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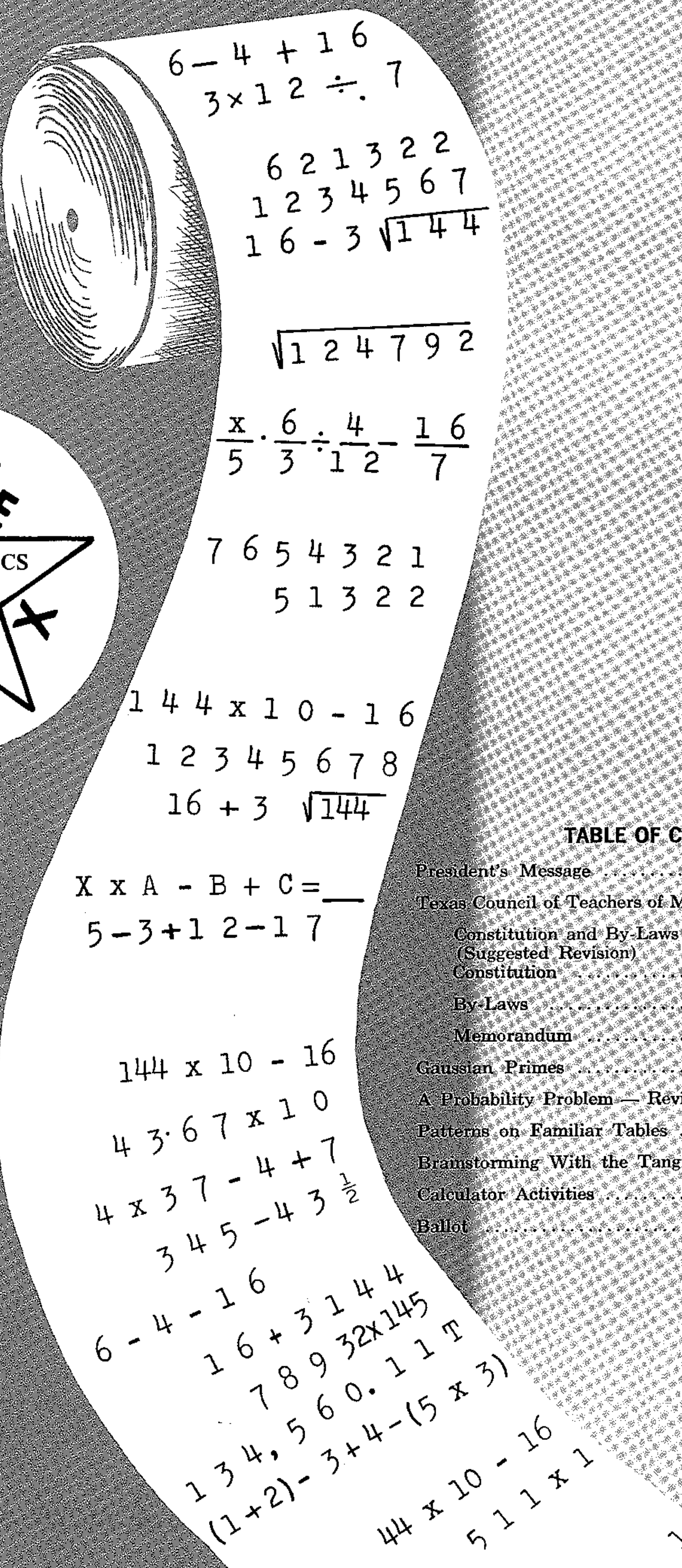
TEXAS MATHEMATICS TEACHER

April 1977



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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

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President's Message

A special TCTM "Thank you!" goes to Bill Ashworth and Josephine Langston for work done beyond the call of duty! Bill has volunteered to be "the Membership Committee." He has done a tremendous job in the past, spending many hours sending out reminders and notices to our members. Josephine has been chairman of the Constitution Revision Committee. She and her members have put a great deal of time and thought into their task. TCTM sincerely appreciates the efforts of both of these special people.

In May you will be asked to elect some new officers. The 1977 Nominating Committee members are: Evelyn Robson, Chairman (Baytown), Sandy White (San Antonio) and Josephine Langston (Garland). This year we will elect a President-Elect and two Vice-Presidents. I am sure the members of the Nominating Committee would certainly appreciate your suggestions . . . just as I appreciate their willingness to serve on this committee.

In less than a year San Antonio will be the host for a regional NCTM meeting. Paul Foerster and the Program Committee are already hard at work. It should be a great meeting! Please remember to put 2-4 February 1978 on your school calendar for next year, and make plans to attend. San Antonio is a lovely city, and I am sure you will enjoy the meeting . . . in and out of the sessions!

SHIRLEY COUSINS

TCTM Constitution, By-Laws

Revised

For many years the Texas Council of Teachers of Mathematics has been operating under a constitution and by-laws which has become outdated. Many parts of the old constitution and by-laws are not being followed because of the many interim legislative actions which have been passed.

Beginінg on page 3 of this issue you will find a copy of suggested revisions for the new constitution and by-laws for TCTM. Also, there is a memorandum of rationale relating to some of the revised parts.

It becomes your responsibility to read this suggested revision very carefully prior to marking your ballot on the inside of the back page and mailing same as indicated.

TCTM members who contributed to this suggested revision are: Kenneth Owens, St. Mark's School, Dallas; Jerry Hall, Elizabeth Chandler, Easter Blount and Diane Podrasky, Richardson; James Rollins, Texas A&M University; Wayne Wilson, New Braunfels; Anita Priest and J. William Brown, Dallas; Shirley Cousins, Houston; and W. A. Ashworth, Jr., Pasadena.

JOSEPHINE LANGSTON
Chairperson, Richardson

TEXAS COUNCIL OF TEACHERS OF MATHEMATICS CONSTITUTION and BY-LAWS

(Suggested Revision)

CONSTITUTION

ARTICLE I — NAME

This organization shall be known as Texas Council of Teachers of Mathematics.

