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TEXAS MATHEMATICS TEACHER

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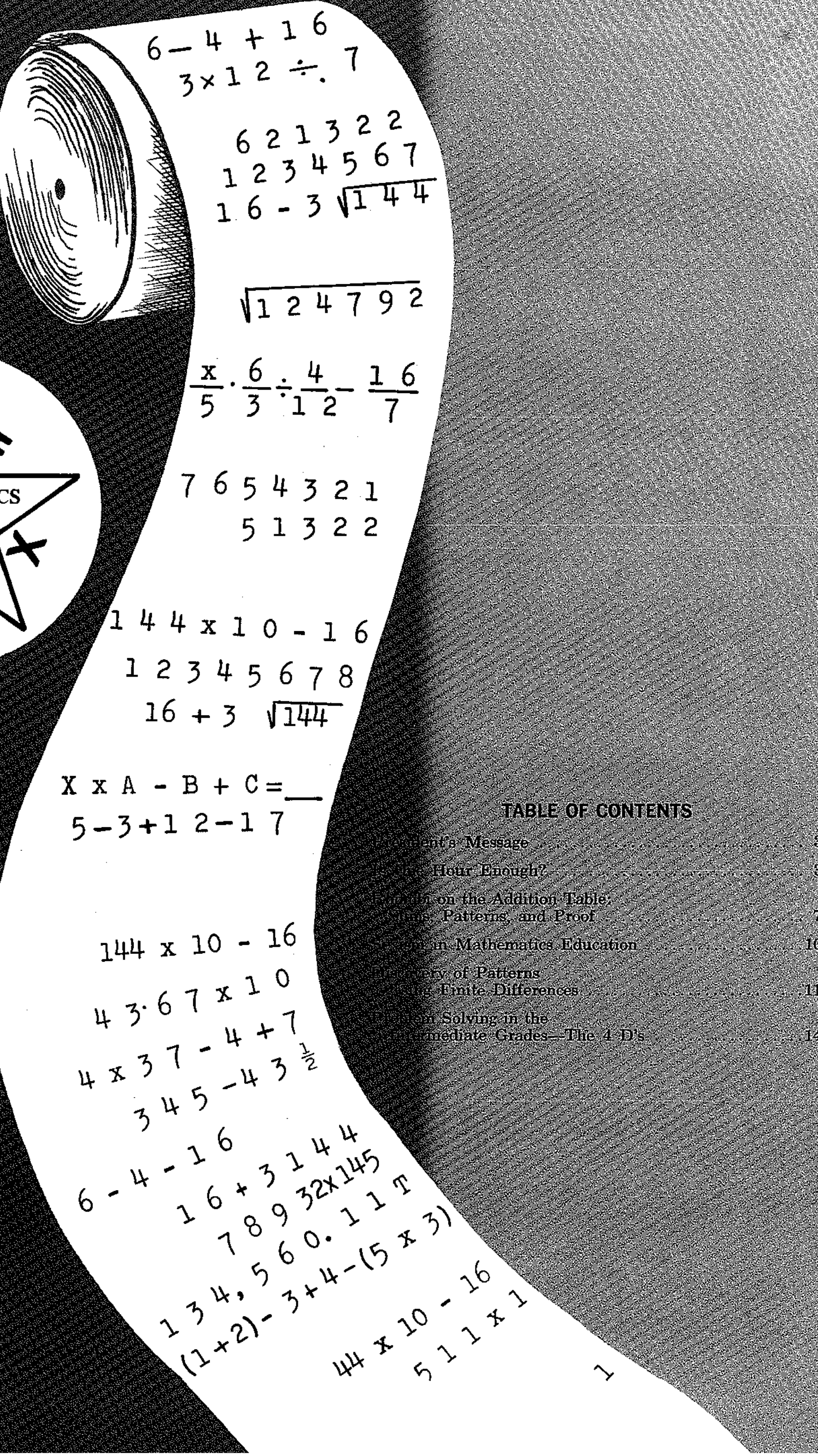

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to be able to redefine them (reducing, conversions, percents, etc.). That's all there is to arithmetic and problem solving involving numbers.

ADDITION What does a student need to know to get *the right answer?* He needs only to know how to count. Anything else? No. (Here is how I count and put things together. 4 fingers and 9 fingers, that's 4 and 5, 6, 7, 8, 9, 10, 11, 12, 13. 13 is the answer. Here is how I count and separate a number into its parts. 13, that's 4 and then 5, 6, 7, 8, 9, 10, 11, 12, 13. 4 is one part and 9 is the other part).

WATCH	4	5	8	squeeze	458
	7	9	7	SQUEEZE-E-E	797
	11	14	15		1255

A. IF you're LEFT-HANDED

```
 →         →
4 5 8  THEN  4 5 8
7 9          7 9 7
--------     -------
11 4         11 14
12           12 5 5
```

In both cases
slide 1
to the
Left

B. IF you're RIGHT-HANDED

```
  1            1 1
 ←            ←
4 5 8  THEN  4 5 8
    7        7 9 7
------       ------
    5        12 5 5
```

SUBTRACTION What does a student need to know to get the right *answer?* He needs to know how to count. Anything else? No. (Be sure he continues to count forward.)

```
WATCH   7 5 2
       -3 9 6
       ------
```

STOP! ! ! !

