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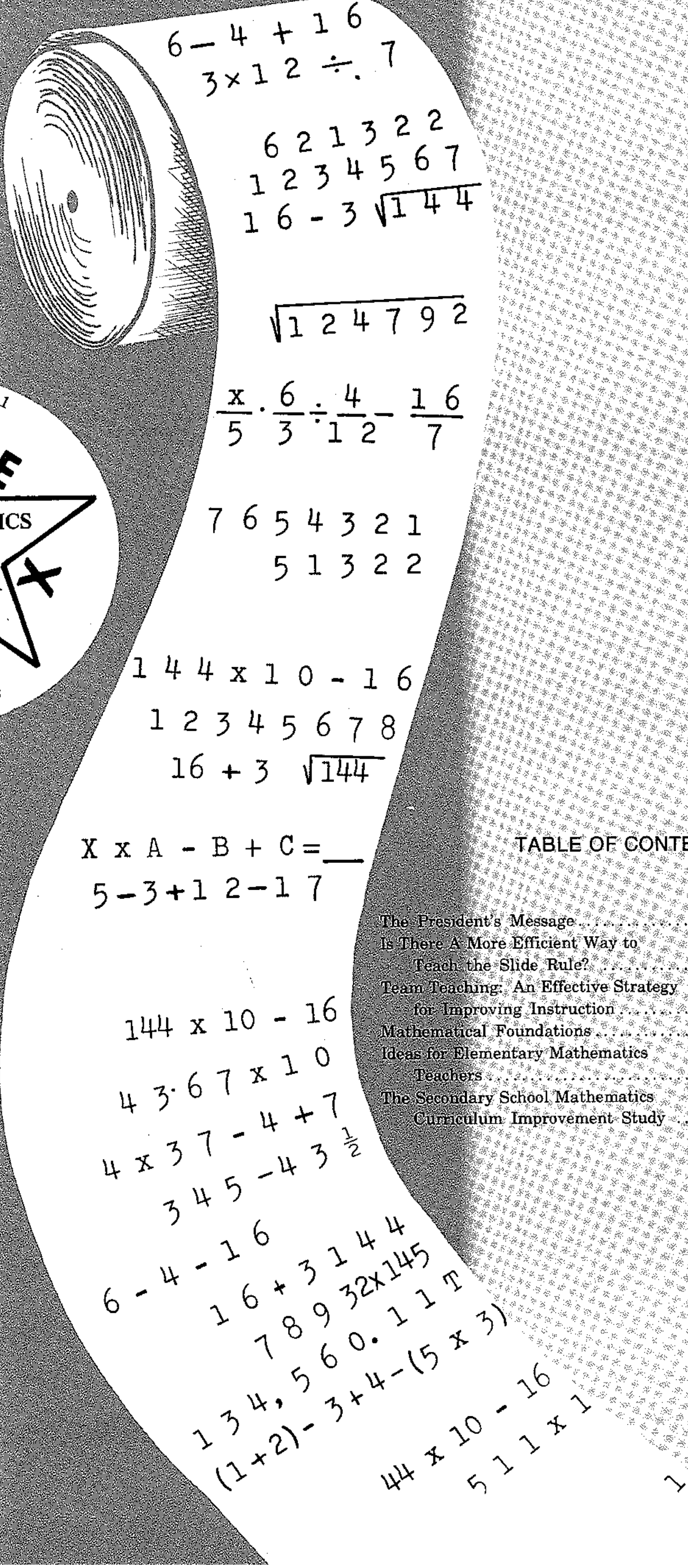

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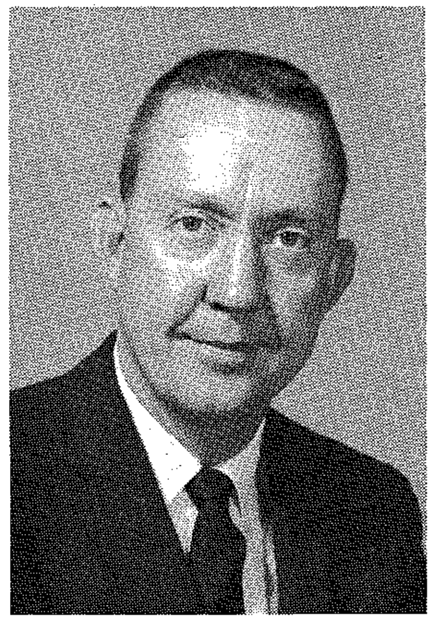
THE PRESIDENT'S MESSAGE

