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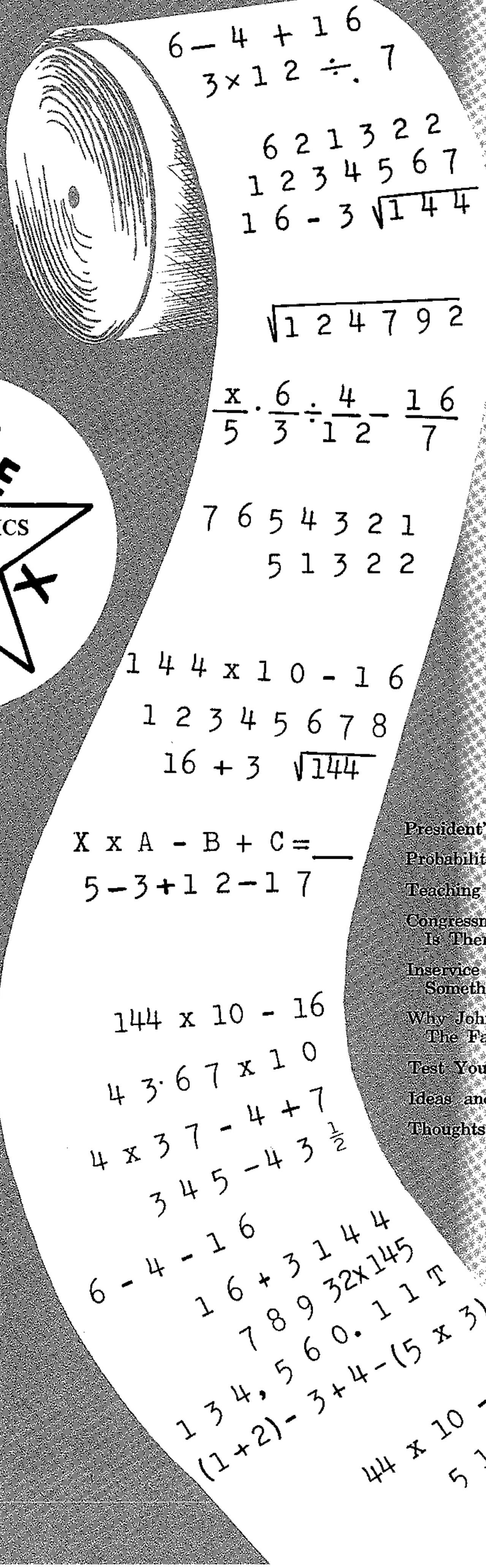

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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

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President's Message

Every school year seems to be more interesting than the year before, and this year has certainly been special. I feel very fortunate to have had the opportunity to meet and work with so many of you. I appreciate all of you and the help you have volunteered.

Next year will be a busy one for mathematics teachers in Texas. I hope you will be involved in as many activities as possible. If you are having a conference in your area, please let me know.

Houston Metro Math Conference, October 1, 1977, Stratford Senior High School, Spring Branch

C.A.M.T., November 3-5, Austin

NCTM Name-of-Site Meeting, February 2-4, 1978, San Antonio

NCTM Annual Meeting, April 12-15, 1978, San Diego, Calif.

Have a lovely summer!

SHIRLEY COUSINS

Probability — Magic or Mathematics? *

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The concept of a finite probability model is well-defined, well-developed and well-understood. High school Algebra 2 textbooks make it very clear that a finite probability model consists of a list or set of possible outcomes of an experiment together with a set of probabilities, one of which is assigned to each of the outcomes in the list. The set of possible outcomes is called the sample space or event set while the elements of the event set are called simple events or elementary events. The probabilities assigned to the simple events must be greater than or equal to zero and less than or equal to one and the sum of the probabilities over all of the simple events must equal 1. An event is a subset of the event set. The probability of an event A is the sum of the probabilities of the simple events in A . If A and B are events, then the event, either A or B, is the event $A \cup B$. While the event, A and B, is the event $A \cap B$. It has been my experience that students can without too much difficulty grasp these ideas. The difficulty for the student often begins when he or she is asked to decide on what probabilities should be assigned to the simple events in the model. In their counting of possible outcomes students are sometimes confused by whether order matters or by whether objects are distinguishable or indistinguishable. Confusion can also arise over the notion of independence and when probabilities should and should not be multiplied. It is my opinion that a good many textbooks do not provide the student with an adequate explanation concerning why one assignment of probabilities is preferred over another. This note is an attempt to supply that explanation.

I. Coin Tossing

Two coins are tossed and the number of heads that appear is recorded. If you, the teacher, ask your students to construct the probability model for this experiment, the answer you expect to receive is model I.2. However, you can usually count on having one student ask, "If the coins are indistinguishable, why isn't model I-1 the correct answer?" The numbers 0,1,2 which constitute the even set here refer to the number of heads.

Model I.1

Event Set	0	1	2
Probability	1/3	1/3	1/3

Model I.2

Event Set	0	1	2
Probability	1/4	1/2	1/4

You can often convince the student of the error of his or her ways by asking what would be their answer if a nickel and a penny were tossed rather than two indistinguishable coins. However, this is only partly satisfactory. Model I.2 is the correct answer because the probabilities given in that model will be seen to be approximated by the long run frequencies of the three elementary results, no heads, one head, two heads if the experiment of tossing two coins is repeated a large number of times. This can be demonstrated in the classroom by computing the frequencies that result when your students toss a pair of coins many, many times. However, a word of warning is in order. Unless you are prepared to have your students toss coins all period, you may be in for some embarrassment. During a talk which was based on this paper given at the El Paso Regional Meeting of the National Council of Teachers of Mathematics, my audience supplied me with a single head frequency half-way between one-half and one-third. Obviously in this case, 84 tosses of a pair of coins was not a large enough number of tosses.

(**Reprinted: Errors in October, 1976, issue*)

The real world phenomenon of long run stability of frequencies is basic to the building of probability models. It must be emphasized, however, that the stability property of frequencies is not a consequence of logical deduction. In the words of J. L. Hodges and E. L. Lehmann [2], "it is quite possible to conceive of a world in which frequencies would not stabilize as the number of repetitions of the experiment becomes large. That frequencies actually do possess this property is an empirical or observational fact based on literally millions of observations. This fact is the experimental basis for the concept of probability."