You *can't* count 6, 7, 8, 2

You *can* count 6, 7, 8, 9, 10, 11, 12

You *can't* count 9, 10, 11, 5

You *can* count 9, 10, 11, 12, 13, 14, 15

```
              →
NOW TRY IT   7 15 12
            —3  9  6   (Slide 1 To The Right)
            ---------
             3  5  6
```

```
                      →          →          →
(In slow motion) (a) 7 5 2 (b) 7 15 2 (c) 7 5 12
  Left-Handed       —3            9             6
                    ----       ------     -------
                     4          4  6       4 6  6
                                3          3 5  6
```

MULTIPLICATION What does a student need to know to get the right *answer?* He needs to know how to count. Anything else? No. (After all isn't multiplication just a short-cut to adding when all the counted objects are the same.)

```
        ←
WATCH 23456
      23456
      23456
      23456
      23456
      -----
```

STOP! ! ! ! Take a short-cut

```
               ←
NOW TRY IT   2 3 4 5 6
                    ×5
             ---------
             11 7 2 8 0
```

(In slow motion)

(a) Count on your fingers *or* add five
0,0
1,5
2,10
3,15
4,20
5,25
6,30
7,35
8,40
9,45
→10,50

(b) Remember your adding and it is easier if you are a right-handed person

```
        ←
    1 2 2 3
(c) 2 3 4 5 6
           ×5
   ----------
   11 7 2 8 0
```

(This tells me I counted correctly, can you see why?)

Want to see me do a big one?????

```
WATCH   9 8 7
          2 3
      ---------
      2 2 7 0 1
```

(I like short-cuts, here's what I did)

Food for Thought:

Mathematics teachers can be divided into three groups: those who make things happen, those who watch things happen, those who wonder what happened. In which category are you?

(In slow motion)

(a) Count

0,0
1,3
2,6
3,9
4,12
5,15
6,18
(7,21)
(8,24)
9,27
10,30

(b) Add

```
 9 8 7
 9 8 7
 9 8 7
------
```

(c)

```
  2 2
  9 8 7
      3
-------
 29 6 1
```

(d) Count

0,0
1,2
2,4
3,6
4,8
5,10
6,12
7,14
8,16
9,18
10,20

(e) *Add*

(f)

```
    1 1
    9 8 7
      2 0
---------
 19 7 4 0
  2 9 6 1
---------
 22 7 0 1
```

(notice that 2 and 20 differ only by 0)

DIVISION What does a student need to know to get the *right answer?* He needs to know how to count. Anything else? No. (Remember division is just subtracting the same object over and over.)

WATCH $\frac{\overrightarrow{4\ 7\ 8\ 9}}{4} = 1\ 1\ 9\ 7\ \frac{1}{4}$

(Isn't that a sneaky way to teach fractions) What did I do? I just subtracted four's. It is nice to be left-handed sometimes. Just look how easy it is when you are.

(a) Count

0,0
(1,4)
2,8
3,12
4,16
5,20
6,24
(7,28)
8,32
(9,36)
10,40

(b) Subtract (c)

```
 (4)
 -4   1
  (7)
 -
   (8)
 -
    (9)
 -
```

(d)

```
   4
  -4   1
  07
 - 4   1
  38
 -36   9
  29
 -28   7
   1
```

I always check this subtract by adding up. Since 1 1 9 7 four's were subtracted, that would be 4788 and then I had 1 leftover that did not make a complete four. So I best put that in too, 4789. Throwing away that 1 is like throwing away a penny. That is why I wrote ¼. Okay just one more and a big one this time.

(a) $\frac{\overrightarrow{3434}}{41}$

(b) Count

(0,0)
1,41
2,82
(3,123)
4,164
5,205
6,246
7,287
(8,328)
9,369
10,410

(c) Subtract

(d)

```
    3
   -0   0
   34
  - 0   0
  343
 -328   8
  154
  123   3
   31
```

(e) Answer

$83\ \frac{31}{41}$

Wasn't that division easy? Let's see those division problems again.

(a) $\frac{14}{7} = 2\ \frac{0}{7}$ (b) $\frac{15}{7} = 2\ \frac{1}{7}$ (c) $\frac{2}{7} = 0\ \frac{2}{7}$

Look, when I divide, my answer has what is known as a WHOLE number and a remainder. It just happens that occasionally the whole number or remainder turns out to be a zero. Is that IMPORTANT? YES, YES, YES.

COUNTING What are numbers for if you can't count with them?????

Count

WATCH

$0\ \frac{0}{3}$

$0\ \frac{1}{3}$

$0\ \frac{2}{3}$ What happens here???

$0\ \frac{3}{3} = 1\ \frac{0}{3}$

$0\ \frac{4}{3} = 1\ \frac{1}{3}$

$0\ \frac{5}{3} = 1\ \frac{2}{3}$ Not Again???

