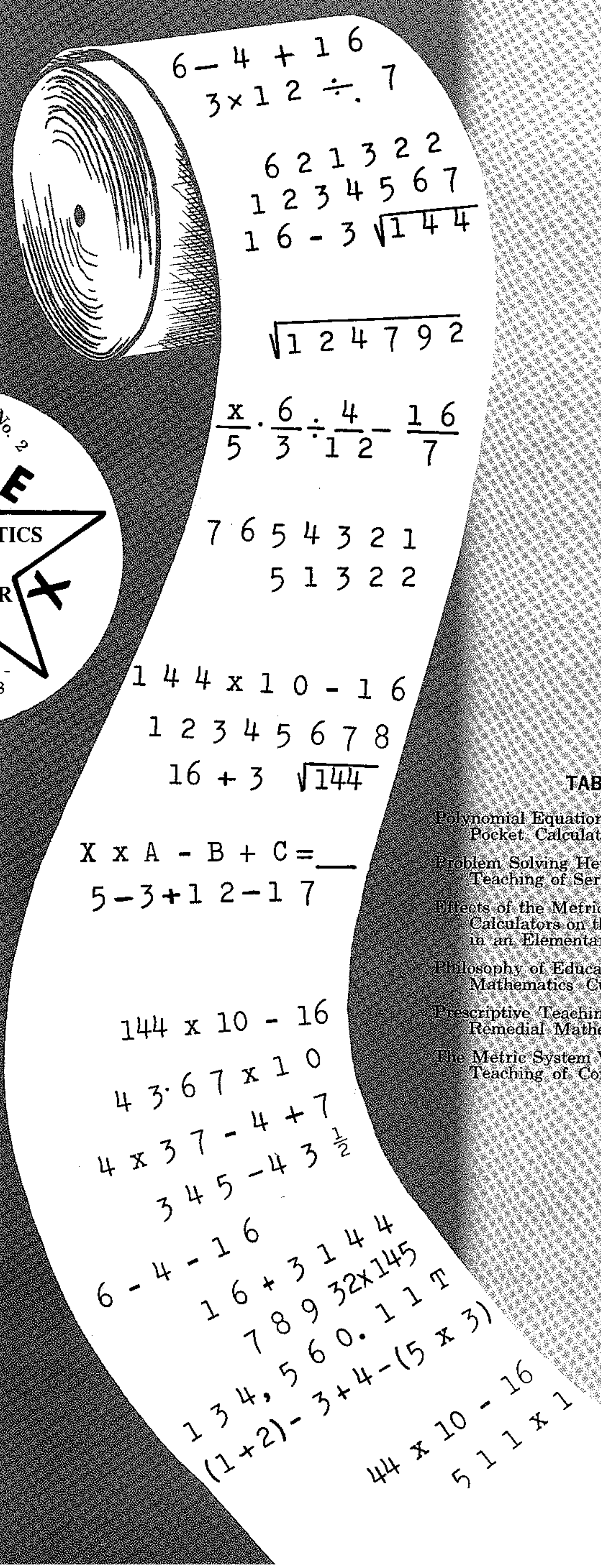
Vol. XXV No. 2
TEXAS
MATHEMATICS
TEACHER
March 1978



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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher,* 100 So. Glasgow Drive, Dallas, Texas 75214.

TEXAS MATHEMATICS TEACHER is published quarterly by the Texas Council of Teachers of Mathematics. Payment of membership fee of $5.00 entitles members to all regular Council Publications.

President's Message

(Substitute Contributor)

Importantly and briefly, let us say . . .

All members should concentrate on the increase in membership in Texas Council of Teachers of Mathematics by encouraging every mathematics teacher in his/her building to send in membership dues now, for next year, to the treasurer, whose address is given on the back of this journal. If membership forms are needed, xerox the form printed herein and have fellow teacher complete to mail. *We need to promote an increase in membership now!! You can help us!!*

Place on your calendar the following date:
November 2-4, 1978 — CAMT, Austin

Polynomial Equations and the Pocket Calculator

Sally I. Lipsey

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The purpose of this paper is to describe two classroom uses of the pocket calculator which I found successful in teaching about polynomial functions. The first is an approach to synthetic division via synthetic substitution, leading to ideas in the theory of equations. The second shows how easy it has become to use a formula for solving cubic equations; in addition, this formula provides an opportunity to demonstrate the usefulness of complex numbers.

We generally solve polynomial equations of degree greater than 2 by a procedure involving repeated substitutions of intelligent guesses. Obviously, the pocket calculator makes the computations more palatable. Students were able to carry out the iterative procedure far enough to find irrational zeros of polynomials to a required decimal place without losing patience before attaining an understanding of the process.

There are other benefits from the use of the calculator. Firstly, the effort of searching for the most efficient method of carrying out the substitutions led us to a useful but unconventional kind of factoring. For instance, one of our problems was an example of substitution with averaging. We were dealing with the equation

$$P_4(x) = x^4 + x^3 - 3x^2 + 3x - 1 = 0.$$

We noted that $f(0) < 0$ and $f(1) > 0$ and concluded, by the Intermediate Value Theorem, that there was a real root in the interval (0,1). Our goal was to locate this root, x, correct to 2 decimal places. We let x_j represent the jth approximation, with $x_1 = 0.5$.

It was a nuisance to compute:

$$(0.5)^4 + (0.5)^3 - 3(0.5)^2 + 3(0.5) - 1$$

on the simple pocket calculator that the students carried, because each term had to be computed separately and then the results combined. We learned to make repeated use of the distributive law as follows:

$$P_4(x = (x^3 + x^2 - 3x + 3)\,x - 1$$
$$= [(x^2 + x - 3)\,x + 3]\,x - 1$$
$$= [((x + 1)x - 3)\,x + 3]\,x - 1$$

This last line, the nested form of $P_4(x)$, gave us an efficient method (synthetic substitution) for finding $P_4(0.5)$ or $P_4(c)$ for any value of c. With synthetic substitution, the keys we touched in succession to find $P_4(0.5)$ were:

0.5, +, 1, x, 0.5, −, 3, x, 0.5, +, 3, 0.5, −, 1, =

Using synthetic substitution, we closed in on the root quickly, substituting the midpoint of each interval in which we found the root must lie: $x_2 = 0.75$, $x_3 = 0.625$, . . ., $x_8 = 0.554$ and $x = 0.55$. Synthetic substitution was, of course, also helpful in finding rational roots.