II. Tossing Two Marbles Into Four Urns

Our main point can also be made by considering the experiment of tossing two marbles into four urns. This experiment might be described by any one of the following four models

Here the two digits in each of the numbers in the event sets correspond to the urns that are occupied. The elementary event 13 corresponds to the result that urn #1 and urn #3 each contain a marble while the elementary event 22 corresponds to the result that both marbles find their way into urn #2. If one of the marbles used is green and the other yellow, then students can usually be expected to answer that model II.1 gives an assignment of probabilities that is appropriate for this experiment. However, if the students initially are told that the experiment is being performed with two red marbles, then some uncertainty may find its way into the minds of your students. If the experiment is to be performed with thimble sized urns that contain room for only one marble, then either model II.3 or model II.4 is appropriate, however you might expect to get some student disagreement over which of these two models should be used. A moments reflection, though, should make it clear to your students that the last two models are equivalent. To the result that both urns #1 and #2 are occupied both models assigned probability 1/6. If the marbles are indistinguishable then we use II.3 with the simple event 12 corresponding to this outcome. If the two marbles are distinguishable then we can use II.4 with the two-element event consisting of the simple events 12 and 21 corresponding to this result. However, confusion concerning II.1 and II.2 is not quite so easy to dispel. When a double occupancy is possible, the fact that the probability of the event that urns #3 and #4 are occupied is correctly given by model II.1 as $1/16 + 1/16 = 1/8$ and not by model II.2 as 1/10, whether the marbles are distinguishable or not, can be definitely established only after an examination of the frequencies associated with a large number of performances of the experiment. However, an approach similiar to the penny-nickel argument of the previous example can usually convince the unsure, thus removing the need for many, many repetitions of the experiment. Doubts, though, still may linger. If the marbles are indistinguishable then the event set for the experiment will have to be the event set of II.2. "What then?" your students might ask. In this situation we use the event set of II.2, but we do not assign equal probabilities to each of the simple events. Instead, we base our assignment of probabilities on model II.1, so that the simple events 11, 22, 33 and 44 are each assigned probability 1/16 while the remaining six simple events are each assigned probability $1/16 + 1/16 = 1/8$. A blindfolded experimenter will expect to find urns #3 and #4 occupied approximately 1/8 of the time whether the marbles are distinguishable or not. Both the coin tossing and marble tossing examples illustrate the general principal that in the macro world of our experiences objects behave as if they are distinguishable even though they may not appear to be so. When probability assignments are based on considerations of symmetry, this general principal should always be kept in mind.

Model II.1

Event Set	11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
Probability	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16	1/16

Model II.2

Event Set	11	12	13	14	22	23	24	33	34	44
Probability	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10	1/10

Model II.3

Event Set	12	13	14	23	24	34
Probability	1/6	1/6	1/6	1/6	1/6	1/6

Model II.4

Event Set	12	13	14	21	23	24	31	32	34	41	42	43
Probability	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12	1/12

With very much larger numbers, the previous models are used in physics. Urns become physical states and marbles become particles. Particles whose behavior can be described by models II.1, II.2, and II.3 are said to possess, respectively, Maxwell-Boltzmann statistics, Bose-Einstein statistics, and Fermi-Dirac statistics. Experimentation has revealed the surprising fact that in the micro world of particle physics there are particles that behave as if they are indistinguishable, that is, there are particles possessing Bose-Einstein statistics. Perhaps an even more surprising fact is that no known particles possess Maxwell-Boltzmann statistics. An assignment of probabilities like that in II.1 which some have argued in the past is inherent to the notion of randomness has been shown through experimentation not to be relevant to the study of the random behavior of physical particles. The fact that for appropriately defined physical states, no two electrons, neutrons or protons can occupy the same state and hence that these particles possess Fermi-Dirac statistics does not contradict our intuitive notion of randomness. However the fact that there are physical forces at work causing protons, nuclei and atoms containing an even number of elementary particles to behave as if they were indistinguishable, that is to possess Bose-Einstein statistics, does make us possibly want to look again at our concept of randomness. To what extent the physics of Bose-Einstein statistics is well understood we will leave to the physicists to debate. Our purpose in discussing the behavior of certain physical particles is only to make again the point that a "correct" assignment of probabilities in a probability model is that assignment that best reflects reality in the sense that the probabilities are well approximated by the relevant real-world long run frequencies. W. Feller [1] makes the statement "no pure reasoning could tell that photons and protons would not obey the same probability laws." With reference to Bose-Einstein and Fermi-Dirac, Feller goes on to say "the justification of either model depends on its success" and that this discussion "provides an instructive example of the impossibility of selecting or justifying probability models by *a priori* arguments."

The counting formulas used to compute the probabilities in II.1, II.3, and II.4 are, of course, a part of the material covered in most algebra 2 units on probability. If there are n urns and r distinguishable marbles and if multiple occupancy is permitted, then there will be n^r possible distinguishable outcomes of the experiment of tossing the marbles into the urns. If no more than one marble can occupy an urn, then the appropriate counting formulas are, for II.3 $\binom{n}{r}$, and, for II.4, $n!/(n-r)!$. In each of these three cases the desired probabilities are simply the reciprocals of the relevant counting formulas. The formula needed in II.2 is that which gives the number of distinguishable arrangements of r indistinguishable marbles in n distinguishable urns, or, equivalently, that which gives the number of distinguishable non-negative integer solutions to the equation $r_1 + r_2 + \cdots + r_n = r$. To see that this is the case think of r_i as the number of marbles in the i^{th} urn. The total number of marbles is then $r_1 + r_2 + \cdots r_n = r$. In II.2, $r = 2$ and $n = 4$. The solution $r_2 = r_3 = 1$, $r_1 = r_4 = 0$ corresponds to the occupancy of urns #2 and #3 and hence to 23 in the event set while the solution $r_1 = 2$, $r_2 = r_3 = r_4 = 0$ corresponds to the double occupancy of urn #1 and hence to 11 in the event set. Enumeration will show that there are 10 distinct non-negative integer solutions to the equation $r_1 + r_2 + r_3 + r_4 = 2$. Discovering the required counting formula can stand as a challenge to your better students. The fact that this formula is given by

$\binom{n+r-1}{r}$ is not immediately obvious although the clever stars and bars proof of this result found on page 38 of reference 1 does provide the reader with some insight. An algebra 2 student who understands the binomial coefficient counting formula will be able to follow the Feller derivation of the $\binom{n+r-1}{r}$ formula.