ARTICLE II — AFFILIATION

This council shall be affiliated with the National Council of Teachers of Mathematics.

ARTICLE III — PURPOSES

Section 1. To encourage an active interest in mathematics.

Section 2. To provide the opportunity to study and keep abreast of any new trends in the teaching of mathematics.

Section 3. To improve teacher training programs for Texas mathematics teachers.

Section 4. To promote professional cooperation and communication of teachers and administrators for the realization of sound educational achievements.

Section 5. To assist Texas mathematics teachers in obtaining the benefits from the National Council of Teachers of Mathematics and the Conference for the Advancement of Mathematics Teaching.

ARTICLE IV — MEMBERSHIP

Section 1. Active membership is available to anyone who is interested in mathematics education, upon payment of dues as provided in the by-laws.

Section 2. Associate membership is available to any student upon payment of dues as provided in the by-laws.

Section 3. Honorary membership may be granted to any person who has rendered service which this council may desire to recognize with a life-time honorary membership, upon the recommendation of the executive committee and elected by a plurality of members voting. An honorary member shall be exempt from paying dues and shall enjoy all privileges of active membership.

ARTICLE V — OFFICERS and EXECUTIVE COMMITTEE

Section 1. The officers of the council shall be a president, a president-elect, an immediate past president, three vice-presidents, a secretary, a treasurer, and a parliamentarian elected by the general membership of the council.

Section 2. There shall be a National Council of Teachers of Mathematics representative, an editor, and a business manager appointed by the president with the approval of the executive committee.

Section 3. The voting members of the executive committee shall consist of the elected officers, the NCTM representative, the editor, the business manager, the mathematics consultants of the Texas Education Agency, and four regional directors. (See ART. IV in the By-Laws.)

Section 4. The non-voting executive committee members may be a representative of any mathematics teachers' organization with fifteen members or more and shall be affiliated with the National Council of Teachers of Mathematics.

ARTICLE VI — MEETINGS

Section 1. There shall be at least one regular meeting of the Texas Council of Teachers of Mathematics during the fiscal year.

Section 2. There shall be at least one meeting of the executive committee during the fiscal year.

ARTICLE VII — AMENDMENTS

This constitution may be amended in the following manner:

A. The proposed amendment or amendments shall be sent to the president or to the secretary at least ninety days prior to the annual meeting.

B. An announcement of the proposed change(s) and a ballot shall be included in the general announcement of the annual meeting.

C. To become effective, any change(s) in the constitution shall be approved by a two-thirds majority of active members who respond to the balloting.

ARTICLE VIII — DISSOLUTION

If, at any time, the Texas Council of Teachers of Mathematics shall cease to carry out the purposes as herein stated, all assets and property held by it, whether in trust or otherwise, shall, after payment of its liabilities, be paid over to the National Council of Teachers of Mathematics to be used exclusively for any project, or educational purposes, as determined by the Board of Directors of that organization.

BY-LAWS OF TEXAS COUNCIL OF TEACHERS OF MATHEMATICS

ARTICLE I — RULES OF ORDER

Robert's Rules of Order, Revised or *Greggs Parliamentary Law* shall be the authority on all questions of procedure not specifically stated in this constitution and by-laws.

ARTICLE II — QUALIFICATIONS, TERMS and DUTIES of OFFICERS and OTHER MEMBERS of the EXECUTIVE COMMITTEE

Section 1. Qualifications of Officers

Officers of this council shall be active members of the Texas Council of Teachers of Mathematics and members of the National Council of Teachers of Mathematics.

Section 2. Terms of Office.

a. All officers shall assume the duties of their elective or successive offices at the beginning of the fiscal year.

b. The president shall serve for a period of two fiscal years, succeeding to office from the office of president-elect. This office, if vacated, shall be filled by the president-elect.

c. The president-elect shall be elected to serve for the fiscal year corresponding to the second fiscal year of the president's term of office.

d. The three vice-presidents shall be elected, one each year, and shall serve for a period of three years. This office, if vacated, shall be filled by an election either by ballot or at the annual meeting of the Texas Council.

e. The secretary shall be elected for a period of two years and shall be subject to re-election. This office, if vacated, shall be filled by an election either by ballot or at the annual meeting of the Texas Council.

f. The treasurer shall be elected for a period of two years and shall be subject to re-election. This office, if vacated, may be filled by the executive committee, or by an election.

g. The parliamentarian shall be elected for a period of two years and shall be subject to re-election. This office, if vacated, shall be filled by the executive committee.

h. The editor shall be appointed by the president with the approval of the executive committee for a minimum of two years and shall be subject to repeated appointment.

i. The business manager shall be appointed by the president with the approval of the executive committee for a period of two years and shall be subject to repeated appointment.

j. The representative to the National Council of Teachers of Mathematics shall be appointed by the president with the approval of the executive committee for a period of one year.

k. The mathematics consultants shall be from the Texas Education Agency.

l. The regional directors shall be elected by members of affiliated groups in each of four geographical regions of the State of Texas, designated as Southeast, Southwest, Northeast and Northwest, for a period of 2 years and shall be subject to re-election.

m. The non-voting members are representatives of groups that are affiliated with the National Council of Teachers of Mathematics.