$0\ \frac{6}{3} = 1\ \frac{3}{3} = 2\ \frac{0}{3}$

As I counted I made some of the words easier to read. Let's count by 3's

WATCH			
0,0	0,0	Use the right hand side only and SQUEEZE	0,0
1,3	1,3		3,3
2,6	2,6		6,6
3,9	3,9		9,9
4,12	4,12		10,10
5,15	5,15		15,15

Hey, Look they divide and give "1". Must be like the dictionary, you know where big and large mean the same, and pretty and beautiful mean the same. What about 3 feet and 1 yard. Gee, must be a lot of things that mean the same...... So to make a word look different and yet say the same thing all I have to do is

(a) Count	Count (b)	Use the right hand side only and Squeeze-e-e		(c) the word is 2
0,0	0,0		6 | 3	
1,6	1,3		12 | 6	
2,12	2,6		18 | 9	
3,18	3,9		24 | 12	
4,24	4,12			

$\frac{6}{3}$ is the word 2. Just like big and large are the same words.

Big begins with the letter "b". Large begins with the letter "l"; "looks are one thing and meanings are another."

That is just too easy. Oh well, let's see what some more look like.

Let's play with $\frac{2}{3}$

(a) Count	Count	(b) Use the right side only and squeeze-e-e
1,2	1,3	
2,4	2,6	
3,6	3,9	
4,8	4,12	

(c)

2	3
4	6
6	9
8	12

(d) These are all the same word

$\frac{2}{3}\ \frac{4}{6}\ \frac{6}{9}\ \frac{8}{12}$

This is just like using the dictionary. I guess the little chart I made should have a name so I'll call it my *mathematical* dictionary. Counting and changing the name is easy so I guess now I should be able to put those objects together or take them apart.

ADDITION and SUBTRACTION Yep, all you have to be able to do is count. Oh yes, let's use the mathematical dictionary.

A. *WATCH* $88\ \frac{1}{3} + 7\ \frac{5}{9}$

(a) Whole numbers	(b) Mathematical Dictionary	(c) remainders	(d) answer
88 + 7 95	1 | 3 ; 2 | 6 ; 3 | 9 5 | 9	3 + 5 8	$95\ \frac{8}{9}$

B. *AGAIN* $\frac{4}{7} + \frac{1}{2}$

(a) Numbers	(b) Mathematical Dictionary	(c) remainders	(d) answer
0 + 0 0	4 | 7 ; 8 | 14 1 | 2 ; 2 | 4 ; 3 | 6 ; 4 | 8 ; 6 | 10 ; 7 | 14	8 + 7 15	$0\ \frac{15}{14}$ OR $1\ \frac{1}{14}$

If you want to subtract the last example, just subtract the remainders instead of adding.

C. *NOW A HARD ONE* $33 - 8\ \frac{2}{5}$

STOP!!!! The problem is written wrong.

GO!!!! $33\ \frac{0}{5} - 8\ \frac{2}{5}$

(a) Whole numbers	(b) Mathematical Dictionary	(c) Remainders
33 − 8 25	0 | 5 2 | 5	0 −2 ??? STOP!!

(d) Mathematical Dictionary	(e) Remainders
5 | 5 Dictionary Slide	5 −2 3
On The Way Down	Answer 2~~5~~ (4) or
See!!	$24\ \frac{3}{5}$

Well that is the end of my story. I hope you enjoyed it. Oh, you have a question about where to look in the dictionary for the word $\frac{14}{35}$ Do you remember the charts we made for multiplication and division? Well, 15 and 35 are both on the same chart. Can you think of which one? If there is no chart, there is no other word with the same meaning, if there is a chart you will find the answer.

WATCH 0,0
1,7
(2,14)
3,21
4,28
(5,35)

ANSWER $\frac{2}{5}$ means the same thing as $\frac{14}{35}$ they are just spelled differently.

What about percents? Oh, that is just using the dictionary. Decimals? Look in the dictionary. See you around the library.

Rhombi on the Addition Table: Sums, Patterns, and Proof

by Bonnie H. Litwiller and David R. Duncan
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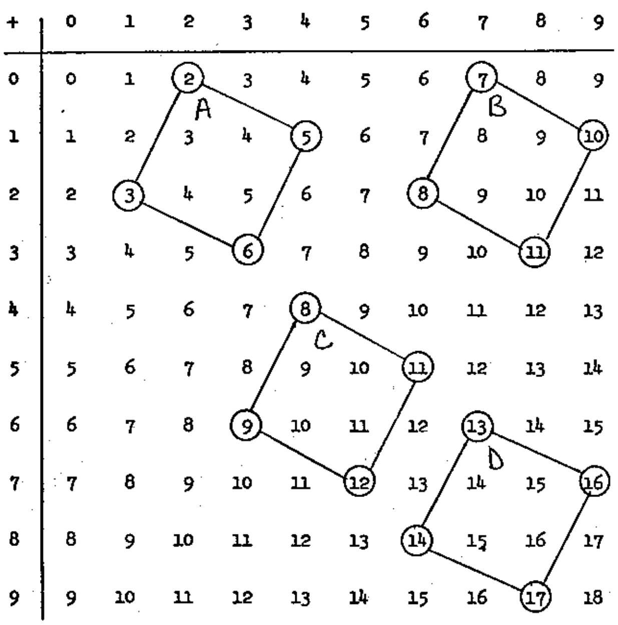
The Addition Table is a rich source of number patterns. Seeking these patterns gives the student experience in formalizing and verifying conjectures. A valuable by product is the maintenance of computational skills.

