Vol. XXIII No. 1
TEXAS MATHEMATICS TEACHER
January 1976

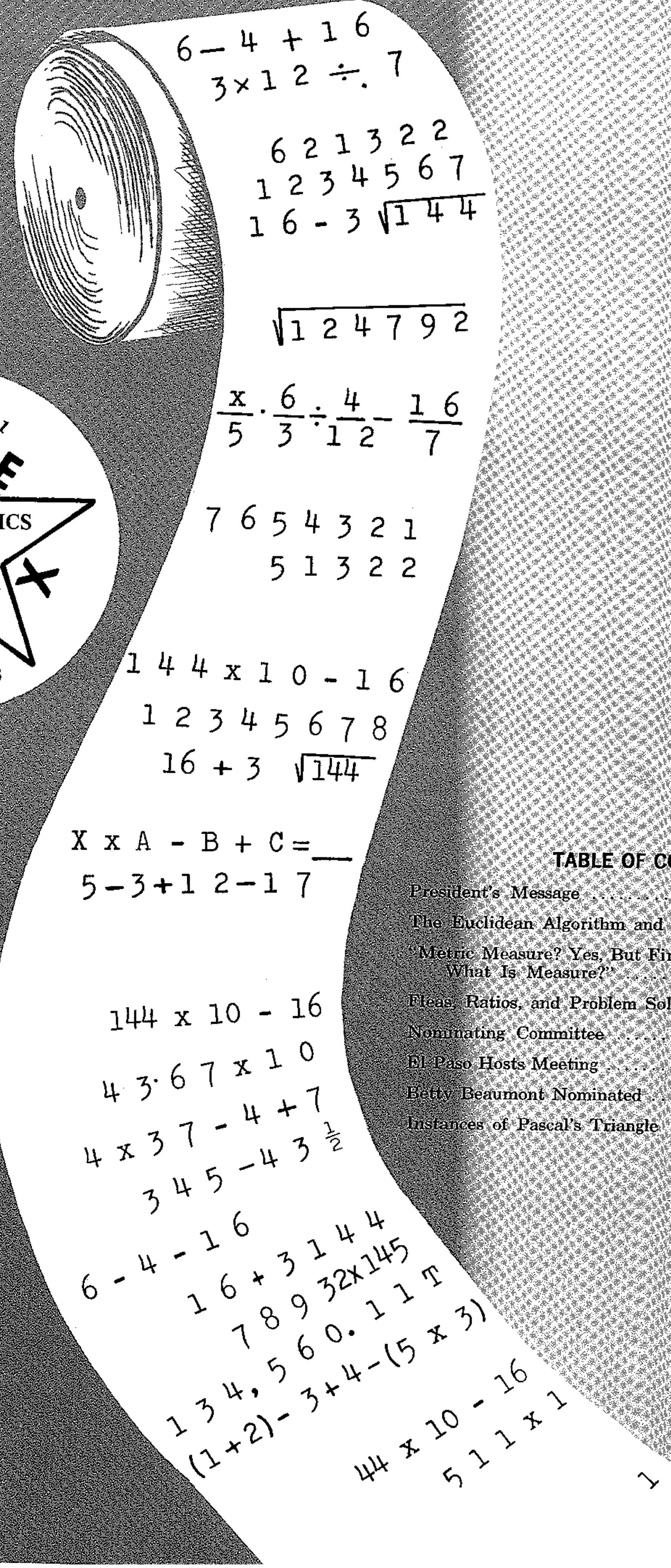


TABLE OF CONTENTS

TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

TEXAS MATHEMATICS TEACHER is published quarterly by the Texas Council of Teachers of Mathematics. Payment of membership fee of $5.00 entitles members to all regular Council Publications.

President's Message

By the time you receive this, CAMT will have come and gone. From the appearance of the program, Nancy Ogden has planned an outstanding program. Our thanks go to her, as well as the other committee chairmen and committee members who spent so much time and effort to make the program and meeting a successful one. We must all be indebted to Alice Kidd for her continuing efforts to coordinate all the activities.

You will receive this Journal during the Holiday Season — a time all of us use to some extent to reflect back upon the year's activities and rejoice in the accomplishments, and resolve to correct our errors. I wish each of you would join me in resolving to make TCTM a growing, vital organization in the year ahead. This can be accomplished as each of us assumes responsibility to be a more enthusiastic and willing worker.

According to the Constitution, it is my obligation to make a report to you concerning the "state-of-the-council" for the year. It is a pleasure to tell you our membership is nearing 1000, our finances are becoming more respectable, and our activities increasing. Our first newsletter will be off the press soon. We hope for at least two more during the year. Membership reminders are also being mailed. Should you have paid your dues and still receive a reminder, please return the card with this information concerning when and where the payment was made.

The journal, as well as the newsletter, will contain membership forms. Please share these with your fellow teachers and encourage them to become a part of our organization. As each of us becomes a membership campaigner, we will become the organization we should be.

On behalf of the officers of TCTM, may the New Year be filled with numerous joys, successes, and dreams being fulfilled beyond your highest expectations.