Section 3. Duties of Officers.

a. The president shall preside at all meetings of the Texas Council and of the executive committee, and he shall administer the affairs of the council. He shall appoint all committees not otherwise provided for and shall be an ex-officio member of all committees. He, or his designated delegate, shall serve as the official delegate to the National Council of Teachers of Mathematics. He shall prepare an annual report covering the activities of the council during administration to be given at the annual meeting and filed as part of the permanent records.

b. The president-elect shall assist by performing duties as assigned by the president, and shall be an ex-officio member of all committees.

c. The immediate past president shall, upon request, assist the president and the executive committee.

d. There shall be three vice-presidents. Each shall serve as chairman of a grade-level section of the Texas Council of Teachers of Mathematics, and may plan a program for that section for the state convention of the Texas State Teachers Association. Each shall serve as the chairman of a standing committee each of the first two years in office, and shall be chairman of the Nominations and Elections Committee during his third year in office.

e. The secretary shall keep all records and minutes of the Texas Council of Mathematics and of the executive committee. He shall be responsible for preserving the annual reports and historical records of the council.

f. The treasurer shall collect all dues and other income of the Texas Council of Teachers of Mathematics. He shall pay all routine bills provided for by the annual budget and such other bills as approved by the executive committee or the president; shall keep all financial records and make a financial report at the executive committee meeting and at annual meeting of the council. A copy of this report shall be filed as a part of the permanent records. With the approval of the executive committee, the treasurer shall determine the CAMT expense allowance for the editor and the NCTM expense allowance for the president. He shall upon request furnish the president, executive committee, or the secretary with a complete listing of all members and their addresses.

g. The state representative to the National Council of Teachers of Mathematics shall be the liaison person between NCTM, TCTM and each of the four regional directors in the State of Texas.

h. The editor shall prepare the *Texas Mathematics Teacher* bulletin for publication and distribute it to the members.

i. The business manager shall be a member of he editorial committee, shall be responsible for solicitation of and the collection of fees for advertising in the *Texas Mathematics Teacher*.

j. Each regional director shall keep a list of all organized council of mathematics groups in his region, and shall be the liaison person between the NCTM state representative and these groups. He shall be responsible for conducting the re-election of a director in his region.

ARTICLE III — EXECUTIVE COMMITTEE

The executive committee shall sit as the governing board of the Texas Council. It shall have power to transact the business of the council; initiate, develop, and determine the policies of the council; work out the budget, and appoint officers as provided in this constitution and by-laws.

ARTICLE IV — GEOGRAPHICAL REGIONS

The State of Texas shall be divided into four geographical regions comprised of areas which coincide with designated TSTA Districts.

Section 1. The NORTHEAST Region consists of districts 6, 7, 8, 10 and 12.

Section 2. The SOUTHEAST Region consists of districts 1, 2, 3, 4, and 5.

Section 3. The NORTHWEST Region consists of districts 9, 11, 14, 16 and 17.

Section 4. The SOUTHWEST Region consists of districts 13, 15, 18, 19 and 20.

ARTCLE V — COMMITTEES

Section 1. Standing Committees

Each of the standing committees shall be composed of a chairman, a member from each of the levels of instruction — elementary, junior high school, and senior high school — college.

a. Research Committee
The chairman shall be the vice-president who is serving his first year in office. This committee shall prepare teaching materials for, and in other ways help mathematics teachers in the state to keep informed and to grow professionally. It shall submit reports to the editorial committee relating to new methods, techniques, and aids in the teaching of mathematics.

b. Membership Committee
The chairman shall be the vice-president who is serving his second year in office. This committee shall, with the support of National Council of Teachers of Mathematics Affiliated Group Representatives to the Texas Council, present the advantages of membership in TCTM to the Mathematics teachers of Texas.

c. Editorial Committee
The chairman shall be the editor. This committee shall be responsible for the procurement of materials for publication and distribution of all bulletins of the council, and shall submit announcements and other pertinent materials to state and national publications.

Section 2. Special Committees

a. Nominations and Elections Comtee
This committee shall consist of five members, the vice-president in his third year of office, and one member from each level of instruction — elementary, junior high school, senior high school and college, with each of the four geographical regions being represented.

1. There shall be at least two nominees for each office to be filled.
2. The nominee for president-elect shall be from a geographical region different from that of the president.
3. The three vice-presidents shall be elected on alternate years. Each should be from a different geographical region.
4. Any National Council of Teachers of Mathematics affiliated group may also nominate a candidate for any office by a petition signed by at least fifteen Texas Council members. This petition shall reach the nominating committee before April 1 preceding the election.
5. The committee's slate and all other nominees shall be presented to the Texas Council members by way of a ballot printed in a bulletin or a newsletter, or by the United States mail.
6. Ballots shall be returned to a designated person, as the editor, who will send them to the Nominations and Election Committee to be tallied.
7. The officers shall be elected by a plurality vote of all ballots received.
8. This committee shall immediately notify all nominees of the election results, and invite the elected officers to the executive committee meeting at the CAMT meeting.
9. A copy of the constitution and by-laws shall be given each offi-

cer and executive committee member.

b. The Auditing Committee
This committee shall audit the financial records before the annual meeting.

c. Other Special Committees
These committees shall assume duties as outlined when appointed.

ARTICLE VI — FISCAL YEAR and MEMBERSHIP DUES

Section 1. The fiscal year and the membership year for the Texas Council of Teachers of Mathematics shall be from September 1 extending through August 31.

Section 2. Membership Dues

a. Annual dues for any active member shall be five dollars.

b. Annual dues for any associate member shall be one dollar.

c. There shall be no annual dues for honorary members.

ARTICLE VII — MEETINGS and QUORUMS

Section 1. One regular meeting of the Texas Council of Teachers of Mathematics shall be on Friday of the annual meeting of the Conference for the Advancement of Mathematics Teaching (CAMT). A quorum shall consist of twenty-five members.

Section 2. One meeting of the executive committee shall precede the annual council meeting at the CAMT meeting. A quorum shall consist of ten members.

ARTICLE VIII — AMENDMENTS

These by-laws may be amended by a two-thirds majority of votes by members of the executive committee.

"Toward the end of a cocktail hour a young mathematician observed: Well a fifth may go into three with none left over, but there may be one to carry."

The best way to learn is to do. The worst way to teach is to talk.

