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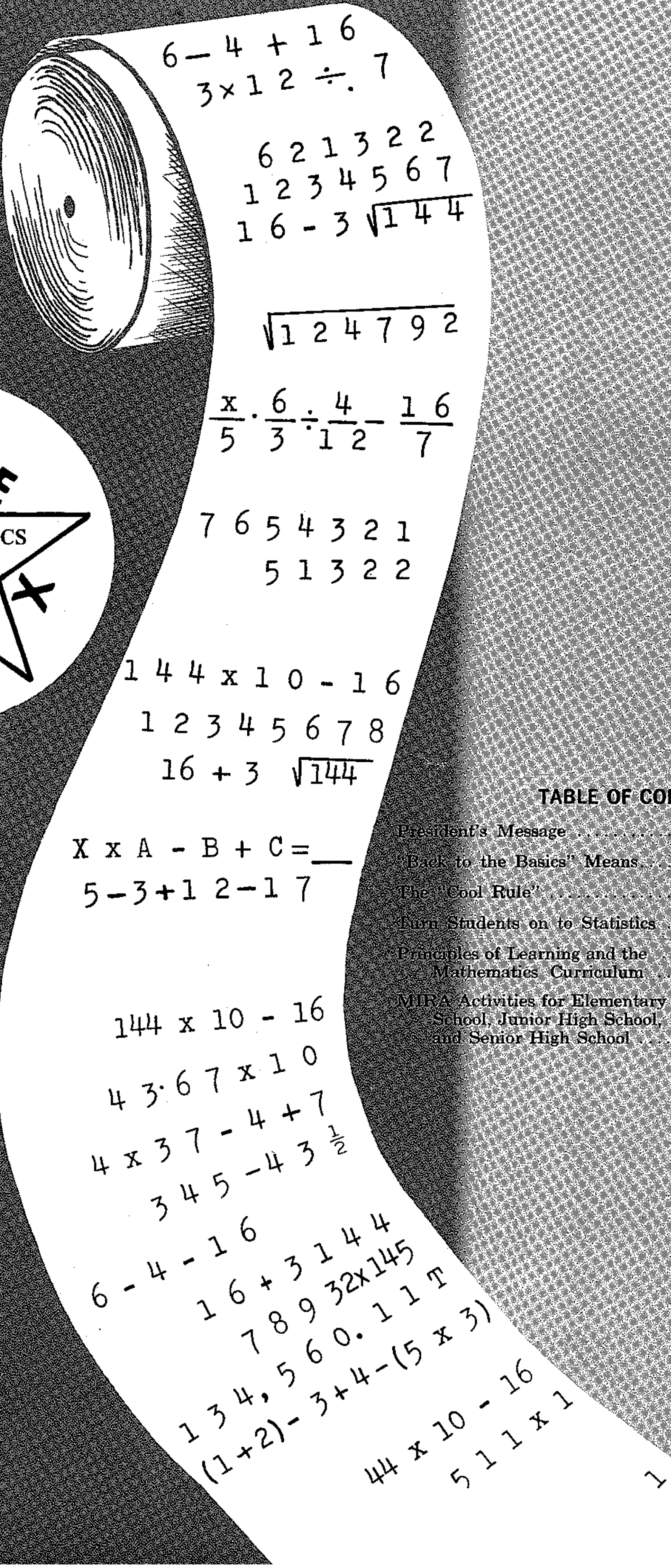

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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher,* 100 So. Glasgow Drive, Dallas, Texas 75214.

TEXAS MATHEMATICS TEACHER is published quarterly by the Texas Council of Teachers of Mathematics. Payment of membership fee of $5.00 entitles members to all regular Council Publications.

President's Message

Dear TCTM Member:

Welcome back for another school year! I hope you are now involved in many mathematical activities, and are looking forward to others. Certainly two such activities worth considering are CAMT, November 3-5 in Austin, and the NCTM Name-of-Site Meeting in San Antonio on February 2-4. If your district gives Continuing Education Units, attendance at either of these meetings may merit credit toward a unit.

Since the end of the last school year, several important things have occurred. TCTM has a new President-Elect, Anita Priest of Dallas, and a new Vice-President, Bill Stanford of Waco. Congratulations! I hope you will give these fine educators your support.

TCTM also has a new constitution. A copy of the constitution appeared in the TEXAS MATHEMATICS TEACHER (April, 1977). One of the important additions to this constitution is the creation of four positions for regional directors. The regional directors will be elected by members of the affiliated groups in each geographical region designated by the constitution. I hope that TCTM and the other Texas councils can agree on an equitable and suitable method for conducting these elections.