III. Independence and Sex Ratios

If A and B are two events and if the real-world outcomes corresponding to A and B are believed not to influence one another, then A and B are said to be independent and the assignment of probabilities within the model is made in such a way that $P(A \cap B) = P(A)P(B)$. The fact that it is the long-run behavior of frequencies that leads us to associate real-world unrelatedness and the multiplication of probabilities is a point that again I feel is not always clearly made in algebra 2 textbooks. Before presenting examples designed to clarify this point, it is necessary to construct a probability model with simple events that correspond to the sex of a newborn child. Then if we label these simple events M for male and F for female, it is tempting, due to our understanding of how sex is determined, to assign equal probabilities of 1/2 to each of the simple events M and F. However, once more there are forces at work which cause frequencies to differ from the probabilities deductively obtained from symmetry considerations. Data on the sex of newborn children suggest that the assignment of probabilities in model III.1 is closer to the mark than is the assignment 1/2, 1/2.

Model III.1

Event Set	F	M
Probability	.485	.515

Model III.2

Event Set	MM	MF	FM	FF
Probability	$(.515)^2$	$(.515)(.485)$	$(.485)(.515)$	$(.485)^2$

Table I

2nd \ 1st	M	F	Total
M	526	498	1024
F	506	470	976
Total	1032	968	2000

Next, let us imagine that we are interested in the sex of the first two children born in your county in the new year. If we decide to ignore multiple births, then our intuitive feeling is that the sex of the firstborn can in no way affect the sex of the second born. If the data in Table I were a record of the sex of the first and second births in 2000 counties across the country for this year, then if our intuition is correct we would expect to find that the ratio 526/1032 is approximately equal to the ratio 1024/2000, that is we would expect to find that among those counties that recorded a male first birth, the proportion of male second births would be approximately the same as the frequency of male second births in all of the 2000 counties. This, in fact, is the case. The approximate equality $526/1032 = 1024/2000$ can be rewritten as $526/2000 = (1032/2000)(1024/2000)$. Hence, if $A = \{MM,MF\}$ (the first birth is male), and $B = \{MM,FM\}$ (the second birth is male), then $A \cap B = \{MM\}$ (both births are male) and the approximate equality tells us that in our model we want $P(A \cap B) = P(A)P(B)$. III.2 should then provide an acceptable model for the experiment of noting the sex of the first and second born children in your county in the new year.

Let us turn next to an example that exhibits a lack of independence. A child is selected at random. Its sex is noted as well as whether or not he or she is color blind. The possible outcomes for this experiment are listed in model III.3. The simple event MC corresponds to the result that the child selected is both male and color blind while the simple event FN corresponds to the result that the selected child is female and normal. MN and FC have similar interpretations.

Model III.3

Event Set			
MN	MC	FN	FC
Probability			
(.93)(.515)	(.07)(.515)	(.995)(.485)	(.005)(.485)

Table II

	C	N	Total
M	72	958	1030
F	5	965	970
Total	77	1923	2000

The assignment of probabilities in III.3 must be based on observed frequencies. Referring to Table II, the ratio 72/77 is not even close to the ratio 1030/2000. Among those who are color blind, the male frequency, 72/77 is much greater than the male frequency in the general population, 1030/2000. Color blindness and sex are related. Hence, if $A = \{MC,MN\}$, the event that the child selected is male, and if $B = \{MC,FC\}$, the event that the child selected is color blind, then $A \cap B = \{MC\}$, the event that the child selected is a color blind male, and we do not want $P(A \cap B) = P(A)P(B)$. This, in fact, is the case in III.3 because (.07)(.515) does not equal (.515) [(.07)(.515) + (.005)(.485)].

Tables like Table I and Table II should, I feel, be used as teaching aids when independence is discussed in the classroom. You can form tables with numbers that are not based on actual population figures as was done in Table I and Table II or you can have your students generate actual data with, say, a coin and a die. Actual data is, of course, preferred but if you choose to use contrived figures, the numbers selected should produce frequencies that are close to frequencies that are based on actual data. The probabilities in models III.1, III.2, and III.3 are close to the frequencies based on a large number of actual observations. The important thing to keep in mind is that arrays of data like those shown in Table I and Table II provide the student with a quantitative bridge joining his intuitive notion of unrelatedness with the multiplication of probabilities within the model. The tables should help to make it clear that the independence of two events means that the frequency of the first event among those trials where the second event occurs must be approximately equal to the unrestricted frequency of the first event.

Summary

The "right" assignment of probabilities within a probability model is that assignment that best reflects the real world. Such assignments are based either on considerations of symmetry backed up by general experience or on a direct use of frequencies observed in long sequences of repeated expriments. In the world of our experience objects behave as if they are distinguishable so that when probabilities are based on symmetry considerations, objects should be treated as if they are distinguishable. The apparent exception to this rule that comes to us from physics is probably best looked at as a motivation for a challenging counting problem that you can toss out to your better students. Long run frequencies, besides serving as the final arbiter, in the assignment of probabilities, can also, if looked at in the right way, provide the student with additional insight into the notion of independence.

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1 Feller, William, *An Introduction to Probability Theory and Its Applications,* New York. John Wiley and Sons, Inc., 1968.

2 Hodges, J. L., Jr. and E. L. Lehmann, *Basic Concepts of Probability and Statistics.* San Francisco, Holden-Day, 1970.

Teaching by Consensus

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A major concern in education today is that effective instructional programs and strategies be devised for strengthening the development of decision-making processes in students at all levels of instruction.

Throughout Alvin Toffler's best selling book, *Future Shock*, it is emphasized that many more decisions need to be made each day in our present society than was true for previous generations of people and also that each decision needs to be made final within a smaller time period because of the increasing rapidity of change within our environment. Toffler concluded:

> For education the lesson is clear: its prime objective must be to increase the individual's 'cope-ability' — the speed and economy with which he can adapt to continual change.[2]

He emphasized that to increase the youth's "cope-ability" he must be put in situations which help him clarify his understanding of personal values, and also that changes are occurring so quickly that often today's fact might have no use for tomorrow. The student, therefore, must not be taught merely facts, but how to predict what will be necessary for the future. If he has been given the opportunity to explore and hypothesize within the classroom, he will become aware of making logical predictions.