Activity I: Consider rhombi drawn on the Addition Table as shown in Figure I.

Figure I



1. Add the four numbers which represent the vertices of each rhombus; call the sum V.

2. Add the four interior numbers of each rhombus; call this sum I.

The results of the above calculations appear in Table I.

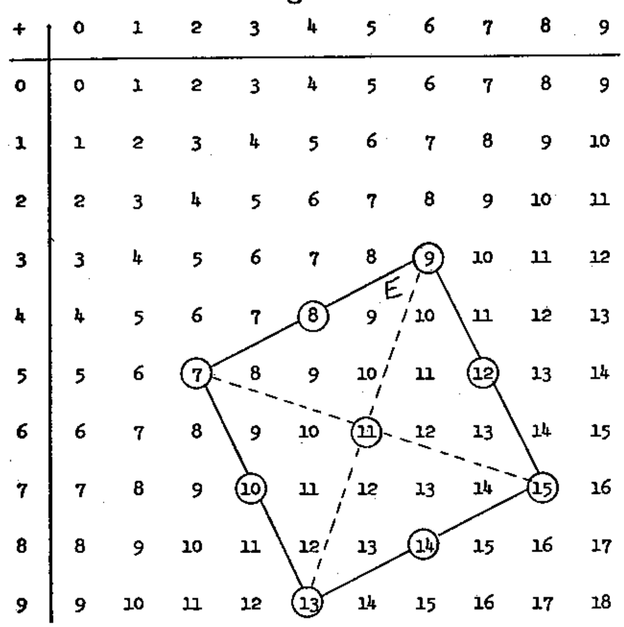
Table I

Rhombus	V	I
A	16	16
B	36	36
C	40	40
D	60	60

Observe that for each rhombus the sums V and I are the same; the ratio of these corresponding sums is 1/1. To see if this pattern is true for all rhombi of this size, check other rhombi positioned differently on the table.

Activity II: Consider the rhombi drawn on Figures II and III. Note that these rhombi have three circled numbers per side.

Figure II



1. Add the four numbers which represent the vertices of each rhombus; call this sum V.

2. Add the eight numbers which lie on the perimeter; call this sum P.

3. Add the seventeen interior numbers; call this sum I.

4. Let C be the "center number" of each rhombus.

5. Find I — C.

The results of these calculations appear in Table II.

Figure III

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6 F	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Table II

Rhombus	V	P	I	C	I − C
E	44	88	187	11	176
F	36	72	153	9	144

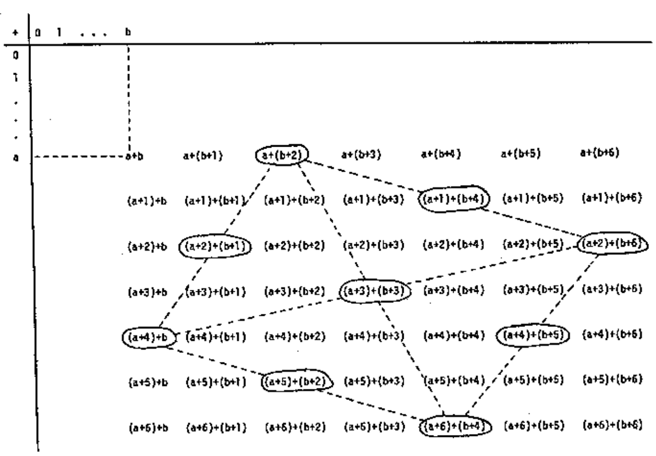
Observations:

1. $V = 4C$ or $\frac{V}{C} = \frac{4}{1}$; the sum of the vertices is four times the center number.

2. $\frac{P}{I} = \frac{8}{17}$. Both 72/153 and 88/187 can be renamed 8/17. Note that there are eight numbers on the perimeter of the rhombus and seventeen numbers in the interior of the rhombus.

3. $\frac{I-C}{C} = \frac{16}{1}$. Both 176/11 and 144/9 can be renamed 16/1. Note that there are sixteen numbers in the interior of the rhombus excluding the center number.

Figure IV: A General Addition Table



Be sure to check these patterns for other rhombi of this size positioned elsewhere on the table.

We shall verify pattern 3; that is, $\frac{I-C}{C} = 16$.

Figure 5

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

The sum I - C is depicted as follows:

$$
\begin{array}{l}
(a+1)+(b+2)\\
(a+1)+(b+3)\\
(a+2)+(b+2)\\
(a+2)+(b+3)\\
(a+2)+(b+4)\\
(a+2)+(b+5)\\
(a+3)+(b+1)\\
(a+3)+(b+2)\\
(a+3)+(b+4)\\
(a+3)+(b+5)\\
(a+4)+(b+1)\\
(a+4)+(b+2)\\
(a+4)+(b+3)\\
(a+4)+(b+4)\\
(a+5)+(b+3)\\
(a+5)+(b+4)\\
\hline
(16a+48)+(16b+48)
\end{array}
$$

$$\frac{I-C}{C} = \frac{16(a+b+6)}{(a+3)+(b+3)} = \frac{16(a+b+6)}{(a+b+6)} = 16.$$

Questions for the reader and his/her students:

1. Do the patterns found in Activities I and II hold on the Multiplication Table, the Subtraction Table, and the Hundred Square?

