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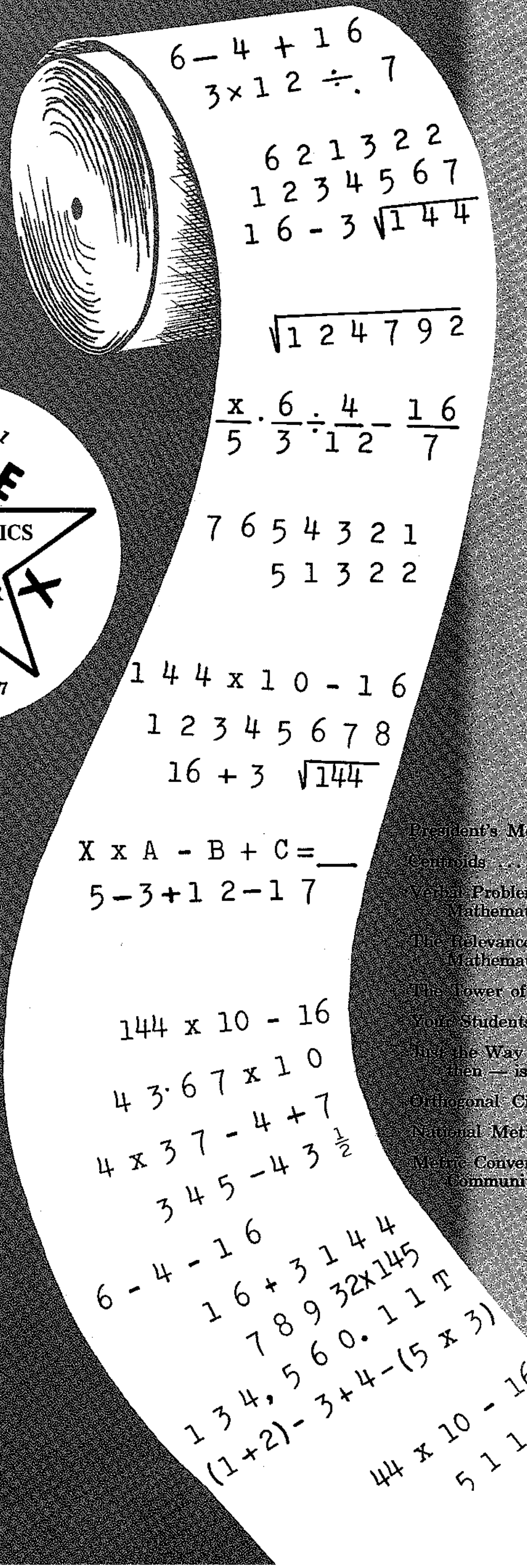

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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

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President's Message

CAMT has become an exciting part of each school year, and this year proved that the excitement can not be dulled by Texas weather. Almost 1200 teachers and student teachers attended this year's conference.

As I talk to the teachers in my building from other departments, I become increasingly aware of how lucky the mathematics teachers in Texas are to have strong leadership at every level. There are so many math educators who are willing to spend their evenings and Saturdays in planning meetings and sessions. Certainly there must be many other teachers in the state who are willing to share with us. Would you, for instance, be willing to serve as a presider for CAMT (3-5 November 1977)? If each of the local affiliates of NCTM could recommend members who would be willing to preside or help in some other capacity, I feel sure Ralph Cain, CAMT Program Chairman, and other committee chairmen would sincerely appreciate your help. If you, or anyone you know, is willing to help, please let me know soon. Don't wait. It is often very difficult to contact teachers during the summer.

Although CAMT was exciting, we can be assured that it is not the only interesting activity of Texas math teachers. In classrooms all over the state there are computer pilot courses, special metric programs, basic skill centers and hundreds of teaching techniques that can be shared with other teachers. Many excellent ideas may come from teachers in the other departments of your school. Some of these ideas may be illustrated in just a sentence or two. Would you please ask the teachers in your building what works for them? Let's share these ideas in an issue of the newsletter. Your experiences may save the day for another teacher!

During a recent meeting of the TCTM Executive Board the topic of textbooks arose. What do you think about the books that are currently available in Texas? What would you like to see in (or out) textbooks for your students? We would appreciate your comments. I think the publishers are really interested in what we think.

It may be lonely at the top, but it certainly isn't in the classroom or school building. Thank goodness! I know how much I need my students, the teachers in my building, my principal, the supervisors and the math organizations. They each provide unending support and encouragement and contribute to my personal and professional growth and experience. I hope that you derive part of your sustenance from our organizations. That is their only purpose!

SHIRLEY COUSINS

CENTROIDS

by Howard J. Rickard, Jr.
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Were you ever amazed by the basketball star that could spin the ball on one finger? Did you ever wonder why a boat may bob in the water, yet manages to stay upright? The answer to these situations is simply the common knowledge that the center of the weight is held in balance.