JAMES E. CARSON



T.C.T.M. seems to be doing quite well. We sent out approximately 1,600 copies of the Journal in October. We want to be sure and keep our membership up because this is our primary source of income. I also think that it is the obligation of every mathematics teacher in the State of Texas to support the Texas organization. We appreciate your membership and if we can do anything to help in any way just let us know. We are looking to the local councils for help and we want to help you.

We appreciate all of the memberships that the local councils have sent in. This is the way that we get things done. Work together!

We had a very fine workshop in September at J. Frank Dobie High School, with more than 200 in attendance. We came out in the black, therefore N.C.T.M. did not have to pick up any of the expenses. I am happy to inform you that Madge Simon and her fine workers of The Coastal Bend Council of Teachers of Mathematics have scheduled another workshop in the name of T.C.T.M. to be January 20, 1973. Thanks a lot, Madge. We are looking forward to seeing many of you at Gregory-Portland High School on that date.

Our annual meeting at CASMT in Austin on February 15-17, 1973 will be here before we realize it. I understand that Shirley Ray has a fine Luncheon speaker lined up for our meeting. The annual meeting of T.C.T.M. will follow after the Luncheon. I hope to see many of you there.

Speaking of the CASMT meeting, T.A.S.M. and T.C.T.M. are making an effort to get Bishop Pitts to have our meeting in the fall beginning next year.

The next thing that we should be thinking about is the N.C.T.M. Annual Meeting which will be held in Houston, April 25-28, 1973. We will be calling on you to help out on many of the committees. If you are willing to work on some committee and are already on one, write to Thelma Hammerling, Houston Independent School District and inform her that you are willing to work. The framework of the convention has been laid, but much remains to be done before April. We are expecting 8,000 people from all over the United States and we are hoping that we can show them what Texans can do by having one of the best ever.

Therefore, I close by saying that I hope that I will be seeing many of you at some of the conventions and workshops.

DATES TO REMEMBER

NCTM Joint Meeting with MAA, January 27-29, 1973, Dallas

CASMT, February 15-17, 1973, Austin (Annual Meeting of TCTM).

51st Annual Meeting, NCTM, April 25-28, 1973, Houston

Name-of-Site NCTM Area Meeting, August 15-17, 1973, Fort Worth

IS THERE A MORE EFFICIENT WAY TO TEACH SLIDE RULE?

by CHARLES TANNERY
Teacher, Pasadena Independent School District
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There are two ways to teach mathematics. One is to take real pains towards creating understanding – visual aids, that sort of thing. The other is the old British system of teaching until you're blue in the face.

James Ray Newman

At times all teachers feel that they have been teaching until they are blue in the face, and a better technique would certainly be welcome. The following system for introducing slide rule has been very effective in participating schools for students from the seventh to twelth grade.

For a student who is learning the use of the slide rule, the most important goal should be accuracy. The way to improve accuracy is to use an instructional program with few and simple rules. If a single procedure is used time after time, the student masters the technique and forms a desirable habit. Here is an instructional procedure with this goal in mind.

Change all multiplication and division problems so that they will fit the pattern of n *5* 1 elements in the numerator and n elements in the denominator.

Example: 24 *7* 51.3 *7* 8450

Rewrite: 24 *7* 51.3 *7* 8450
1 *7* 1

Example: .0025
81.6

Rewrite: .0025 *7* 1 *7* 1
81.6

Certainly multiplying or dividing by additional "ones" will not change the answer, and now any problem that contains multiplication and /or division can be performed using a single rule. Before explaining the rule, some terms will be renamed. There are three parts to a slide rule: the body, the slide, and the indicator or cursor. To make instruction more descriptive, one should refer to the slide as the "divider" and the cursor as the "mutiplier".