BILL ASHWORTH

The Euclidean Algorithm and Its Applications

by Manuel P. Berriozabal (Berri)
Professor of Mathematics
University of New Orleans
New Orleans, Louisiana 70122

One technique to compute the greatest common divisor (GCD) of two nonzero integers is through the use of the Euclidean Algorithm. In fact, by use of this algorithm one can show that any two nonzero integers have a GCD and the GCD can be expressed as a linear integral combination of the two integers.

For example 2 is the GCD of 74 and 58 and (1) $2 = 74(11) + 58(-14)$, the equality showing that 2 can be expressed as an integral combination of the two integers 74 and 58. Of course, to compute the GCD of two nonzero integers, it is usually easier to examine the prime factorizations and find the product of the common prime powers of each factorization. The GCD is this product if it exists; otherwise, the GCD is 1. However, this latter technique will not determine an appropriate linear integral combination. To compute (1), one can employ a trial-and-error method which is cumberson, disorganized, and not always productive, or one can use a systematic computational device like the Euclidean Algorithm which we will now illustrate in the first example.

Example 1. Compute GCD (74,58) and write it as a linear integral combination of 74 and 58.

Solution

$$(2)\quad \begin{aligned} \boxed{74} &= \boxed{58}\cdot 1 + 16\\ \boxed{58} &= \boxed{16}\cdot 3 + 10\\ \boxed{16} &= \boxed{10}\cdot 1 + 6\\ \boxed{10} &= \boxed{6}\cdot 1 + 4\\ \boxed{6} &= \boxed{4}\cdot 1 + 2\\ \boxed{4} &= \boxed{2}\cdot 2 + 0 \end{aligned}$$

In the first step of the computation, we applied the division algorithm to 74 and 58 and obtained a quotient of 1 and a remainder of 16. In the second step, we applied the division algorithm to 58 and 16 and obtained a quotient of 3 and a remainder of 10. We repeat the division algorithm for the pair 16 and 10, then for 10 and 6, and finally for 6 and 4 where through this technique we end up with the last non-zero remainder of 2. *This last non-zero remainder is the GCD of 74 and 58.*

Next, by using (2), we show how one writes 2 as a linear integral combination of 74 and 58. From (2), we see that

$$(3)\quad \begin{aligned} 16 &= 74(1) + 58(-1)\\ 10 &= 58(1) + 16(-3)\\ 6 &= 16(1) + 10(-1)\\ 4 &= 10(1) + 6(-1)\\ 2 &= 6(1) + 4(-1) \end{aligned}$$

Starting with the last equality and working backwards with a series of substitutions, we obtain the following results

$2 = 6(1) + 4(-1) = 6(1)\ [10(1) + 6(-1)]\ (-1) = 6(2) + 10(-1)$

$= [16(1) + 10(-1)](2) + 10(-1) = 16(2) + 10(-3)$

$= 16(2) + [58(1) + 16(-3)](-3) = 16(11) + 58(-3)$

$= [74(1) + 58(-1)](11) + 58(-3) = 74(11) + 58(-14)$

Thus we have a systematic means (namely the Euclidean Algorithm) of writing GCD(74,58) as an integral linear combination of 74 and 58. Furthermore, this method will work for any two non-zero integers. The fact that the last non-zero remainder is equal to the GCD of 74 and 58 can be seen from the following argument which is applicable to any pair of non-zero integers.

Starting from the bottom and working upward in (2), we see that 2 divides 4. Going up to the next equation, since 2 divides 2 and 2 divides 4, then 2 divides 6. Going up to the next equation, since 2 divides 4 and 2 divides 6, then 2 divides 10. Going up to the next equation, since 2 divides 6 and 2 divides 10, then 2 divides 16. Since 2 divides 10 and 16, then 2 divides 58. Since 2 divides 16 and 58, then 2 divides 74. Thus 2 is a common divisor of 74 and 58. Now let k be any common divisor of 74 and 58. To show that 2 = GCD (74,58), it suffices to show k divides 2. Here we use (3). We start with the first equality and work down to the last equality to arrive at the conclusion that k divides 2.

Since k is a common divisor of 74 and 58, then k divides 16.
Since k divides 58 and 16, then k divides 10.
Since k divides 16 and 10, then k divides 6.
Since k divides 6 and 4, then k divides 2.
Thus 2 = GCD(74,58).

Another question we can easily dispose of is whether GCD(74,58) can be written uniquely as a linear integral combination of 74 and 58. The answer is NO. In fact, the following gives a systematic method of finding infinitely many such combinations.

$2 = 74(11) + 58(-14) = 74(11) + 58(-14) + 0$

$= 74(11) + 58(-14) + 74(58) + 58(-74)$

$= 74(11 + 58) + 58(-14 + -74) = 74(69) + 58(-88)$

Thus 2 is written as another linear integral combination of 74 and 58. A general formula for finding infinitely many solutions is $2 = 74(11 + 58n) + 58(-14 + -74n)$ where n is any integer. A similar formula applies to any pair of non-zero integers.

