Vol. XXII No. 2

TEXAS MATHEMATICS TEACHER

March 1975

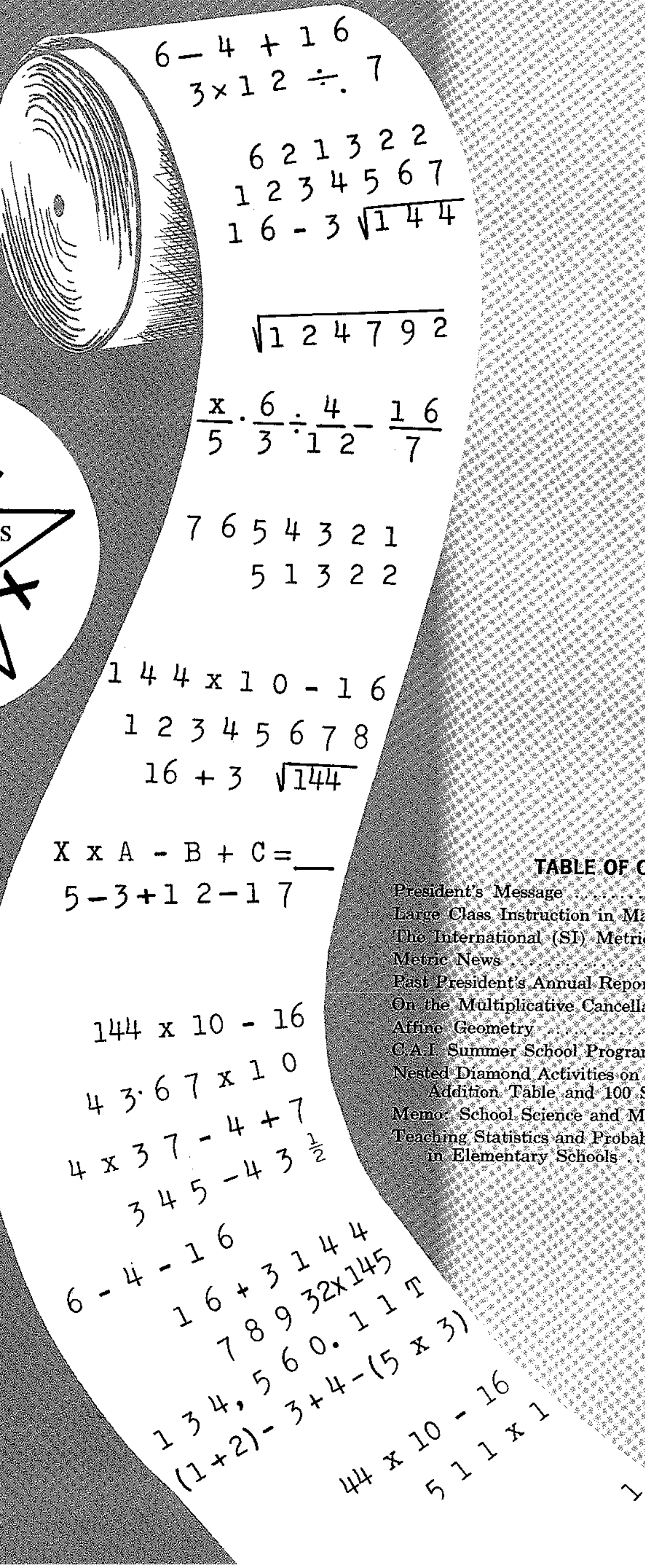


TABLE OF CONTENTS

TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

President: W. A. Ashworth, Jr.
J. Frank Dobie High School, PISD
1307 Cottonwood
Pasadena, Texas 77502

Vice-Presidents: John Savell
Lanier High School, AISD
3602B Las Colinas Dr.
Austin, Texas 78731

Mrs. Evelyn Bell
8445 Valdespino
El Paso, Texas 79907

Dr. Harry Bohan
Sam Houston State University
Huntsville, Texas 77340

Secretary: Mrs. Josephine Langston
127 West Harvard Drive
Garland, Texas 75041

Treasurer: Dr. James Rollins
1810 Shadowwood Drive
College Station, Texas 77840

Parliamentarian: Kenneth Owens
St. Mark's School of Texas
10600 Preston Road
Dallas, Texas

Editor: Mr. J. William Brown
Woodrow Wilson High School
3632 Normandy Street
Dallas, Texas 75205

Journal Business Manager: Shirley Cousins
4515 Briar Hollow Pl. #218
Houston, Texas 77027

Past President: Mrs. Shirley Ray
Elementary Coordinator
Corpus Christi
Independent School District
3443 Olsen Drive
Corpus Christi, Texas 78411

N.C.T.M. Rep.: Helen Gascoyne
9455 Viscount Blvd.
El Paso, Texas 79925

TEXAS MATHEMATICS TEACHER is published quarterly by the Texas Council of Teachers of Mathematics. Payment of membership fee of $5.00 entitles members to all regular Council Publications.

President's Message

Some of you have probably already heard by way of the grapevine the dates for CAMT — 1975 have had to be changed. These dates are now December 4-6. There were conflicts with hotel accommodations which caused the change. In view of this, dates are being firmed up for the 1976 meeting which will be in October as so many of you have requested.

Many of you by now have received a membership renewal notice. Should you have paid your dues already, please do not be offended by this. The mailings were based on the membership information current to the CAMT meeting in November. Often when dues are paid through Area Councils there is some delay in the memberships being forwarded to TCTM. So that no one will be slighted, the Journal will be mailed to you four times from the Treasurer's receipt of your membership.

The newsletter you have received was the result of numerous requests for activity materials, more current news, recognition of new programs, and current news concerning area councils. Again our thanks to Judy Tate, Harris County Department of Education for preparing this for us. It has been suggested that if such a newsletter, which will in no way replace the Journal, is the desire of the membership it could be mailed bimonthly to complement the Journal.

If this is the course we are to take we have four needs:

1) more membership income — new members — to assist in financing the project;

2) someone who has access to a school "career-printing" facility to secure printing economically;

3) someone to serve as coordinator of the activity (volunteer anyone?);

4) contributions for the newsletter. This is vitally important.

I would appreciate whatever input you wish to contribute to these suggestions. We will be guided by your reactions as to the planning of another newsletter.

Byron Craig has decided to decline to be NCTM Representative. He will be replaced by Helen Gascoyne of El Paso. Thank you, Helen, for assisting us in this way.