In finding P (c), no recording of intermediate results was necessary. It was interesting to see, however, that when we did record the intermediate results, further helpful information became available. As an example, let

$$P_3(x) = 2x^3 - 17x^2 + 38x - 15.$$

Then $P_3(c) = [(2c - 7)c + 38]c - 15$, in nested form. The intermediate results of the computation were 2c, 2c—17, (2c — 17)c, etc. We wrote the coefficients in order and recorded the results of the operations indicated. The following table emerged:

C	2	−17	38
		2c	(2c−17)c
	2	(2c−17)	(2c − 17)c + 38
			−15
			((2c−17)c + 38)c
			−[(2c − 17)c + 38]c − 15 = P_3 (c)

By long division of $2x^3 - 17x^2 + 38x - 15$ by $x - c$, we "discovered" that $P_3(c)$ was also the remainder of the division process and that the coefficients of the quotient had been found automatically.

$$\begin{array}{l|llll} & 2x^2 & +\ (2c-17)x & +\ [(2c-17)c+38] & \\ \hline x-c & 2x^3 & -17x^2 & +38x & -15 \\ & 2x^3 & -2cx^2 & & \\ \hline & & (2c-17)x^2 & +\ 38x & \\ & & (2c-17)x^2 & -\ (2c-17)cx & \\ \hline & & & [(2c-17)c+38]x & -15 \\ & & & [(2c-17)c+38]x & -[(2c-17)c+38]c \\ \hline & & & & [(2c-17)c+38]c-15 \end{array}$$

The connection between substitution and division was further illuminated by writing $P_3(x) - P_3(c)$ as a product of $(x-c)$ and $Q(x)$. Thus $P_3(x) = 2x^3 - 17x^2 + 38x - 15$ and $P_3(c) = 2c^3 - 17c^2 + 38c - 15$. Hence

$$\begin{aligned} &P_3(x) - P_3(c) \\ &= 2(x^3 - c^3) - 17(x^2 - c^2) + 38(x - c) \\ &= (x - c)[2(x^2 + xc + c^2) - 17(x + c) + 38] \\ &= (x - c)[2x^2 + (2c - 17)x + ((2c - 17)c + 38)] \\ &= (x - c)\ Q(x). \end{aligned}$$

Now $P_3(x) = P_3(c) + (x-c)\,Q(x)$. Finally this showed that the quotient and the remainder resulting from the division of $P_3(x)$ by $(x-c)$ may be found by the synthetic substitution (hence also called synthetic division) process. Formal proofs of the remainder theorem and the factor theorem were then interesting and reasonable.

Iterative procedures are, of course, unnecessary in the case of quadratic equations since we have a convenient formula for finding the roots directly. Now, with the help of the pocket calculator, the Tartaglia-Cardan formula for the solution of cubic equations is no longer inconvenient to use, at least for those students who have learned the elementary operations with complex numbers. In a more advanced class, it was possible to show, without resort to differential equations or physical applications for which the students were unprepared, that *imaginary* numbers are useful in solving problems whose real solutions involve *real* numbers only.

Let $a_3t^3 + a_2t^2 + a_1t + a_0 = 0$, where $a_3 \neq 0$. If we divide each term by a_3 and replace t by $x - a_2/3a_3$, we reduce the original to an equation with no second degree term. Thus every cubic equation can be written in the form $x^3 - cx + d = 0$. It is easy to solve this equation. Let $x = u - v$, where $uv = -c/3$. Replacing x by $u - v$ and simplifying, we have $u^3 - v^3 + d = 0$. But $v = -c/_3u$, so that $27u^6 + 27\,du^3 + c^3 = 0$. By the quadratic formula,

$$u^3 = -\tfrac{1}{2}d + \sqrt{\tfrac{1}{4}d^2 - c^3/27.}$$

Now $v^3 = u^3 + d$ may be found. Since $x = u - v$, we have the following solution for every cubic equation of the form $x^3 - cx + d = 0$.

$$x = \sqrt[3]{-\tfrac{1}{2}d + \sqrt{Q}} + \sqrt[3]{-\tfrac{1}{2}d - \sqrt{Q}},$$

where $Q = \tfrac{1}{4}d^2 - c^3/27$. If Q is negative, imaginary numbers will always appear in the process of finding the solution.

Example 1. Let $x^3 - 3x + 1 = 0$. Thus $c = 3$, $d = 1$, and $Q = -\tfrac{3}{4}$.

Then $x = \sqrt[3]{-\tfrac{1}{2} + \sqrt{-\tfrac{3}{4}}} + \sqrt[3]{-\tfrac{1}{2} - \sqrt{-\tfrac{3}{4}}}$
(substitution for d and Q)

$= \sqrt[3]{-\tfrac{1}{2} + i\sqrt{3}/2} + \sqrt[3]{-\tfrac{1}{2} - i\sqrt{3}/2}$
(Simplification of square roots)

$= \sqrt[3]{\cos 120^\circ + i \sin 120^\circ}$
$+ \sqrt[3]{\cos 120^\circ - i \sin 120^\circ}$ (polar form)

$=$
$(\cos 40^\circ + i \sin 40^\circ) + (\cos 40^\circ - i \sin 40^\circ)$
(principal root)

$= 2\cos(40^\circ + k \cdot 120^\circ), k = 0, 1, 2.$
(general solution)

The decimal values of the 3 roots are 1.532, —1.879, and 0.347. A check is easily done, using the nested form. In each case, $(x^2 - 3)x + 1 = 0.00$

Example 2. Let $x^3 - 5x + 1 = 0$. Thus $c = 5$, $d = 1$ and $Q = 4.380$.

Then $x =$
$\sqrt[3]{-\tfrac{1}{2} + \sqrt{-4.380}} + \sqrt[3]{-\tfrac{1}{2} - \sqrt{-4.380}}$

$= \sqrt[3]{-0.5 + 2.093i} + \sqrt[3]{-0.5 - 2.093i}$

$= \sqrt[3]{2.152\ (\cos 103.433^\circ + i \sin 103.433^\circ)} +$

$\sqrt[3]{2.152\ (\cos 103.433^\circ - i \sin 103.433^\circ)}$

$=$

$(1.291)\ [(\cos 34.478^\circ + i \sin 34.478^\circ) +$
$(\cos 34.478^\circ - i \sin 34.478^\circ)]$

$= 2(1.291)\cos(34.478^\circ + k \cdot 120^\circ).$

Here, the 3 roots are 2.128, —2.330, 0.202. Checking, we find that $(x^2 - 5)x + 1 = 0.00$.

Having had success in using the pocket calculator as an aid in motivating and expediting the study of topics in the theory of equations and elsewhere, I am convinced that it is going to be integrated into the curriculum at a rapid rate. I believe that the calculator will give us a new and refreshing perspective; the result is likely to be a more stimulating curriculum.

References

Boyer, C., *History of Mathematics,* John Wiley and Sons, Inc., N.Y., 1968.

Higgins, G. Albert, *The Elementary Functions: An Algorithmic Approach,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1974.