2. Can you prove the other patterns?

3. Can you find similar patterns on the following nested rhombus configuration? There are four circled numbers per side. There is no center number; there is a center rhombus.

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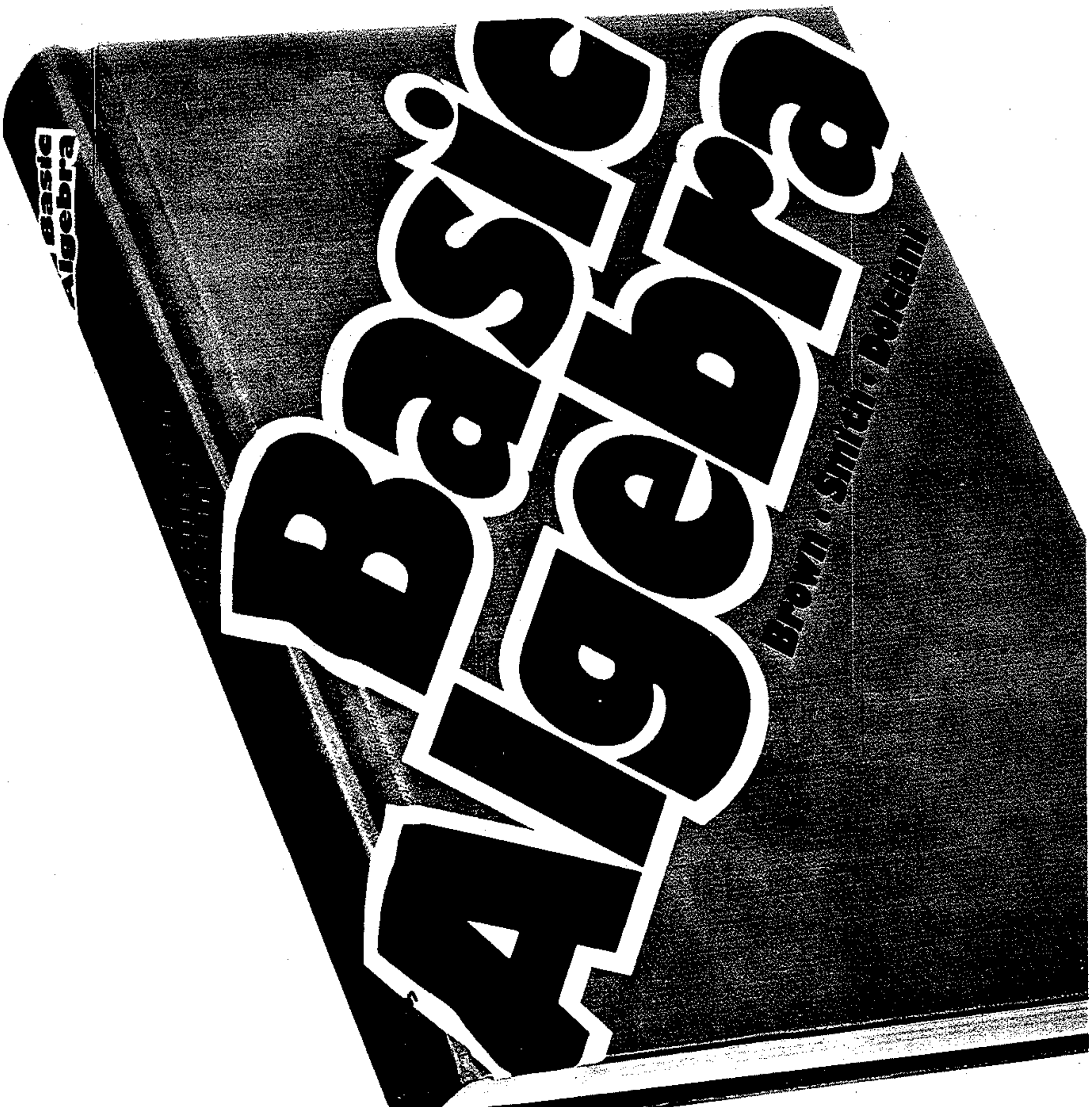

Sexism in Mathematics Education

by Floyd Vest
North Texas State University

Men tell jokes about their wives and other women who cannot balance their checkbooks, divide a restaurant check among themselves, make change, and do other minor calculations. The joke has been overdone, but many women do have problems with mathematics. Widely conducted studies have shown that after about the seventh grade, on the average males score higher on standardized mathematics tests than females. There are numerous explanations as to why the girls and boys, in general, don't display equal abilities in mathematics. Some of the causes may be stereotyping by textbooks and by teachers, limited advice from counselors and parents, biological differences including anxiety, and pressure from society influencing self-image.

The National Assessment of Educational Progress demonstrated that girls slowly but steadily lose ground to boys of ages 9-17[9]. Girls also trail on college entrance examinations. Last year girls scored forty-six points lower than boys on a six hundred-point scale in the mathematics component of the college board admission test. Even on verbal abilities, the girls scored six points below the boys[1]. Females tend to do more poorly than males at all ages in spatial relations[5]. According to the NAEP survey in 1975, at age seventeen females score considerably lower than males on consumer cost problems than males[9].