Many teachers are utilizing some type of group process approach in attempting to attain decision-making goals. Where now there is resistance by many people to authoritarian direction and pronouncements, it seems that involving students in some form of group decision-making will be beneficial. By using the interaction process students are given greater opportunity to understand material which in turn helps them to make more cautious and correct predictions. It also teaches them to review all information more thoroughly before making a judgment.

Emory and Niland have written, "it has been said that more decisions are made through habits than by any other method."[2] If utilizing group processes enhances the formation of better individual habits in arriving at decisions, those processes become viable habit-forming strategies for each individual within a given group. Practice through debate enhances the willingness to search for and weigh more possibilities when called upon to make individual judgments since in group decision-making there is a broader background of experience to produce more varied suggestions. This promotes practice in the analysis of a wider range of alternatives.

It is important, also, that the educational system help the student develop a system of personal values. It is also important that he be helped to understand the values of others. Interchange of ideas and beliefs within a group helps to promote such an understanding. Making a choice, then, becomes an exercise in decision-making in which the students must know his previous values, identify the alternatives, investigate the consequences of those alternatives, and make a choice with which he is happy and which he can defend.

Of the various types of group activities utilized within a classroom, decision by consensus seems a favorable strategy to promote decision-making skills. Consensus has been utilized in higher education in the preparation of school administrators for decision-making with such groups as school boards, faculty staff, and the public has received limited publicity supporting its usage in other areas of instruction.

If consensus is to be used as an instructional strategy, however, viable avenues for acceptable implementation must be explored and analyzed. One basic factor that must be considered in any use of consensus is that students should first be given an opoprtunity to formulate individual opinions concerning a problem or situation prior to being placed in a group to arrive at consensus. A person cannot defend or argue about that which he has no preconceived concept.

One strategy utilizing consensus as a verbal problem solving instructional strategy was devised by Dr. Robert Blomstedt for a dissertation study in 1974. The conceptual framework for utilizing consensus in this study was designed to compare the effects of two treatments on two identified sets of students, the consensus (experimental) set and the expository feedback (control) set, on verbal problem solving ability and attitudes toward mathematics.

Ten problems were selected from a master problem list from which problems were adapted as a pretext. Each of the ten problems was of a different type problem as outlined in a problem type description. A ten-problem posttest was developed consisting of the same problems as those used in the pre-test with only names and/or numbers changed.

A sequence of problem exercises was developed for administration during the study proper. Each of these exercises contained five problems of one of the problem types used in the pretest and posttest. One problem on each of these exercises was the same as a pretest and posttest problem except that names and/or numbers were changed. Problems were generally adapted, according to problem type, from problems on the master problem list. Any other problems utilized were selected from the same fifth or sixth grade mathematics textbooks.

An introductory session on the use of consensus was conducted with the consensus set during the week immediately prior to the administration of the pretest. During this introductory session, all aspects for the use of consensus were thoroughly explained and any questions answered. The following week all sixth grade students in the school were administered the verbal problem-solving pretest and the Dutton attitude scale. During the succeeding ten weeks all sixth grade students in the school were administered, on a designated day, one exercise per week of the ten-set problem exercise sequence. Each student completed each problem exercise individually in duplicate. One copy was retained by the individual student for reference during the following activities:

1. The consensus set divided into groups (predetermined by the teacher) for each exercise and, using their individual work for reference, each group arrived at a consensus solution to each problem. Each group consisted of four students, whenever possible, and a recorder was named for each group to record solutions and to keep discussion flowing.

2. Students in the expository feedback set referred to their individual work while the teacher explained at least one possible solution to each problem. No special instruction relative to a particular problem exercise was given by the teacher prior to the administration of that exercise.

Time allocations for each set of students for each problem exercise were as follows:

1. *Consensus:*

Check roll and pass out problem exercises	5 minutes
Individual work and recopy	15 minutes
Collect individual work original copy and arrange class in groups	5 minutes
Work problem exercises by consensus	15 minutes
Collect group solution and put room back in order	5 minutes
Total	45 minutes

2. *Expository Feedback:*

Check roll and pass out problem exercises	5 minutes
Individual work and recopy	15 miuntes
Collect individual work original copy	2 minutes
Teacher explanation of problem solutions and discussion	20 minutes
Put room in order	3 minutes
Total	45 minutes

During the twelfth week all sixth grade students in the school were administered the verbal problem-solving posttest and again administered the Dutton attitude scale. The results of this study indicated that consensus produced significantly greater gains in achievement. There was no significantly greater gain produced by consensus in student attitude over this period of time.

It is felt that results indicated the need for further study of the consensus strategy as an instructional tool, not only in mathematics, but for other curriculum areas as well. It is hoped that sufficient motivation for additional research in the area of decision-making instructional strategies will be generated to create an ongoing exploration of phenomena that tend to create a more logically analytical and decision conscious society.

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Congressmen and the Zodiac: Is There a Relationship?

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Many people are interested in the signs of the Zodiac and believe these various signs influence the lives of persons born under them. They believe that distinct personality traits are associated with persons born under these different signs; they also eagerly read horoscopes in newspapers and magazines for their particular sign's predictions.

We tested the significance of the signs of the Zodiac in one particular instance, that is, the births of the U.S. Congressmen (the representatives and senators). Using Chi-Square, we tested whether or not *disproportionate* numbers of congressmen were born under the different signs of the Zodiac. Our null hypothesis was: The numbers of congressmen born under the various signs of the Zodiac are proportional to the numbers of people in the general population born under these signs.

To do this we consulted congressional records to learn the birth dates of the members of the 94th Congress (1975-1977); birthdates were listed for 496 of the 535 members. From this we determined the number of congressmen born under each sign of the Zodiac.

To determine the *expected* number of congressmen to be born under each sign of the Zodiac, we acquired census data listing the numbers of registered births per month during the years (since 1914) when most of the congressmen were born. Totaling the monthly data for all these years and assuming that births within each month were uniformly distributed, we computed the numbers of registered births for each sign of the Zodiac during the years in question.

This procedure yielded the ratios which were used to compute expected numbers of congressmen born under each sign of the Zodiac. For example, 3,271,258 of the 37,455,239 recorded births were born under Leo (assuming the births within each month were uniformly distributed). Hence we would expect there to be $(\frac{3,271,258}{37,455,239}) \cdot 496$ or 43.3 congressmen born under Leo; there were, in fact, 45.

The table below displays the number of registered births for the signs of the Zodiac (given our assumptions), the actual and expected number of congressmen born under each sign, and the ratio needed to compute the Chi-Square statistic.