Problem Solving Heuristics in the Teaching of Series

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In the study of sequences and series, typically, students are given little time to develop formulas as they are deluged with symbols for the first term, last term, number of terms, common difference, common ratio, and sum. Almost as soon as a sequence is defined, the student is virtually given the nth term formula for an arithmetic sequence, and soon after he is shown the standard derivation of the sum formula for an arithmetic series. Usually the students are told the story about young Gauss who obtained the sum of an arithmetic series in a twinkling but they themselves share little in development of such formulas. Students then practice the use of the formulas which essentially involve manipulations of symbols, n, a, d, l, r, S. There is little opportunity to study patterns, make conjectures, generalizations, or otherwise engage in mathematical thinking and problem solving.

Providing the students with such formulas is an efficient method for obtaining answers. It fails to develop problem solving skills.

The main goal of this article is to show how Series sums can be found using problem solving heuristics. Polya made the term *heuristic* prominent in mathematics instruction in his widely read work, *How to Solve It* (1945). It has been interpreted in different ways, but Polya defines heuristic reasoning as "reasoning not regarded as final and strict but as provisional and plausible only, whose purpose is to discover the solution of the present problem." Higgins (1971) cites Geleunter and Rochester on the difference between algorithm and heuristic. "A heuristic may lead us to a blind alley . . . If a method does not have the characteristic that it may lead us astray, we would not call it a heuristic but an algorithm."

A natural beginning for the study of sums of series is the series of natural numbers. Let the students play with, think about, and search for different ways to find the sums of series like:

(1) $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$

(2) $1 + 2 + 3 + \ldots + 98 + 99 + 100$

(3) $1 + 2 + 3 + \ldots + n$

Some students less gifted than Gauss may discover shortcuts to find sums of such series. They may make use of the average value of the terms. Some may even notice the equal sums of symmetrical pairs of terms. Both methods are forms of Polya's heuristic of symmetry.

A logical successor to a natural number of series in a series of even numbers:

(4) $2 + 4 + 6 + 8 + 10 + 12 + 14 + 16$

Nothing need be said about arithmetic series or common differences or last term. Students may see the analogy between this series and the natural number series so the methods for discovering a shortcut for this sum include those used in the natural number series. Students may observe that the terms of the second series are twice the corresponding terms of the first series. They will conjecture the second sum to be twice that of the first. More generally, if they have discovered that the sum of n terms of the first n natural numbers $\frac{n^2 + n}{2}$, it will be evident that the sum of the first n even numbers ought to be $n^2 + n$. When students make conjectures, they are motivated to prove conjectures. Again, by symmetry and analogy students should have time to discover a shortcut for the summation of:

(5) $1 + 3 + 5 + 7 + \ldots + 15$

and more generally for:

(6) $1 + 3 + 5 + \ldots + 2n - 1$.

When they obtain the sum of the first n odd numbers as n^2, they again have the opportunity to compare the result with the analogous series of even numbers. The result is n less than the sum of the first n even numbers. Is this reasonable? Note that each term of the odd numbered series is one less than the corresponding even numbers, and since there are n such numbers, the answer should be less by n.

At this point, if students have been given enough thinking, guessing, and pattern-observing time, they could be presented with any arithmetic series and produce the correct sum without the "symbolic noise" that produces a kind of mathematical nausea among students who have been bulldozed into formulas too rapidly.

Series with general term $\frac{1}{p^n}$ provide useful vehicles for teaching other problem solving heuristics.

Consider first the special case:

(7) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} \cdots + \frac{1}{2^n}$.

The useful heuristic here is extreme cases. We reduce the problem without destroying the gist of the problem. Let students be given time to sum each of these series in turn:

(a) $\frac{1}{2}$ Sum is $\frac{1}{2}$

(b) $\frac{1}{2} + \frac{1}{4}$ Sum is $\frac{3}{4}$

(c) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$ Sum is $\frac{7}{8}$

Allow students to observe the pattern of sum and invite them to make a conjecture about the "next" series sum. Then let them test the conjecture. It seems reasonable to expect that students can generalize from the pattern of partial sums to obtain a conjecture for the sum for series (7). Students would treat the result $\frac{2^n - 1}{2^n}$ as entirely credible and perhaps be more activated to prove the conjecture. The result may be appreciated more than the formula $S = \frac{a - ar^n}{1 - r}$ which is usually derived and given to the students in a lump too much to swallow with too little time to digest.

It is now natural to move to analogous cases like series with general term $1/3^n$, $1/4^n$, etc. The aim would be to eventually make a generalization of series with general term $1/p^n$. Consider:

(8) $1/3 + 1/9 + 1/27 + \ldots + 1/3^n$

and

(9) $1/4 + 1/16 / 1/64 + \ldots + 1/4^n$.

For (8) the sequential pattern of partial sums (heuristic of extreme cases) is

$1/3, 4/9, 13/27, \ldots$

and for series (9) it is

$1/4, 5/16, 21/64, \ldots$

Given time to observe and think, one can expect students to volunteer the next terms in each pattern of sums. Powers of 3 and 4 in the denominators are almost obvious. The numerators are not obvious but students should have the chance to make conjectures. All conjectures should be tested. After several successful conjectures comes the generalization conjecture. We can summarize the conjectured sums for series (7), (8), and (9) as follows:

(7) $\text{Sum} = \frac{2^n - 1}{2^n}$

(8) $\text{Sum} = \frac{1}{2} \quad \frac{(3^n - 1)}{3^n}$

(9) $\text{Sum} = \frac{1}{3} \quad \frac{(4^n - 1)}{4^n}$

Inductive reasoning leading to generalization conjectures has been used to obtain each of these formulas. Another conjecture could now reasonably be made or the series with general term $1/5^n$, leading to a conjecture for series with terms $1/p^n$. Students should be cautioned not to infer certainty from plausibility. Confirmation of a conjecture is not a proof.

Having made a generalization we can sometimes gain even more from the solution of a problem by using the heuristic of extension, which goes beyond the solution to consideration of a related but more difficult problem or to a broader generalization. An important extension of results with series of type $1/p^n$ is the consideration of series of type p^n. We first use the heuristic of special cases and look at the series:

(11) $1 + 2 + 4 + 8 + 16 + \ldots + 2^{n-1}$,

Next, using successive extreme cases of this special case we obtain the pattern of the sequence of sums as:

$1, 3, 7, 15, \ldots$

By inductive reasoning students may conjecture that the fifth partial sum is 31 and that the nth partial sum is $2^n - 1$. The easy solution for series (11) should encourage students to sum the analogous series:

(12) $1 + 3 + 9 + 27 + \ldots + 3^{n-1}$.

Here students will meet some frustration in using the heuristic of extreme cases which previously worked so well. Again, we are reminded that a heuristic may lead us to a blind alley. We are solving a problem, not using a can't miss algorithm in an exercise. The student's first guess might be a conjecture that series (12) has sum $3^n - 1$. It is a reasonable conjecture based on analogous series (8) but it is soon checked out and rejected. Where do we go from here? Perhaps we could use Polya's heuristic of "looking back and re-examine the result." The initial conjecture of $3^n - 1$ does not work as with $n = 4$, the result is 80 whereas the actual sum of four terms is 40. But the conjecture is precisely twice as big as required. Try $\frac{3^n - 1.}{2}$ Does it work for $n = 5$?