By the time you receive this another school year will be almost gone. Budgets for next year will be upon us; and spring will be in the air. Most of us will have begun counting the days until school is out — some of our students have been at this for a long time. In the midst of all this, perhaps we will be able to find the key which will open the door of understanding for that one student who has not been reached — perhaps we will ourselves get a "second wind" to really achieve more of our potential as the teachers we can and should be. Perhaps we can, with the time remaining in the year, lose sight of our excuses for doing little and find reason for giving our best to achieve something. Perhaps!

BILL ASHWORTH

Dates to Remember

APRIL 23-26, 1975 : NCTM 53rd Annual Convention, Denver, Colorado

AUGUST 20-22, 1975 : Honolulu NCTM Meeting

DECEMBER 4-6, 1975 : CAMT (TCTM) Austin, Texas

LARGE CLASS INSTRUCTION IN MATHEMATICS

Floyd Vest and B. G. Nunley
North Texas State University

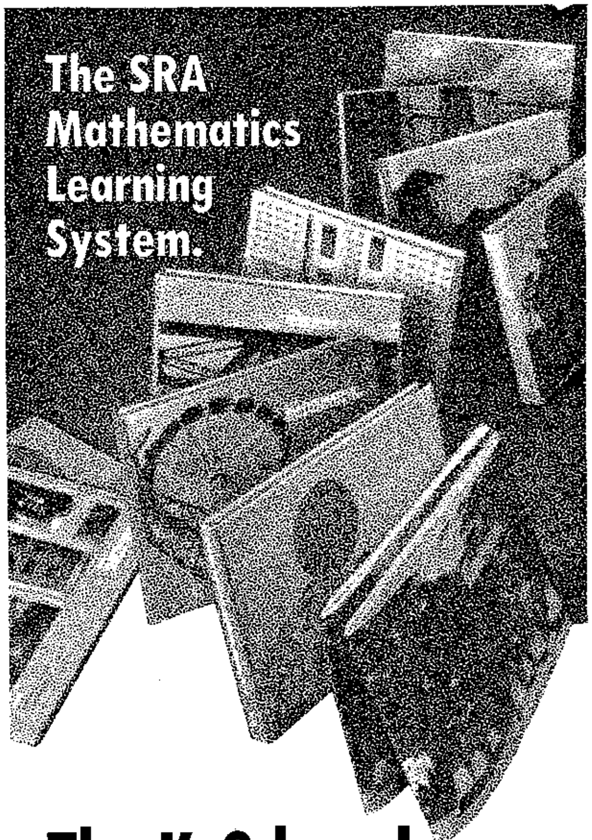
Many pressures contribute to the need for efficient use of qualified and interested teachers of mathematics courses for prospective elementary teachers. Although it appears that no studies of large class instruction have been conducted in the area of mathematics for prospective elementary teachers, extensive experimental comparisons of large and small class instruction have been reported at Miami University (Macomber and Siegel, 1957) and other studies are reported in several sources (De Cecoo, 1964; Harris, 1960; Nelson, 1959).

Problem

The purpose of this study is the comparison of large class instructional procedures with conventional small class procedures for a mathematics course for elementary education majors (Mathematical Association of America, 1964). The treatments were compared on the basis of mathematical achievement and attitude toward arithmetic. Relationships between mathematical achievement, attitude toward arithmetic, initial attitude toward the experimental treatment, and mental ability were investigated.

Procedure

The experiment involved two experimental large classes, two control small classes, and two instructors as described in Table 1. The control classes were taught by the conventional method — one teacher for each class; classes met three clock hours per week for the usual testing, lecture, and discussion procedure. The experimental classes attended lecture for two clock hours per week and a recitation session of about twenty students each



The K-8 basal for teachers who'd like to love teaching Math.

Ask your SRA representative to show you the SRA Mathematics Learning System. How colorful, functional and fun it is. How it helps kids love to learn math.

And how it helps teachers love to teach it.

SRA © 1974, SRA

SCIENCE RESEARCH ASSOCIATES, INC.
A Subsidiary of IBM

Table 1
EXPERIMENTAL DESIGN

Semester	Experimental Group	Control Group
Fall	Instructor A 98 students	Instructor B 17 students
Spring	Instructor B 77 students	Instructor A 26 students
	Total 175	Total 43

for one clock hour per week. Instructor A, Instructor B, and one other instructor taught all the recitation sessions in which homework and topics from the textbook were discussed. The large lecture sessions were taught by the usual lecture, discussion, and testing procedures as in the small classes with somewhat less discussion. The instructors in both types of classes used only the traditional chalkboard for instruction.

With the exception of a dozen or so students, all the subjects were female elementary education majors of sophomore and junior standing. All subjects studied the first seven chapters of the same textbook (Peterson and Hashisaki, 1967). Homework and attendance were treated the same in all classes. All students were informed that the results of the experimental evaluative instruments would not count toward course grades. Since all class meetings except recitations were scheduled at 12 noon, it was possible to attempt random assignment of students to treatments as they appeared at the registration table. However, this procedure was interfered with to a small extent due to administrative difficulties. The analysis of the pre- and post-data was conducted in terms of residualized gains.

The evaluative instruments were:

Mathematics Inventory I and II (MI) — a multiple-choice teacher-made test of recall of the mathematical facts and principles of the course with a split-half corrected reliability coefficient of .87.

LC Scale — a Thurstone-type attitude scale which measures attitude toward the experimental treatment with a reliability coefficient of .83 based on the administration of two equivalent forms. Larger scores on this eleven unit scale represent more positive attitudes toward the treatment.

Dutton Scale for Attitude Toward Arithmetic (D) — a Thurstone scale for measuring attitude toward arithmetic with a reliability coefficient of .94 based on test-and-retest. Larger scores on this eleven unit scale represent more positive attitudes toward arithmetic.

Otis Quick-Scoring Mental Ability Test (Otis) — a commercial standardized test of mental ability.

At the beginning of each semester, Mathematics Inventory I, LC Scale, Dutton Scale for Attitude Toward Arithmetic, and the Otis Quick-Scoring

Mental Ability Test were administered to all subjects. At the end of each semester, Mathematics Inventory II, LC Scale, and Dutton Scale for Attitude Toward Arithmetic were administered.