MEMORANDUM

Below are some suggestions for the rationale relating to some of the revised parts:

1. Constitution, ART. V, Sect. 3. It is thought that 4 regional directors would insure that all regions of the state would be represented by at least one vote on the executive committee.
2. Constitution, ART. VI, Sect. 2. It is hoped that there will be more than one meeting each year. For example, one at the state meeting of TSTA in the spring.
3. Constitution, ART. VIII. For any organization there should be provisions for its dissolution.
4. By-Laws, ART. II, Sect. 2. 1. It would be easy to divide this state into four equal geographical regions, using TSTA district numbers to identify the areas.
5. By-Laws, ART. V, Sect. 2 a. It is felt that 5 persons representing the different levels of instruction and the geographical regions can nominate for office qualified persons who will be distributed all over the state, rather than concentrated in any specific area.
6. By-Laws, ART. VI, Sect. 1. Since it was requested last year that membership dates be the same for all members, it seems that the fiscal year dates would be better. Teachers have a tendency to join the professional organizations at the beginning of the school year rather than the calendar year. We need more members, so this seems a better plan.
7. By-Laws, ART. VII, Sec. 2. Provision is made for at least 17 members on the executive committee, so 10 is slightly over half; we need business transacted by more than 6 people.
8. By-Laws, ART. VIII. By-laws should be more easily amended than the constitution, therefore when a change is needed quickly it may be done by the executive committee.

What Are You Doing?

Is there some classroom technique in any level of mathematical education of which you are proud? Write it down and send it to this editor NOW. We would like to publish it!!!

Metrics
Analyze
Techniques
Hands On
Enrichment
Manipulatives
Activities
Tangrams
Individualize
Calculators
Sequence

Gaussian Primes

William E. Briggs
University of Colorado, Boulder

Karl Friedrick Gauss (1777-1855), the German mathematician, is considered by many along with Archimedes and Newton as one of the three greatest mathematicians in history. He had made significant contributions in number theory by the age of 19 and later produced singular advances in such diverse fields as algebra, non-Euclidean geometry, elliptic functions, differential geometry, astronomy, and electromagnetism. He was the first to identify vectors with complex numbers and initiated the theory of algebraic numbers. The term "Gaussian integer" arises from this last field of investigation in which he considered the arithmetic properties of the set G of complex numbers of the form $a+bi$ where a and b are ordinary rational integers. The set G of all Gaussian integers forms an integral domain with the operations of addition and multiplication; that is G has the same algebraic properties as the set of rational integers with the same operations. In particular while G is closed under multiplication, the multiplicative inverse does not always exist. For example, there is no element in G which when multiplied by $1+i$ will give the unity element 1. Elements which do have multiplicative inverses are called "units" and it is not hard to determine that the units of G are ± 1 and $\pm i$. Consider the reciprocal of a Gaussian integer $a+bi$, that is, $1/(a+bi)$ and "realize," rather than rationalize, the denominator by multiplying numerator and denominator by $a-bi$ to obtain $(a-bi)/(a^2+b^2)$. This in turn is a Gaussian integer if and only if $a^2+b^2=1$.

Let us now consider factorization in G and ask whether a given element in G can be expressed as the product of two other elements in G. We are not at all familiar with the arithmetic of G so that we do not know any simple test for divisibility by say $1+i$ as we do for divisibility by 2, 3, or 5 in the set of rational integers. We can say immediately that $6+4i=2(3+2i)$ but a question still remains as to whether the factors 2 and $3+2i$ can be factored in G. Without knowing about any algorithm for factorization in G, we could follow the definition for rational integers and say that any Gaussan integer which cannot be factored in G except by using a unit as one of the factors will be called a "Gaussian prime." Having done this, we must ask whether in fact there is such a thing as a Gaussian prime, and if so, how can we tell?

Since all rational integers are Gaussian integers (since $a=a+0i$), we might ask rather naturally whether the rational prime 5 is a Gaussian prime. The answer is negative because $5=4+1=2^2+1^2=2^2-i^2=(2+i)(2-i)$ and this is a non-trivial factorization. (That is, neither factor is a unit.) From this it is immediately clear that any rational prime which is the sum of two squares can be written as the product of two Gaussian integers and therefore is not a Gaussian prime. Other examples are $13=3^2+2^2$, $17=4^2+1^2$, $29=5^2+2^2$, $37=6^2+1^2$, $41=5^2+4^2$. (In case you have noticed, it *is* significant that each of these primes is one more than a multiple of 4 and it is true that any such prime can be so expressed.) But we have not yet found any Gaussian prime. Trying 2 is unproductive since $2=1^2+1^2=1^2-i^2=(1+i)(1-i)$ so let us try next the rational prime 3. At this point an important algebraic fact will be very helpful. Recall that the absolute value of a complex number is defined as its distance from the origin in the complex plane. The important fact is that the absolute value of the product of two complex numbers is the product of the absolute values. Because the square root is involved, it is sometimes simpler to use the square of the absolute value. Let $\alpha=a+bi$ be a complex number and let $N(\alpha)=|x|^2=a^2+b^2$ be the "norm" of α. It is important to note now that if α is a Gaussian integer, a and b are rational integers so that likewise $N(\alpha)=a^2+b^2$ is a rational integer. So if a Gaussian integer α can be expressed as the product $\beta\gamma$ of two elements in G, we will have $N(\alpha)=N(\beta)\cdot N(\gamma)$ where each of these numbers is a rational integer. (For example, from a previous calculation $N(5)=25$ and $N(2+i)=N(2-i)=5$.)

Now back to the number 3. Let us suppose that 3 can be factored as $\beta\gamma$ so that $N(3)=9=N(\beta)N(\gamma)$ where $N(\beta)$ and $N(\gamma)$ are rational integers. Since any norm is non-negative, we must have factors of 1 and 9 or 3 and 3. But the only elements in G with a norm of 1 are the units and this combination would give us a trivial factorization such as $3=(3i)(-i)$. Thus we have to explore the possibility of a norm of 3 which would mean that there exists an element of G say $\beta=a+bi$ with $N(\beta)=a^2+b^2=3$. But neither a nor b could be too large since the sum of their squares would exceed 3. At the same time none of 0^2+1^2, 1^2+1^2, 0^2+2^2 equals 3 so that there is no Gaussian integer with norm 3. Thus the rational prime 3 is also a Gaussian prime and we now know that at least one such creature exists.