In 1973 at the University of California at Berkeley, fifty-seven per cent of the entering males had completed four years of high school mathematics, but only eight per cent of the entering females had the same preparation. Therefore, ninety-two per cent of the women in the freshman class were not eligible for courses such as calculus and statistics even though all but five of the twenty majors at Berkeley required calculus or statistics. Women students congregated in the other major fields — elementary education, guidance and counseling, humanities, music, and social work. Perhaps this pattern occurs not only because of sex-role vocational specialization but also because of failure to complete college preparatory mathematics[11].

Many educators feel the reason that girls' scores are lagging is not because they are any less intelligent. Bernice Sandler, Chairperson of the U.S. Office of Education's Advisory Council on Women's Programs, said "If anything, research shows that girls reach mental and physical maturity faster than boys"[1]. The cause for the lag probably results from a combination of factors such as career expectations, counseling, testing practices, and textbooks.

Mathematics textbooks are often found to contain sex bias. They generally overlook outstanding female mathematicians and their work. Females are often characterized as preoccupied exclusively with domestic chores such as buying sugar or cloth, and measuring ingredients for a recipe. The males calculate distances, construct buildings, and compete in sports. They also "help out" the females who aren't good at mathematics. The men manage large sums of money and do exciting things. The women spend less money and participate in less dramatic activities[10].

Some educators believe that the way mathematical knowledge is tested is often alien to women and that women's scores improve when a problem is placed in a familiar context. Karen McCarty, Director of Testing and Evaluation for the White Plains, New York, school system admits that "feminine-biased" questions may perpetuate stereotypes but says they also may be the only way that women can have fair treatment on achievement tests[2]. Many feel that it is all right to use purchasing and cooking examples for women, but textbooks should have the men doing some of the purchasing and cooking, and let the women also travel, have interesting jobs, and be intelligent. Household mathematics should still be included, but without stereotyping them as "feminine."

Parents, teachers, and guidance counselors often take a narrow view of the career possibilities open to girls. They may guide a girl into a "suitable career" such as secretarial studies, nursing, or teaching while failing to stress the availability of other interesting pursuits. One example is that of a high school counselor who advises a girl not to take an advanced mathematics course since she "already has all the math a girl needs." Girls at the high school age often have not decided on a career and succumb to such pressures and avoid mathematics and science. If at a later date they consider a technical career, many feel that it is too late to go back and catch up on the necessary mathematics prerequisites.

Biological factors haven't been ruled out as explanations for sex differences in academic abilities. Researchers have discovered that women are more anxious about mathematics and their feelings affect their performance more than men. In a study of the physical processes during testing (heartbeat, blood pressure, etc.), women were more nervous when doing the mathematics section and men were more nervous when doing the verbal section[2]. In another study, a mathematics test was followed by a discussion on how attitudes affect success, the women's scores on the subsequent test improved significantly while the men's did not.

Professor Jerome Kegan of Harvard found that children display inhibitions about learning subjects they feel are inappropriate for their sex[2]. Dr. Lewis Aiken, Professor of Psychology at Gillford College, states that, "The notion that mathematics is for boys can play an important role in a girl's conception of herself as not interested or competent in mathematics"[2]. In one survey students were asked why they did poorly on a particular mathematics exam. The girls generally answered that they lacked the ability while the boys answered that they didn't work hard enough[4]. Many women avoid mathematics; they immediately respond "I can't" when confronted with words such as "fraction, inequality, curve, or exponent." Our culture seems to assume that mathematics ability is a masculine attribute and tends to punish women for doing well in mathematics. Society excuses women by saying they don't have mathematical minds.

Girls generally make better grades than boys, but this may be because they are expected to be neat and "ladylike," which in turn pleases the teacher. However, attributes such as these do not have such a significant effect on standardized test scores. In elementary school, boys are expected to be rowdy and then settle down and learn in high school in order to prepare for college. For girls, the pressures from society are reversed. In elementary school, girls are expected to be teachers' pets, but in high school, they can't act too smart when it is time to acquire sexual acceptance from boys of their own age.

In one study of nearly 9000 American students from the fifth through the eleventh grades, researchers concluded that part of the reason for the disparity in mathematics test scores for the sexes was the feeling on the part of girls that mathematics wouldn't be useful to them in the future[2].

Dr. Kagan of Harvard suggests that "both parents and children must look on vocational requirements of mathematics and science as appropriate for women as well as for men. Only then will you have equal achievement"[2].

Steps are being taken to provide equal opportunity for the sexes. The most significant law passed in this area, Title IX of the Education Amendments of 1972, states that institutions receiving federal aid may not discriminate on the basis of sex. As a consequence more courses are open to both males and females. A change in the attitudes of teachers and counselors is expected. Unnecessary stereotypes in textbooks are being eliminated. Such programs as Pennsylvania's Program of Sexism in Education have set guidelines for textbooks as well as for counseling, sports, pupil services, teacher training, and evaluation[3].

The sex bias pattern is not just an academic problem. Increasing numbers of new jobs, especially the high paying ones, are in scientific and technical fields. Preparation for even the more traditionally "feminine fields" often now require statistics and calculus. Both vocational and academic fields are utilizing more mathematics.