Another question we may pose is whether we can give a characterization of integers which can be written as a linear integral combination of 74 and 58. For example, can both 15 and 18 be written as such a combination, or in other words, do there exist pairs of integers a,b and c,d so that 15 = 74a + 58b and 18 = 74c + 58d? In the case of the second equality, the answer is yes. Since 2 = 74(11) + 58(−14), then multiplying each side by 9 we obtain 18 = 74(99) + 58(−126). Let us now consider whether the first equality can exist. Suppose there exist integers a and b so that 15 = 74a + 58b. Since 2 is a common divisor of 74 and 58, then 2 must divide 74a and 58b, that is, 2 must divide 15, this is a contradiction. Consequently, the first equality cannot exist. These two examples illustrate the fact that an integer j is a linear integral combination of 74 and 58 if and only if 2 divides j, that is to say, GCD(74,58) divides j.

In summary we state the following general results.

If a and b are two non-zero integers, then GCD (a,b) exists. Also, there exist integers x and y such that GCD (a,b) = ax + by. Furthermore, an integer j can be written as a linear integral combination of a and b if and only if GCD(a,b) divides j.

Let us now examine the following examples.

Example 2. Compute GCD(74,−58) and write it as a linear integral combination of 74 and −58.

SOLUTION

Clearly GCD(74,−58) = GCD(74,58) = 2.

Using example 1, 2 = 74(11) + 58(−14) = 74(11) + (−58)(14), this last expression being a desired integral combination of 74 and −58.

Example 3. Compute GCD(−74,−58) and write it as a linear integral combination of −74 and −58.

SOLUTION

Again, GCD(−74,−58) = GSD(74,58) = 2. Also, 2 = 74(11) + 58(−14) = 74(−11) + −58(14), this last expression being a desired integral combination of −74 and 58.

We summarize the content of examples 2 and 3 as follows: *if a and b are two non-zero integers, then GCD(a,b) = GCD(|a|,|b|). GCD (a,b) can be written as a linear integral combination of a and b by expressing GCD(a,b) as an appropriate linear integral combination of |a| and |b| and making suitable adjustments of signs in this expression.*

Finally we consider the case of finding the GCD of three non-zero integers and writing it as a linear integral combination of the three integers. Before we consider this situation, we need to establish the following result. *If a, b, c are three non-zero integers, then GCD(a,b,c) exists and GCD(a,b,c) = GCD(GCD(a,b), c).* In the same manner of easily finding the GCD of two non-zero integers, we can find the GCD of three non-zero integers by examining the prime factorizations of each and then compute the product of the common prime powers

(Continued on Page 6)

of each factorization. The GCD is this product if it exists; otherwise the GCD is 1. Now let k = GCD(a,b,c) and d = GCD(GCD(a,b), c). In order to prove k = d, it suffices to show k divides d and d divides k. Since k = GCD(a,b,c), then k divides a, k divides b, and k divides c. Since k divides both a and b, then k divides GCD(a,b). Thus k divides GCD(GCD(a,b), c) that is, k divides d. Next, we will show d divides k.
Now d divides GCD(a,b) and d divides c.
Thus d divides a and d divides b and d divides c.
Hence, d divides GCD(a,b,c), that is, d divides k.
Consequently, k = d.

Example 4. Compute GCD(54, 78, 83) and write it as a linear integral combination of 54, 78, 83.

SOLUTION

By the previous result, GCD(54, 78, 83) = GCD (GCD(54,78),83) = GCD(2,83) = 1.

By using the Euclidean Algorithm, we write 1 as a linear combination of 2 and 83.

$83 = 2 \cdot 41 + 1$
$41 = 1 \cdot 41 + 0.$

Thus $1 = 83(1) + 2(-41)$.

Next, by using the Euclidean Algorithm, we write GCD(54,78) as a linear integral combination of 54 and 78. Since the work has been done in example 1, we use that result and we have $2 = 74(11) + 58(-14)$.

Substituting in the last equation of (3), we obtain

$$1 = 83(1) + [74(11) + 58(-14)](-41)$$
$$= 83(1) + 74(-451) + 58(574).$$

Thus GCD(54,78,83) = 1 is written as a linear integral combination of 54, 78, and 83.

An observation to be made from the last example is that the GCD of any finite nonempty subset of non-zero integers can be written as a linear integral combination of these integers.

"Metric Measure? Yes, But First What is Measure?"

Dr. Richard A. Little
Kent State University
Canton, Ohio

In the last two or three years, the switch to Metric Measure which lies ahead for the United States has received extensive attention by educators. Some mathematics educators have attempted to extricate their souls from the criticisms being heaped on the "New Math" by flinging themselves thoughtlessly onto the calliope wagon, blaring a tune called "Metrication." Those already aboard were disheartened by the vote in the U.S. House of Representatives on May 7, 1974. On that day, our representatives applied a little understood parliamentary procedure to block any amendments to House Bill 11035. The process was employed to gain additional prospective on the bill and its amendments (in this day, let us occasionally give our legislators the benefit of the doubt). The action by the U.S. House did *NOT* defeat the bill (as many of us initially supposed), but only postponed the deliberations on it. Its final disposition will undoubtedly occur soon. Even if the bill is defeated, we can still look forward to a Metric America in less than 15 years because many of our industrial giants have already established metrication schedules. We in education should not dispare of our lawmakers' action but thank them for the additional time they have provided. Let us use the time to answer the central question at hand; "What is Measure?" Let us not ride a calliope called "Metrication" into oblivion. After allowing "New Math" to be wrenched from us and exploited to the detriment of sound education, let us not lose this opportunity to redeem ourselves.