Table I

Sign of the Zodiac	Nos. of Births	Expected no. of Congressmen (E)	Observed (Actual) no. of Congressmen (O)	$\frac{(O - E)^2}{E}$
ARIES *(Mr 21-Ap 20)*	3,226,960	42.7	37	0.76
TAURUS *(Ap 21-May 21)*	3,158,039	41.8	42	0.00
GEMINI *(May 22-June 23)*	3,165,472	41.9	37	0.57
CANCER *(June 22-July 21)*	3,329,078	44.1	35	1.88
LEO *(July 24-Aug 23)*	3,271,258	43.3	45	0.07
VIRGO *(Aug 24-Sept 23)*	3,265,974	43.2	57	4.41
LIBRA *(Sept 24-Oct 23)*	3,023,681	40.0	39	0.02
SCORPIO *(Oct 24-Nov 22)*	2,918,819	38.7	30	1.96
SAGITTARIUS *(Nov 23-Dec 21)*	2,783,944	36.9	47	2.76
CAPRICORN *(Dec 22-Jan 20)*	3,023,612	40.0	35	0.62
AQUARIUS *(Jan 21-Feb 19)*	3,165,129	41.9	36	0.83
PISCES *(Feb 20-Mr 20)*	3,123,273	41.4	56	5.15
TOTALS	37,455,239	495.9	496	$19.03 = X^2$

Using the .05 level of significance and 11 (or 12 — 1) degrees of freedom, the table Chi-Square is 19.68. The null hypothesis is not rejected. The evidence does not warrant the conclusion that disproportionate numbers of congressmen were born under various signs of the Zodiac.

One might conjecture from our data that no one sign of the Zodiac is associated with congressional personality traits. If, in fact, there are differences between persons born under different Zodiac signs (a question which our test did not resolve), it would appear as though these traits are represented in Congress in proportions similar to that of the general population.

Birth dates of famous persons are available in many reference sources in libraries; census reports can usually be located through large libraries. The reader is invited to construct and execute other tests related to differences between persons born under various signs of the Zodiac.

Inservice Training— Something Extra for Everyone

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An effective teacher develops an awareness of his students' attitudes, feelings, and expectations and tries to relate to them. A resourceful way to foster this awareness is for the teacher to put himself in the place of the student by returning to the classroom himself. Chances are he will find himself to be a very different kind of student after having taught and, subsequently, will likely be a better teacher upon returning to his own teaching situation. It is unfortunate that many of us teachers do not have the opportunity to continue our studies formally and so are unable to experience the revival that this type of student-teacher role switching effects.

Recently, my colleagues and I participated in an activity similar to inservice training for teachers and had the opportunity to reassume the position of students. I found it a perfect time to reflect, not only on the feelings and attitudes of my own students, but also on those of the teachers around me. Here are some of my observations:

1. The attitude of teachers in attendance at various required inservice training programs is very likely the same attitude manifested by the students who enter their classrooms every day.
2. The teacher's opinion that his seasonal bulletin board merits more attention than a metric workshop is very like his student's opinion that the playground equipment merits more attention than the multiplication tables.
3. The teacher's restlessness prompted by required listening to a lengthy speaker is not unlike that of his student dying to share some gossip but unable to talk for the duration of a lengthy class.
4. The teacher's interest in having the topics of sessions relate directly to his classroom activities with nothing extra is much like his student's interest in learning only what is practical for him.
5. The teacher's preference for conducting rather than watching a demonstration is very much like his student's desire for active rather than passive classroom participation.
6. The teacher's introduction to new procedures is not unlike the abundance of new material presented to his students by several people each day.
7. The teacher's preference for a session director who is well organized and able to interject clever remarks and cute stories relevant to the topic at hand is no different from his student's preference for a teacher who enlivens Monday mornings and Friday afternoons with some degree of "showmanship".

These parallels between teachers and students were made readily apparent by the activity of my colleagues and I was fortunate enough to be part of. Our experiences have elicited this plea. Teachers, let us take a look at ourselves and our attitudes to better see how our students feel. Let us put ourselves in our students' desks and then ask ourselves, "Do we measure up as a teacher to what we expect of our own teachers?"

Note: I am indebted to Ann Griffin for help in preparing this article for publication.

Why Johnny Can't Visualize— The Failures of the Behaviorists

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INTRODUCTION

The following occurrences can be recalled by most college mathematics instructors: Trigonometry students are unable to set up the problem about telephone poles casting shadows down a hillside; Calculus II students find the riddle about drilling a hole through the center of a ball bearing incomprehensible; Algebra students find it difficult to derive an equation from the riddle about rowing with and against the current; Calculus III students are unable to get the proper limits of integration when changing from rectangular to polar coordinates. The instructor usually comes to their rescue, sketches out a picture that represents the situation, labels the parts of the drawing and sets up the appropriate equation. The student usually makes a comment such as: "Why couldn't I set that up?"; "How simple it is when you do it!"; "How did you know how to draw that diagram?"; or "I don't understand." The next problem attempted is equally as baffling to the student and the patient tutor may spend a semi-infinite amount of time sketching out the diagrams that are implicit in the applied exercises. Why is it that so many students are unable to come up with any diagram that even remotely resembles the situation described in the problem? The student is unable to associate any visualization with the data related in the "story problem". This phenomenon we will refer to as "poor visualization".

Mathematics instructors refer to poor visualization by many different phrases. Some examples are: "The student can't read."; "He is unable to understand."; "He doesn't put in any effort."; "He needs to be spoon fed."; "He is immature."; "He doesn't care."; or "He is plain stupid." These remarks indicate a lack of understanding of the situation and are brutally unfair and insensitive to the student afflicted with poor visualizations. Certain equally brutal remedies are advanced, such as: "Fail the lot of them."; "Give them more to do."; "Have them hire a tutor."; "Stay with it until they get it."; or "Solve by the step process, first . . ." Most instructors find some way to avoid coming to grips with the situation such as: "Skip the story problems."; "Ask only about basics."; or "Stress the formulas." If some instructors persist in trying to remedy the situation by some means or seek to punish the students for their inability they are quite often reminded (perhaps by the Dean of Engineering) that "Our students don't need this theoretical math. What we want is mastery of the basic formulas." or (perhaps by the Department Chairman) to "Avoid these trivial applications. We waste too much time because the students are not able to set up the problems." One student was even heard to complain to a faculty member "I have always had trouble with story problems. Besides I have come here to learn not to think."

