140	10	5696
1141-1160	14	5710
1161-1180	11	5721
1181-1200	9	5730
1201-1250	43	5773
1251-1300	36	5809

Table I (cont.)

Number of Hits	Number of Players	Cumulative Number of Players
1301-1350	36	5845
1351-1400	26	5871
1401-1450	17	5888
1451-1500	31	5919
1501-1550	25	5944
1551-1600	23	5967
1601-1650	16	5983
1651-1700	18	6001
1701-1750	17	6018
1751-1800	17	6035
1801-1850	17	6052
1851-1900	12	6064
1901-1950	12	6076
1951-2000	10	6086
2001-2050	7	6093
2051-2100	15	6108

Table I (cont.)

Number of Hits	Number of Players	Cumulative Number of Players
2101-2150	13	6121
2151-2200	7	6128
2201-2250	5	6133
2251-2300	8	6141
2301-2350	9	6150
2351-2400	3	6153
2401-2450	4	6157
2451-2500	6	6163
2501-2600	6	6169
2601-2700	9	6178
2701-2800	5	6183
2801-2900	7	6190
2901-3000	7	6197
3001-3500	7	6204
3501-4000	2	6206
4001-5000	1	6207

It is very surprising that approximately 10% of these 6,204 players never attained one hit! Of course, most of them had very brief major league careers, probably for obvious reasons. Zero is the most frequent number of hits and is thus called the mode. Consequently, since the mean, median, and mode are each frequently called "the average," zero then could be called "the average" number of hits per player. Is this a good use of the word "average"? The median occurs in the 56-60 range, while the mean is 298.0 hits per player.

To better visualize these data, suppose that each player's height were proportional to the number of hits he achieved. Let the mean (298.0) represent a 6 foot player. Suppose that all the 6,207 players were able to walk by you in single file in one hour (noon to 1 p.m.), all players walking at the same constant speed. Table II displays pertinent data of specific players as they walk by you.

Table II

Time	Ordinal Number of Player	Number of Hits of Player	Height of Player Player	Name of a Representative Player*
12:00	—	—	—	—
12:05	517	0	0	Joe Campbell
12:10	1034	2	½ in.	Walt Linden
12:15	1552	5	1¼ in.	George Mangus
12:20	2069	13	3¼ in.	Bob Lennon
12:25	2586	28	6¾ in.	George Hunter
12:30	3104	(56-60)	1 ft. 2 in.	Bill Hart
12:35	3621	(106-110)	2 ft. 2 in.	Roy Elsh
12:40	4138	(191-195)	3 ft. 11 in.	Josh Clarke
12:45	4655	(331-340)	6 ft. 9 in.	Hap Myers
12:50	5172	(601-620)	12 ft. 3 in.	Billy Myers
12:55	5690	(1121-1140)	22 ft. 9 in.	Mike Mitchell
12:56	5793	(1251-1300)	25 ft. 8 in.	Eddie Miller
12:57	5897	(1451-1500)	29 ft. 8 in.	Frank Malzone
12:58	6000	(1561-1700)	33 ft.9 in.	Lu Blue
12:59:00	6104	(2051-2100)	42 ft.	Chuck Klein
12:59:15	6129	(2201-2250)	45 ft.	Joe Di Maggio
12:59:30	6155	(2401-2450)	49 ft.	Mickey Mantle
12:59:45	6181	(2701-2800)	55 ft.	Lou Gehrig
12:59:60	6207	4191	84 ft.	Ty Cobb

*Biographical Data of the Named Players:

Name	Playing Years	League or Club	Position	Batting Average
Joe Campbell	1967	Cubs	Outfield	0 for 3 (.000)
Walt Linden	1950	Braves	Catcher	2 for 5 (.400)
George Mangus	1912	Phillies	Outfield	5 for 25 (.200)
Bob Lennon	1954, 1956-57	New York Giants Cubs	Outfield	13 for 79 (.165)
George Hunter	1909-1910	Brooklyn Dodgers	Outfield	28 for 123 (.228)
Bill Hart	1943-1945	Brooklyn Dodgers	3rd base Shortstop	56 for 270 (.207)

Roy Elsh	1923-1925	White Sox	Outfield	106 for 404 (.262)
Josh Clarke	1898, 1905, 1908-9, 1911	Teams in both leagues	Outfield	193 for 809 (.239)
Hap Myers	1910-1915	Teams in both leagues	1st base	335 for 1251 (.268)
Billy Myers	1935-1941	Reds & Cubs	Shortstop	616 for 2399 (.257)
Mike Mitchell	1907-1914	Teams in both leagues	Outfield	1137 for 4094 (.278)
Eddie Miller	1936-1950	National league	Shortstop	1270 for 5337 (.238)
Frank Malzone	1955-1966	Red Sox & Angels	3rd base	1486 for 5428 (.274)
Lu Blue	1921-1933	American league	1st base	1969 for 5904 (.287)
Chuck Klein	1928-1944	National league	Outfield	2076 for 6486 (.320)
Joe Di Maggio	1936-1951	Yankees	Outfield	2214 for 6821 (.325)
Mickey Mantle	1951-1968	Yankees	Outfield	2415 for 8102 (.298)
Lou Gehrig	1923-1939	Yankees	1st base	2721 for 8001 (.340)
Ty Cobb	1905-1928	Tigers	Outfield	4191 for 11,429 (.367)

Observe (Table II):

1. Until 12:06:16 the players walking by are not visible; they have zero height.
2. At 12:30 (halfway through the hour), the height of the player is only 1 foot 2 inches.
3. The first player who is 6 feet in height appears between 12:40 and 12:45. Question: At what exact time does he appear?
4. The "2,000-hit-players", the elite in baseball, do not appear until slightly before 12:59.
5. The last player is Ty Cobb whose height is 84 feet. The Lincoln Tomb superstructure is *80 feet high,* 188 feet long, and 118 feet wide.

Questions for discussion:

1. Find the heights of other "2,000-hit-players." (Their names can be found in most almanacs.)
2. Rod Carew won the 1977 American League batting championship with 239 hits. How many feet would he have grown during the 1977 baseball season?

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