The point at which the weight may be considered to be concentrated is commonly referred to as the "center of gravity". For brevity, let us call these "balance points" simply centroids.

The concept of centroids is very old. It antedates Archimedes, who used it in the third century B.C. without, however, explicitly defining the term.

How does one find a centroid? Algebraically one needs to solve at least two equations of rotational equilibrum. An object will be in "balance" when the algebraic sums of all torques upon it is zero. The details of such calculation involve some mathematical processes not at the command of the average underclassman.

There must be an easier way! Indeed there is. The alternative method requires only that one possess the ability to construct the medians of a triangle.

For this discussion, let us consider convex plane figures bounded by straight lines. Figure 1 illustrates how one would locate the centroid of an obtuse triangle. Note how the intersection of two medians provide the centroid.

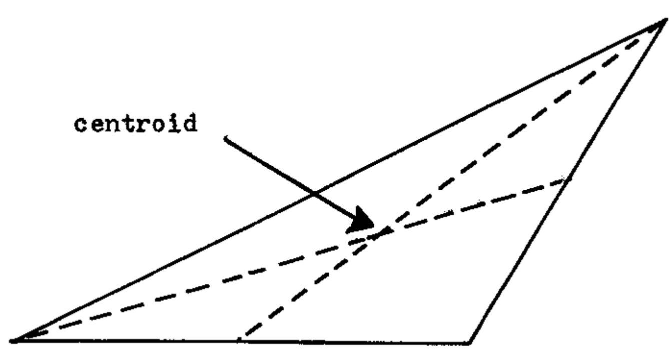

Fig. 1

Next, let us locate the centroids of a convex quadrilateral. Since we know how to find the centroid of a triangle, let us partition the quadrilateral into triangles, and find the centroid of each triangle formed. (see fig. 2) activity. This approach implies that verbal problems, demanding new skills or concepts for solution, should introduce the unit of instruction. In other words, the objective for the unit should be the ability to solve problems of the nature presented by the verbal problems themselves. Learning activities or procedures would be geared toward the teaching of all needed items—computational skills, vocabulary, etc.—that are essential to the goal of solving the prescribed verbal problems.

Let us suppose, for example, that a new concept to be learned is the process of division of whole numbers by two digit divisors. The unit instructional objective might be stated:

> Given 10 *verbal problems* involving division of whole numbers by two digit divisors, the student will correctly solve a minimum of 8 problems within a thirty minute period of time.

A verbal problem pretest would be constructed of the nature prescribed in the objective. Each of these problems would be designed to convey as much personal meaning to the learners as possible as prescribed in the theory promoted by Dr. Arthur Combs and other psychologists that any idea or new piece of information will be learned to the degree that it takes on personal meaning. Students are then instructed concerning the objective for the unit and administered the pretest which is designed to develop within each student a personal need for learning the concept. The remainder of the unit is devoted to the development of teaching/learning procedures necessary to the attainment of the goal outlined by the objective, including review of prerequisite concepts and skills, and the learning of needed new vocabulary and skills.

Such an approach to curriculum development reverses the traditional approach. Instead of being last, verbal problems would be the first item in each unit developed. Such a change would seem especially advantageous in the upper elementary, middle school, and secondary school programs. It is hoped that curriculum developers will give the concept consideration.

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The Relevance of the Nature of Plato's Mathematics or Why Study Mathematics?

by Anthony C. Maffei
Columbia, S.C.

Without warning a high school mathematics teacher's whole career can be put to the test when asked by a student, usually an inquisitive and restless average or above average one, "Why study mathematics?" Besides this writer's typical responses of 1) you need it for college or 2) it's logical or 3) it's challenging or 4) it has many practical applications in life, a sense of failing to provide an adequate answer was still felt by both teacher and student. The desire for a more comprehensive and deeper answer to the student's query led to a study of the wisdom of the ancients. The philosophy of Plato seemed appealing at the time and an answer to the general question, "Can Plato's notion of mathematics provide us with some type of universal justification for studying mathematics today?", was sought. The research follows.

Although Plato wrote no formal texts in mathematics, his insistence on precise definitions, clear assumptions, and logical proof laid the groundwork for Euclid a century later.[1] He believed that mathematical reasoning gave structure to our thinking:

> . . . measuring and numbering and weighing come to the rescue of the human understanding . . . and the apparent greater or less, or more or heavier, no longer have the mastery over us, but give way before calculation and measure and weight.[2]

Mathematics for Plato comprises mainly of three areas: arithmetic, geometry, and astronomy. Each area deals with a practical and theoretical knowledge. In *The Republic* Plato focuses his attention on the importance of the theoretical knowledge over the practical knowledge. Although a study of arithmetic or numbers has pragmatic significance, it is actually a means for attaining absolute truth:

> . . . for the man of war must learn the art of number or he will not know how to array his troops, and the philosopher also, because he has to rise out of the sea of change and lay hold of true being, and therefore he must be an arithmetician.[3]