It turns out that a similar argument can be applied to any rational prime which is 3 more than a multiple of 4 such as 7, 11, or 19. For example if $11=\beta\gamma$, $N(11)=121=N(\beta)N(\gamma)$ and if neither β nor γ is a unit, we must have $N(\beta)=N(\gamma)=11$. But again we cannot find rational integers a and b such that $a^2+b^2=11$ because this expression can never take on a value which is 3 more than a multiple of 4.

Use of the norm is helpful in finding non-real Gaussian integers which are Gaussian primes. For instance, if $\alpha = c + di \epsilon G$ and if $N(\alpha) = c^2 + d^2 = p$ a rational prime (which from above cannot be 3 more than a multiple of 4), then α must be a Gaussian prime since $N(\beta) \cdot N(\gamma) = p$ is not possible except in a trivial way. Thus each of $1+i$, $-1+i$, $-1-i$, and $1-i$ is a Gaussian prime since each has norm 2. (These numbers are called "associates" of each other since one can be obtained from another by multiplying by a unit.) We have referred previously to the fact that any rational prime p which is one more than a multiple of 4 can be written as the sum of squares. Thus it follows from $5 = 2^2 + 1^2 = (2+i)(2-i)$ that $2+i$ and $2-i$, and their associates are Gaussian primes since their norms are 5. Similarly from writing $13 = 3^2 + 2^2$ we obtain $3+2i$, $3-2i$ and all their associate Gaussian primes. It turns out that all non-real Gaussian primes can be found in this way.

To summarize, the Gaussian primes fall into the following three classes:

1. all rational primes of the form $4k+3$ and their associates.
2. $1+i$ and its associates.
3. Gaussian integers of the form $a+bi$ or $a-bi$ and their associates where a^2+b^2 is a rational prime of the form $4k+1$.

It is interesting to look at the Gaussian primes geometrically by using the usual two-dimensional coordinate system representation of complex numbers. The elements of G have coordinates which are rational integers and form what is known as a lattice. The heavy dots in Figure 1 represent Gaussian primes.

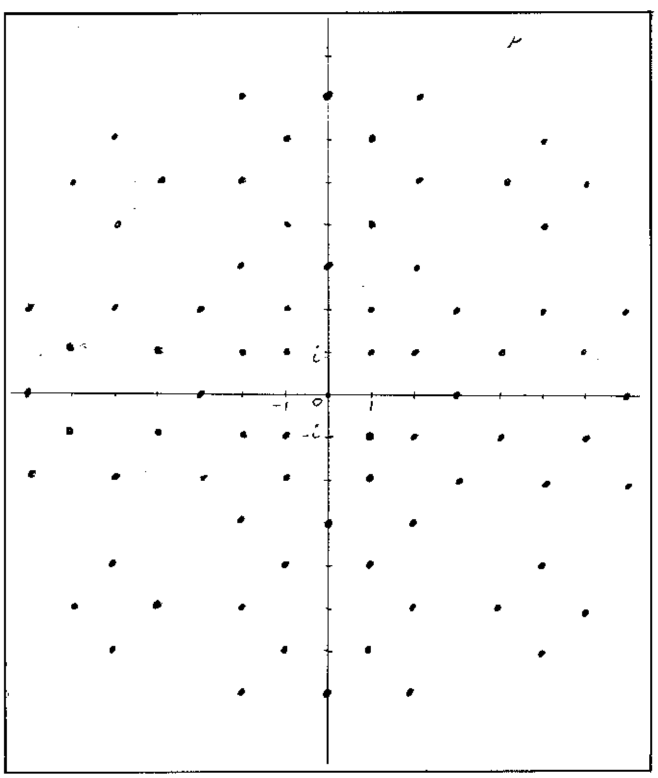

Figure 1

It is clear that interesting patterns and symmetries emerge as the Gaussian primes are plotted in the plane. These patterns become more graphic if each point representing a Gaussian prime is enclosed by a unit square with the point at its center, and then darkened as in Figure 2.