Results

A 2 x 2 analysis of variance was conducted with residualized gains resulting from pre- and post-administration of Mathematics Inventory as the dependent variable. As independent variables, students were stratified on two levels of mental ability (upper 50 per cent and lower 50 per cent as determined by the Otis Quick-Scoring Mental Ability test) and the two treatments consisting of experimental and control. These results are reported in Table 2. As can be seen from the table, the F ratio indicating difference in mathematical achievement between the groups with high and low mental ability was significant as might be expected. The F ratio indicating difference in mathematical achievement between treatments was also significant. The mean residualized gain for the experimental group was —.81 while that of the control group was 3.39 indicating greater mathematical achievement on the part of the control group. The F ratio indicating interaction between level of mental ability and treatment was not significant.

Table 2
Summary of Analysis of Variance on Residualized Gains on Mathematics Inventory; Treatments; and Mental Ability

Source	Sum of Squares	df	Mean Square	F
Treatment	622.20	1	622.20	14.04***
Mental Ability	636.55	1	636.55	14.36***
Interaction	18.21	1	18.21	.41
Within Cells	9486.68	214	44.33	

***p < .001

In order to study the effect of treatment on the important dependent variable of attitude toward arithmetic held by future elementary school teachers, a 2 x 2 analysis of variance was conducted with residualized gains resulting from pre- and post-administration of the Dutton Scale for Attitude Toward Arithmetic as the dependent variable. As independent variables, students were stratified on the basis of experimental and control treatments and two levels of mental ability (upper and lower 50 per cent as determined by the Otis). The results are reported in Table 3. The F ratio which indicates difference in achievement in attitude toward arithmetic between treatments was significant at the .05 level. The mean residualized gain on the Dutton Scale for Attitude Toward Arithmetic for the experimental group was —.07 and the mean for the control group was .27 indicating a more favorable change in attitude on the part of the control group. The other F ratios in Table 3 were not significant. All groups involved had means of difference scores based upon pre- and post-administration of the Dutton Scale which were positive. A positive difference score arises from a more favorable attitude score on the post-administration than on the administration at the beginning of the course.

Table 3
Summary of Analysis of Variance on Residualized Gains on Attitude Toward Arithmetic; Treatments; and Mental Ability

Source	Sum of Squares	df	Mean Square	F
Treatment	4.29	1	4.29	4.12*
Mental Ability	.15	1	.15	.14
Interaction	.10	1	.10	.10
Within Cells	222.67	214	1.04	

*p < .05

One might conjecture that the initial attitude toward the experimental treatment would have a significant effect on the mathematical achievement for those in the experimental group. As can be seen in Table 4, the upper and lower 50 per cent of the students in the experimental group, as determined by Pre LC scores, were not significantly different nor did one achieve significantly more than the other in mathematical knowledge. These results indicate that the initial attitude toward the experimental treatment did not have a significant effect on the mathematical achievement of those in the experimental group.

Table 4
Means on Otis and Residualized MI of Groups with High and Low Initial Attitude Toward the Experimental Treatment

	Upper 50% on Pre LC in Exp. Group N = 87		Lower 50% on Pre LC in Exp. Group N = 87		*t*
	Mean	SD	Mean	SD	
Otis	55.08	7.80	54.98	8.98	.08
Res. MI	-.09	6.92	-1.53	6.28	1.43

Summary

On the basis of these findings, the following general summary seems warranted:

1. Elementary education majors enrolled in mathematics courses taught by an experimental large class procedure seemed not to achieve as well in mathematical knowledge and attitude toward arithmetic as those taught by conventional small class procedures.

2. The two procedures did not seem to produce a differential effect or interaction with levels of mental ability, and levels of initial attitude toward the experimental treatment.

Such findings as these tend to indicate that the large class procedure adopted in this study is inferior to the conventional small class procedure.

NCTM

THINK DENVER in '75

APRIL 23-26

THE INTERNATIONAL (SI) METRIC SYSTEM

A WORD OF CAUTION TO EDUCATORS!!

Delaware educators have been responding positively and enthusiastically to the State Board of Education's resolution on the International Metric System of Measurement. The State Board's resolution calls for all schools to be providing instruction in the metric system commencing with the 1976-77 school year. By the year 1980, the conversion of all measurement language to the International Metric System of Measurement shall be provided in all phases of public education.

A recent survey by the Metric Information Office of the National Bureau of Standards indicated that 19 states have undertaken some kind of action in metric education through their state department of education or legislature. Forty-three states reported other formal, state level activity, such as workshops, formation of metric committees, and development of metric materials.

School districts in Delaware are moving ahead in providing instruction in metrics. Metric workshops provided through the State Department of Public Instruction, the Del Mod System and local districts, have reached approximately 1,200 teachers.

School districts are purchasing and will continue to purchase metric instructional materials for use in the classroom. This is where a word of caution is needed!! The metric system that will be used in the United States is the International System of Units, or SI (Le Système Internationale d' Unites), established by the Central Conference of Weights and Measures in 1960 and interpreted or modified for the United States by the Secretary of Commerce. Because SI is an international language of measurement, it is important that all countries use the same base units, derived units, prefixes, symbols and rules. The symbols, not the written words, are part of the international language of SI. The spelling of the words change from country to country, but the symbols remain the same. Both prefixes as well as units are expressed in symbols.

Some schools have purchased material with symbols used as abbreviations and where units are omitted. There is so much conflicting material being offered to schools that many schools have delayed purchasing as well as their programs.

Many educational suppliers have not done their homework. If a date on printed material is before 1972, it may be obsolete material, but, a recent date does nothing to establish that it was not copied from older information. There are other metric systems being used in the world. Not only are

References

DeCecco, J. P. Class size and co-ordinated instruction. *British Journal of Educational Psychology,* 1964, XXXIV, 65-74.

Dutton, W. H. Measuring attitude toward arithmetic. *Elementary School Journal,* 1954, LV, 24-31.

Dutton, W. H. Attitude change of prospective elementary school teachers toward arithmetic. *Arithmetic Teacher Journal,* 1962, IX, 418-424.

Harris, C. W. (Ed.) *Encyclopedia of educational research.* (3rd ed.) New York: Macmillan, 1960.

Macomber, F. G. & Siegel, L. A study in large-group teaching procedures. *Educational Research,* 1957, XXXVIII, 220-229.

Mathematical Association of America, Committee on the Undergraduate Programs in Mathematics. *Course guide for the training of teachers of elementary school mathematics.* (Fourth draft) Berkeley: MAA, 1964.

Nelson, W. B. An experiment with class size in teaching elementary economics. *Educational Record,* 1959, XL, 330-341.

Peterson, J. A. and Hashisaki, J. *Theory of arithmetic.* (2nd ed.) New York: John Wiley, 1967.

countries changing from the use of their customary non-metric units to SI, but "metric" countries of long standing, like France, Holland, and the USSR, have changed, or are changing to the "new" system.