I look forward to seeing you at the TCTM luncheon during CAMT.

"You can't do today's job with yesterday's tools and be in business tomorrow." . . . Helen Macintosh.

SHIRLEY COUSINS

DATES TO REMEMBER

November 3-5, 1977:	CAMT, Austin, Texas
February 2-4, 1978:	NCTM Name-of-Site Meeting, San Antonio, Texas
April 12-15, 1978:	NCTM 56th Annual Meeting, San Diego, California

VOTER APATHY

During the school year 1976-77, members of Texas Council of Teachers of Mathematics had the privilege to vote on the revision of the constitution and in the election of officers.

But, the response was not up to expectations. The percentage of members who voted was lower than it should have been.

TCTM members will have future opportunities to vote. It is their duty to vote in any TCTM election, and they should take advantage of it.

"Back to the Basics" Means . . . .

by the MESA Executive Committee

Walter W. Leffin, Editorial Director

Educational fads and frills during the last two decades reduced instructional standards and played havoc with student morality is a claim advocated by many parents, businessmen, professional people, students, and educators.

A reaction to such criticism seems to be another educational fad term, "back to the basics."

However, in the area of mathematics there seem to be many different interpretations of "back to the basics." Some people interpret the phrase as meaning "let's go back to having students memorize that which I consider important." Others insist that there should be exclusive emphasis on computation. A more conservative interpretation is that there should be increased attention given to skills and their applications. Most of these types of interpretations seem to share the feeling that being trained to perform skill oriented tasks is much more important than understanding the processes involved.

Back to the basics means . . .

MESA endorses the position which defines "back to the basics" in terms of the program components which lead to mathematical literacy.

The mathematics curriculum, and the program based upon it, needs a balance which equates components such as arithmetic and measurement skills; knowledge and understanding of concepts; and applications of concepts and skills in real life or problem solving situations.

Students at all levels of mathematics need a special sequence of learning stages or experience for every topic. This sequence involves five stages, *readiness, understanding, mastery, application,* and *retention.*

Readiness means that the learner has all of the necessary prerequisite capabilities which guarantees meaningful learning. For example, long division cannot be learned in a meaningful way unless the prerequisite skills involving multiplication, subtraction, and estimation have already been mastered.

Understanding of each mathematical process is an important goal of developmental instruction. To memorize the fact that $3 \times 4 = 12$ does not guarantee that the learner understands multiplication. Recognizing that three groups of four objects, or that the points of intersection of three horizontal and four vertical lines are physical models of the multiplication fact demonstrates an understanding of the multiplicative process.

Mastery of learned skills, once they are understood, is an important goal of a well balanced mathematics program. *Mastery*, here, as related to basic facts, means immediate and accurate recall of any given fact. Performance criteria cannot be established for *mastery* except at the 100 percent level. For example, a student who can recall multiplication and addition facts with only 80 or 90 percent accuracy can *never* come up with correct answers to questions such as "what is the product of 7963 and 4588?"

Application of mathematical knowledge and skills while solving problems cannot be done unless mastery and understanding are considered prerequisites. Lack of such readiness inhibits the problem solving process.

Retention of learned knowledge and skills is essential if students are to continue study of mathematics or use mathematics already learned in problem solving situations. The well balanced mathematics program includes continued, periodic review and reinforcement; and teachers must carefully plan and implement this facet of the program.

MESA recommends incorporation of the readiness, understanding, mastery, application, and retention sequence of student learning experiences for every mathematics topic, kindergarten through college. This should happen within a program which demonstrates a reasonable balance between concept development, emphasis on mastery of arithmetic and measurement skills, and applications of mathematical processes.

What is MESA?

MESA (Mathematics Education Specialists of America) is a professional organization dedicated to studying current issues in mathematics education and publishing position statements on these issues. Prior to publication of MESA position statements, members approve through consensus their contents and authorize the MESA executive committee[1] to prepare the statements for publication.