The conjectured formula gives 121 for $n = 5$ which is the actual sum of the first 5 terms. At this stage it will be interesting to see what conjectures they will make for:

(13) $1 + 4 + 16 + 64 + \ldots + 4^{n-1}$.

All conjectures should be tested, even unreasonable ones. When conjectures are successfully tested, students may be ready to generalize to series with general term p^{n-1}. If they have been given time to conjecture and confirm in several cases, observation of the special cases for $p = 2$, 3, 4, and 5 should strongly suggest a conjecture for the general case. The sums for $p = 2$, 3, 4, and 5 are respectively:

$$\frac{2^n - 1}{1}, \frac{3^n - 1}{2}, \frac{4^n - 1}{3}, \frac{5^n - 1}{4}$$

This sequence of sums should readily suggest the conjecture $\frac{p^n - 1}{p - 1}$ for the general case $1 + p + p^2 + p^3 + \ldots + p^{n-1}$.

What has been suggested so far are thinking, reasoning, conjecturing, confirming, and generalizing experiences with series. However, although we have looked only at arithmetic and geometric series thus far, we have *not* given the students *the formulas* for arithmetic or geometric series. We have tried to give students the opportunity to discover some formulas to increase their motivation to observe, search, and conjecture, instead of making them manipulators of symbols in formulas. It is therefore quite natural to introduce series which are neither arithmetic nor geometric, yet may be approached by heuristics similar to those already used. At this stage the teacher may choose a class of series such as the following:

$$(14)\quad \frac{1}{1 \times 2}+\frac{1}{2 \times 3}+\frac{1}{3 \times 4}+\frac{1}{4 \times 5}+\cdots+\frac{1}{n(n+1)}$$

$$(15)\quad \frac{1}{2 \times 4}+\frac{1}{4 \times 6}+\frac{1}{6 \times 8}+\frac{1}{8 \times 10}+\cdots+\frac{1}{2n(2n+2)}$$

$$(16)\quad \frac{1}{1 \times 3}+\frac{1}{3 \times 5}+\frac{1}{5 \times 7}+\frac{1}{7 \times 9}+\cdots+\frac{1}{(2n-1)\ (2n+1)}$$

$$(17)\quad \frac{1}{1 \times 4}+\frac{1}{4 \times 7}+\frac{1}{7 \times 10}+\frac{1}{10 \times 13}+\cdots+\frac{1}{(3n-2)\ (3n+1)}$$

The sums of these series do not at first glance appear to be promising for easy summation. Yet each of them yields readily to the heuristics of extreme cases, with conjecturing, verifying, and generalizing to the nth sum, and extension to different but analogous series. Series (14) with partial sums ½, 2/3, ¾, invites conjecture almost immediately. Series (15) may suggest several approaches. The extreme cases give partial sums of 1/8, 1/6, 3/16, 1/5 which looks unpromising. It would be more promising if written in the form 1/8, 2/12, 3/16, 4/20 but a more reasonable approach to series (15) would be using the heuristic of analogy or related problem. A comparison of denominators should reveal that denominators in series (16) are 4 times denominators in series (15).

A look at series (16) shows the analogy with either series (15) or (14). Extreme cases lead to the partial sums pattern:

1/3, 2/5, 3/7, 4/9, . . .

from which a plausible conjecture easily arises. It is interesting to note that if a calculator were used to find partial sums, the decimal sums would not suggest reasonable conjectures as obviously as sums in fraction form.

For series (17) we can use Polya's heuristic "use a result to solve another problem", another form of extension. We note that series (17) is analogous to series (14) and (16). Look at the results for (14) and (16), and conjecture the sum of (17),

$$(14)\quad \frac{1}{1 \times 2}+\cdots+\frac{1}{n(n+1)}=\frac{n}{n+1}$$

$$(16)\quad \frac{1}{1 \times 3}+\cdots+\frac{1}{(2n-1)\ (2n+1)}=\frac{n}{2n+1}$$

$$(17)\quad \frac{1}{1 \times 4}+\cdots+\frac{1}{(3n-2)\ (3n+1)}=???$$

It is not implied that investigations of series always produce reasonable conjectures for sums so readily. The following series all are related in some way to series (14) through (17), and yet the heuristics used thus far would not enable the investigator to find the sums.

$$(18)\quad \frac{1}{1 \times 2}+\frac{1}{3 \times 4}+\frac{1}{5 \times 6}+\cdots+\frac{1}{(2n-1)\ (2n)}$$

$$(19)\quad \frac{1}{1 \times 1}+\frac{1}{2 \times 2}+\frac{1}{3 \times 3}+\cdots+\frac{1}{n \times n}$$

$(20)\ 1 - 1/2 + 1/3 - 1/4 + 1/5 - \ldots + 1/n(n-1)^{n-1}$.

Yet, although the actual sums are different to find, students can use the results of the related series (14) — (17) to determine bounds for the sums of each of the series (18) — (20). They also provide experiences in obtaining partial solutions to problems.

Presenting the student only with risk free algorithms and formulas inhibits his problem solving stance. When teachers provide experiences with problem solving heuristics, students may achieve more success in solving problems. A study by Mendoza (1975) demonstrated that students who were taught the heuristic of cases and analogy used these heuristics in novel problems significantly more than control groups who were taught content without heuristics.

References

Higgins, Jon L. *"A New Look at Heuristic Teaching."* Mathematics Teacher, (October 1971), 487-495.

Pereira-Mendoza, Lionel. *The Effect of Teaching Heuristics on the Ability of Grade 10 Boys to Solve Novel Mathematical Problems.* Unpublished doctoral dissertation, University of British Columbia, 1975.

Polya, George. *Mathematical Discovery: On Understanding, Learning, and Teaching Problem Solving.* Vol. I. New York: John Wiley and Sons, Inc., 1962.

Effects of the Metric System and Hand-Held Calculators on the Sequencing of Concepts in an Elementary Mathematics Program

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Today mathematics curriculum development focuses on issues largely ignored during the 1955-1970 period of "modern math." Responding to the concerns of classroom teachers, as well as educators and laymen interested in the basic goals of general education, attention has now shifted to the impact of metrication and the extensive use of minicalculators on traditional sequencing of math concepts at the elementary level. Greater concern is being directed toward programs for the less able students, for minimal mathematical competencies for effective citizenship, and to the interaction of mathematics and its field of application (NIE and NSF, 1977).