The road to ultimate truth in arithmetic is in our ability to distinguish between the abstract idea and its physical representation. Today mathematics teachers also point out the difference between a number (the idea) and the numeral (what we perceive by the senses). It was interesting to find this distinction in Plato's *Theaetetus* around 2,400 years ago and emphasized recently by a mathematics educator:[4]

> . . . a mistake may very likely arise between the eleven or twelve which are seen or handled, but that no similar mistake can arise between the eleven and twelve which are in the mind.[5]

One contemporary philosopher views this distinction by Plato as his way of contrasting between pure ideas and particular objects of the senses.[6]

The real purpose of studying geometry is to gain knowledge, that is, ". . . knowledge of the eternal and not of aught persisting and transient."[7] The ideal circle and ideal square are more important areas of investigation than their imperfect material counterparts of our senses. We become increasingly aware that for Plato ultimate reality are real ideas which are partly discovered by studies in geometrical forms and numbers.[8] The world of our senses is changing, unreal, and imperfect, whereas the world of ideas, as approached in a study of mathematics, is changeless, real, and perfect.

Similarly, real astronomy is not a study of the movements of the stars and planets of our changing physical world. This realm is relegated to applied astronomy. Rather, pure astronomy directs us to lasting knowledge by compelling ". . . the soul to look upwards and leads us from this world to another."[9] The starry heavens are only a reflection of the ". . . true motions of absolute swiftness and absolute slowness, which are relative to each other, and carry with them that which is contained in them, in the true number and in every true figure."[10] Consequently, studying mathematics for Plato necessarily implies the interrelated pure disciplines of astronomy, geometry, and arithmetic.

Plato readily admits the difficulty involved for most people when studying these pure sciences. However, the fruits of these studies lead to a knowledge that is perfect and unchanging:

> . . . I quite admit the difficulty of believing that in every man there is an eye of the soul which, when by other pursuits lost and dimmed, is by these purified and re-illumined; and is more precious far than ten thousand bodily eyes, for by it alone is truth seen.[11]

However, mathematics for Plato involves only a study of the ideas of sensible objects as ends in themselves without ever actually being ultimate truth. The mathematician, therefore, remains mainly within the world of pure ideas of the imperfect sensible objects. The dialectician, on the other hand, uses the pure ideas of sensible objects as further hypotheses to an eventual knowledge of ultimate reality, that is, the true. This is knowledge ". . . which reason herself attains by the power of dialectic, using the hypotheses not as first principles, but only as hypotheses—that is to say, as steps and points of departure into a world which is above hypotheses, and . . . by successive steps she descends again without the aid of any sensible object, from ideas, through ideas, and in ideas she ends."[12] For Plato, then, dialectic is the culmination of man's reasoning capacity since it embraces ultimate knowledge. As one philosopher says, absolute truth for Plato is arrived at when the dialectician, after establishing hypotheses upon hypotheses, reaches an ultimate that is unhypothetical.[13]

Somewhere in between fleeting sensible knowledge of objects and absolute knowledge of truth, there exists the knowledge of the pure ideas of physical objects—the knowledge of mathematics. Mathematics for Plato is actually a means to the dialectician's end of arriving at the absolute. Plato's emphasis on sound logical proof as well as clear definitions constitutes the methodology of mathematical reasoning. In fact, the procedure of the Socratic method, which Plato employs in his writings, is a faith in deductive implications and an avoidance of contradiction and it is also the method of mathematics.[14]

Plato's conception of mathematics seems to be very modern and potentially motivating for high school students to study mathematics. Plato sees mathematics as dealing with the relative truth of on area in geometry or in arithmetic, as opposed to the dialectician who employs the ideas and methods of the mathemetician to attain knowledge of absolute truth. Similarly, mathematics is currently taught in the classrooms as a means to particular truth depending upon the types of definitions, terms, and assumptions made within that theory. For example, in Euclidian geometry the sum of the angles of a triangle equal 180°. This is not the case in Lobachevskian and Riemannian geometries where different terms and postulates are used to arrive at different conclusions from those of Euclid. Nonetheless, Euclidian and non-Euclidian geometries are true because of the valid reasoning employed within the domain of their systems.

Also, a correlation exists between Plato's stress on sound reasoning skills to arrive at relative truth in mathematics and a recent report by a National Council of Teachers of Mathematics committee on the nature of mathematics. The latter postulates that a training in mathematics attempts to enable us to distinguish between a valid and invalid argument in theorems by proceeding from assumptions to conclusions by means of a series of logical implications.[15]. However, logical reasoning is not merely relegated to mathematics alone. Plato employed his rigorous reasoning in such fields as politics and ethics to arrive at the notions of an ideal state and the good. Accordingly, reasoning abilities of mathematics have been used in such non-mathematical fields as English and history as long as clear definitions and assumptions were logically followed.