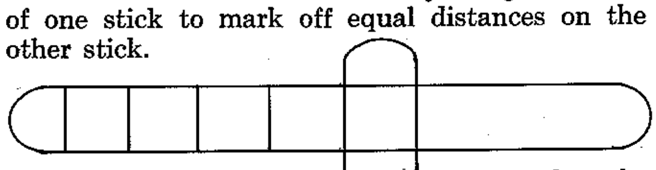
[1]MESA Executive Committee: *President Elden Egbers, Washington State Mathematics Consultant; Executive Vice President George L. Henderson, former Illinois and Wisconsin State Mathematics Consultant; Editorial Director Walter W. Leffin, University of Wisconsin, Oshkosh; Vice President (Central) John D. Aceto, Director of Mathematics, Unified Schools, Racine, Wisconsin; and Vice President (West) Ron Gutzman, Nevada State Mathematics Consultant.*

The "Cool Rule"

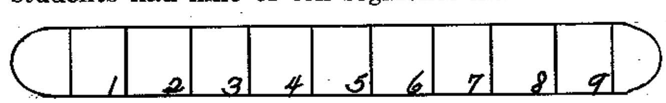
by Helene Silverman
Lehman College

The major portion of the undergraduate mathematics methods course at Lehman College is spent in urban elementary schools. Each student is required to prepare weekly lessons for a group of three to six children. The fieldwork is supervised by the college methods instructor and the classroom teacher. The methods students each bring all materials for the particular lesson of the day.

For one particular lesson, a Lehman student intended to teach non-standard measurement using popsickle sticks to a group of third grade children. After a few minutes, it became obvious that the lesson which had been prepared was too easy and the children in the group became very disruptive. It became necessary for me as the College Supervisor to redirect the lesson, without departing too radically from what the undergraduate student had prepared. I picked up from some of the clowning of the students, who began to call the popsickle sticks "cool rules". The students were asked to create smaller units of measure by using the width of one stick to mark off equal distances on the other stick.



Then the students were asked to number the distances away from their first mark. Most students had nine or ten segments marked.



One child decided to measure the segments with a "real ruler" and found that they were 1 centimeter!

The lesson proceeded naturally as the children asked to measure things with their "cool rule". It was decided that they should trace their hand and measure each of their fingers. Each child was asked to keep a record:

My longest finger is my_________. It is_________ centimeters long.

My shortest finger is my_________. It is_________ centimeters long.

The following fingers are the same size. They are _________centimeters long.

The sum of the lengths of all my fingers is_________ centimeters.

The children became curious about the measurements others had taken. So the student teacher created a chart:

Name	Thumb	Pointer	Middle	Index	Pinky	Total
Felix	5	6	7	6	5	29
Milagros	4	5	6	5	4	24
Dexter	5	6	7	5	4	27
Myrta	6	7	8	7	5	33
Carmen	4	5	6	6	5	26

The children were asked to generate comparisons. Most comments were in the order of: "My thumb is the same size as Dexter's" and "I have the most centimeters." One child noticed that the middle finger was the longest for everybody.

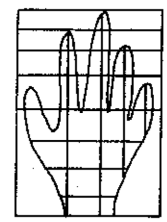
Carmen noticed that she had used her right hand while everyone else used his/her left hand. She insisted that both hands be measured. Carmen predicted that all right hands would be smaller. Unfortunately, there was not enough time to follow-up on Carmen's prediction, and the student teacher did not follow up at the next visit.

Students frequently copy from each other, and the following week another student repeated the lesson with a fifth-grade class. The children argued over what to do when the measurement did not fall at a line. The following rule was made: "If more skin shows, it counts; if less skin shows, it does not count." In practice this meant, round to the nearest whole unit.

Because the children in the fifth-grade needed practice with division, the children were also asked to find the average length of their fingers. One child decided to find out how far from the average each of his fingers were.

One child decided to find out the average thumb-length for the class. She also decided to find out if there was a difference for left hands and right hands.

The following week this student-teacher decided to follow-up the lesson. The children were asked to find the area and perimeter of their hands. The children were amazed to learn that they could outline their hands with string and then measure the string. Many of the children had to be taught "what to do when they ran out of ruler".



For area the children were asked to count the number of boxes their hands covered on graph paper ruled into 2 cm. squares. The children easily approximated irregular areas. However, they immediately concluded that the area they had found was in square centimeters.

Luckily, the teacher supplied Cuisenaire Rods, which the children knew to be centimeter cubes. The children found the graph paper contained four Cuisenaire Rods in each box. Some children immediately multiplied the area they had found by four. Others attempted to cover the hand with Cuisenaire Rods. I supplied a grid ruled into square centimeters (on reprocessed xray). The children placed the grid on top of their drawings and began to number units on the grid.

One child decided to measure her feet. This led into a discussion over whether the children with the largest hand measures would have the largest foot measures.