Some Suggestions for Educators to Use in the Transition to Metrics

- Learn the International System of Units (SI) before acquiring instructional material and before teaching someone else.
- Select a committee or individual to be responsible for the metric authority in your school?
- Check any material against the SI before using or purchasing for classroom use.

— *Borrowed from Delaware Council of Teachers of Mathematics* NEWSLETTER, *December, 1974*

METRIC NEWS

1. In reading articles about the metric system and in studying the metric system, the letters SI appear frequently. SI refers to the universal abbreviation for Le Systēme Internationale d' Unites (the International System of Units).
2. The trend seems to be to spell metre and litre rather than meter and liter.
3. In pronunciation the accent is on the prefix, such as in kilowatt.
4. All prefixes, except tera, giga, and mega, use a small letter as a symbol.
5. The prefix and unit name should not be separated by a hypen (kilometre not kilo-metre).
6. There is no period after the symbol unless it is at the end of a sentence.
7. All symbols should be used in the singular form (15 metres = 15 m not 15 metres = 15 ms).
8. SI unit names do not start with a capital letter unless they are at the beginning of a sentence.
9. The numerical value associated with a symbol should be separated by a space (16 mm not 16mm).
10. Use a space to separate large numbers into groups of three. Do not use a comma (1 020 123 not 1,020,123).
11. Avoid the use of fractions. Express a partial unit as a decimal with a zero before the decimal marker (0.25 km and not ¼ km).
12. Use decimal notations for computation and recording measurements in the metric system.
13. A prefix should not be used alone (kilogram not kilo).
14. The use of m^2 and m^3 is preferred to square metres and cubic metres.
15. The word "mass" is used in preference to "weight." Weight is a force and requires force units. Grams and kilograms measure "mass" while newtons measure "force." Use "What is the mass?" rather than "What is the weight?" if we are using grams or kilograms.
16. Metric measurements should be recorded as a single unit. For example, 1 metre and 41 centimetres should be recorded as 1.41 metres or 141 centimetres.
17. The symbol for Celsius is a capital "C."

GIVE THE 2.5 CENTIMETRES AND THEY WILL TAKE 1.6 KILOMETRES.
1.57 METRES AND EYES OF BLUE.

Recommendations For Equipment For Teaching the Metric System of Measurement — write to J. B. West, President, Spectrum Educational Supply Limited, 8 Devism Street, Markham, Ontario, Canada L3R 2P2.

—IBID, *February, 1975*

Denver NCTM Highlights

★ **23 April 1975** 8:00 p.m.
Opening address: Eric McPherson, University of Manitoba, Winnipag, Canada. "The Neo-Orthodox Position."
Delegate Assembly during the day.

★ **24 April 1975** 7:30 p.m.
Presidential Address: E. Glenadine Gibb, University of Texas at Austin, Austin, Texas and NCTM President "Aspirations — Actualities — Anticipations."

★ **25 April 1975** 7:30 p.m.
Banquet: The speaker is Lt. Col. Ben M. Pollard, United States Air Force Academy, Colorado Springs, Colorado. "Aerodynamics, Hanoi Hilton Style."

★ Over 200 section meetings, classified as elementary, junior high, secondary, two-year college, general interest, teacher education, and research in mathematics education. Section meetings will be held during the day on April 24th and 25th and on the morning of April 26th.

★ Approximately 115 workshops, including computer workshops. Workshops will be held during the day from noon of April 23rd through noon of April 26th. Workshop spaces are available through pre-registration.

WHAT IS YOUR SCHOOL DOING?

Inform the Editor for inclusion in the *JOURNAL*.

If you were *there* and heard a talk which impressed you, could you send us a summary? Our readers would appreciate sharing your reactions and impressions. Let's hear from you.

ARTICLES WANTED

Is there some technique of which you are particularly proud? Would you be willing to write about it? Send it to the editor. Even a short anecdote or a paragraph will do. We are particularly interested in articles dealing with all levels of mathematical education. Include snapshots if possible.

Past President's Annual Report

November 22, 1974

The Texas Council of Teachers of Mathematics had many successes during the 1973-74 Council years.

The year began following the annual meeting November 16, 1973. The conference, at which this meeting was held, was co-sponsored by Texas Council and was a tremendous success. This was the first such conference conducted by the co-sponsoring groups under the direction of a steering committee. We had hoped for 500 or 600 participants. The final count was 949, and the evaluations indicated it was the best state mathematics convention many had attended.

Local school districts and mathematics councils who co-sponsored workshops again this year were

- Coastal Bend Council of Teachers of Mathematics and Corpus Christi Independent School District. The Education Service Center, Region II also co-sponsored this workshop
- Pasadena Mathematics Council and Pasadena Independent School District

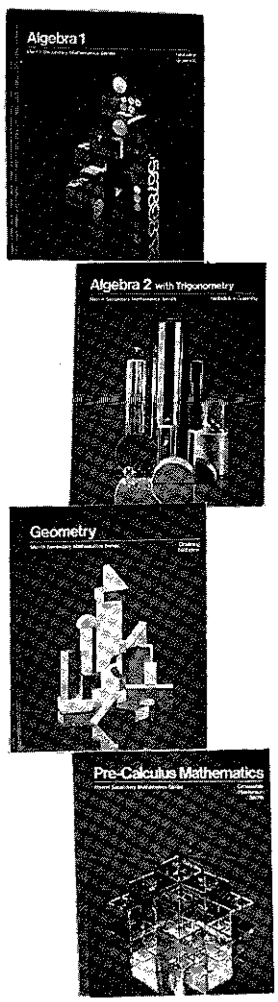
TCTM presented a mathematics program on the quarter system at the state TSTA convention in Fort Worth.

Speakers and workshop leaders represented Texas Council across the state during the year.

TCTM has been able to strengthen its affiliation with NCTM through meetings with the representative of the Committee on Affiliated Groups (CAG). Dr. Sarah Burkhart conducted two meetings with local council presidents to provide a vital communication link among NCTM, TCTM, and local councils.

Texas continues to receive recognition for its outstanding endeavors toward improving mathematics for boys and girls and youths in our state. TCTM provided many opportunities to share in this endeavor. Many teachers across the state took advantage of these opportunities during 1973-74.

Shirley Ray
President, 1973-74



Do your students really understand mathematics . . .

or do they think of it as just a bunch of numbers and symbols? The MERRILL SECONDARY MATHEMATICS SERIES can help you and your students discover how mathematics relates to the world outside the classroom. This program contains numerous problems and special pages featuring applications that turn numbers and symbols into *real math* for the *real world.*

To learn more about how the MERRILL SECONDARY MATHEMATICS SERIES can help you teach math for the real world, contact the Merrill representative in your area.