The simultaneous appearance of the metric system and hand-held calculators on the educational scene in the USA has mandated a careful rethinking of the content and the sequencing of that content in a mathematics program for elementary school children. For years mathematics teachers have been attracted to the elegant simplicity of the *Systeme International d' Unites* (SI), commonly known as the "metric system." The adoption of SI (the Metric Conversion Act) in the United States on December 23, 1975, and the subsequent implementation of the metric system involve the use of a new set of base units (meter, liter, gram) which are interrelated by multiples of ten and are easily expressed by the decimal system. The shift to decimal representation of measurements suggests a striking change in the importance of traditional arithmetic skills with the fraction symbol. Traditionally, operations with rational numbers expressed as fractions require skills that are so often stumbling blocks for students through junior high school. On the other hand, arithmetic skills with decimals are fairly easy for elementary students because decimals build on the concept of place value and algorithms for whole number operations. The shift in the sequencing of decimals before fractions is long overdue. Psychologically, as well as mathematically, the earlier introduction to and greater emphasis on numbers represented as decimals, with corresponding delay and de-emphasis on the fractional representation numbers, would be an excellent move in the right direction. Concepts involving the many different types of fractions, least common denominators, equivalent fractions, rewriting fractions in simplest form, and so on, would be postponed until the student is chronologically older and mentally better able to handle mathematical abstractions required to attain skill with operations on rational numbers expressed as fractions.

We urge all school systems to give serious attention to implementation of the metric system in measurement instruction and that they re-examine the current instructional sequence in fractions and decimals to fit the new priorities (NACOME, 1975, p. 44 and p. 139). A similar recommendation was made by Belletallii (1977) to de-emphasize fractions and emphasis decimals, introducing them earlier.

The metric system is based on the decimal system, like our decimal system of currency. Multiples and submultiples of any given unit are always related by powers of 10. Changing from a greater metric unit to a lessor metric unit and vice versa simply means moving a decimal point. No fractions are involved (Consumer Close-Ups, 1976).

In order to remain competitive in world trade, the United States had to go metric. By 1978 United States exporters to the nine Common Market countries will be required to indicate dimensions in metric units. Trade with other industrial nations also necessitated conversion to metric measurements; at present, 43 countries are in various stages of transition to the metric system, including the U.S., Great Britain, Australia, New Zealand, Canada, and South Africa.

Surely no other device has had more potential for influencing instruction in mathematics than the advent of the hand-held calculator and its use in the elementary school classroom. The challenge to traditional instructional priorities is clear. The elementary school mathematics curriculum will be restructured to include much earlier introduction to and greater emphasis on decimals, place value expressed as powers of 10, scientific notation, integers, squaring numbers, and extracting the square root of numbers, with corresponding delay and de-emphasis on operations and algorithms using the fraction symbol. This change is appropriate to match the language of instruction to the language of calculators (NCTM 1977 Yearbook).

While students will quickly discover decimals as they experiment with calculators, they will also encounter concepts and operations involving positive and negative integers, exponents, square roots, scientific notation — all commonly topics of junior high school instruction. For instance, students may discover from the calculator that the product of two negative integers is a positive integer. Thus computational facility with integers (using the calculator) will precede, rather than follow, the careful conceptual development of these concepts.

Personally, I am always pleasantly surprised and favorably impressed by the reactions of children when they are asked to use a calculator to verify the quotients of division examples that have remainders. If the children enter the dividend, then the sign of operation, the divisor, and then the equal sign, the quotient registers on the display in decimal form. If the children are not already familiar with the decimal symbol, this so-called verification is meaningless. At this point in time, the children who recognize the relation of multiplication and division will suggest an alternate way of checking the answer. They decide that if they multiply the divisor (given factor) by the quotient (missing factor) and add the remainder, they should obtain the dividend (product). This problem solving technique will help to verify the accuracy (or lack of accuracy) of papers and pencil computations. Children who do not understand the relation of an operation to its inverse operation will be unable to suggest an alternate method of verifying a quotient with a remainder when confronted with the display in decimal form.

With the increasing availability of inexpensive calculators, adults will have less need for paper and pencil arithmetic computations in the future. The time that is currently spent teaching elaborate "long" multiplication, "long" division, and complicated division, and complicated addition and subtraction of rational numbers expressed as fractions requiring complicated least common denominators (often with little success) could be more wisely spent on more relevant, interesting, rewarding, and motivating topics (NIE, 1976).

The fact that arithmetic proficiency has commonly been assumed as an unavoidable prerequisite to conceptual study and application of mathematical ideas has condemned many low achieving students to a succession of general mathematics courses that begin with and seldom progress beyond drill in arithmetic skills. Providing these students with calculators has the potential of opening a rich new supply of important mathematical ideas and at the same time breaking down self-defeating negative attitudes acquired through years of arithmetic failure. The re-assurance of being able to verify the answer to a troublesome basic fact is sufficient motivation for a slow learner of math to pursue the algorithms.

The fascination of discovering patterns which emerge from otherwise "messy" and lengthy paper and pencil computations is unhampered by the inevitable drudgery if hand-held calculators were not available. An analysis of positions that people hold regarding the use of calculators in schools is reflected over and over in published articles. The most frequently cited reasons for using calculators in the schools are:

1. They remove drudgery and save time on tedious calculations.
2. Low achievers find computations less frustrating.
3. Calculators encourage estimation, verification, and approximation.
4. They facilitate understanding and concept development.
5. They help and enlarge the scope of problem solving.
6. They motivate and encourage curiosity, positive attitudes, and independence (an instant answer key).
7. They exist and are here to stay—so we cannot ignore them.

The last reason is perhaps the most compelling one.

Perhaps the greatest concern and one that is most frequently expressed by parents and by other members of the public, as reflected by newspaper articles, is the fact that children will not memorize the basic facts for all four fundamental operations. However, very few educators believe that calculators should be used by children before they have memorized the basic facts for the fundamental operations. Perhaps the greatest disadvantage of using calculators in schools is the fact that they lead to maintenance and security problems; however, the advantages of using calculators far outweigh this single disadvantage.

To date most research studies about the use of hand-held calculators in the schools have been exploratory. Some of the "hardest" data come from studies conducted by calculator manufacturers. Not surprisingly, the findings indicate that students (a) can use the calculator with a variety of content and (b) achieve well when using the calculator. Many schools are checking data on

References

Bell, M., Estey, Payne, J. N., and Suydam, M. N. Hand-Held Calculators: Past, Present, and Future. In the *NCTM 1977 Yearbook* (ch. 13). Reston, Virginia: NCTM, Inc., 1977.

Consumer Close-Ups. America Goes Metric. New York State, Department of Consumer Economics and Public Policy. April/May, 1976.

Hill, Shirley, *et al. Overview and Analysis of School Mathematics, Grades K-12.* (the NACOME Report). Washington, D.C.: National Advisory Committee on Mathematical Education, Conference Board of the Mathematical Sciences, 1975.

National Council of Supervisors of Mathematics. NCSM Position Paper on Basic Mathematical Skills. Minneapolis Public Schools, 1976.

National Institute of Education. *Conference on Basic Mathematical Skills and Learning* (Euclid Report). Volumes 1 and 2. Los Alamitos, California: SWRL Educational Research and Development, 1976.