Unfortunately, time was up, and the final session was separated by a two-week interval.

Subsequently, I have used the cool rule with other groups of children. One particular group began to study descriptive statistics as they found the range, average, median and mode of various finger measurements. Another group created books of personal measurements which included fists, ankles, biceps, elbows, and knees. Still another group created a contest book which included: the biggest smile, the longest nail, and the longest distance between thumb and pointer.

It is obvious that this inexpensive, easy to transport tool is easy to implement in a classroom and can be used at a variety of levels of understanding of measurement and computation.

Turn Students on to Statistics

by Gerda I. Rosenbaum
School District of Philadelphia
Supervisor, Mathematics Education

The study of statistics can become a highly interesting activity when it is based on surveys done by the students themselves on topics which are of interest to them. This article describes a procedure used successfully with middle school youngsters. Steps involve deciding data on which to collect information and selecting the sample population; gathering the data, presenting them in statistical form, and summarizing them. Students of this age-group are quite capable of handling descriptive statistics tasks, but leaving inferential statistics to more mature students.

In today's society, statistical data are used increasingly to study current trends and forecast future developments. Surveys are taken in many areas such as opinion polls, product preferences, business sales, etc. Graphs are printed in the press and data read over TV, yet it is questionable how much of this information can be adequately interpreted by the average person. The NACOME Report[1] states that on the average less than 50% of the teachers treat the topic of graphs and statistics, as it's not found in their textbooks, is not part of their system objective, nor part of their testing program. If material is found at all it usually concerns itself with data with which the learner is not concerned: test scores of classes other than his own, sales of unknown companies, temperature readings of strange cities; thus working with this material has little relevance to the youngsters and does not act as a motivator. If, on the other hand, the students do surveys of topics they are interested in and then use the data for their own statistics, the activity will become more relevant and act as a motivator to drill and/or learn many skills.

Teaching a unit such as described in this article might take about a week or more. The sequence of suggested steps follows:

I. Preliminary Activities

Use available material to teach and/or review types of graphs, or better yet, prepare a graph concerning an item of interest to the total group. Students should be familiar with line, bar and circle graphs, and possibly pictographs. Present vertical and horizontal bar graphs, single and double line graphs, the concept of intervals; discuss the importance of legends. When working with circle graphs, be sure the students understand operations involving percent, cross-multiplication to convert percent to degree readings, use of the protractor and compass. Ask questions to check on the understanding of and ability to interpret the graph, concentrating on the measures of central tendency: mean, median, mode and range. These preliminary steps will establish readiness for the activities to follow.

II. Selecting The Topic—Some Suggestions

Individual students' height, weight, hair color, shoe size, shirt size, number of siblings in the family.

Favorite singer, type of music, time of day, TV show, spectator or participant sport, reading, food, pizza topping, color.

Other information: how many cars are in the family; source of news; job or educational aspirations; opinion poll regarding current events or school issues.

III. Collecting the Data

Prepare a class-list, leaving an empty space with a line next to each name (see IV). The student will circle his/her name to indicate who is making the survey, and list the title of the survey at the top of the paper.

Students should be urged to limit choices to five or six, or the graph will become too complex. One way to get around this difficulty is to include the category "other". Example: Preferred News Sources: choices could be: newspaper, radio, word of mouth, TV, magazines, other.

It is important that a procedure be established for the orderly passing of the survey papers. Each student responds to the survey question on the line next to his/her name.

IV. Making Up a Frequency Distribution

After a student's survey has been completed, he must present his data in a more orderly fashion than they appear on the sheet in order to have them make sense.

Example: How many siblings in the family?

List 0, 1, 2, 3, 4, more than 4

Adams, Charles	1
Burns, Adelaide	3
Camos, Juan	4
Demarest, Joan	2
	•
	•
	•

A frequency distribtuion sheet should now be prepared.

Sample:

Number of Siblings	Tally	Frequency
0		2
1		10
2		8
3		5
4		3
More than 4		4

V. Deciding on the Type of Graph

Depending on the information to be graphed, and the ability of the individual student, the decision must now be made. Students will need a great deal of individual attention at this stage. Allow youngsters to refer back to any available graphs, help them to decide on the scale and intervals. Help them with computation. Usually the more advanced students will decide on the circle graph, slower ones will stay with bar graphs.

VI. Preparing the Graph

Have available: graph paper, plain paper, rulers, compasses, protractors, different color felt-tipped pens or pencils, etc. Stress the importance of neatness, labeling, listing of the student's name, indicating the legend, etc.