> If a given statement follows logically from a set of definitions and assumptions, then the statement is true within the framework of this set of definitions and assumption.[16]

What Plato meant by his idea of absolute truth is unimportant here. As with any philosopher, he is entitled to review reality as he sees it. The significance of Plato for mathematics students and educators is his stress on pure reasoning, as exemplified in mathematics, to formulate a structured thought system. The connection, therefore, is between refined reasoning abilities and the creation of a particular system of thought. The influence and appeal of Plato's thought remains widespread today. What better testimony can we give to Plato than to point to his development of sound reasoning principles? Of what duration and appeal would a philosopher or any person have whose thoughts were based upon loose logic, unstated assumptions, and unclear terms? In fact, when we think of some great philosophers, such as Aristotle, Descartes, and Bertrand Russell, we find that they either espoused the study of mathematics or were mathematicians. Of course, it is possible to reason correctly and to develop valid logic without ever studying mathematics. However, Plato and other philosophers would probably agree today that the creation of any valid system of thought is basically derived from the force of refined reasoning specifically learned through a study of mathematics.

After reading a particular passage from Plato, this writer was convinced that he had a more lasting response to the student's question of why we should study mathematics. The essential message is a distinction between valid reasoning versus nonvalid reasoning:

> Would the art of measuring be the saving principle or would the power of appearance? Is not the latter that deceiving art which makes us wander up and down and take the things at one time of which we repent at another, both in our actions and in our choice of things great and small? But the art of measurement is that which would do away with the effect of appearance, and showing the truth, would fain teach the soul at last to find rest in the truth, and would save our life.[17]

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The Tower of Hanoi Revisited

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The "right" assignment of probabilities within a probability model is that assignment that best reflects the real world. Such assignments are based either on considerations of symmetry backed up by general experience or on a direct use of frequencies observed in long sequences of repeated experiments. In the world of our experience objects behave as if they are distinguishable so that when probabilities are based on symmetry considerations, objects should be treated as if they are distinguishable. The apparent exception to this rule that comes to us from physics is probably best looked at as a motivation for a challenging counting problem that you can toss out to your better students. Long run frequencies, besides serving as the final arbiter, in the assignment of probabilities, can also, if looked at in the right way, provide the student with additional insight into the notion of independence.

There is a legend that at the beginning of time a group of monks in a monastery high in the Himalayas were presented with a device consisting of three diamond needles on one of which were 64 graduated discs of pure beaten gold. The monks were charged with the task of transferring the pile of discs from one needle to another according to the following rules: *1. only one disc can be moved at a time; 2. no disc can ever be placed over a smaller disc; 3. and all was to be accomplished in the minimum number of moves.*

This activity was to continue day and night. Upon completion of the task, the world will come to an end; the good will be rewarded and the evil punished.

Assuming that scientists are correct in estimating the age of the earth at about four billion years and assuming that the monks move one disc every second, is there any cause for alarm?

By using a puzzle piece similar to the drawing, students can complete the accompanying table showing the relationship between the number of discs (n) and the minimum number of moves (m).

This puzzle provides a good example of the use of mathematical induction. It is apparent that to move a pile of $n + 1$ discs, it is necessary to move a pile of n discs, then move the bottom disc, then move the n discs back again. For example, to move three discs, one must first move the top 2 $(m = 3)$, then move the bottom disc (1), then replace the top 2 $(m = 3)$ for a total of seven moves. The computations to the right of the table show this process.

Now, the number of moves for $k + 1$ moves needed in the proof by mathematical induction must be $(2^k - 1) + 1 + (2^k - 1)$ which equals $2 \cdot 2^k - 1$ or $2^{k+1} - 1$.

Now, back to our story. When $n = 64$, then $m = 2^{64} - 1$. Using a calculator or logs, students can compute the value of 2^{64}.

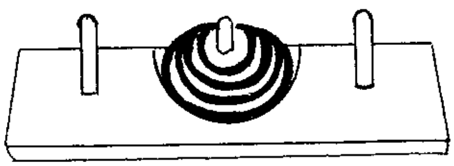
$$\log 2^{64} = 64 \log 2 = 64(.301) = 19.264$$

$$\log N = 19.264, N = 1.84 \times 10^{19}$$

N gives us the number of moves and since the monks move the discs at the brisk pace of one per second, we have number of seconds. Converting this figure into years yields

$$\frac{1.84 \times 10^{19}}{3.15 \times 10^{7}} = 5.85 \times 10^{11} \text{ years}$$

so we can relax for the next 581 billion years!



n	m	
1	1	
2	3	1 + 1 + 1
3	7	3 + 1 + 3
4	15	7 + 1 + 7
5	31	15 + 1 + 15
⋮	⋮	
k	$2^k - 1$	

Your Students: Spectators or Participants?

by E. Anne Wood
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Mathematics is a very abstract subject. Even the concept of the counting numbers—one, two, three . . . is an abstraction of the common quality of sets of objects containing that number of members. However, most of the abstractions of mathematics originated in concrete examples such as the sets of one, two or three. As mathematics teachers, we sometimes forget this and endeavor to teach abstract concepts without concrete experiences. Learning theorists tell us that learning must proceed from the concrete to the semi-concrete (models or picture) and then to the abstract. Particularly at the high school level, we are guilty of expecting students to skip the first two phases and proceed directly to the abstract level of learning. Only the brightest are capable of such a feat.