The whole of the concept of *measure* can be summed up in three sequential ideas; (1) a collection of people has an *object* which they wish to ascertain the size or value of (i.e., they want to measure it); (2) They must determine a *unit* of size or value which is compatible with the object to be evaluated; (3) They must determine the *number* of *units* in the *object*. Thus, "*to measure*" is "*to count the number of agreed upon units in the given object*," and the measure of the object is a *number*. Hence, we see why mathematics teachers ought to be concerned with measure in general, and metrication in particular. Let us explore each of the three basic concepts of measure.

An *object* which we wish to evaluate. Measure deals primarily with things, not with feelings. Researchers are still at a very primitive level in their pursuit of measuring human affections or thought processes. So, we can concentrate on the items of commerce in our classroom pursuit of measure. Indeed, the very genesis of measure was in commerce. In today's intercontinental commerce, complex machines are assembled from subcomponents constructed on two or three continents (e.g., television sets), and we should easily grasp the need for world-wide standard units of measure.

A *unit* of measure must be compatible with the object to be measured. Most often, the unit of measure most compatible to a given object is an abstract model of the object to be measured. Thus, a meter, itself a length, is a standard unit for measuring length. A square meter, itself a region, is a standard unit for measuring the size of a region. A standard unit of measure did not have to be very "standard" when commerce rarely went beyond the village of the fief. As commerce expanded, the need for more widely accepted standard units of measure became evident. As one studies the history of measurement, the constant progress of more widely accepted units of measure parallels the progress of intercontinental commerce. Today, only Canada and the Untied States among the world trade leaders remain outside of the system of world commerce that is totally metric. The metric system is definitely superior to the U.S. Standard System, so we are in no position to do anything but to adopt *metric units*. Common sense not only dictates this route, but a little known agreement among the countries of the European Common Market to accept only those products that are in metric units and sizes after 1977, also indicates that there is no choice. Worldwide commerce necessitates worldwide standard units and the metric system provides the best and most comprehensive system yet devised by humans. The United States must change.

The third idea related to measure is the one to which teachers and students must give their closest attention. How do we determine the *number* of standard units in a given object? Here is where the laboratory approach *must* be employed. We must not tell pupils about measure; we must provide them with experiences in measure. The student should be provided with innumerable experiences in measuring. These measuring experiences should establish *models* for a liter, a meter and a kilogram. These models should become pictures in the students' minds. By first guessing then measuring, and later approximating then measuring, the student will gain understanding of measure and proficiency in the four arithmetic operations using decimals. (See recent issues of the ARITHMETIC TEACHER for excellent suggestions of metric measure activities for youngsters.)

The only major weakness in the "New Math" curriculum was its pursuit of theory at the experience of "hands-on" experiences. We are fortunate that metrication provides us with the opportunity of "hands-on" applications for measure activities in mathematics class. Let us stress the three basic concepts of measure at all times;

1. An *object* suitable for measure (usually an object of commerce)
2. A *standard unit* of measure compatible with the object to be measured.
3. A method for determining the *number* of standard units in the given object.

Fleas, Ratios, and Problem Solving

by Arthur A. Hiatt, Ph.D. *Associate Professor,*
Education and Mathematics
California State University, Fresno
Fresno, California

Introduction. The following problem was used not so much to teach content as it was process. In other words, this type of problem helps the student develop the ability:

1. to make observations.
2. to organize observations (data)
   a) to recognize patterns.
   b) to make conjectures (guess).
3. to specialize and generalize.
   a) to use inductive reasoning.
   b) to reason by analogy.
4. to invent symbolism to express mathematical ideas.
   a) to accept conventional symbolism.
5. to prove conjectures.
   a) to invent or accept an axiomatic structure.

As all experienced teachers know, the more bizarre or ridiculous a problem, the more it captures the imagination of the students. However, for such problems the teacher must carefully set the stage.

Setting the stage. You enter your general mathematics class and place a flea, held captive in a jar, on your desk. Now, even the least inquisitive student will find this scene a curious one. The writer has motivated the problem in many grades from fourth to high school by such an act. Without failure, some student has always asked, "What are you doing with that flea in the jar?" From this response many exciting lessons take place. What follows will be a greatly condensed version of what can happen in a classroom over a period of several weeks.

The problem. After much discussion, the students are led to believe that the flea is being trained to jump on command. Students do not believe that fleas can be trained to perform. This requires some library work on flea circuses. Since fleas are known to be good jumpers, the following question is natural:

If a man could jump as well as a flea, in proportion to his weight, how far could he jump?

In most classes we first considered the jumping ability of a cat. A good, ten pound, jumping cat can jump about twelve feet. We then construct a table as follows:

A 10 lb. cat can jump 12 feet

A 20 lb. cat should be able to jump ? feet.