Representatives serving you are:

Ralph Carrington
5307 Athens
Tyler, Texas 75701
(214) 597-8740

Fred H. Ellis, Jr.
1138 Challenger
Austin, Texas 78746
(512) 261-5413

James W. Roberts
P. O. Box 26
Fulshear, Texas 77441
(713) 346-1548

CHARLES E. MERRILL PUBLISHING CO.
A BELL & HOWELL COMPANY
1300 Alum Creek Drive • Columbus, Ohio 43216

BELL & HOWELL

ON THE MULTIPLICATIVE CANCELLATION PROPERTY

JAMES R. BOONE
Department of Mathematics
Texas A&M University

The multiplicative cancellation property for the integers states that: if a, x and y are integers such that $a = 0$ and $ax = ay$, then $x = y$. In dealing only with integers, this property is not verified by dividing both sides by the number a. It is the result of a fundamental property of the integers that is often referred to and usually never established.

This important property of the integers is: (\$) if a and b are integers and $ab = 0$, then either $a = 0$ or $b = 0$. (This is sometimes stated as: the integers have no non-zero divisors of zero. It is this property that makes the ring of integers an integral domain.)

I will establish that the integers have property (\$) and then use this property to prove the multiplicative cancellation property. Recall that a whole number m is defined as the property which is common to all sets in the equivalence class generated by a set M under the matching relation on the class of all sets. This is denoted by $m = n(M)$. For example, $2 = n([\{p,q\}])$ and the common property of the equivalence class generated by $[\{p,q\}]$ is "twoness".

The binary operation of multiplication is defined on the whole numbers as follows: if a and b are whole numbers and A and B are sets such that $a = n(A)$ and $b = n(B)$, then $ab = n(AxB)$, where AxB is the set of all ordered pairs (r,s) where r is an element of A and s is an element of B.

To prove (\$), let a and b be whole numbers such that $ab = 0$. Let A and B be sets such that $a = n(A)$ and $b = n(B)$. Then $ab = n(AxB)$. Since $ab = 0 = n(AxB)$ and $0 = n(\emptyset)$, the empty set $\emptyset$ and the set AxB must be in the same equivalence class under the matching relation. The only set which can be matched to the empty set is the empty set. Hence $AxB = \emptyset$. The only way AxB can be empty is to either have no first elements or to have no second elements. Thus either $A = \emptyset$ or $B = \emptyset$. Accordingly, either $a = n(A) = 0$ or $b = n(B) = 0$. Hence property (\$) is established. That is, for any integers a and b, if $ab = 0$, then either $a = 0$ or $b = 0$.

To show that (\$) implies the multiplicative cancellation property, suppose $a \neq 0$ and $ax = ay$. By adding $(-ay)$ to both sides, $ax + (-ay) = 0$. By the distributive property, $a(x - y) = 0$. Since $a \neq 0$ and $a(x - y) = 0$, (\$) implies $x - y = 0$ or $x = y$. Accordingly, if $a \neq 0$ and $ax = ay$, then $x = y$.

The logical connection between (\$) and the multiplicative cancellation property is stronger than I have indicated so far. In fact, these properties are equivalent. In view of what was proven in the preceding paragraph, to verify this equivalence, I need only to show that the multiplicative cancellation property implies property (\$). To do this, let a and b be integers such that $ab = 0$. Suppose $a \neq 0$. Since $ab = a0$, by the multiplicative cancellation property, $b = 0$. A similar argument shows that $b \neq 0$ implies $a = 0$. Thus in either case, either $a = 0$ or $b = 0$. Hence the multiplicative cancellation property implies property (\$).

AFFINE GEOMETRY

by Kenneth Cummins
Kent State University

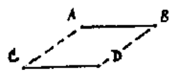
The underlying idea of "transformational geometry" or "geometry through transformations" is that different geometries are characterized by what properties remain invariant under different kinds of transformations. The properties which remain invariant under rigid motion transformations in the plane [translation and rotation] make up the geometry of Euclid. A more recent approach to this geometry has been through transformations known as *isometries* which are length- and angle-preserving but which include translation, rotation *and* reflection. It can be shown that these three transformations can be based on reflections only. The isometries as a class preserve length of line segment, angle-size, parallelism, midpointness, cross-ratios and, indeed, all geometric properties except orientation. Although there are formulae which can express isometries there exist also *postulate systems*, as we all know, from which can be deduced the same geometry as that which arises as the set of properties invariant under transformations which are isometries.

$$T: \begin{cases} \bar{x} = ax + by + c \\ \bar{y} = dx + ey + f \end{cases}$$
$$\{ae - bd \neq 0\}$$

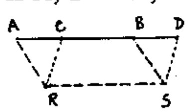
which is called an *affine* transformation is not length- or angle- preserving but it does preserve parallelism, ratio of lengths of segments, equality of lengths of line segments if they are parallel and incidence properties of points and lines as well as other projective and topological properties. *Postulationally* affine geometry is a consequence of primitive statements on incidence and the Playfair postulate on parallels — that through a given point P not on line L there can be drawn one and only one line parallel to L.

It is interesting to note that many theorems in high school geometry are really *affine* theorems and hence we should be able to prove them in an *affine* postulate framework. For example, in proving the well-known theorem that the diagonals of a parallelogram intersect at their mid-points we use (as did Euclid) the congruence of angles and segments which properties belong to euclidean geometry. Since this theorem concerns only the affine properties of parallelism and midpointness it should be possible to prove it with affine methods. Herein lies a challenge.

In affine geometry we construct and define a very useful concept — that of *parallel congruence.* The segments AB and CD are defined to be parallel congruent. (A, B ≡ C, D) if AB || CD and AC || BD.

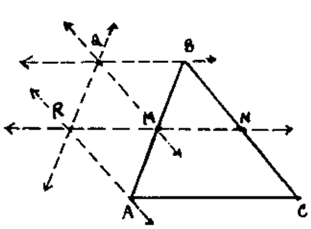


If the points A, B, C, D are collinear then A,B ≡ C,D is defined to mean that there exists a segment RS such that AB || RS, AR || BS, CD || RS and CR || DS. It follows immediately that if A, B ≡ C, D then A, C ≡ B, D; also parallel congruence is either proved or taken to be an equivalence relation. Too, we define P to be the *midpoint* of AB if A, P ≡ P, B.