VII. Interpreting the Graph

Have students interpret the graph orally or in written form. Stress measures of central tendency.

VIII. Displaying the Graph

Post the graphs on the class or hall bulletin board, or collect them in a loose-leaf binder. Have some graphs prepared on poster board for display purposes. Publicize the project within the school.

IX. Extended Activities

Have students do the identical surveys using different populations in school, or out-of-school, different in age and/or socio-economic background. Compare the results and have students become aware of the fact that frequently a different sample will yield different results.

It is hoped that the teaching of this unit will help youngsters understand how statistics come about, how they are interpreted, and the importance of the make-up of the sample population. In addition, going through the steps outlined will give students experience in following directions, doing arithmatical computations, using tools of geometry, and working closely with members of the group. It will help them to get to know each other better, and help them see that the teacher cares about them and their interests. It will increase their understanding of graphs and statistics which will be helpful with school assignments and, most importantly, in later life. The entire unit can be an exciting and worthwhile part of their math program.

[1]Conference Board of the Mathematical Sciences, National Advisory Committee on Mathematical Education. *Overview and Analysis of School Mathematics, Grades K-12.* Washington, D.C. 1975, p. 13,

Principles of Learning and the Mathematics Curriculum

by Marlow Ediger
Northeast Missouri State University

There are selected principles of learning that need adequate emphasis in teaching-learning situations in the mathematics curriculum. These principles or guidelines should aid pupils in attaining optimal achievement. Which principles of learning should the teacher stress in ongoing units of study?

Purpose in Learning

Pupils need to perceive purpose or intent to learn. Thus, if learners, for example, are studying a unit on "Uses of Graphs," adequate time should be taken to develop reasons for participating in ongoing learning activities. Pupils could be guided in developing a picture graph pertaining to visitors to the school-class setting during American Education Week or National School Lunch Week. Experiences that pupils have personally in these situations can be shown on the picture graph. A cutout or a drawing for each visitor can be put in the picture graph for each of the days of the week—Monday through Friday.

Pupils may be guided to notice that the contents of a picture graph can simplify information for readers. Thus, the reader of the completed graph may quickly notice trends in terms of visitors, for each of the days of the week, coming to the school-class setting.

Sequentially, as pupils progress through the public school years, more complex learnings may be stressed in developing diverse kinds of graphs such as in the following learning activities:

1. Showing population figures of diverse countries being studied in ongoing social studies units on picture, line and bar graphs.
2. Presenting data pertaining to growth in the Gross National Product (GNP) covering several decades within the framework of appropriate kinds of graphs.
3. Using relevant graphs portraying data on inflation covering selected years.

Intent to learn or reasons for attaining selected understandings and skills need to be emphasized when initiating a unit as well as when developmental and culminating activities are in evidence. Appropriate attitudes may be a significant end result when pupils perceive purpose in learning.

Interest in Learning

Interesting learning experiences need to be provided for pupils. Thus, learners may attend to and extract relevant information as well as abilities from ongoing activities. There are several methods to utilize in emphasizing the concept of interest in the mathematics curriculum. The teacher, for example, may utilize a variety of activities such as markers, place value charts, an abacus, filmstrips, slides, films, pictures, as well as content from reputable mathematics textbooks to stimulate pupil interest in learning. Varying learning experiences are necessary to develop and maintain interest in ongoing units of study. Attempts also need to be made to determine present achievement levels of each learner in the mathematics curriculum. Continuous progress then is possible when new objectives, related learning activities, and evaluation procedures are sequentially perceived by learners. Pupils may lose interest in learning if content to be learned is excessively complex or easy. Learning experinces need to be challenging but not overwhelming!

Problem Solving in the Mathematics Curriculum

Learners need to have ample opportunities to engage in solving realistic problems. Situations in life demand that human beings become proficient in problem solving. Thus, pupils should have ample opportunities to engage in the solving of real problems. Pupils with adequate background knowledge could solve problems such as the following:

1. A miniature supermarket could be housed in the class setting. Learners may bring empty cereal boxes, fruit and vegetable containers, candy bar wrappers, flour sacks, and sugar bags. These items should be placed on a counter, properly labeled and priced. Pupils may "buy" needed items using toy money. Thus, needed addition, subtraction, multiplication, and division facts may be learned in this manner.
2. A "cafeteria" could also be set up in the class setting. Cutouts of appropriate food items may be pasted on paper plates. Each food item would need to be priced meaningfully. Learners again may use toy money to purchase selected food items in the "cafeteria."
3. The mathematics laboratory concept of teaching and learning can well become an important facet of the mathematics curriculum. Thus, pupils may measure areas, distances, and determine volumes of specific containers in actual problem solving situations utilizing the English as well as metric systems of measurement.
4. Realistic problems may also be solved by pupils within the framework of the use of reputable textbooks, films, filmstrips, slides, video-tapes, and life-like situations in society.