A 170 lb. cat should be able to jump ? feet.

Although the students understand that this is ridiculous, they enjoy the problem. From the above, we organized the data into ordered pairs and graphed them. From this experience, formulas relating length of jump to weight were invented. Many opportunities were afforded the students to verbalize their findings before developing formal symbolism.

Total understanding of the cat problem enables students to solve the flea problem. Students research the necessary information. (Several books on fleas were checked out of the public and college library and brought into class.) Some of the facts were:

1. Eighteen well fed fleas weigh one grain.
2. The average flea can jump eight inches.
3. The average man weighs 170 lbs.

Now, Archimedes Beamon, the captive flea, was not average; he could jump 13 inches. His weight figured out to be approximately .0000079 lb. since one grain is 1/7000 lb.

To make a long story short, if a 170 lb. man could jump as well as a champion flea, he would jump about 4,415 miles in one leap. Imagine that, in one leap from Seattle, Washington to Miami, Florida with room to spare. Or to look at it differently, the distance is greater than all the freeways in Texas put end-to-end.

Summary. The foregoing was a brief account of some lessons. It would be impossible to list all the ideas that came up in the various grades. Of course, it is easy to see that the following topics are possible.

1. Ratio
2. Proportion
3. Formulas
4. Equations
5. Graphing
6. D = RT
7. Metric measures
8. The curve a jump follows (parabolas)
9. Projectiles
10. Velocity
11. Tangents
12. Area
13. Functions
14. Similarity
15. Conversion tables
16. Use of hand calculators (good old fashioned drill)
17. Rates of change
18. Gravity
19. Large and small numbers
20. Geography
21. Amount of concrete used in streets

Certainly, one could develop enough ideas to last a lifetime from a problem such as the above. In addition to the above list, a team of fourth graders developed a formula for determining the length of jumps of local animals in the city zoo and made graphs. A sixth grade class designed human bodies that were more efficient for jumping. They used a flea as a model.

The seventh graders tended to be more sophisticated. One wanted to know how long the jump would take. This led to trying to time the jumps of grasshoppers. We probably had more fun with this problem than any others. Another student wanted to know if the jumper drops faster, once he reaches the top of the jump. This led to some research on the works of Galileo. Since it was beyond us, we did not pursue it. However, a few of them did learn to evaluate Galileo's formulas for the path of a projectile.

Once we learn that students love to mathematize the world, get over our fears of not being able to answer all their questions, perhaps, then we can assist them in learning mathematics, especially process. Try a problem tomorrow, have fun, and with your students mathematize the world around you.

Nominating Committee

At the meeting of the TCTM Executive Committee during CAMT, December 4-6, 1975, the following nominating committee was appointed:

Madolyn Reed, Houston ISD, Chr.
Mrs. Bennett Touchstone, Sinton
Jim Bezdek, NTSU, Denton

Offices to be filled in the fall of 1976 are vice-president (secondary) and treasurer. If you have suggestions for names of nominees, please send them to a member of the committee NOW. The slate must be submitted to the editor for publication by March 1, 1976.