The gap between boys and girls in the traditionally male dominated areas may never be closed completely, but as Ms. Bernice Sandler who chairs the U.S. Office of Education's Advisory Council on Women's Programs predicts, "People are going to be surprised what girls and guys can do once they get over the notion that certain things are 'tomboyish' or 'sissy' "[1].

Note: The Texas Section of the Mathematical Association of America is conducting a pilot visiting lectureship program for high schools in selected regions of Texas. For information contact Vivian Mays, Mathematics Department, Baylor University, Waco, Texas.

Free limited quantities of the brochure "The Math in High School . . . You'll Need for College" can be obtained from the Mathematical Association of America, 1225 Connecticut Avenue, N.W., Washington, D.C. 20036. Larger quantities may be purchased at a reasonable rate.

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Discovery of Patterns Using Finite Differences

by Sister M. Geralda Schaefer
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The story is told of Carl Friedrich Gauss (1777-1855) that at the age of nine he discovered a pattern for calculating the sum of the first one hundred counting numbers. He mentally "folded the list in half" and noted that there were fifty sums of 101 each, which gave a total of 5050.

$$\begin{array}{ccccc} 1+ & 2+ & 3+ & \ldots + & 50 \\ 100+ & 99+ & 98+ & \ldots + & 51 \\ 101 & 101 & 101 & & 101 \end{array}$$

At the age of twelve, he proved that the general term for the sum of the first n counting numbers is

$$\frac{n(n+1)}{2}$$

Another method of arriving at this formula utilizes finite differences. When a table is prepared and differences indicated, the following pattern emerges:

n	Sum	1st difference	2nd difference
1	1		
		2	
2	3		1
		3	
3	6		1
		4	
4	10		1
		5	
5	15		

Since the common difference appeared in the second subtraction, the general term is of quadratic form. A proof of the fact that a common difference in first subtraction leads to a linear general form, in second subtraction leads to a quadratic general form; in third subtraction, a cubic general form, can be found in Betty L. Baker's article, "The Method of Differences in Determination of Formulas", in *School Science and Mathematics*, April, 1967.

In our example, then, the general form of the sum is $S = an^2 + bn + c$. From the table we can write the following equations:

$$\begin{aligned} \text{When } n=1 \quad & a+b+c=1 \\ n=2 \quad & 4a+2b+c=3 \\ n=3 \quad & 9a+3b+c=6 \end{aligned}$$

Solution of this system gives $a = \frac{1}{2}$, $b = \frac{1}{2}$, and $c = 0$, so $S = \frac{1}{2}n^2 + \frac{1}{2}n$ or $S = \frac{n(n+1)}{2}$

As an alternative to writing and solving the system of equations, one could devise a table for the general quadratic equation $f(x) = ax^2 + bx + c$.

x	f(x)	1st difference	2nd difference
0	c		
		$a+b$	
1	$a+b+c$		$2a$
		$3a+b$	
2	$4a+2b+c$		$2a$
		$5a+b$	
3	$9a+3b+c$		$2a$
		$7a+b$	
4	$16a+4b+c$		

One-half the second difference is a; then the value of b can be determined by substitution in the expression, $a + b$; and the value of c is $f(0)$.

This pattern can be utilized in a motivational "trick". The student is asked to think of a quadratic equation of the form $f(x) = ax^2 + bx + c$ and then calculate $f(0)$, $f(1)$, $f(2)$ and give the results to the teacher.

For example, the students thinks of

$$f(x) = 2x^2 - 3x + 5 \text{ and calculates}$$
$$f(0) = 5$$
$$f(1) = 4$$
$$f(2) = 7$$

These three values are given to the teacher who quickly proceeds to find first and second differences:

5		
	−1	
4		4
	3	
7		

where $2a = 4$, $a = 2$; $a + b = -1$, so $b = -3$; and $c = 5$. The teacher then announces the original quadratic. Hopefully, the curosity of the students will motivate them to discover how the teacher performed the "trick".

Suppose our problem is to find the number of squares on a 100×100 checkerboard. As we analyze the situation and look for a pattern on the familiar 8×8 board, we discover the following:

size of square	number of squares
8×8	1
7×7	4
6×6	9
5×5	16
4×4	25
3×3	36
2×2	49
1×1	64

The problem is that of finding the sum of squares: $1 + 4 + 9 + 16 + \ldots + n^2$. Again we construct a table and compute differences.

n	Sum	2nd difference	1st difference	3rd difference
1	1			
		4		
2	5			
			5	
		9		2
3	14			
			7	
		16		2
4	30		9	
		25		2
5	55		11	
		36		
6	91			

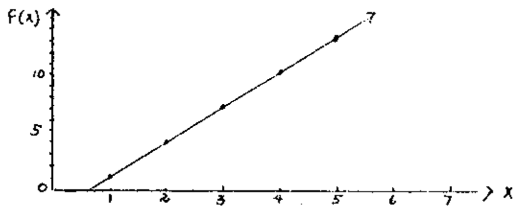
Since the common difference appears in the third subtraction, the common term is a cubic expression, $S = an^3 + bn^2 + cn + d$. A system of equations could be

$$\begin{aligned} \text{When } n=1 \quad & a+b+c+d=1 \\ n=2 \quad & 8a+4b+2c+d=5 \\ n=3 \quad & 27a+9b+3c+d=14 \\ n=4 \quad & 64a+16b+4c+d=30 \end{aligned}$$

Solution yields $a = 1/3$, $b = 1/2$, $c = 1/6$, and $d = 0$. So the common term is $S = \frac{2n^3 + 3n^2 + n}{6}$

or $\frac{n(n+1)(2n+1)}{6}$. When $n = 8$, $S = 204$, the number of squares on an 8×8 checkerboard. When $n = 100$, $S = 338{,}350$ squares on a 100×100 checkerboard.