Poor visualization is not new but an increasing number of students seem to be afflicted with this disease. Why is it some astute students have developed good visualization and others are visually bankrupt? It is the purpose of this discussion to identify some of those facets of educational development that contribute to poor visualization.

INGREDIENTS FOR GOOD VISUALIZATION

In order to have a good visualization a person would have to have a sufficient "memory bank" of experiences to draw from and some practice in forming visualizations. The remembered experiences have to be of sufficient quality to warrant a vivid accurate recollection. These experiences are best realized if they consist of a concrete nature rather than an abstract one. The person should have been an active participant rather than a spectator. Experiences are best remembered if they produce a pleasurable reaction as opposed to a frightening, baffling or arduous reaction. In addition, experiences that are relevant, rather than those artifically contrived or intricate, yield a better chance of creating a visualization.

The problem posed must be appropriate to the developmental level of the student. Consistent experiences with problems beyond the student's level of abstraction causes the student to believe the attempts are futile. The problems should be clearly stated and as explicit as possible. The more closely related to the student's interest and experience the more likely a successful visualization of the problem will occur.

In addition to possessing the necessary background and having a good problem, the student should have an adequate reason for wanting to visualize the problem. In short, what reward can the student expect for attempting to visualize the problem? Direct rewards such as grades, acceptance of peers and/or instructor, or perhaps even the inner experience of intellectual contentment are important. Note here we stress a visualization of the problem, not necessarily the ability to solve the problem as visualized. Instructors could help encourage students to visualize by being willing to mete out great amounts of partial credit for visualizations and even attempted visualizations.

ENCOURAGEMENTS FOR POOR VISUALIZATION

Very early in life a child is encouraged to avoid visualization. The typical three-year-old is anxious to please the parents. Eating and toilet habits are beginning to conform to the ones desired by the parents. Most parents take pleasure in showing off the superior intellect of their child. The child is often encouraged to say verbal expressions that sound like "one, two, three, etc." The child has learned a verbal pattern that is encouraged and rewarded. He has not learned to count! Counting implies enumerating objects but the child does not relate the sounds he is making to the counting of objects. This is usually the first "mathemetical fraud" performed by the child. The child gives the adult the response desired, passes it off as mathematical understanding and receives a reward for "faking" the knowledge.

To cite a few other instances where a preschool child is encouraged to be a "mathematical fraud", consider the poem "1 and 1 is 2, 2 and 2 is 4, 4 and 4 is 8, etc." or counting backwards a la Sesame Street. Indeed we have heard that more preschool children can count backwards from ten than can count backwards from eight. During this period the child begins to be trained in answer emphasis. When asked to respond to a problem like "What is 3 and 3?" a wrong answer, say 4, might be given. When that answer is not accepted the child may begin to rapidly give out random answers in hopes of satisfying the questioner. For example, the responses might come 7, 2, 11, 8. This is one of the first indications that the child is beginning to realize that the answer is to be rewarded and not the reasoning that produces the right answer.

When the child enters the school system, a greater variety of adult forces begin to come into play. One which occurs very often is the rush to symbols and their manipulation. Many learning theorists claim that the child should still be operating at the concrete level well into first grade yet we find many teachers working on number concepts without the benefit of manipulatives. A great deal of pressure is put on the child to be an answer producer. The answers are the rote memorized responses to verbal sayings and quite divorced from the reality of the situation. Indeed, the ones most in need of the concrete experience are penalized for being "slow". (These are usually boys.) The child willing to play games with symbols that have no reality is highly rewarded and praised. (These are usually girls.) Many a time a teacher can be observed doting over the child who answers quickly and uses no manipulatives. The comment often goes something like "Sally can do the problems without the beans!" Many of these children, thought to be advanced by their teachers, are initiating the course that will eventually crumble when visualization skills become necessary.

Even if the child survives the first grade with good concrete experiences and a fair ability to produce visualizations, the pressures will continually try to drive the child off course. Second grade teachers have been heard to comment, for example: "Put down the 3 and carry the 1."; "Cross out the 3, change it to 2 and put the 1 over by the 5."; and "If you can only answer faster you will get into my three-minute club." The first two of these comments concern algorithms and are particularly conducive to memorization of symbol manipulation rather than reasoning and visualization. The last comment is referring to memorization of basic addition concepts where now the student is virtually forced to respond without thinking, i.e. make a conditioned response. Again, those performing the least desirable trait, mere memorization, are likely to be the ones rewarded while the visualizers and the reasoners receive little recognition for their efforts. Comments of third grade teachers are similar: "Multiply the 6 by the 7 and get 42, put down the 2, carry the 4, 6 times 3 is 18, add the 4 getting 22, put that down, and the answer is 222."; "We will keep doing this page until we can all get it in less than two minutes."; "Always in these story problems subtract the smaller one from the larger one." At this level, the teacher is running into the visualization problem. The last comment was a signal that the teacher was no longer going to ask the child to read the problem and figure some method of solving via a visualization but would accept an answer produced by any means. The suggestion of a rule for all problems on that page completely discourages the student from getting a clear understanding of the problem. A fourth grade teacher instructs: "Always divide the larger number by the smaller."; "When it says 'more' you subtract."; and "Cross out the 4, put a 9 above the 0 and a 1 by the 3." A fifth grade teacher sets the answer producing strategies: "Divide the numerator and denominator by 3."; "Don't ask how many 29's in 8763 but ask how many 3's in 8." or "Put the 2 under the 8 of the multiplier and carry the 3.' A sixth grade teacher encourages: "Don't pay any attention to 'ladder' division. This short cut will work better."; "Invert the divisor and multiply."; or "Slide the decimal point over two places to the right and add the % sign." A seventh grade teacher shares the secret of successful computation: "Divide the 'is' by the 'of' and you will get the right answer." Algebra instructors suggest: "Take the 5 to the other side and subtract."; "Line up the formula of 'rate x time = distance' and then plug in the values."; or "Memorize the quadratic formula then you won't have to try to factor." Some high school teachers say: "There is one way to work these problems. The first step is . . . . ., second step . . . . ., third step . . . . ., etc."

All of the above expressions are encouragements for the child to avoid thinking about the problem, to avoid coming up with a vivid visualization, and instead to come up quickly and somewhat magically with the proper answer. Succumbing to these suggestions leads the child into a fraudulent behavior of generating answers for problems he does not understand.