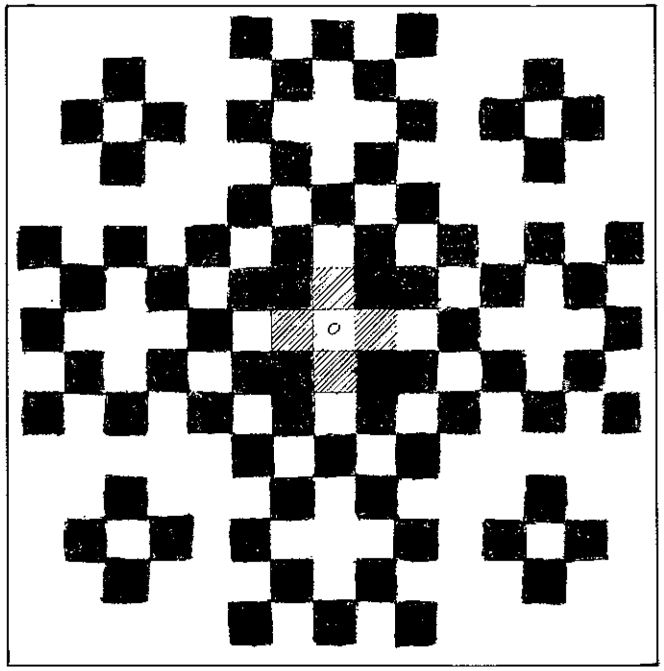

Figure 2

Many more things can be done with Gaussian integers and primes of a recreational and instructional nature. The pattern of blocks of Gaussian primes in the complex plan can be extended and various arrangements of blocks observed. (For diagrams of some interesting configurations, refer to a paper by Jordan and Raburg in the February 1976 issue of *Journal of Number Theory.*) This can be done of course by determining Gaussian primes in classes 1 and 3 listed previously. While we have not said much about factorization, it is possible to investigate the factorization of Gaussian integers into Gaussian primes. Using the norm is very helpful when obvious factors do not appear. For example $5+3i$ is not a Gaussian prime and so must be factorable. $N(5+3i) = 5^2 + 3^2 = 34 = 2 \cdot 17$ so that one factor must have norm 2 and the other norm 17. The Gaussian prime $1+i$ has norm 2 and when $5+3i$ is divided by $1+i$, the quotient is $4-i$ which has norm 17 and hence is a Gaussian prime. So the Gaussian prime factorization of $5+3i$ is $(1+i)(4-1)$. Finally more calculations of this kind can lead to a demonstration that factorization into Gaussian primes is unique except for order and the use of units. This will follow from the fact that there is unique factorization of rational integers into rational primes. Other interesting geometric patterns of blocks arise by considering all the multiples (by all Gaussian integers) of a particular Gaussian integer such as 2 or $1+i$.

A PROBABILITY PROBLEM—REVISITED

by Joe Dan Austin
Emory University

Rudd (1974) gives an interesting probability problem and a clever solution given by one of his students. This article gives a similar problem, but one where the student's method can lead to an incorrect answer.

In Rudd's problem two players alternately draw without replacement balls from an urn with player A drawing first. Initially there are 99 white balls and one red ball. The player drawing the red ball wins. The problem is to find the probability that player A wins. The student's solution is that player A has the draws 1, 3, 5, . . ., 99 to get the red ball while player B has the draws 2, 4, 6, . . ., 100 to get the red ball. Therefore since there are 50 possible draws for player A, the probability A draws the red ball is $\frac{50}{100}$ or $\frac{1}{2}$. Rudd shows this is the correct answer.

The same problem was considered in a course for high ability high school students taught by the author and several students used the same reasoning to get the answer $\frac{1}{2}$. Next the same problem was considered but assuming that the selection was made *with* replacement. Thus initially there were 100 balls — 99 white and one red. The two players drew with replacement alternately until the red ball was drawn. Player A still drew first. The winner was the player who drew the red ball first. Again the problem is to find the probability that player A wins. Here the students were quick to apply the same logic that worked previously. They argued that Prayer A can win on draws 1, 3, 5, . . . and player B can win on draws 2, 4, 6, ... Then since there is the "same number" of even and odd numbers, the probability that A wins should still be $\frac{1}{2}$. While some students did not think this was the best way to solve the problem, none argued that the answer was incorrect. Here we show that this answer is incorrect.

To solve this problem note that player A can win on draws 1, 3, 5, . . . and these wins are mutually exclusive. Therefore

$$(1) \quad P[\text{ A wins }] = P[\text{ wins on 1 }] + P[\text{ wins on 3 }] + \ldots$$

The probability A wins on 1 is $\frac{1}{100}$. The probability A wins on 3 is $(\frac{99}{100})^2 \cdot \frac{1}{100}$ because for A to win on the third draw, no player can be permitted to draw the red ball on the first two draws. [Note the drawing is with replacement.] Similarly the probability A wins on the fifth draw is $(\frac{99}{100})^3 \cdot \frac{1}{100}$

Thus

$$(2) \quad P[\text{ A wins }] = \frac{1}{100} + (\frac{99}{100})^2 \cdot \frac{1}{100} + (\frac{99}{100})^4 \cdot \frac{1}{100} + \ldots = \frac{1}{100}[1 + (\frac{99}{100})^2 + (\frac{99}{100})^4 + \ldots].$$

The probability A wins is an infinite series, namely a geometric series

$$(3) \quad 1 + a^2 + a^3 + \ldots = \frac{1}{1-a} \text{ for } -1 < a < 1$$

and $a = (\frac{99}{100})^2$. This gives from (3) that

$$(4) \quad P[\text{ A wins }] = \frac{1}{100}\left[\frac{1}{1-(\frac{99}{100})^2}\right] = \frac{100}{199}$$

Player A wins more than half of the time! [Player A has an advantage as he goes first.]

In general if there are n ball in the urn — n — 1 white and one red — and player A draws first, the

$$(5) \quad P[\text{ A wins }] = \frac{n}{2n-1}.$$

The probability in (5) is decreasing in n and goes to $\frac{1}{2}$ as n goes to infinity.

This paper illustrates that the intuitive solution strategy given by Rudd's student does not always work if the student is careless.

References

Rudd, D. "A Problem in Probability." *Mathematics Teacher*, 1974, *67*, 180-181.

Patterns on Familiar Tables

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Teachers are quite interested in activities which provide practice in the maintenance of computational skills. It is an added bonus if patterns may also be discovered in these activities. We shall develop a few such activities using basic number tables.

Figure I: Hundred Square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Activity I

Figure I is the well-known Hundred Square. Give each of your students a copy of this square and have them:

1. Circle any number (33).
2. Add the number immediately to its right (34) and the number immediately below (43). The sum is 77.
3. Add the "next number" to the right (35) and the "next number" below (53) The sum is 88.
4. Continue this process as far as the table extends. These sums are:

$$36 + 63 = 99$$
$$37 + 73 = 110$$
$$38 + 83 = 121$$
$$39 + 93 = 132$$

5. Observe that the consecutive differences between the sums (steps 2-4) are 11.

77 > 11
88 > 11
99 > 11
110 > 11
121 > 11
132

Repeat this entire process with other beginning numbers; the consecutive sum differences are always 11.