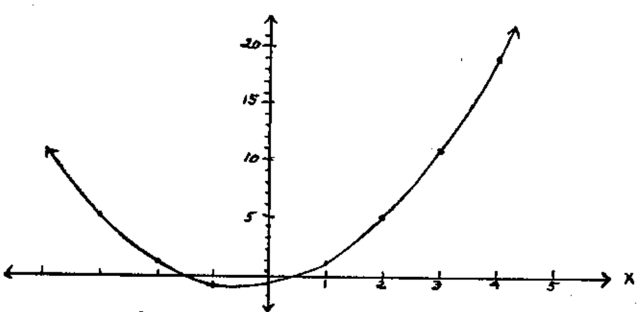
Another interesting application of finite differences—one related to analytic geometry—deals with finding the equation of a curve shown on a graph. For example, from the following graph, a table of values can be prepared.



x	1	2	3	4	5
f(x)	1	4	7	10	13
	3	3	3	3	

A common difference on the first subtraction signifies a linear expression, so $f(x) = ax + b$ and $a + b = 1$ and $2a + b = 4$. Solution of this system yields $a = 3$ and $b = -2$, so the equation is $f(x) = 3x - 2$.

Similarly, the equation of the accompanying parabola can be determined.



x	1	2	3	4	5
f(x)	1	5	11	19	29
	4	6	8	10	
	2	2	2		

The equation has the form $f(x) = ax^2 + bx + c$. The accompanying system of equation for

$x = 1, 2, 3$, is:

$$a + b + c = 1$$
$$4a + 2b + c = 5$$
$$9a + 3b + c = 11$$

Solving this system or using the general form above, it is determined that $a = 1$, $b = 1$, $c = -1$, so the equation is $f(x) = x^2 + x - 1$.

Other interesting problems that lend themselves to solution by means of finite differences are:

1. Find the relation between N, the number of points on a circle, and p, the number of points of intersection formed by chords determined by these points, where p is a maximum.
2. In a circle find the relation between C, the number of chords, and n, the maximum number of regions formed by the chords.
3. Find the sum of a series; e.g. $1 + 3 + 6 + 10 + 15 + \ldots + \frac{1}{2}n(n+1) = S$
4. Every year, before the first session of the U.S. Supreme Court opens, each justice shakes hands with the other justices. If there are 9 justices, how many handshakes take place? Suppose this practice took place in the House of Representatives? How many handshakes would there be?

This pattern discovery approach to problem solving is an illustration of a means of sparking interest in recreational mathematics which can add zest to mathematics instruction and learning. However, a word of caution is in order. Pattern recognition is fraught with danger since there are many examples wherein an apparent pattern breaks down at later stages. Hence, the careful teacher will present this strategy with the warning that it is not a substitute for a deductive proof.

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Problem Solving in the Intermediate Grades — The 4 D's

By Charles E. Lamb
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Problem solving is one of the major goals of instruction in the elementary school mathematics classroom. As such, the topic has received much emphasis in literature addressed to the teachers of young children. This brief article will present a new look at a possible way to motivate children to solve problems.

Polya (1957) suggested the following outline of strategy for problem solving:

I. Identify the problem — understand it;
II. Devise a plan for solution;
III. Try out the plan; and
IV. Reflect on the solution.

Often, teachers point out this sequence of steps for children to follow as they consider mathematical problems. The procedure to be suggested here will be a new adaptation of this long-used method.

Most children in the intermediate grades have developed reasonably good verbal communication skills. It is this strength which will be used as a foundation for developing problem-solving skills. The 4 D's of problem-solving are:

I. *D*etermine — What is being asked?
II. *D*ecide — What procedure?
III. *D*o — Solve the problem?
IV. *D*igest — What did I learn?

This sequence presents a new wording scheme for the Polya suggestions. It seems to be the case that a new way of wording or looking at things often gets kids turned on to ideas. It should be noted again that children need to have good verbal skills to work with words such as determine and digest.

This use of these words in this sequence has been informally field-tested with third-graders and found to be very successful. The use of an easily remembered "trick" seems to get the children excited and they they are "locked" into a method for looking at problems. It also appears that the kids get a "kick' out of using the "big" words such as determine and digest.

Teachers should be encouraged to use devices like the one suggested here. Tricks of the trade help to make the mathematics classroom an exciting and eventful place for learning to take place. Pupils tend to learn well when the learning process is fun and enjoyable.

Reference

Polya, G. *How to solve it.* New York: Doubleday and Company, 1957.

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