"Mathematics is not a spectator sport." Teachers say this to students over and over. Too many times we mean that they should participate in the sport of mathematics by working the problems which they have been assigned—a page of addition, a page of long division, a page of factoring, a page of solving equations. Too few times we mean that the students should be directly involved in the sport of mathematics. The typical mathematics teacher wants to tell the students what to do, show them how to do it; then have them go and do likewise. It is the rare and sensitive teacher who leads his students through a learning experience by having them participate in the development of a technique, has each student decided for himself how to do the work at hand, and then do it in the manner best suited for him at that moment in his mathematical development. This kind of teaching requires more preparation than paper grading; more involvement by the teacher with his students; and more involvement by the students in learning.

To participate in a sport means to be active, not passive; to have some input concerning the outcome; to be involved in the game, not just the final score. Yet we teach students to listen passively; to do what *we* say in *our* prescribed manner; to be concerned with answers, not processes. This makes the teacher the participant and the student only a spectator.

There are many ways to turn the spotlight from the teacher to the students. An easy way is to turn the customary teacher-lecture into a student-discussion. Since mathematics is a sequential subject, each topic to be taught must be based on some prior knowledge. Since teaching mathematics uses the spiral approach, the students may already be familiar with the topic. Socrates said that a good teacher does not just impart knowledge; he makes the student aware of what he already knows.

Another method is to devise a series of active learning experiences for students to follow as a class, as small groups, or as individuals. This may be formalized with learning centers for individuals or groups. It may be a list of activities for each student to perform. Or it may be just a teacher-conducted class activity. The important thing is for the students to be directly involved—not just listening. Activities designed by the classroom teacher are the best, but there are commercially prepared activity cards to use directly or as models.

Another technique is to substitute a game or an activity for drill exercises. The old-fashioned spelling bee had its good points. In using a game, it is important not to embarrass poor students, to provide practice for all students, and for some element of chance to prevent the good student from always winning.

At a time when more of our students are dropping out of school, when truancy is on the increase, when too many students are present in body only, teachers must conduct classes that encourage students to stay enrolled in school, to attend regularly, and to be a participant in class, not a spectator. Is yours such a class? Are your students participating in the sport of mathematics, or are they merely spectators?

Just the way it reads: "If__of__, then__is__of__?"

by William E. Goe
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As a writer of textbooks, one is forced into unholy association with those low creatures known as *editors,* whose job it is to ask embarrassing questions about manuscripts. The other day this note appeared on an exercise in some edited material:

"What sense will this make to the student?"

The exercise amounted to this:

"Mr. Boss left the hose across the sidewalk one night after he had finished watering his lawn. That night a lady tripped over the hose and broke her leg. She sued Mr. Boss and the court awarded her $5,000. Mr. Boss earns $8,000 a year.

a. If Mr. Boss was saving 8% of his income, how many years' savings would it take to pay the $5,000?
b. If personal-liability insurance costs $12 a year for $50,000 protection, how many years' premiums could Mr. Boss have paid with the amount of money the court required him to pay?"

Some quick calculations show that the answer to *a* is about 8 years, an amount of time which is realistic. The answer to *b*, however, is 416+ years, a time segment extended sufficiently to approach absurdity.

One has to agree with the editor. The whole point of the problem was to illustrate that insurance costs are cheap when compared to large losses. Yet this point was lost by the writer's not asking the right question. A better question would have been:

"If Mr. Boss had paid twelve premiums since he owned the house, how much more was the court award than the total of the premiums?"

Shortly after this experience, the following example was noted in a sixth grade arithmetic book:

"Tony said he was going to paint his room using this plan:

$\frac{1}{2}$ of the room today, $\frac{1}{4}$ of the room tomorrow, $\frac{1}{8}$ of the room the next day, and so on.

a. How much will he have painted afted 4 days? After 5 days? After 6 days?
b. Will he ever finish painting the room?"

Granted that problem-solving examples involving the use of fractions are hard to come by, but this one seems totally unrealistic. Imagine the setting. Tony measures his room carefully in order to determine what half of it will amount to. Then he paints and cleans up. The next day he measures and makes the mess again. What is truly insufferable is that theoretically he will never finish. This is not the object in painting a room. The thought would be enough to keep Tony from even stirring the paint.

If one looks at the problem from standard practice, painting belongs mathematically in an area setting. The process of painting entails computing wall area in order to know how much paint to order. It also includes elimination of time to complete the job. Tony deserves to be set in a similar situation.