Meaningful Learnings in the Curriculum

Pupils need to understand and attach meaning to learnings obtained in ongoing units of study. For learnings to be meaningful to pupils, the following criteria may well need to be in evidence:

1. Adequate emphasis placed upon manipulative materials, and semi-concrete materials before emphasizing abstract learnings in the mathematics curriculum.
2. Sequential experiences perceived by learners need to be inherent in teaching-learning situations.
3. Adequate readiness experiences to progress to increasingly more abstract levels of learning.
4. Pupil-teacher planning being a part of the mathematics curriculum.

If objectives are excessively easy to attain, selected learners generally will feel a lack of challenge in learning. Also, if the objectives in ongoing units of study are excessively complex, pupils may not have needed background experiences to make adequate progress. Thus, for each learner, new objectives to achieve need to be in evidence; however, pupils individually need to be successful in their attainment.

Providing for Individual Differences

There are diverse ways to provide for individual differences in achievement in the mathematics curriculum. The following, among others, are ways to provide for diverse achievement levels in the mathematics curriculum:

1. Use the mathematics laboratory concept in teaching-learning situations. Thus, pupils on an individual basis sequence their own progress through the actual weighing of selected items, measuring of surfaces, as well as finding the volume of selected containers.
2. Utilize learning centers in the school-class setting. Learners sequentially choose the task to work on at a particular center. Ideally, the task or learning activity provides for new challenging experiences.
3. Utilize problem solving methods. Pupils with teacher guidance may select realistic problems to solve on an individual basis. These problems must be on the present achievement levels of individual learners.
4. Pretest pupils using a reputable series of mathematics textbooks. Each learner is then at a different place within the confines of the textbook in terms of achievement. Pupils individually progress as rapidly as possible in satisfactorily completing sequential learnings in the textbook. The teacher gives explanations and guidance to learners as the need arises. Continual help is also given to diagnose errors made by learners in specific problematic situations. Remedial aid

is given to learners to overcome identified deficiencies.

5. Use contracts in the mathematics curriculum. Pupils with teacher guidance write up in contract form what the former are to achieve within a given period of time, such as a few days or a week. The level of accuracy of completed work may also be spelled out in the contract. Both pupil and teacher sign the agreement or contract. If the contents of the contract later appear too difficult for the learner to achieve, needed modifications can then be made.
6. Have pupils individually achieve objectives of diverse levels of achievement. Thus, for example, pupils who achieve at a less complex level, as compared to fast achievers, may be guided in attaining objectives suitable to their optimal level of development.

The teacher needs to think of and implement the principle of providing for each level of achievement in the mathematics curriculum. Only then, can learners attain optimal achievement.

In Summary

There are selected principles of learning which need adequate emphasis in the mathematics curriculum. These include:

1. Pupils perceiving purpose in learning.
2. Learners being involved in the solving of problems.
3. Meaningful learning experiences being inherent in the mathematics curriculum.
4. Provisions being made to guide each learner in achieving optimal gains in ongoing units of study.

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2. Grossnickle, Foster E., & John Reckzeh. *Discovering Meanings in Elementary School Mathematics.* Sixth Edition. New York: Holt, Rinehart & Winston, 1973.
3. Swenson, Esther J. *Teaching Mathematics to Children.* Second Edition. New York: The Macmillan Company, 1973.

MIRA Activities for Elementary School, Junior High School, and Senior High School

by Lynn H. Brown

Mathematics Department

Illinois State University Normal, Illinois 61761

The Mira is a recently developed mathematical teaching aid which has been gaining in popularity with teachers in elementary school, junior high school, and senior high school. It is made of red plastic (see Fig. 1) and it is used to obtain the reflection of a set of points in a line. For example, place the bottom (beveled) edge of the Mira on line l, as illustrated in Fig. 1, and look into the

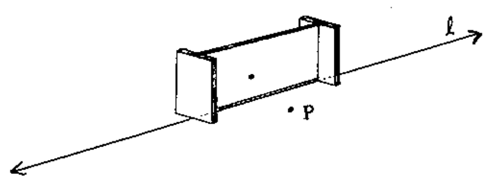

Figure 1

Mira from the side containing point P. The image of point P will appear in the Mira and it is possible to reach across the Mira and mark the image of the point P. It is best to use a very sharp pencil.