El Paso Hosts Meeting

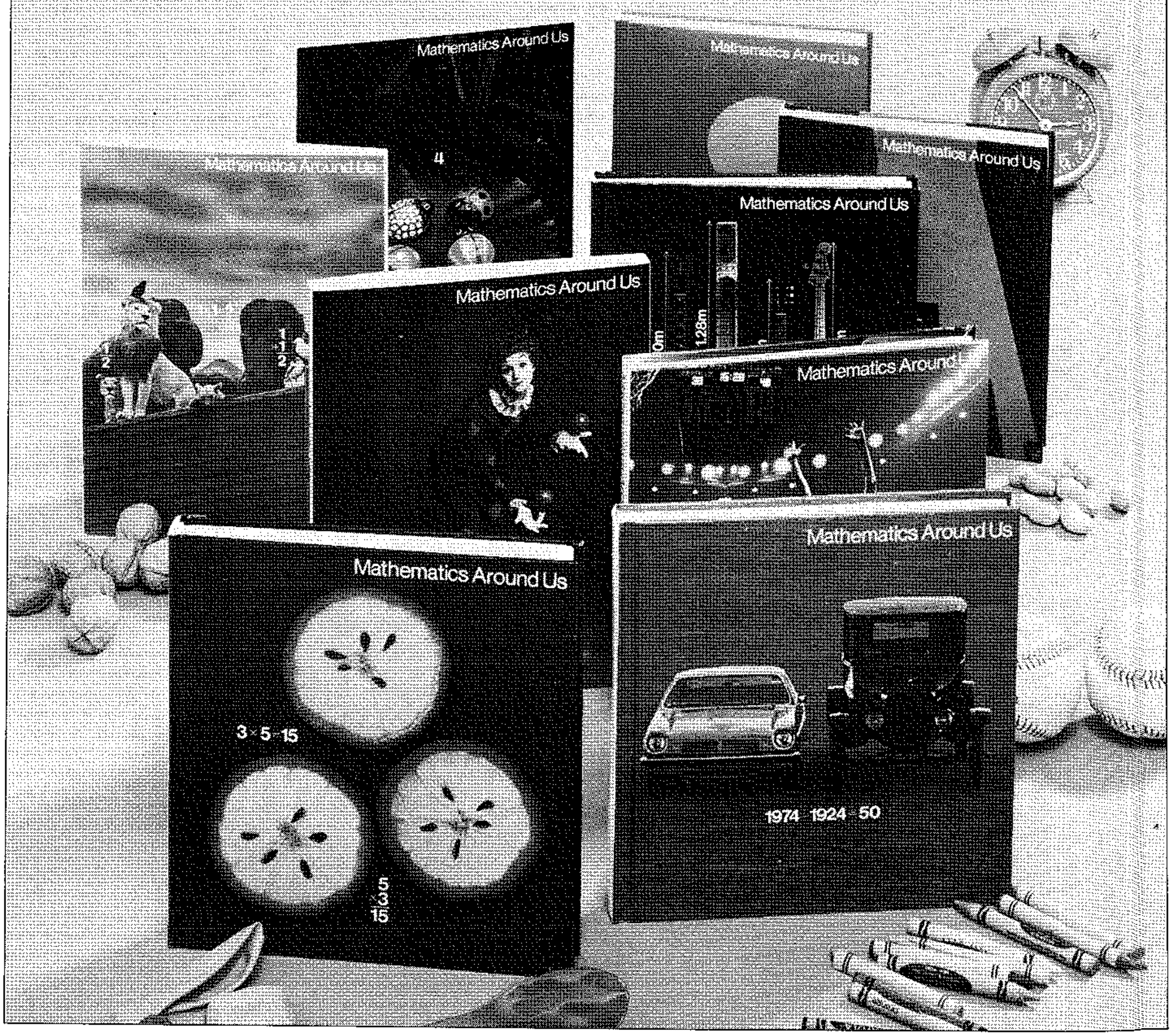
The El Paso Council of Teachers of Mathematics will host a NCTM Meeting on February 26-28, 1976. There will be more than 135 section meetings and more than 35 workshops stressing quality teaching at every level. Speakers are coming from over thirty states and provinces, including British Columbia, Florida, New York, Chihuahua, Washington and California. Join us as we honor Glenadine Gibb and Irene St. Clair for their numerous contributions to mathematics education.

The program will begin on Thursday, 26 February, at 1:30 P.M. and end on Saturday, 28 February, at 12:15 P.M. All convention activities will be held at the El Paso Convention and Civic Center, with the exception of the reception, which will be held at the Hotel Paso del Norte.

Betty Beaumont Nominated

Congratulations to Betty Beaumont. Her name appears among the nominees of NCTM directors to be elected in 1976. We urge all Council members of NCTM to support Betty when the election ballots arrive in January.

Mathematics Around Us



The real math program for 1976

Highlights of Mathematics Around Us Grades 1-6:

- Practical, real-world applications
- Heavy emphasis on basic computational skills
- High visual impact, minimum reading
- Thorough development of the metric system
- Extensive individualizing features
- Complete management system
- Efficient testing program

Texas representatives: Claude Campbell, Bill Brunson, Max Harper, Jim Kaml, Clark McPherson, Al Walker, Ron Wilkins, Dean Wilkinson
consultants: Pat Shaver, Sue Wagley

Scott, Foresman and Company 11310 Gemini Lane Dallas, Texas 75229

Instances of Pascal's Triangle

by David F. Robitaille
University of British Columbia

Pascal's Triangle (see Figure 1) is a fascinating example of a mathematical pattern. The Triangle was used by Blaise Pascal (1623-1662) in connection with his work in the field of probability. In spite of the fact that the arrangement bears his name, Pascal was not its discoverer. Boyer (1968) has traced the origins of the Triangle back to the twelfth century in Chinese works where it was used to find co-efficients of binomial expansions. In these as well as in some later works, the arrangement is called the Arithmetic Triangle.

Row 0	1					
Row 1	1	1				
Row 2	1	2	1			
Row 3	1	3	3	1		
Row 4	1	4	6	4	1	
Row 5	1	5	10	10	5	1

Figure 1 — Pascal's Triangle

One intriguing aspect of Pascal's Triangle is the number of seemingly different instances of it that can be found. Upon close examination, one can discern a common base underlying several such instances, but this does not detract from the surprise students feel upon seeing Pascal's Triangle emerge from so many situations.

1. Coefficients of the Powers of a Binomial

The most familiar instance of Pascal's Triangle arises from the numerical coefficients of the powers of a binomial. We compute the non-negative integral powers of a binomial, beginning with the zero power, and detach the numerical coefficients of the resulting expansions. This procedure is illustrated in Figure 2.

$(x+y)^0 = 1$	1				
$(x+y)^1 = x+y$	1	1			
$(x+y)^2 = x^2 + 2xy + y^2$	1	2	1		
$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$	1	3	3	1	
$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$	1	4	6	4	1

Figure 2 — Binomial Coefficients

The coefficients, when displayed in the usual triangular arrangement, form the rows of Pascal's Triangle. In this instance, the rows of the Triangle may be used to determine the numerical coefficients of a binomial expansion and vice versa.