Every teacher could contribute several similar examples. Who hasn't heard his students remark, "What a stupid problem!" or "A better way would have been ——."

What are some guidelines for writing good math problems?

1. The problem setting should be in the realm of the *students' experiences*. It is really not too practical to talk about consumption of gasoline in miles per gallon to students who do not drive. Nor is it realistic to talk about purchasing material to make a dress to students who know nothing about sewing, or about building a patio to those who have lived in his-rise apartments all their lives.

2. The problem setting must be in the realm of *students' current interests*. One must be aware of youngsters' interests. Most teen-agers relate to areas involving dating, owning cars, sports, buying clothes, movies, earning money, recreation and travel. Many elementary school pupils are taking part in organized groups such as athletic teams or scouts. They care for pets, make models, go on trips, carry papers, do lawn work and the like.

3. Supply *reasonable data*. Make sure that facts are those which might expectably fall within pupils' experiences. Very few elementary school pupils, for instance, earn more than 75¢ to $1.50 an hour working at any job, and most secondary school students earn money within the range of $1.50 to $3.00. Therefore, it does little to ground the problem in reality to talk about earning $7-8 an hour with time-and-a-half for overtime. Similarly, sporting-event statistics that approach world records (9.2-second hundred-yard dashes and 8-foot-plus high jumps) have less verisimilitude for pupils than those which prevail at high school events.

4. Ask a *question* whose *logic* can be justified. Be sure the question asked is a typical one for the data given. It is not justifiable to give the weights lifted by the gold-medal, silver-medal and bronze-medal winners at the Olympics and then ask, "What was the average weight lifted?" *Average* has no meaning in this situation. A proper question would involve some relationships about the weights. "How many more kilograms of weight were lifted by the gold-medal winner than by the silver-medal winner?"

   One good technique in writing word problems is to begin with the question. This is normally the way problem-solving situations actually occur—initiated by a need to solve a problem. As an example:

   "Does John have enough money? He has $8.00. He plans to take Jane to a movie whose admission is $3.00 per seat and have pizza and cokes afterwards. A pizza will cost $1.95 and two cokes will cost 80¢."

   This is really John's problem. He wouldn't worry if he had $100. The fact that he knows his level of expenses and his resources to meet them gives a hard, realistic edge to his worrying.

5. If the setting becomes too contrived to support a problem, make a drill example or an algebraic equation out of it. The problem of Tony's painting his room was contrived in order to practice adding fractions. It should simply have been:

   $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = x$

Nothing has been said here about some of the other standard emphases of publishers of textbooks: keeping the reading level below the current level of the student by one or two years, working for precision in the use of words, conveying ideas in terse sentences. Such are other concerns of writers, of course.

An abiding concern of a teacher, however, is keeping up with his pupils. This writer recently visited Room 107 in an elementary school and noticed perplexity furrowing the brow of one second-grade girl.

"What's the matter?"

"I'm working this problem," she said. "Mary had sixteen stamps and on the way home she lost five, and I'm trying to figure out how many she had when she got home."

"The thing that bothers me is seeing Mary going down the street saying to herself, 'I lost one, I lost two, I lost three ——.' Seems to me she would *know* how many she got home with and *wonder* how many she lost!"

Orthogonal Circles

by Floyd Bowling
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Since we deal with orthogonality in many places in mathematics, this article will deal with orthogonal circles:

One definition of two orthogonal curves may be stated:

> By the angles of intersection of two coplanar curves at a point which they have in common is meant the angles between the tangents to the curves at the common point. If the angles of intersection are right angles, the two curves are said to be orthogonal.

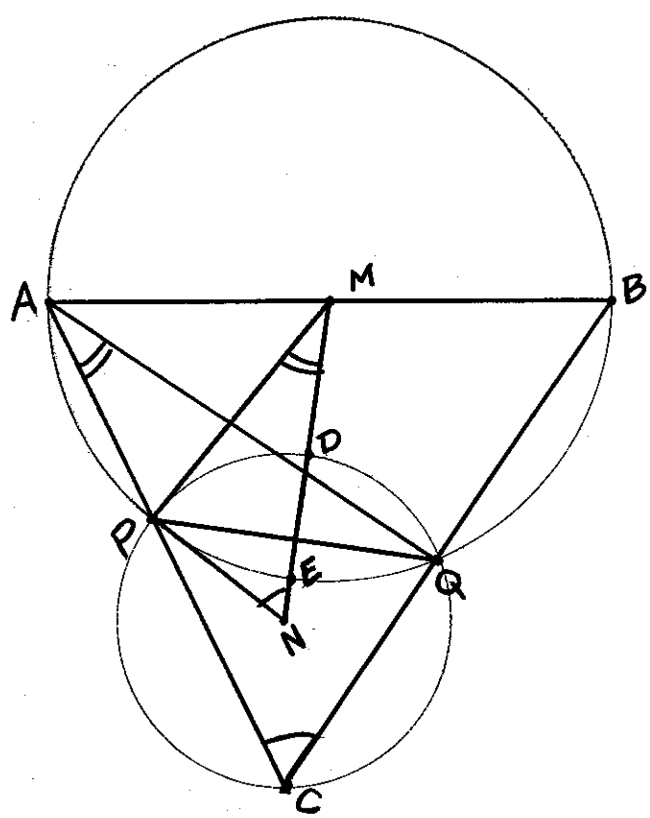
The theorem to be discussed here is:

If AB is a diameter of a circle and if any two lines AC and BC meet the circles again at points P and Q respectively, show that circle CPQ is orthogonal to the given circle.