Activity II

1. Circle any number (58).
2. Add the number immediately to its left (57) and immediately below (68). The sum is 125.
3. Add the "next number" to the left (56) and the "next number" immediately below (78). The sum is 134.
4. Continue this process as far as the table extends. These sums are:

$$55 + 88 = 143$$
$$54 + 98 = 152$$

5. Note that the consecutive differences between the sums are 9.

125 > 9
134 > 9
143 > 9
152

Activity III

What is the consecutive difference between the sums if this process proceeds to the left and up from the circled number? To the right and up from the circled number?

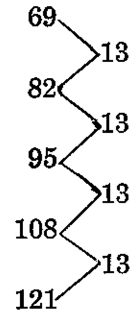
Activity IV

Figure II is a 12 × 8 number rectangle. Do the same process as in Activity I.

1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	(28)	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96

Figure II: Number Rectangle

1. 28
2. $29 + 40 = 69$
3. $30 + 52 = 82$
4. $31 + 64 = 95$
   $32 + 76 = 108$
   $33 + 88 = 121$
5. Note that the consecutive differences are 13.



What are the consecutive sum differences if the directions of Activities II and III are used on the number rectangle? What happens if these directions are used on other number arrangements such as the addition and multiplication tables?

Challenge to the reader: Why do these patterns hold?

BRAINSTORMING WITH THE TANGRAM

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Using Figure 1, connect the following pairs of points with line segments: (1A & 5A), (1A & 1E), (1E & 5E), (5E & 5A), (5A & 1E), (1A & 4D), (5C & 3E), (4B & 4D), (2D & 3E). If you did this correctly, you should have generated a large square which has been divided into five isosceles, right triangles, one small square and one parallelogram. Cutting up the large square as indicated creates what is called a tangram. The origination of this puzzle is usually attributed to the Chinese, although its history is vague.

By rearranging the seven pieces of this puzzle, an almost endless array of shapes can be formed. A few of these are shown in Figure 2. You might enjoy trying to form these keeping in mind that all seven pieces are to be used in each figure. In addition to such purely recreational uses of the tangram, what else can be done with it in a mathematics classroom? This article will suggest several possible activities which are suitable for junior or senior high school students.

Activity 1 — Using all seven pieces of the tangram puzzle, form a: square, triangle, trapezoid, parallelogram, and rectangle.

Activity 2 — In Figure 3, call the area of the piece labeled "b" one unit.

A. What is the area of the remaining six pieces?

B. Suppose we let shape "a" have an area of one unit. What is the area of the remaining six pieces?

C. Let shapes "c," "d," and "g" be the units of area in turn, and calculate the areas of each of the remaining shapes.

Activity 3 — The solution of Activity 2 necessitates discovering a relationship between the areas of shapes "a" and "b" and "a" and "d". What is this relationship? What theorem in plane geometry is suggested by this relationship?

Activity 4 — Call the side length of shape "b" in Figure 1 one unit.

A. Determine the perimeter of each of the seven pieces of the tangram puzzle.

B. Determine the perimeter of each of the figures formed in Activity 1.

C. Which shape has the smallest perimeter?

Activity 5 — Let the hypotenuse of shape "a" be one unit. Now do parts A and B of Activity 4.

Activity 6 — The areas of "a" and "f" are also related in a certain way.