At the elementary school level the Mira is useful in teaching the concepts of congruence and symmetry. Students are first given experiences finding images of various figures such as fish, airplanes, and cars. They should be instructed to place their Mira facing the fish (see Fig. 2), draw the reflection-image of the fish, and then draw the Mira-line along the beveled edge of the Mira. The concept of congruence can be taught by having the students use the Mira to reflect a figure onto a second figure. If they are successful, then the figures are congruent. For example, in Fig. 3, it is

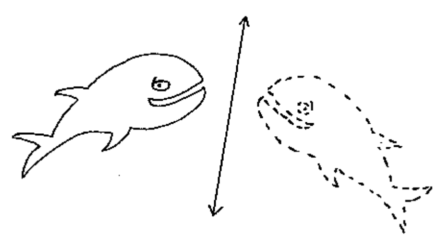

Figure 2

possible to reflect one figure onto the other. Request the students to draw the Mira-line if they are successful in their search.

Line-symmetry can be taught by having the students try to find a reflection-line which divides a figure into two congruent parts. In Fig. 4, by placing the Mira in the indicated position, the Mira-line reflects one-half of the figure onto the other half. Investigations enable students to see that circles have many lines of symmetry, squares

Figure 3

have four, rectangles have two, and isosceles triangles have one. Also, a triangle with three lines of symmetry is called an *equilateral triangle* and a triangle with no line of symmetry is called a *scalene triangle.*

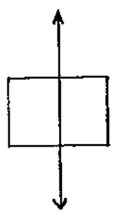

Figure 4

In the upper elementary and junior high grades the Mira is useful in constructing a perpendicular from a point to a line, the perpendicular at a point on a line, the perpendicular bisector of a line segment, and the bisector of an angle. To construct the perpendicular from a point P to a line l, place the Mira so that the Mira-line will contain point P. Move the Mira until the part of line l on one side of the Mira is reflected onto the part of line l on the other side of the Mira. Draw the Mira-line. This line is the desired perpendicular. If the point P is on line l, the construction is done in the same manner.

To obtain the perpendicular bisector of a line segment, place the Mira so that the Mira-line looks as if it is the perpendicular bisector of the segment. Move the Mira as much as necessary so that one endpoint of the line segment is reflected by the Mira onto the other endpoint. Draw this Mira-line which is the required perpendicular bisector.

The bisector of an angle is constructed by placing the Miro so that the Mira-line appears to be the bisector of the angle. It will be if one side of the angle is reflected onto the second side. If it is not, move the Mira as necessary. The Mira line must contain the vertex of the angle. When the necessary conditions are satisfied, draw the Mira-line. This line is the bisector of the angle.

These constructions are done much more quickly with the Mira than with a compass and straightedge. Once students are able to do these basic constructions, they can be guided to discover that the three altitudes of a triangle are concurrent, the three angle bisectors of a triangle are concurrent, the three perpendicular bisectors of the sides of a triangle are concurrent, and the three medians of a triangle are concurrent. These constructions can also be done with paper-folding[1] but not as quickly nor as accurately.

Transformation geometry is taught in many high schools today. It is also taught in the junior high schools but there it is often called motion geometry. The Mira is useful for students studying these topics since the basic transformation used is the reflection in a line. Students can be guided to the discovery of many important ideas such as the invariance (preservation) of distance, angle measure, lines, betweenness of points, triangles, perpendicularity, parallelism, and orentation under certain transformations. Worksheets might include exercises which instruct the student to use the Mira to find the image of several given line segments in a given line and then to find the lengths of the given segments and the lengths of their images. Space should be provided on the worksheet for the student to make a conjecture about the lengths. Later these conjectures provide the motivation for a good class discussion. A second worksheet could consist of exercises which ask the student to find the reflection-images of several different angles and then to use a protractor to find the measures of the angles. Again, a conjecture should be made. Similar worksheets may be used to guide students to the discovery that the reflection of a line in a line is a line, and that betweenness of points, perpendicularity of lines, and parallelism of lines are preserved.