2. Binomial Probability

The rows of Pascal's Triangle may be generated by answering the question, "What are the probabilities of obtaining 0, 1, 2, 3, . . ., n heads in one flip of n coins?" For example, in the case where n is 3, we are asking for the probabilities of obtaining 0 heads, 1 head, 2 heads, or 3 heads in one flip of 3 coins. These probabilities are $\frac{1}{8}$, $\frac{3}{8}$, $\frac{3}{8}$, and $\frac{1}{8}$ respectively as can be seen from an examination of the sample space for this experiment:

$$S = \{HHH, HHT, HTH, HTT, THH, TTH, TTT\}$$

The numerators of the four fractions are the elements of Row 3 of Pascal's Triangle, and any row of the triangle may be constructed in a similar way. Construction of the top row of the Triangle, i.e., Row 0, by asking "What is the probability of obtaining 0 heads in one flip of 0 coins?" should provoke some lively discussion among your students.

The data obtained from answering the coin-flipping questions might be displayed as shown in Table 1.

Number	Number of Heads						
of Coins	0	1	2	3	4	5	...
0	1						
1	$\frac{1}{2}$	$\frac{1}{2}$					
2	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$				
3	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$			
4	$\frac{1}{16}$	$\frac{4}{16}$	$\frac{6}{16}$	$\frac{4}{16}$	$\frac{1}{16}$		
5	$\frac{1}{32}$	$\frac{5}{32}$	$\frac{10}{32}$	$\frac{10}{32}$	$\frac{5}{32}$	$\frac{1}{32}$	

Table 1 — Probability of Obtaining n Heads

3. Subsets of a Finite Set

Textbooks at the junior high school level frequently include an enrichment section dealing with the number of subsets, proper and improper, or a finite set. Such a section is usually designed to have students discover the generalization that a set consisting of n elements has 2^n subsets. However, this topic may also be used to provide another instance of Pascal's Triangle if we ask a further question: "How many subsets of 0 elements, 1 element, 2 elements, . . ., n elements does a set consisting of n elements have?" For example, given a set with 4 elements, how many subsets does it have that consist of 0, 1, 2, 3, and 4 elements? The answers, which can be found in a number of ways, are 1 subset of 0 elements, 4 subsets of 1 element, 6 subsets of 2 elements, 4 subsets of 3 elements, and 1 subset of 4 elements. These numbers (1,4,6,4,1) constitute Row 4 of Pascal's Triangle and any row of the Triangle may be found in a similar way. The data obtained might be presented as in Table 2.

Number of Elements in the Set	Number of Subsets with n Elements						
	0	1	2	3	4	5	...
0	1						
1	1	1					
2	1	2	1				
3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	

Table 2 — Subsets of a Finite Set

4. Networks

A network, such as that shown in Figure 3, provides yet another instance of Pascal's Triangle.

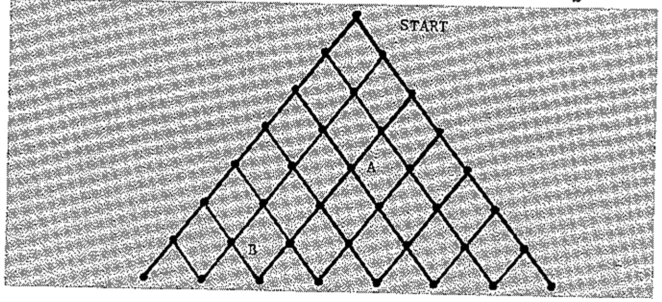


Figure 3

"How many ways are there to get from START to point A?" to Point B?" For convenience we denote the rows of the network in the same way as the rows of Pascal's Triangle: START is Row 0, the next line is Row 1 and so on. There is exactly 1 way of reaching either of the two points in Row 1 from START. We can record this on the network as shown in Figure 4.

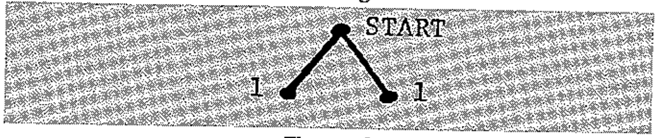


Figure 4

Continuing to Row 2, it is clear that there is only 1 way to reach either end point of Row 2 (or of any subsequent row for that matter) from START and that there are 2 ways of reaching the remaining point of Row 2. (see Figure 5.)

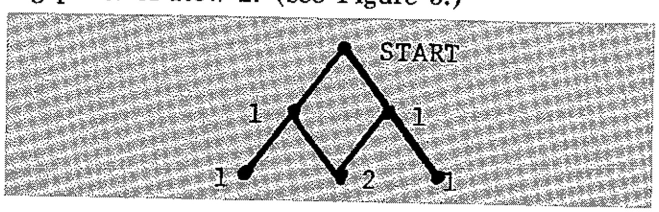


Figure 5

The familiar pattern becomes evident again. Continuing to Row 3, we see that there are 1, 3, 3, and 1 ways of reaching these four points from START. For the sake of completeness, we place a 1 at START and obtain yet another instance of Pascal's Triangle.