We are not against memorization of facts, algorithms and formulas but we are against divorcing this memorization from the essence of mathematical development. One of the greatest encouragers for avoiding visualization is the standardized time frame test. The plight of the fifth grade teacher on Long Island was described as follows:

> I know our textbook series stresses the "ladder" method of long division but it is too slow to use on the standardized tests (PEP) given to the children. We have decided at this school to teach the traditional algorithm rather than bother with the "ladder" method.

Here the format of the test has dictated a curriculum change which opts for the expediency of producing answers. This curriculum change can only help to convince the child that visualization is unimportant. We doubt the wisdom of any curriculum change that uses as its main objectives producing answers on standardized tests.

Standardized tests have in common the feature of aiming at superficial knowledge and never digging into the depth at which a student may actually comprehend the concepts. It is a shame that society places such a high value on the correct answers generated by these low level cognitive skills. We firmly believe that one of the best climates for improvement of real education would be one free of standardized time frame tests.

FORCES OPPOSED TO IMPROVEMENT

We believe that the behavioral psychologists are at the root of this problem and responsible for maintaining this answer-oriented and anti-visualization strategy. The bill of goods they have sold the school administration and teachers has the schools virtually concerned with pre-tests, post-tests, recording the level of skills, moving down the prescribed path, evaluating, rewarding and charting. They pay little attention to individual human needs, modality preferences, esthetics, problem solving, individual investigations, or, in short, all the aspects that are uniquely human. With the school administration and teachers dangling on the string of the behaviorists we see the producers of textbooks rushing to create materials that can be used by the teachers. One representative of a publisher informed us that they now have revised their texts so you can avoid the following: Missing addends, brain teasers, ladder method, stacked divisors, regrouping and metrics. Why? Because the teachers requested that these difficult topics be avoided. The students can't work them anyway so they may as well use a method that will be successful. Notice that all of the topics to be avoided are building blocks for good visualization.

PARTIAL SOLUTIONS FOR IMPROVING VISUALIZATION SKILLS

What would happen if you confronted an integral calculus class with the following instructions: "Below are five definite integrals. Write up two different real life practical problems which would eventually use the first integral in the solution of the problem. Form two problems for the second integral, etc." Might we have a better picture of whether or not the student understood integral calculus by his responses rather than by asking him just to evaluate these integrals?

Could we do something similar with fifth graders who have been given seven long division problems? Could we ask them to build story problems in which the division problems would be an intermediate step to the answer?

Could we ask first graders to tell stories about "3 plus 5" or "8 minus 3"? Could we ask preschoolers to count the objects won in games and compare that number with the objects won by their competitor?

Numerous analagous examples can be generated for every conceivable case. At the very least, problems of this type demonstrate to the student that there is value in visualization of the problem as well as in generating the answer.

REMARKS

Since mathematical skills, as measured by instruments designed by the behaviorists, have declined dramatically, we can conclude that the behaviorists' philosophy of teaching mathematics has failed. They have practically eliminated visualization skills from the educational process. Yet even with their continuing failures they somehow dominate school administrators, influence most teachers and are able to dictate the textbooks that are available for mathematics in the schools. Can we not call them to task for their past failures and ban them from influencing the mathematical development of children? Probably not. Their vested interest is too strong. Perhaps the few visualizers that survived and those that will continue to survive, despite the efforts of the behaviorists, will lead the way in providing opposition and alternatives for the mathematical education of our youth. The problem is always that people of vision have many demands on their time. The priority of the education of our youth may not be high enough to warrant their attention. It should be.

TEST YOUR METRICS

The following is a "Metric Shopping Trip" you might use to test your skills in the metric system. It won't be long before everyone has to start "thinking metric." The story below appeared in the Kansas Home Economics Association newsletter and also the Dallas Independent School District's *Homemaking Happenings*.

On a warm fall day with a temperature of 25 degrees ______, you drive 45 ______ to town, slowing down for a school zone as the caution sign reads 56 ______ per hour, you stop and buy 30 ______ of gas. Your next stop is the fabric store where you buy 4.5 ______ of fabric and a 15 ______ zipper. You borrow a tape measure from the sales clerk and recheck your measurements, bust 87 ______, waist 65 ______, and hips 92 ______ before buying a pattern. On the way out you notice some scales and decide to check your weight. You read the scales with pleasure, only 50 ______.

On to the drug store where you purchase film for your 35 ______ camera and some medication which has 325 ______ of acetaminophen. While at the drugstore you decide to have 500 ______ of soft drink at the fountain. A friend stops by and shows you the birth announcement for a new niece. The new baby's weight is 3.1 ______ and length is 49 ______.

One more stop and your shopping trip is complete. On to the grocery store where you pick up a ______ of milk, 454 ______ of coffee, 2 ______ of apples, a dozen eggs, one bunch of carrots, and a loaf of bread.

You can fill in the blanks using the following words: celsius, millimeters, centimeters, meters, kilometers, milligrams, grams, kilograms, liters and millimeters. And, of course, if you're really sharp, you can convert the figures.

1977 Nominating Committee Report TCTM

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Borrowed from The Greater Dayton Council of Teachers of Mathematics • January 20, 1977

WHICH WAY TO THE BASICS?

(Excerpts from the Summary of the National Council of Supervisors of Mathematics Position Paper on Mathematical Skills)

Mathematics supervisors are concerned that as a result of the "back-to-the-basics" movement, today in many schools there is too much emphasis on computation and not enough stress on other important mathematical skills. To respond to this trend, the National Council of Supervisors of Mathematics (NCSM) set up a twelve-member task force to write a position paper on basic mathematical skills. The position was first written in July, 1976, and later revised on the basis of ideas from supervisors throughout the country.

The position paper urges that we move forward, not "back" to the basics. The skills of yesterday are not the ones that today's students will need when they are adults. They will face a world of change in which they must be able to solve many different kinds of problems. The NCSM position paper lists ten important skill areas that students will need:

Problem Solving: Students should be able to solve problems in situations which are new to them.

Applying Mathematics to Everyday Situations: Students should be able to use mathematics to deal with situations they face daily in an ever-changing world.

Alertness to Reasonableness of Results: Students should learn to check to see that their answers to problems are "in the ball park."

Estimation and Approximation: Students should learn to estimate quantity, length, distance, weight, etc.