Translations (slides) and rotations (turns) are also easily developed with the Mira since a translation consists of two reflections in parallel lines and a rotation consists of two reflections in intersecting lines. A worksheet might instruct the students to find $\triangle A'B'C'$ which is the reflection-image of $\triangle ABC$ in line l and then to find $\triangle A''B''C''$, the image of $\triangle A'B'C'$ in line m, where l and m are parallel lines (see Fig. 5). The students are then instructed to find the lengths of AA'', BB'', CC'', and XY. After doing this with several different triangles or other polygons, the students are asked to make a conjecture. Accurate constructions and measurement will guide them to the fact that $AA'' = BB'' = CC'' = 2(XY)$.

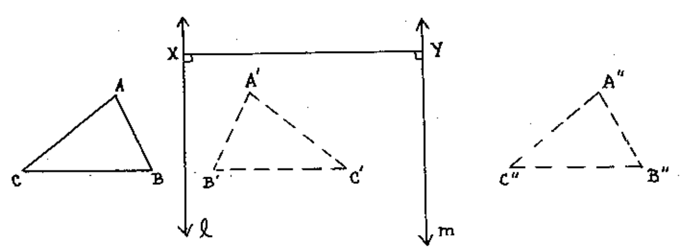

Figure 5

Also, the three triangles are congruent and orientation is preserved for $\triangle ABC$ and $\triangle A''B''C''$. If the lines l and m intersect in point P as in Fig. 6, again ask the students to find $\triangle A'B'C'$ which is the image of $\triangle ABC$ in line l and then to find $\triangle A''B''C''$ which is the image of $\triangle A'B'C'$ in line m. Instruct the students to use a protractor to find the measures of angles APA'', BPB'', $CPC''C$ and XPY. Accurate measurement will help them discover that $m\triangle APA'' = m\triangle BPB'' = m\triangle CPC'' = 2(m\triangle XPY)$. Again, the three triangles are congruent and the orientation of triangles ABC and $A''B''C''$ is the same.

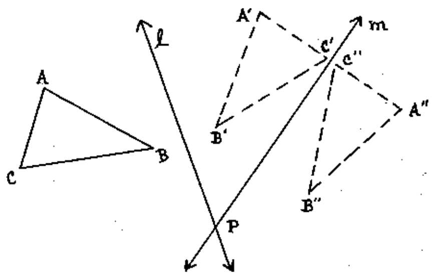

Figure 6

My last example of a use for the Mira is the construction of conic sections at the senior high school level. The parabola, ellipse, and hyperbola can be constructed by folding paper (wax paper is excellent for a bulletin board display) but use of the Mira is much faster. Use a worksheet consisting of a line and a point not on the line. Instruct the students to place the Mira so that the point F is reflected onto line l. Draw the Mira-line and then move the Mira slightly to reflect F onto a different point of line l and again draw the Mira-line.

If this is continued at least twenty times the result should look like Fig. 7. A circle with a point F in the interior is used to construct an ellipse. Reflect the point onto the circle and draw the Mira-line as before. If this is done twenty to thirty time, moving the reflected point entirely around

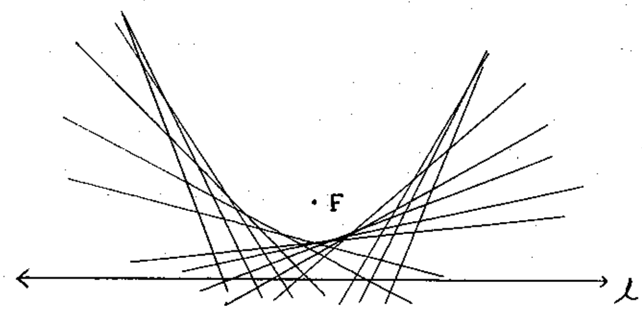

Figure 7

the circle, the result will be an ellipse. The focus points will be point F and the center of the circle. A hyperbola can be constructed by reflecting a point which is in the exterior of a circle onto the circle again making certain that the reflected point is moved entirely around the circle. In all three constructions it can be demonstrated that the definitions of parabola, ellipse, and hyperbola in terms of distances involving focus points are satisfied.

There are many more ways in which the Mira can be used effectively in K-12 classrooms. I challenge teachers at all levels to experiment with it. I am certain many will create new activities.

MIRA MATH FOR ELEMENTARY SCHOOL, Creative Publications, 1973.

MIRA ACTIVITIES FOR JUNIOR HIGH SCHOOL GEOMETRY, Creative Publications, 1973.

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