5. Light Switches

Given a row of n light switches, in how many ways can we have 0, 1,2, . . ., n switches on? For example, given a bank of 3 switches and using the numeral 1 to represent a switch in the ON condition and 0 to represent OFF, we see that eight possible arrangements of the switches are

000, 001, 010, 011, 100, 101, 110, 111

(Note that these are the binary representations of the numbers 0 through 7.) Counting the possibilities, we have 1 way to have 0 switches ON, 3 ways to have 1 switch ON, 3 ways for 2 switches, and 1 way for 3 switches. As before the data may be summarized in a table.

Number of	Number of Switches ON						
Switches	0	1	2	3	4	5	...
1	1	1					
2	1	2	1				
3	1	3	3	1			
4	1	4	6	4	1		
5	1	5	10	10	5	1	

Table 3 — Arrangement of Light Switches

6. Partitions

Given a positive integer n, how many ways are there to express n as the sum of 2 or more positive integers where the order of addends is important? For example, how many ways are there to express the number 5 as the sum of 2 or more positive integers. Tabulating the possibilities we have:

1+4	1+1+3	1+1+1+2	1+1+1+1+1
4+1	1+3+1	1+1+2+1	
2+3	3+1+1	1+2+1+1	
3+2	1+2+2	2+1+1+1	
	2+1+2		
	2+2+1		

If, for the sake of completeness, we consider 5 by itself to be a partition of 5 we then have 1,4,6,4, and 1 ways of writing 5 as the sum of 1,2,3,4, and 5 positive integers respectively. This, of course, gives us Row 4 of Pascal's Triangle and any other row may be generated in an analogous way.

7. Mathematical Spelling

In Figure 6, how many different paths result in the correct spelling of each word?

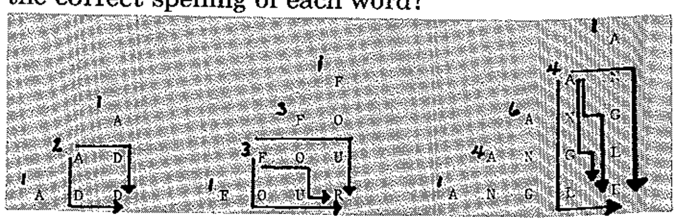


Figure 6 — Mathematical Spelling

Once more we have an instance of Pascal's Triangle, this one being a slightly disguised version of the network instance discussed earlier.

More than one mathematician has said that mathematics is the study of patterns. One of the intriguing aspects of mathematical patterns such as Pascal's Triangle is that the same pattern seems to occur in many different situations. Your students may enjoy looking at these various instances of the Triangle and some may be inclined to pursue that interest even further. Such students might be interested in investigating the close relationship that exists between Pascal's Triangle and the Fibonacci Sequence, for example.

References

Association of Teachers of Mathematics. *Notes on Mathematics in Primary Schools.* London: Cambridge University Press, 1967.

Boyer, Carl B. *History of Mathematics.* New York: John Wiley & Sons, 1968.

Wisner, Robert J. *A Panorama of Numbers.* Glenview, Illinois: Scott, Foresman and Company, 1970.

PROFESSIONAL MEMBERSHIP APPLICATION

Date:________ School:________ School Address:________

Position: ☐ teacher, ☐ department head, ☐ supervisor, ☐ student,* ☐ other (specify)________

Level: ☐ elementary, ☐ junior high school, ☐ high school, ☐ junior college, ☐ college, ☐ other (specify________

Other information________

		Amount Paid
Texas Council of Teachers of Mathematics	☐ New membership ☐ Renewal membership	5.00
Local ORGANIZATION:________	☐ New membership ☐ Renewal membership	
OTHER:________	☐ New membership ☐ Renewal membership	

Name (Please print)________ Telephone________

Street Address________

City________ State________ ZIP Code________

National Council of Teachers of Mathematics

Check one: ☐ New membership ☐ Renewal membership

$11.00 dues and one journal ☐ *Arithmetic Teacher* or ☐ *Mathematics Teacher.*

16.00 dues and both journals

5.50 student dues and one journal* ☐ *Arithmetic Teacher* or ☐ *Mathematics Teacher*

8.00 student dues and both journals*

6.00 additional for subscription to *Journal for Research in Mathematics Education* (NCTM members only)

.60 additional for individual subscription to *Mathematics Student Journal* (NCTM members only)

The membership dues payment includes $4.00 for a subscription to either the *Mathematics Teacher* or the *Arithmetic Teacher* and 25¢ for a subscription to the *Newsletter.* Life membership and institutional subscription information available on request from the Washington office.

* I certify that I have never taught professionally________ *(Student Signature)* *Enclose One Check for Total Amount Due* →

Fill out, and mail to Dr. Donald Perry, 2209 Kilgore, Baytown, Texas 77520

NOW!

TEXAS MATHEMATICS TEACHER

J. William Brown, Editor
Texas Council of Teachers Of Mathematics
Woodrow Wilson High School
100 S. Glasgow Drive
DALLAS, TEXAS 75214

NON-PROFIT ORGANIZATION
U. S. Postage
Paid
Dallas, Texas
Permit #4899