National Institute of Education and National Science Foundation. Report of the Conference on Needed Research and Development on Hand-Held Calculators in School Mathematics. Washington, D.C.: 1977.

Werner, Marijane. *Teaching the Set of Integers to Elementary School Children.* Dubuque, Iowa: Kendall/Hunt Publishing Company, second printing, 1976.

their own students to determine the effect of the use of calculators. Reports of these studies indicate that using calculators generally results in achievement at least as high as that which results when calculators are not used. In some instances, computation scores are significantly higher when calculators are used, and in others problem solving scores are significantly higher (Bell et alii, 1977).

As each of you reflects upon the curricular changes that are mandated by the advent of the metric system and hand-held calculators, I sincerely hope that the convictions which result from your reflections evolve into dynamic actions to change and to update mathematics instruction in order that the children we are now teaching will be more adequately prepared to function effectively in the 21st century.

Philosophy of Education And the Mathematics Curriculum

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Diverse schools of thought in the philosophy of education have much to offer in terms of objectives, learning activities, and appraisal techniques for pupils in ongoing units of study in the curriculum area of mathematics. Teachers and supervisors need to study, analyze, and implement selected strands from diverse philosophical schools of thought.

Experimentalism and the Curriculum

A teacher emphasizing experimentalism as a philosophy of teaching and learning may well emphasize generalizations such as the following:

1. pupils are to solve realistic problems. These problems identified by learners need to be life-like and emphasize what is relevant in society.
2. interesting experiences are important for pupils which then sustain effort in learning.
3. subject matter is learned to solve problems and is not an end in itself.
4. solutions related to problems are tentative and subject to change.

Experimentalism, as a philosophy of education, would definitely not recommend the following:

1. pupils being required to work content on each page sequentially in a reputable series or multiple series of mathematics textbooks.
2. learners working on basic operations and story problems within the framework of a reputable mathematics workbook. These situations do not provide opportunities for pupils to identify and attempt to solve relevant problems.
3. the teacher determining objectives, learning activities, and appraisal techniques for pupils.

Experimentalism then emphasizes that problem solving be the heart of the curriculum. Real problematic situations existing in society are important. Learners should have ample opportunities to work within the committee framework on interesting activities and experiences. Active involvement in learning is to be wholeheartedly stressed in ongoing units of study. The consequences of each deed or act is important to consider.

Existentialism and the Curriculum

Existentialists emphasize the following generalizations in the curriculum:

1. pupils individually actively making choices in terms of what to learn and the means of learning.
2. individual choices are to be stressed as being important rather than group or committee decisions.
3. individuals lose their humanness if choices are not made on a personal basis.
4. alienation is possible, of course, when choices are made.
5. situations in life may appear to be ridiculous or absurd.
6. creative, products, acts and deeds of individuals are important! Conformity behavior is definitely not an end goal.

Existentialists would not emphasize the following:

1. the teacher selecting what pupils are to learn (ends), the activities to achieve these ends, and means of appraising learner achievement.
2. pupils completing sequentially pages of content from a mathematics textbook or workbook.
3. pupils exhibiting conformity behavior in terms of learnings gained.

Realism and the Curriculum

Realists emphasize precise, exact learnings which learners may acquire. Many realists stress the curriculum areas of science and mathematics as being more important generally for pupils as compared to other curriculum areas. Thus, the real environment can be known as it truly exists. Mathematical concepts and generalizations may be utilized to express precise content pertaining to reality.

A second set of realists may emphasize the importance of using precise, measurable objectives in the curriculum. These educators believe that what exists can be identified in terms of behaviorally stated objectives. Learning activities may then be provided for pupils to attain these desired ends. Ultimately, it can be measured if these chosen objectives have been achieved by pupils. Thus, all curriculum areas may be emphasized adequately in terms of balance in the school-class setting. Cognitive objectives (use of intellect) and psychomotor objectives (use of neuro-muscular skills) may receive primary emphasis in teaching-learning situations. Affective or attitudinal ends are more difficult, of course, to state in measurable terms as compared to either cognitive or psychomotor goals.

Realists, then, may emphasize the following generalizations:

1. pupils can know the real environment as it truly exists.
2. learnings gained by pupils individually can

be measured in terms of gains made.

3. selected realists believe that science and mathematics should be emphasized more than other curriculum areas in the school-class setting.

4. values in life would reflect reality in the environment. Thus, pollution, for example, if its many forms would hinder the beauty and goodness within nature. This would suggest that the natural environment be protected and nurtured to truly reflect positive affective ends in society.

Realists deemphasize the following:

1. ideas, concepts, and generalizations being separated from what the human can truly know in terms of nature and the natural environment.

2. cognitive, psychomotor, and affective goals that cannot be identified with precision and in measurable terms.

3. pupils being heavily involved in determining what to learn (the objectives), the means of learning (activities to attain desired ends), and appraisal techniques. The teacher is in a much better position to determine these three parts of the curriculum.

4. many abstract learnings to the detriment of concrete and semi-concrete experiences for the pupils.

Idealism and the Curriculum

Idealists in teaching-learning situations would tend to stress abstract concepts and generalizations as being highly significant in terms of pupil attainment in ongoing units of study. The learner, as well as all individuals, cannot know the real world as it truly exists. However, pupils can acquire ideas about that which is real, actual and life-like.

Thus, pupils would be guided in achieving the following, as advocated by idealists:

1. relevant abstract content, meaningfully presented.

2. universals in terms of broad emcompassing ideas, presented as challenging ideas for learner attainment.

3. deductive learnings being emphasized for pupils in terms of methods of teaching utilized by the instructor. Clear, meaningful presentations given by the teacher then are important within the framework of ongoing activities.

In Closing

Teachers and supervisors need to study, evaluate, and ultimately implement relevant strands from diverse schools of thought in the philosophy of education. The philosophy or philosophies of education adopted may well make for an improved mathematics curriculum in the school-class setting.

Selected References

Brubacher, John. *A History of the Problems of Education.* New York: McGraw-Hill Book Company, 1966.

Ediger, Marlow. *Relevancy in the Elementary Curriculum.* Kirksville, Missouri. Simpson Publishing Company, 1975.