Given: Circle M with diameter AB, point C outside circle M.

To Prove: That circle CPQ (circle N) is orthogonal to circle M. Or that MPN is a right angle.

Proof: Construct circle M a convenient size with AB as a diameter and locate C outside circle M at a convenient distance away. Draw AC and BC cutting circle M in P and Q respectively. Now construct circle N through points C, P, and Q. (Such a construction is permitted in Euclidean geometry). Draw line of centers MN, chords PQ and AQ and radii PM and PN.



(1). Angle PCQ is measured by one-half the arc PDQ (An inscribed angle is measured by one-half of the subtended arc)
Now arc PD = arc DQ for radius ND bisects chord PQ and arc PQ.
Thus, angle PNM is measured by arc PD. (Central angle is measured by arc subtended).
But arc PD = one-half arc PDQ, then

(2). Angle PCQ = angle PNM
Now reasoning in a similar manner, Angle PAQ is measured by one-half of arc PEQ and angle PMN is measured by arc PE which is as follows
Arc PE = one-half arc PEQ, then

(3). Angle PAQ = angle PMN.

(4). However, angle PNM+angle PMN = angle PCQ+angle PAQ.
(By adding (2) and (3))

(5). In triangle CAQ, angle PCQ+angle PAQ+ angle CQA = 180°
(sum of interior angles of a triangle equal 180°)
Therefore, angle PNM + angle PMN = 180° — angle CQA
(by using (4) and (5) above)

(6). In triangle MNP, angle PNM + angle PMN + angle MPN = 180°
(sum of interior angles of a triangle)

(7). In triangle ABQ, angle AQB = 90°
(angle inscribed in a semicircle)

(8). Also, angle CQA = 90°, since angle CQA + angle AQB = 180°
From (5) we have,

(9). Angle PNM + angle PMN = 180° — angle CQA = 180° — 90° = 90°
Therefore, triangle PNM is a right triangle with the right angle at angle MPN since angle PNM and angle PMN are not right angles.
Thus, the radii of circles N and M are PN and MP respectively.
Therefore, circles M and N are orthogonal. (If two radii passing through a point common to two circles are perpendicular, then the two circles are orthogonal)

National Metric Week Activities

MAY 9-13, 1977

WHEREAS, The United States and Canada are now adopting the metric system of measurement; and

WHEREAS, Today's children, as adults, will live in a largely metric world; and

WHEREAS, It is the responsibility today of our schools to educate their students for the future; and

WHEREAS, Parents should have an awareness and an understanding of their schools' metric program;

Now, therefore, the National Council of Teachers of Mathematics declares that the week of 9-13 May shall be designated National Metric Week; and encourages each person to develop and maintain an awareness of the metric system; and admonishes all persons to use the metric system where applicable in all phases of daily life.

Single copies of a list of Community Awareness Activities for National Metric Week are available free from the National Council of Teachers of Mathematics Headquarters Office.

A summary of 1976 National Metric Week activities appears on the back side of this release.

The NCTM Metric Implementation Committee received responses from schools in eighteen states regarding Metric Week activities. National Metric Week appeared to be successful in all school reporting. The dates chosen, 10-14 May, 1976, were satisfactory with 91 percent of the schools reporting. All schools indicated an interest in having NCTM sponsor National Metric Week for several more years. More advance notice was requested by 20 percent of the people responding. Fifty percent of those responding thought the suggested activities were excellent ideas and helpful to them.

Among the activities found successful during National Metric Week were the following:

1. Metric airplane-flying contest
2. Metric field day
3. Metric poster contests
4. Scale drawings of schools in metric measures
5. Scavenger hunts
6. Estimation contests
7. Daily school announcements in metric terms, such as weather reports, etc.
8. Parent workshops
9. Community metric awareness workshops
10. Design contest for auto bumper stickers
11. Writing metric limericks and designing puzzles
12. Distributing metric recipes

Metric Conversion In The Agricultural Community

The Agricultural communities of Saskatchewan are experiencing the process of metric conversion. Most communities have been offered short courses in metric conversion and these have been received with varying degrees of acceptance. The general attitude seems to vary from enthusiastic response to that of indifference.