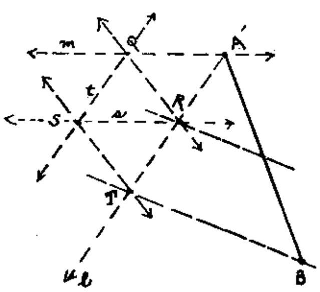
Let us now use these ideas in proving that if a line is drawn through the midpoint of one side of a triangle parallel to another side then it bisects the third side.

P. Let M be the midpoint of AB and let the line through M parallel to AC intersect BC at N. Through M and B draw lines parallel to BC and MN respectively to get point Q. Through Q draw a line parallel to AB which meets MN extended in R. Draw AR. (The above steps are based on the Playfair postulate and on incidence postulates.) Now by definition of parallel congruence Q, R ≡ B, M and by definition of midpoint M B, M ≡ M, A and hence Q,R ≡ M,A. Now Q,M ≡ B,N and therefore RA || BN || NC and therefore R,A ≡ N,C and B,N ≡ N,C and hence N is the midpoint of BC. (The reader might devise an easier way to prove this.) Q.E.D.



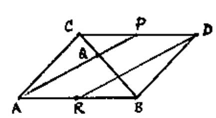
The reader might like to consult Chapter 9 of Prenowitz and Jordan's *Basic Concepts of Geometry* (Waltham, Mass.: Blaisdell Publishing Company, 1965) for basic ideas in affine geometry.

The proof of the above theorem suggests a way to construct the midpoint of any given line segment by a method other than constructing a parallelogram with the given line segment as a diagonal. By repeated use of Playfair's postulate and the concept of parallel congruence we can do the following: Let AB be the segment to be bisected. Through A draw lines L and M as shown. On M take a point Q and through Q draw a line to meet L at, say, R. At R draw a line S parallel to M. Through Q a line T parallel to L and get point S. Through S draw a line parallel to QR and get point T. R is the midpoint of AT. Now draw TB and through R draw a line parallel to TB and where this intersects AB will be the midpoint of AB. This method can be extended to trisect and "n"-isect line segments. Incidentally all this can be done with the *parallel ruler* — the tool of affine geometry.

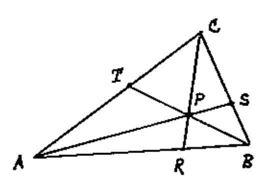


It might be interesting now to attempt to prove other affine theorems with the method demonstrated above. Some suggestions, other than the first affine theorem stated above, are;

(a) if in the parallelogram shown below P is the midpoint then Q trisects BC and also AP;
(b) if P and R are midpoints then AP || RD;



(c) the medians of a triangle are concurrent;
(d) the theorem of Ceva which says that



We see that Euclid did not need all his postulates for some of his very important theorems since part of his structure belongs to the less demanding affine geometry. It hence becomes a very fascinating challenge for students and teachers alike to try to prove these theorems with less strong postulate systems.

C. A. I. SUMMER SCHOOL PROGRAM 1974

By TCTM President

Computer assisted instruction (CAI) was incorporated into Pasadena Independent School District's Intermediate Summer School Program in 1974. Pasadena's summer program is traditionally for remedial work only.

Prior to the close of the 1973-1974 school year, intermediate school counselors and mathematics teachers were asked to submit the names of sixth and seventh grade students who were at least one year behind in mathematics achievement. Letters were sent to the parents explaining the program and from the responses returned, 87 students were enrolled in the program. Only one of the students enrolled failed to complete the program.

The students attended class five hours each day for three weeks. The students were divided into four classes, two morning and two afternoon groups.

Eleven teletypewriter terminals, connected by telephone lines, were linked to Pasadena's Hewlett-Packard 2000F Time Sharing Computer, and Hewlett-Packard's "Mathematics Drill and Practice" curriculum was used. Textbooks were not issued, nor assignments given for work outside class. Activities such as mathematics puzzles and games were available for use when the students were not using the terminals. Parents were informed periodically of student progress and were encouraged to visit the classes. Many parents visited the classes, some returning several times.

A criterion referenced test, Tests of Achievement in Basic Skills, Level B (TABS), was administered at the beginning of the program to identify the specific needs of the students. As these areas were encountered by the students, individual assistance was given each student.

The TABS test measures achievement in three areas: arithmetic, geometry, and modern concepts. Thirty-three concepts are tested in arithmetic, 25 in geometry, and 11 in modern concepts. The results of the test, in terms of percent of these concepts mastered, are as follows:

Percent of Concepts Mastered

Group	No. of Students	Arithmetic	Geometry	Modern Concepts	Total
1st term, morning	25	61.4	45.1	51.3	53.7
1st term, afternoon	18	66.8	61.3	63.6	64.3
2nd term, morning	22	59.6	51.8	47.1	54.8
2nd term, afternoon	22	55.2	52.8	50.8	53.7
Total	87	60.5	52.1	52.6	56.2

Basically, the curriculum was drill in computation skills. Each "year" of the curriculum was divided into 24 blocks. The "year" of curriculum in the material does not coincide with our curriculum guides as prescribed by the Texas Education Agency. The six "years" as developed in the computer curriculum closely parallels the arithmetic concepts prescribed through the eighth grade.

Each block contains five levels of proficiency. The student is tested at the beginning of each block. If he makes no errors in the pretest, he is moved to the next block immediately. The level he is assigned within the block is determined by his score on the pretest. Each student remains in the same block and level until he scores 85% or better. Those scoring below 70% on any level are periodically given review lessons over those concepts. At the completion of the fifth level of each block, the student is given a post-test. A score of 70% or lower on the post-test of any block also caused the student to receive review lessons in those concepts.

At the conclusion of each term the students were given another criterion reference test in the same series, TABS. The results, again in percent of concepts mastered, are as follows:

Percent of Concepts Mastered

Group	No. of Students	Arithmetic	Geometry	Modern Concepts	Total
1st term, morning	24	66.1	51.8	54.5	58.8
1st term, afternon	18	71.2	58.8	72.2	66.9
2nd term, morning	22	62.1	52.2	51.6	56.9
2nd term, afternoon	22	68.5	55.3	61.5	62.6
Total	86	66.9	54.3	59.2	61.1

In terms of increase of improvement, the following results were obtained by comparing percentage of mastery on pre-test and post-test:

Percent of Increase

Group	Arithmetic	Geometry	Modern Concepts	Total
1st term, morning	8.6	14.9	6.4	9.5
1st term, afternoon	6.5	—4.0	13.4	4.2
2nd term, morning	4.2	1.0	9.7	3.8
2nd term, afternoon	23.9	5.2	21.1	16.6
Total	15.6	4.1	12.6	8.7

Of the 86 students completing the program, 4.6% did not complete one-half year's work, 39.6% completed one-half to one year's work, 51.0% completed more than one full year but less than two full years' material, and 14.7% completed two full years' work.