Prescriptive Teaching in the Remedial Mathematics Laboratory

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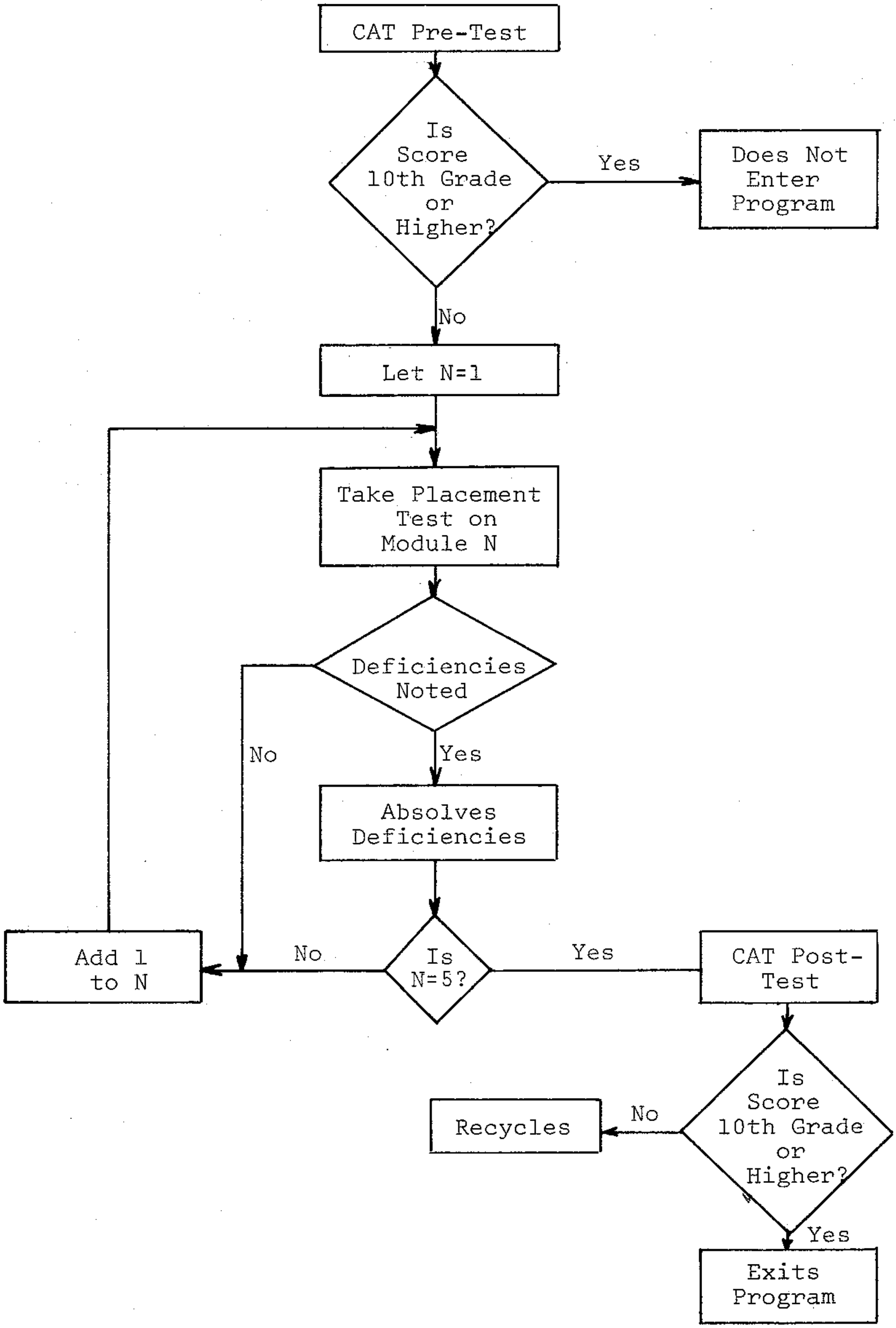
Remedial arithmetic at Alvin Community College is taught in a self-paced program conducted in a laboratory setting. Initially, the program was a paradox in that prescriptive study, one of the strengths of a self-paced program, was not utilized. Presently, however, a student is placed into the remedial arithmetic program only after an analysis of the California Achievement Test (CAT) scores indicate a deficiency in arithmetic. The program now being used is the McHale-Witzke *Arithmetic Module Series* published by Addison-Wesley. Each module contains a placement test that is prescriptive in terms of the units in that module. The following flow chart illustrates the procedure used for each module. The student performs only that work in which he is deficient.

As each unit is completed, a test is given over that unit. The student must demonstrate a proficiency of at least 85% on the unit test before being allowed to go on to the next required unit. Once all modules are completed, the initial screening instrument, the CAT, is again administered to the student.

There are two reasons for giving the CAT as both a pre-test and a post-test. First, a performance change is documented by which program and instructor accountability can be established. The second reason is one which sometimes goes unnoticed in programs that permit retesting for mastery, as does the program at Alvin Community College. When students are permitted to retest several times over the same material, their scores may reflect practice more than understanding. Hence, if the program is effective, a comprehensive post-score should exceed the initial minimum entrance screening score. A pre-test score indicative of a mathematical understanding less than the tenth grade results in the student being required to enroll in the remedial arithmetic program; hence, a post-test score of tenth grade or greater is required to complete the program. To date, no student has scored less than the tenth grade level on the post-test.


CAT Pre-Test
Is Score 10th Grade or Higher?
Yes
Does Not Enter Program
No
Let N=1
Take Placement Test on Module N
Deficiencies Noted
No
Yes
Absolves Deficiencies
Add 1 to N
No
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The Metric System
-versus-
The Teaching of Common Fractions

by Kathy B. Hamrick
Augusta College

According to the report by NACOME (1975), "the elementary school curriculum will be reconstructed to include much earlier introduction and greater emphasis on decimal fractions, with corresponding delay and deemphasis of common fractional notation and algorithms." (p. 41) The reasons for this predicted change in the school mathematics curriculum of the United States were the impending adoption of the metric system and the increasing availability and use of calculators. However, a different picture of the future emerges from a comparison of the treatment of decimal fractions and common fractions in the elementary curriculum of the United States with the treatment of the same topics in the elementary curriculum of Germany, a country that has used the metric system for over a century. The results of this comparison provide some information concerning the validity of the NACOME prediction since the prediction is based in part on the effect of the adoption and use of the metric system in the United States.

In the Federal Republic of Germany, all students attend primary school (Grundschule) for grades 1, 2, 3, and 4. After grade 4, the students are channeled into Hauptschule, Realschule, or Gymnasium. The Gymnasium primarily prepares students for college and entails nine years of study. Students study at the Realschule for six years then usually go to a professional school which is not on a university level. Students who do not go to Gymnasium or Realschule remain in elementary school, now called Hauptschule, for five or six years. Most of these students enter a trade or take up work in a factory.