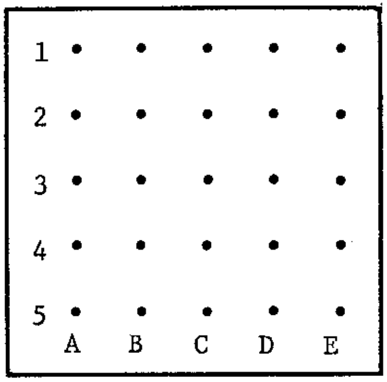

Figure 1

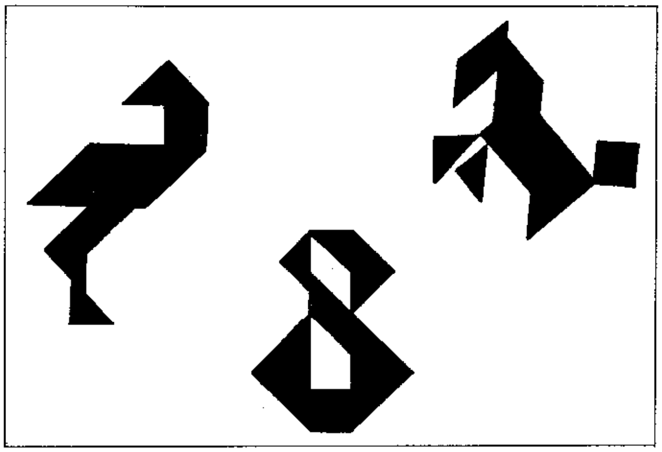

Figure 2

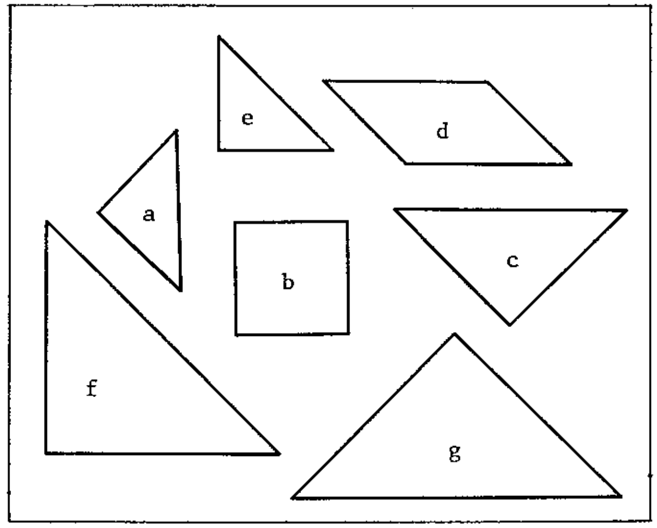

Figure 3

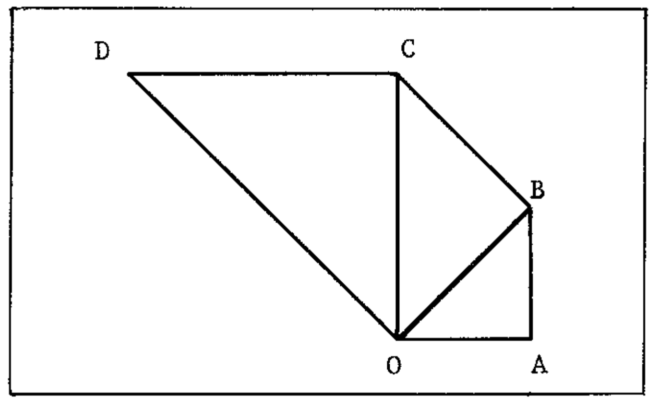

Figure 4

A. Find and state this relationship.
B. Do you think this relationship will be true for triangles other than right triangles?

Activity 7 — Triangles "a," "c," and "f" are related in another interesting way. They are similar. By examining Figure 1, determine the three ratios of similitude that exists between the pairs of triangles.

Activity 8 — Activity 1, required that five polygons be formed. Each of these polygons is convex. There are at least eight more convex polygons that can be formed using all seven pieces of the tangram. See how many you can find; make drawings to record your answers.

Activity 9 — Form several shapes (you need not use all seven pieces of the tangram.) which possess one or more of the following types of symmetry:
A. Line (or bilateral)
B. Point (or central)
C. Rotational
Record your answers indicating all lines and points of symmetry as well as the degrees of rotational symmetry.

Activity 10 — Show how pieces "a," "b," and "e" can be used to justify that the area of a parallelogram is found by multiplying the length of its base times its height.

Activity 11 — The three different sizes of right triangles can be used to form a spiral as shown in Figure 4. Add some more triangles to the spiral. Given that the length of $\overline{OA}$ is one unit:
A. Calculate the lengths of OB, OC, OD, etc. Find a general expression for finding the length of the hypotenuse of the nth right triangle.
B. Find the length of AB, AB + BC, AB + BC + CD, . . . Find a general expression for finding the sum of the lengths of the legs of the first n triangles.

Why not do some brainstorming yourself with the tangram? See what kinds of activities or problems you can discover for use in your junior or senior high school classes. Maybe your students will appreciate a break from the routine of daily assignments from the textbook. Mine do! I would appreciate learning of some of your ideas for using the simple and inexpensive tangram puzzle.

MIDDLE AND HIGH SCHOOLS

Instructional Seminars

Teachers have suggested a new method for obtaining input for the Baseline units in order to spend more seminar time in workshops.

On Friday, February 25, each teacher brought his/her suggestions for improving the units. Revisions were placed in the suggestion box at registration.

Course or Level	Objectives
7	P through DD
8	N through Z
Algebra I	O through CC
Algebra II	O through PP
Geometry	G through K
MOCE	A through Z
FOM I	L through N
FOM II	J through O
IA I	J through Q
IA II	F through I
Elem. Anal.	A through D
Calculus	A through PP
Computer Math	A through Z

Essential Skills and Mathematics Literacy

The State of Texas has decreed that by 1981 all students graduating from high school will be proficient in basic skills of mathematics and reading.

During February a literacy test was administered to Dallas IndependentSchool District eighth graders. DISD high school seniors were given the test in March.

The central mathematics staff is currently studying the development of an essential skills program in grades K-8 to lay the foundation for mathematics literacy at the secondary level. Also being studied are means of remediating literacy skills in grades 7-12.

Geometry Textbooks for 1977-78

The Central Textbook Committee of the Dallas Independent School District has made its recommendations to the Board of Education. The recommendation for Geometry is a multiple adoption with one textbook for honors and high academic classes and a different textbook for regular Geometry classes.

High school mathematics teachers and department chairpersons should work with their principal and counselors to form high academic classes for next fall.

Local and Regional Science Fairs

Local building science and mathematics fairs were scheduled during the weeks of February 28-March 11 in the Dallas Independent School District. The Dallas Regional Science Fair was set for March 31-April 3 at Dallas Memorial Auditorium. Exhibits were held on Saturday, April 2, from 10 a.m. to 6:30 p.m. and Sunday, April 3, from 12 noon to 5 p.m.

National Council of Teachers of Mathematics

The 55th annual meeting of the NCTM will be held in Cincinnati, Ohio on April 20, 21, 22, and 23, 1977.

Calculator Activities

Activity One

Squaring numbers ending in 5.

5 x 5 =	35 x 35 =
15 x 15 =	45 x 45 =
25 x 25 =	55 x 55 =

Do you see a pattern? Test it—

65 x 65 =
75 x 75 =
85 x 85 =

Activity Two

1. Pick 3 digits from the following: 1, 2, 3, 4, 5 6, 7, 8, 9.
2. Write all six two-digit numbers that can be formed from these 3 digits.
3. Add the three original numbers.
4. Add the six new numbers.
5. Divide the sum of the six numbers by the sum of the three original numbers.
6. Try it again with 3 different numbers.
7. Compare your work with someone else. Can you explain these results?

Activity Three

Find a pattern that will help you predict the answers. You may find the results of Activity One useful.

66 x 64 =	42 x 48 =
51x 59 =	91 x 99 =
33 x 37 =	

Test your pattern!

Activity Four

Convert to decimal form and find a pattern in the answers.

1/7 =	1/9 =	1/11 =
2/7 =	2/9 =	2/11 =
3/7 =	3/9 =	3/11 =
4/7 =	4/9 =	4/11 =
5/7 =	5/9 =	5/11 =
6/7 =	6/9 =	6/11 =
	7/9 =	7/11 =
	8/9 =	8/11 =
		9/11 =
		10/11 =

—Borrowed: DISD MATHNEWS, Vol. IV, No. 2, February 16, 1977

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