At the conclusion of each term, attitude surveys were given parents and students. The positive response was almost unanimous, with only one parent and one student, the same family, giving negative responses. Follow-up was made concerning the response and there were circumstances unrelated to the program contributing to their attitude.

The most noteworthy results of the program are not easily reduced to statistics. The changes observed in the students in each of the classes must be mentioned. Open hostility, evident in many of the students, gave way to reluctance and then to genuine interest and eagerness. Further, even more difficult to measure, is the effect on the teacher. Even though the long hours caused extreme fatigue, there was an increasing feeling excitement and satisfatcion in observing the changes in the students.

The second afternoon class was started a week late, and thus went beyond the time of other summer school classes. Both afternoon classes were given more freedom to work at their own pace and take more frequent breaks. The second afternoon class produced the most dramatic results which I feel are directly attributable to the more relaxed atmosphere in the classroom. This class was informed they would be released when they completed the time requirements or completed the curriculum. Of the nine students completing two full years curriculum, five were in this class. Of the four who completed less than a half year's work, only one was in this class. Thirteen completed more than a full year of curriculum. The building was opened on Saturday morning before the final week began and ten of the twenty-two students spent up to four hours working to complete their work. One student, a boy with a slight visual perception problem, completed two full years material in this Saturday morning session. His display of confidence and joy in success was extremely gratifying.

Reflecting upon the overall program and drawing some conclusions from it, the following suggestions are made:

1. Computer-assisted instruction has a very high motivational effect. Most definitely this mode of instruction is not presented as the ultimate answer to all our needs in mathematics education. However, when most teachers are feeling various degrees of frustration in the lack of student motivation, this approach must not be overlooked.
2. When many teachers are reluctant to venture into the "unknowns" of individualized instruction because of too many students, too little time and material, and other various reasons, my experience tells me it does work. Not easily, but it works.
3. Much of the structured atmosphere we have in the traditional school setting is for the benefit of someone other than the student. The student response to the more relaxed atmosphere in this class was, with very few minor exceptions, one of accepting the freedom with responsibility and respect.
4. The role of the teacher undergoes a dramatic change in such a program. It is no longer "what may I teach you?", but "what may I help you learn?", and "how may I help you beat the machine'?" The teacher and student become partners or co-workers in this process. There are some possible "dangers" inherent in such an environment, the principle one being the temptation of becoming too attached to the students and to claim some of their success as your own.

NESTED DIAMOND ACTIVITIES ON THE ADDITION TABLE AND 100 SQUARE

Bonnie H. Litwiller and David R. Duncan
Associate Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

Teachers of mathematics are interested in activities which provide for the maintenance of skills. It is a "bonus" if these activities also provide opportunities for the discovery of patterns. The standard addition table and 100 Square are two rich sources of practice and discovery activities.

Figures I, II, III, and IV display nested sequences of diamonds drawn on the addition table (Figures I and II) and the 100 Square (Figures III and IV). The number in the *center* of each of the nested sequences of diamonds is circled.

Using Figure I, have your students do the following:

1. Find the sum of the entries which lie on the perimeter of each diamond. Also, record the number of entries that were used in finding each sum.
   For diamond A, the sum of the 4 entries is $9 + 11 + 11 + 9 = 40$.

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 1
ADDITION TABLE

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 3
100 SQUARE

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Figure 2
ADDITION TABLE

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Figure 4
100 SQUARE

For diamond B, the sum of the 8 entries is $8 + 10 + 12 + 12 + 12 + 10 + 8 + 8 = 80$

For diamond C, the sum of the 12 entries is $7 + 9 + 11 + 13 + 13 + 13 + 13 + 11 + 9 + 7 + 7 + 7 = 120$

2. Divide each of these sums by the circled center number; in Figure I, this center number is 10.
   In diamond A, $40 \div 10 = 4$.
   In diamond B, $80 \div 10 = 8$.
   In diamond C, $120 \div 10 = 12$

Note that in each case, the quotient is the number of entries that lie on the perimeter of the diamond. Does this pattern hold for a different placement of nested diamonds on the addition table? Table I summarizes the results using Figure II.

Table 1

Diamond	Sum of the entries on perimeter (S)	Number of entries on perimeter	Center Number (C)	S ÷ C
D	36	4	9	$36 \div 9 = 4$
E	72	8	9	$72 \div 9 = 8$
F	108	12	9	$108 \div 9 = 12$
G	144	16	9	$144 \div 9 = 16$

Again, observe that the sum of the entries on the perimeter divided by the center number ($S \div C$) yields the number of entries which lie on the perimeter.

Exactly the same set of instructions was followed for the 100 Square using the nested diamonds on Figures III and IV. The results of these computations are summarized in Table II.

Table 2

	Diamond	Sum of the entries on perimeter (S)	Number of entries on perimeter	Center Number (C)	S ÷ C
Figure 3	H	176	4	44	$176 \div 44 = 4$
	I	352	8	44	$352 \div 44 = 8$
	J	528	12	44	$528 \div 44 = 12$
Figure 4	K	184	4	46	$184 \div 46 = 4$
	L	368	8	46	$368 \div 46 = 8$
	M	552	12	46	$552 \div 46 = 12$
	N	736	16	46	$736 \div 46 = 16$

A general pattern appears for these types of nested diamonds on the addition table and 100 Square: The sum of the entries which lie on the perimeter divided by the center number yields the number of entries which lie on the perimeter of the diamond. Note that these are consecutive multiples of 4. Would this pattern hold if nested diamonds were similarly drawn on the standard multiplication table?

Memo From School Science and Mathematics Association:

The School Science and Mathematics Association has just published a special 128 page issue dealing with integrated science and mathematics.

Some of the questions addressed by articles in this publication are the following: How did single discipline oriented education originate? Are there rational bases for seeking the integration of the disparate content areas founded on such factors as the nature of the universe, the nature of knowledge and the nature of learners? What are some of the historical events leading to this new "search for unity?" What are some possibilities for styles and modes of integration of knowledge? Do differing epistomologies, ways of knowing, among disciplines hinder integration of them? What is the extent and nature of unified courses of study being utilized in elementary and secondary schools both in the United States and abroad? What have evaluation studies had to say about the efficacy of unified courses of study?