Appropriate Computational Skills: Students should be able to use the four basic operations with whole numbers and decimals and they should be able to do computations with simple fractions and percents.

Geometry: Students should know basic properties of simple geometric figures.

Measurement: Students should be able to measure in both the metric and customary systems.

Tables, Charts and Graphs: Students should be able to read and make simple tables, charts and graphs.

Using Mathematics to Predict: Students should know how mathematics is used to find the likelihood of future events.

Computer Literacy: Students should know about the many uses of computers in society and they should be aware of what computers can do and what they cannot do.

The role of computation is put into its proper place. Long computations will usually be done with a calculator, but computation is still important. Mental arithmetic is a valuable skill. Computational skills by themselves are of little use, but when used with other skill areas they give the learner basic mathematical ability. School systems which try to set the same requirements for all students should be aware of requirements which either are too difficult or which stress only low-level skills.

Rather than using only a single method such as drill and practice for learning basic mathematical skills, many different methods should be used. Hands-on experiences with physical objects can provide a basis for learning basic mathematical skills. Standardized tests are usually not suitable for measuring individual student progress. Instead, the tests used should be made especially to measure the mathematical skills being taught.

(TO OBTAIN THE NCSM POSITION PAPER, send a stamped, self-addressed business envelope to Ross Taylor, NCSM Basic Skills, Minneapolis Public Schools, 807 Broadway, N.E., Minneapolis, Minn. 55413.)

Summary of NCTM Position Statement on Basic Skills

The NCTM is encouraged by the current public concern for universal competence in the basic computational skills. The Council supports strong school programs that promote computational competence within a good mathematics program, and urges all teachers of mathematics to respond to this concern in positive ways.

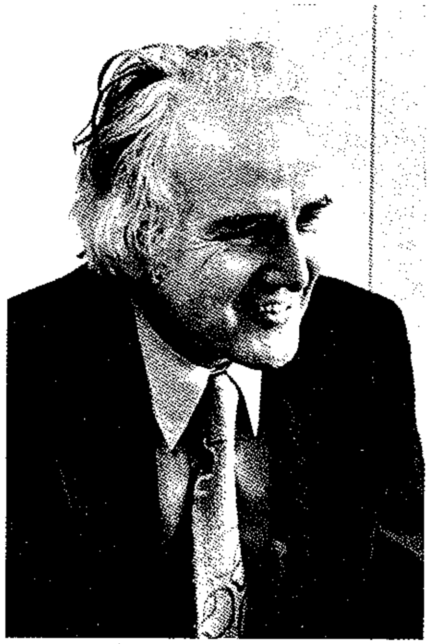
They are deeply distressed, however, by the danger that the "back to basics" movement might eliminate teaching for mathematical understanding. It will do citizens no good to have the ability to compute if they do not know what computations to perform when they meet a problem. The use of the hand-held calculator emphasized this need for understanding—one must know when to push what button . . . Computational skills in isolation are not enough; we must address skills, but we must address them within a total mathematics program . . . Students need to develop geometric intuition and the ability to interpret data, along with many other mathematical understandings to be mathematically functional . . . let us stress basics, but let us stress them in the context of total mathematics instruction.

Source: Excerpted from *NCTM Newsletter*, December, 1976.

Thoughts on Teaching the Slow Learner

Source: NCTM Newsletter, *March, 1977*

JOHN EGSGARD
NCTM President



ONE of my most enjoyable and challenging educational experiences in recent years has been the teaching of students classified as "not interested in mathematics." Most of my twenty-seven years in the classroom has been spent teaching the college-bound student at the junior and senior high school level. Such students accept mathematics as an important part of their curriculum. The majority of them have above-average intelligence. Therefore they are easier to motivate to study mathematics than the slower learner or non-college-bound student.

Five years ago, when I began to teach at my present high school, I encountered classes of non-college-bound students for the first time. I discovered certain important differences between these and the college-bound students. Many of these students did not want to be in school. Some did not want to learn, and others did not seem to be able to learn. Many knew very little mathematics. Most did not study at home. The whole classroom atmosphere was different. The noise level was higher. The students were unwilling to start working without first spending some time talking about things that were not related to mathematics. Many took delight in discussing with me whether or not I would give them a spare period, even though they knew from experience that I never gave "spares." Yet I found that *if* I could get them started, these students would work well, even though they could seldom concentrate for very long. Despite these differences, I was soon enjoying the slower classes, and so were the students!

Now some people claim that one of the most important things in teaching mathematics is to help each student enjoy the subject. I agree. But I have begun to realize that there is an even more important element that must precede enjoyment: each student needs to be successful. To teach for success, I have discovered that it is necessary for me to make mathematics as easy as possible by assessing the ability of a class as a whole and teaching material that each student can do. (I do not mean "trivial" mathematics such as the addition and multiplication facts. I allow the students to use the hand calculator.) Thus, I select only those parts of our textbook that are within the range of my students' ability. After I have decided which problems to assign, I teach the mathematics they must know in order to do these problems. I accomplish this best by having the class as a whole discuss problems of exactly the same type. After the discussion, I summarize *clearly* and *simply*.

I find that although I can present very little new work each day, I must review constantly. I cannot assume that my students know much mathematics. I begin each class by having them do problems from the chalkboard that review the work they did the previous day. We discuss these in detail to make certain everyone understands what is to be done. When we are ready, other problems on the board lead us to a discussion of the new work.

Because these students do not study at home, I must give them as much time as possible in class to work, usually over half the period.

Slow learners need much praise and encouragement. They constantly want to know how they are doing. Thus, during the work period, I am continually going from one to another assuring them that their work is correct or showing them where they have made an error. All of us work better if we are praised. There is no place for sarcasm or criticism in any classroom but especially in the classroom of the slow learner.

Because of their short attention span, slow learners find it difficult to work for a whole class period. For this reason I try to do something different with them at the end of three out of five classes, such as playing a game, discussing a puzzle, or the like.

I have also discovered that this type of student learns best by being tested, and so I give a test every five to eight days. We review for a couple of days beforehand by doing questions exactly the same as the ones on which they will be tested. Thus, the problems on the test are essentially the same ones that the students have been working on all along.

I was apprenhensive when I first entered a class of slow learners. Yet, I know now that my basic philosophy of teaching applies to the slow learner as well as to the college-bound student. Students will enjoy mathematics if they are treated with love and respect and taught at their level of intelligence so they can be successful.

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