The comparison between the decimal fraction and common fraction aspects of the curricula of the two countries was conducted by comparing an elementary (grades 1-6) mathematics text series from the United States with several similar text series from Germany. The series chosen to be representative of the United States curriculum was the *Silver Burdett Mathematics* (1976). Separate German series were selected as representative of Grundschule, Hauptschule, Realschule, and Gymnasium. The following German text series were identified by Dr. Hendrick Radatz of the Bielefield University in Germany as commonly used in each of the four schools:

Grundschule: (1) *Mathematik in der Grundschule 1*
(2) *Wir Lernen Mathematik I and II*
(3) *Welt der Mathematik*

Hauptschule: (1) *Die Welt der Zahl-Neu*
Realschule: (1) *Neue Mathematik*
Gymnasium: (1) *Mathematik Heute I and II*
(2) *Mathematik B6*

Books one through four of the Silver Burdett text series were compared with the German texts used in Grundschule. Books five and six of the German text series used in Hauptschule, Realschule, and Gymnasium. Comparisons were made in each of the following topics:

(1) Concepts of common fractions and decimal fractions
(2) Equivalent common fractions and decimal fractions
(3) Conversions between common and decimal fractions
(4) Operations of addition and subtraction of common fractions and decimal fractions
(5) Operations of multiplication and division of common fractions and decimal fractions
(6) Measurement with common fractional and decimal fractional parts of units
(7) Story problems requiring the use of common fractions or decimal fractions

Summary — Grades One Through Four

The number of pages that contain work on common fractions and on decimal fractions are listed in Table 1 for each text examined. A page was counted if it contained at least one exercise or example involving one of the above seven topics.

There is an earlier introduction and greater emphasis on common fractions in the Silver Bur-

Table 1

Text	Number of Pages	
Germany	Common Fractions	Decimal Fractions
Mathematik in der Grundschule	2	0
Wir Lernen Mathematik Book I	0	0
Welt der Mathematik Book I	0	0
Die Welt der Zahl-Neu Book I	0	0
Neue Mathematik Book I	0	0
Wir Lernen Mathematik Book II	0	0
Welt der Mathematik Book 2	2	0
Die Welt der Zahl-Neu Book 2	0	0
Neue Mathematik Book 2	0	0
Welt der Mathematik Book 3	3	10
Die Welt der Zahl-Neu Book 3	0	10
Neue Mathematik Book 3	2	9
Welt der Mathematik Book 4	3	7
Die Welt der Zahl-Neu Book 4	5	12
Neue Mathematik Book 4	5	20
United States	Common Fractions	Decimal Fractions
Silver Burdett Book 1	7	0
Silver Burdett Book 2	9	0
Silver Burdett Book 3	21	1
Silver Burdett Book 4	45	10

dett texts than the German texts. In the Silver Burdett text series, common fractions are introduced in Book 1 and are increasingly developed in Books 2, 3, and 4, and extended to the topics of addition and subtraction of like fractions, reducing fractions to lowest terms, and rewriting unlike fractions as like fractions. In contrast, except for a brief section defining the fraction "½" as the opposite of "double", concepts of common fractions are not developed until Book 4 of the German texts. Even then, the main sections on common fractions consist of two or three pages at the end of the text.

Neither the Silver Burdett text series nor the German texts contain sections specifically related to decimal fractions. However, the texts of both countries contain examples and exercises requiring the use of decimal fractions in the form of money and metric units of measurement. The German texts contain more of these examples than the Silver Burdett texts. However, the differences between the texts of the two countries is not nearly so great in the treatment of decimal fractions as in the treatment of common fractions.

The NACOME prediction of later introduction and de-emphasis of common fractions appears somewhat supported by the evidence from the comparison of the texts of the two countries. There is not much evidence to support the predicted earlier introduction and greater emphasis on decimal fractions.

Summary — Grades Five and Six

The number of pages in each text and the percent of the pages of each text directly related to common fractions and decimal fractions are listed in Table 2. The Silver Burdett text contains much greater emphasis than the German texts on common and decimal fractions in Book 5. The differences between texts in Book 6 are not as noticeable.

Summary and Conclusions

The German texts definitely introduce common fractions much later than the American text series. In the Silver Burdett text series, common fractions are introduced in Book 1 and are an integral part of the scope and sequence of Book 2 through 6. In contrast, although there is some mention of common fractions in Books 1 through 5 of the German series, except for the text *Die Welt der Zahl-Neu Book 5*, the fractions are not an integral part of the scope and sequence of the texts until Book 6. The series *Die Welt der Zahl-Neu* contains three pages on common fractions at the end of the book. A comparison of Book 6 of each series shows the German texts contain the same emphasis, if not more than the Silver Burdett text, on common fractions.

In the German text series, decimal fractions are not formally introduced earlier than in the American text series. In both series, decimal fractions are formally introduced in Books 5 and 6. In the German series, however, there are exercises as early as Book 3 that are concerned with decimal forms of money and metric measurement. There is not much difference between the texts of the two countries in the emphasis or treatment of decimal fractions in Book 6.

The results of the comparison of several German text series with the Silver Burdett text series indicates the following:

(1) There is evidence to support the prediction of later introduction of common fractions.

(2) There is no evidence to support the prediction of deemphasis of common fractions with corresponding emphasis of decimal fractions.

(3) There is no evidence to support the prediction of an earlier introduction of decimal fractions.

Limitations

This investigation was only a comparison of text series. Before the data listed in Tables I and II was collected, the above conclusions had been reached from the examinations of the texts. The data did tend to support the conclusions.

The conclusions and the validity of the entire investigation are dependent on the assumptions that:

(1) A country's text series reflects what is being taught in it's curriculum.

(2) The chosen texts for this investigation are representative of the texts for the respective countries.

No studies were found pertaining to these assumptions.

During the investigation, it was found that in the German texts common fractions are treated as operators similar to function machines. This different concept of a common fractions could have accounted for some of the differences between the texts of the two countries.

References

Athen, H. & Griesel H. (eds.). *Mathematik heute.* Hannover: Hermann Schroedel Verlag KG, 1971.

Conference Board of the Mathematical Sciences National Advisory Committee on Mathematics Education. *Overview and analysis of school mathematics grades K-12.* Washington: Conference Board of Mathematical Sciences, 1975.

Fricke, A. & Besuden H. *Mathematik in der Grundschule 1.* Stuttgart: Ernst Klett, 1976.

Griesel, H. & Sprockhoff W. *Welt der Mathematik* (4 texts). Hannover: Hermann Schroedel Verlag KG, 1974.

LeBlanc, J. F. (et.al.) *Silver Burdett Mathematics* (6 texts). Morristown: Silver Burdett Company, 1976.

Neunzig, W. & Sorger P. *Wir Lernen Mathematik* (2 texts). Herder Verlag Freiburg im Breisgun, 1971.

Roller, E. (et.al.) *Mathematik B6.* Stuttgart: Ernst Klett Verlag, 1976.

Oehl, W. & Palzkill, L. (Eds.). *Die Welt der Zahl-Neu* (6 texts). Hannover: Herman Schroedel Verlag KG, 1971.

Winter, H. & Ziegler T. *Neue Mathematik* (6 texts). Hannover: Hermann Schroedel Verlag KG, 1969.

Bibliography

The Press and Information Office of the Federal Republic of Germany. *Facts about Germany.* Bonn: Author, 1972.

Meldau, R. *Kleines Deutchlandbuch fur Auslander.* Munchen: Max Heuber Verlag, 1958.

Meet Germany. Hamburg: Atlantik-Brucke, 1969.

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