Single copies of this publication are available for $1.50 from the School Science and Mathematics Association, Indiana University of Pennsylvania, Indiana, PA 15701.

Teaching Statistics and Probability in Elementary Schools

John J. Sullivan
New York State Education Department

It is well known that our lives are increasingly dominated by statistics. Clearly, statistics are important in education, insurance, medicine, biology, and physics. It is also evident that statistics are becoming a dominant force in politics, manufacturing, farming, and ecology. The suspicion is unavoidable that complex activities can be managed in an orderly fashion only through the use of statistics.

A number of national curriculum groups have recommended that statistics and probability be included in school mathematics programs at all levels. At the elementary-school level, of course, the program should be informal, concrete, and intuitive.

This may not be a propitious time to suggest changing elementary-school mathematics programs, particularly if change involves adding new topics. Fortunately, many statistical ideas are already in school programs. Furthermore, statistics and probability for elementary schools require few mathematical concepts and skills not already in most programs. The study of fractions, for example, fits in nicely with statistics and probability. It is no exaggeration to say that all the work done to develop accurate concepts about fractions, at all levels, directly advances the teaching of statistics and probability. Probability estimates, of course, are stated as fractions or as percentages. We often hear statements like, "The probability of rain today is 70%."

Arrays, widely used to illustrate fractions and multiplication, are extremely useful in teaching statistics and probability. In the array below

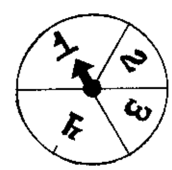
o o o . . . .
o o . . . . .

it is clear that 5/14ths of the elements are circles and 9/14ths are dots. It is just a small step to a question about the probability of an element, selected randomly, being a circle. Arrays are invaluable for arranging data in tables, an important activity in statistics.

Coordinates are taught in many programs precisely because they are so useful in illustrating many mathematical ideas and relations. This holds true for statistics and probability. Below is a coordinate array illustrating the sample space for the toss of a pair of dice, a familiar exercise in studying probability:

	6	1,6	2,6	3,6	4,6	5,6	6,6
	5	1,5	2,5	3,5	4,5	5,5	6,5
DIE	4	1,4	2,4	3,4	4,4	5,4	6,4
# 2	3	1,3	2,3	3,3	4,3	5,3	6,3
	2	1,2	2,2	3,2	4,2	5,2	6,2
	1	1,1	2,1	3,1	4,1	5,1	6,1
		1	2	3	4	5	6
				DIE #1			

Important topics in statistics such as averages (especially arithmetic means) and graphs (bar graphs, circle graphs, and line graphs) are already in elementary-school mathematics programs. Skill in reading tables and graphs is an important educational objective.



Geometry can assist the teaching of statistics and probability. Probability calls for plenty of work with circular spinners, such as the one illustrated.

An understanding of the use of compasses and protractors can help improve the understanding of spinners and circle graphs. Children should know how to construct accurate circle graphs.

A teacher illustrating fractions using parts of a circle is a familiar classroom scene. It would require just a bit more discussion to make this situation a valuable probability exercise.

Dice are useful for studying probability. For dice we can use a variety of polyhedra, not just the familiar hexahedron. This opens the door to many fascinating investigations of vertices, faces, and edges. The waste box in any woodshop can furnish an abundance of useful polyhedra.

The study of probability and statistics is integrated easily with activity-oriented mathematics laboratory programs. Probability theory was born out of an interest in games of chance. There are many opportunities to use games to stimulate an interest in statistics and probability.

Data-collection and analysis can be related to questions that really concern children: questions about television, school, sports, clothing, and physical characteristics, to name a few examples.

Three decided advantages are attached to the study of statistics and probability in elementary school:

1. It can use many interesting and enjoyable games and puzzles.

2. It can be related closely to things going on in the school and community.

3. It can be correlated easily with many topics already in school mathematics programs, and with other subjects such as science and social studies.

The Educational Division of the Institute of Life Insurance (277 Park Avenue, New York, New York) can provide teachers with free copies of an excellent booklet, *Sets, Probability and Statistics*. This brief summary (35 pages) can provide elementary-school teachers with enough background to begin introducing some questions about statistics and probability into their lessons.

Reprinted in part, from New York State *Mathematics Teachers' Journal*, Vol. 24, No. 1, January, 1974; Pp. 39-41.

PROFESSIONAL MEMBERSHIP APPLICATION

Date:__________ School:__________ School Address:__________

Position: ☐ teacher, ☐ department head, ☐ supervisor, ☐ student,* ☐ other (specify)__________

Level: ☐ elementary, ☐ junior high school, ☐ high school, ☐ junior college, ☐ college, ☐ other (specify______

Other information__________		*Amount Paid*
Texas Council of Teachers of Mathematics	☐ New membership ☐ Renewal membership	5.00
Local ORGANIZATION:__________	☐ New membership ☐ Renewal membership	
OTHER:__________	☐ New membership ☐ Renewal membership	

Name (Please print)__________ Telephone__________

Street Address__________

City__________ State__________ ZIP Code__________

National Council of Teachers of Mathematics	Check one: ☐ New membership ☐ Renewal membership	
	$11.00 dues and one journal ☐ *Arithmetic Teacher* or ☐ *Mathematics Teacher.*	
	16.00 dues and both journals	
	5.50 student dues and one journal* ☐ *Arithmetic Teacher* or ☐ *Mathematics Teacher*	
	8.00 student dues and both journals*	
	6.00 additional for subscription to *Journal for Research in Mathematics Education* (NCTM members only)	
	.60 additional for individual subscription to *Mathematics Student Journal* (NCTM members only)	
	The membership dues payment includes $4.00 for a subscription to either the *Mathematics Teacher* or the *Arithmetic Teacher* and 25¢ for a subscription to the *Newsletter.* Life membership and institutional subscription information available on request from the Washington office.	
* I certify that I have never taught professionally__________ *(Student Signature)*	*Enclose One Check for Total Amount Due* →	

Fill out, and mail to Dr. James Rollins, 1810 Shadowwood Drive, College Station, Texas 77840

NOW!!

TEXAS MATHEMATICS TEACHER
J. William Brown, Editor
Texas Council of
Teachers Of Mathematics
Woodrow Wilson High School
100 S. Glasgow Drive
DALLAS, TEXAS 75214

NON-PROFIT
ORGANIZATION
U. S. Postage
Paid
Dallas, Texas
Permit #4899