Regardless, the conversion goes on. Many commodities such as sugar, toothpaste, ice cream, etc. have completed the three-step development. They are sold in metric-sized container and labeled in metric measurement only. Various other products are at the different levels of the three-stage development. Milk, for example, is still in stage one. It is sold in a two-quart container and bears the two-quart label with the metric 2.28 litres added beneath. Farmers who sell milk and cream are still paid for a number of pounds of butterfat.

Some cereals sold in supermarkets have reached level two of the three-stage conversion. That is, the metric measure is given first—then the traditional English measure. The package size has not yet been converted to metric size. For example, a soft drink bottle in this stage would contain 284 millilitres or 10 fluid ounces. The metricated soft drink bottle will contain an even 300 millilitres. This will be stage three.

Grain sales are rather interesting. They are scheduled to have undergone complete conversion by August 1, 1977. A farmer selling his grain to the elevator still sells it in bushels and pounds. However, the agent sells the grain to the terminal in metric tons. Metric conversion in grain sales is beginning at the national level and moving toward the local elevators.

Some information that most farmers find interesting and essential for yield calculation but have difficulty finding is the mass/unit volume of the various grains:

Grain	Mass/m^3
Wheat	.75 t (metric tons)
Flax, Rye	.7 t
Rapeseed	.625t
Barley	.6 t
Oats	.425t

While bulk oil and gasoline sales, nuts and bolts, etc. are still sold in the traditional English units, some industrial parts that we have been importing for tractors, cars, etc. have been metric-sized for quite some time. An interesting advantage of metric conversion is the standardization of nuts and bolts. By becoming metric we can expect an 80% saving in the number of kinds of connections we need in some jobs.

As products are introduced in metric units and we continue to use them, we are gradually becoming metrically converted. By slowly introducing products in metric units, the government facilitates a slowly changing attitude and acceptance of the metric system of measurement!

— *Saskatchewan Mathematics Teachers' Society, May, 1976*

MCTM Endorses Hill Candidacy

Shirley Hill, UMKC, has been nominated for the position of president of the National Council of Teachers of Mathematics. Ballots for the election will be mailed to all NCTM members in January. The Executive Committee of the Missouri Council has endorsed her candidacy.

Hill is a MCTM member and has spoken at many Missouri Council meetings. She served as consultant to the State Leadership Conference and was a member of the local committee for the Kansas City Name of Site meeting. She spoke at the Saint Louis meeting in 1975.

She taught elementary school in Kansas City, received her M.A. from the University of Kansas City and later her Ph.D. from Stanford. She is currently Professor of Education and Mathematics, UMKC.

Author, editor, lecturer, advisor, she was named UMKC Professor of the year in 1972 and is listed in *Who's Who of American Women* and *The World Who's Who of Women.* If elected she would serve as leader and spokesperson for NCTM's 50,000 members from April, 1978-April 1980.

She has agreed to give the luncheon address at the Spring Meeting of MCTM, April 2, 1977.

NOTE:

It might be stated that for an organization to be functional and informative to its members, it must have the support of as many members as possible. Therefore you are asked to become a salesman for your TCTM and approach teachers and libraries in your area with a copy of the membership form. This is one way of increasing our membership, ensuring a stronger council and enabling you to participate in the work of our organization.

Quote and Unquote

"I met a man recently who told me that, far from believing in the square root of negative one, he did not even believe in negative one. This is at any rate consistent attitude."
— E. C. Titchmarsh

"Because both the metric system and the hand-held calculator utilize decimals, common fractions will be as obsolete in 25 years as Roman numerals today."
— Virginia Mathematics Teacher

"Any mathematical trick that has been used at least twice becomes a method."
— George B. Thomas

PROFESSIONAL MEMBERSHIP APPLICATION

Date:__________ School:__________ School Address:__________

Position: ☐ teacher, ☐ department head, ☐ supervisor, ☐ student,* ☐ other (specify)__________

Level: ☐ elementary, ☐ junior high school, ☐ high school, ☐ junior college, ☐ college, ☐ other (specify__________

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National Council of Teachers of Mathematics	Check one: ☐ New membership ☐ Renewal membership	
	$12.00 dues and one journal ☐ *Arithmetic Teacher* or ☐ *Mathematics Teacher*	
	17.50 dues and both journals	
	6.00 student dues and one journal* ☐ *Arithmetic Teacher* or ☐ *Mathematics Teacher*	
	8.75 student dues and both journals*	
	7.00 additional for subscription to *Journal for Research in Mathematics Education* (NCTM members only)	
	1.00 additional for individual subscription to *Mathematics Student Journal* (NCTM members only)	
	The membership dues payment includes $4.00 for a subscription to either the *Mathematics Teacher* or the *Arithmetic Teacher* and 25¢ for a subscription to the *Newsletter.* Life membership and institutional subscription information available on request from the Washington office.	

Note New Membership and Subscription Fees

* I certify that I have never taught professionally__________ *(Student Signature)*

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