Using these terms, the rule for multiplying and/or dividing is to make the first setting with the multiplier, the next setting with the divider, and alternate their use throughout the problem. The final step will always be to use the multiplier since the problem has been constructed to have n *5* 1 elements in the numerator. The answer will always be found under the hairline after the last setting. For all problems, then, the first setting and the answer will be on the D scale, and *all* other setting will be made on the C scale.

Now one could consider the advantages of introducing the slide rule to students in this manner. Every time they pick up the slide rule, they will always move the multiplier to a number in the numerator first; then they will *always* move the divider to a number in the denominator next. With a relatively short period of practice, the procedure will become automatic, and this automatic reaction will eliminate most careless errors.

In determining the decimal, many good methods are available. One method involves using each number's characteristic and observing the use of the right index. The characteristic of a number refers to the exponent of the number when written in scientific notation.

Example: 362,000 *9* 3.62 *7* 10^5
The characteristic of 362,000 is 5

The decimal placement for the answer is determined by totaling the characteristic of each number in the problem. The student adds the characteristic of numbers in the numerator and subtracts the characteristic of numbers in the denominator. The other factor in determining the decimal placement for the answer is to note when the right index or more descriptively the "right one" was used. Each time the multiplier is set on the "right one," the total should be decreased by one. Each time the divider is set on the "right one," the total should be increased by one.

Example: .0495
.337 *7* .889

The characteristic of:
.495 is *6* 2
.337 is *6* 1
.889 is *6* 1

total characteristic *9* (*6*2) *6* (*6*1) *6* (*6*1) *9* 0

Next the the effect of the "right ones" must be determined. When the problem is rewritten

.0495 *7* 1 *7* 1
.337 *7* .889

it can be seen that multiplication by one will occur at two different times. Now by using the slide rule to actually multiply out the problem, one will be forced to use the left index for the first one and the right index for the second one. Using the left index has no effect but multiplying by the right index will deduct one from the characteristic total. Since the total was 0, the answer's characteristic will be 0 *6* 1 *9* *6* 1 and could be written as 1.65 *7* 10 [1]

If, in addition to accuracy, the teacher is interested in developing speed, he should consider how speed is increased when teaching other manual skills. In typing, for example, rhythm is a dominate factor, and rhythm indicates the key to building speed with the slide rule. When pressed to work more problems in a given time interval, it is a natural instinct to make the hand move more quickly resulting in a jerky motion. It is more effective to try to eliminate pauses between settings and to develop a smooth even pace. In order to obtain a constant pace one might try using a timer that makes a tone at regular intervals, for instance, every five to ten seconds, and having his students pace their settings by the tone.

Correct posture is also helpful in increasing accuracy and speed. One should encourage his students to hold their slide rules in a position where the problem can be read and the setting made with a minimum of head and eye motion. This will cut down on students' losing their place while working. Additional speed can be gained by keeping a running total of the characteristics instead of calculating the total after the problem is worked.

Hopefully some of these suggestions will be useful, but the real secret to efficiency on the slide rule is hard work. No matter how effective the teaching method becomes, no one can master the slide rule without practice.

TEAM TEACHING: AN EFFECTIVE STRATEGY FOR IMPROVING INSTRUCTION

BY
JOHN L. CRESWELL and DAVID FITZGERALD

The Problem of Teacher Time

It is generally conceded by authorities that knowledge doubles every seven years. It is also generally believed that knowledge has at least quadrupled since 1900. Much of this explosion of knowledge has occurred in mathematics and in areas related to mathematics.

These events have confronted the elementary and secondary schools of this country with an awesome responsibility. In addition, the Cambridge Report has recommended that the mathematics content which is now taught in twelve years of public school and the first three years of college should be compressed into the K-12 curriculum.[1]

In order to appreciate this burden which has been placed upon the American school system, one must realize that all this new knowledge must be taught or rather, learned, in about the same amount of school time as in 1900. Basically, the length of our school day has changed very little in the past 70 years; today's students are in school approximately six hours per day, five days per week, and the school year is roughly 180 days.

Since there is so much more mathematics which must be taught in approximately the same amount of time, what, then is the solution to this dilemma?

It would seem that mathematics teachers, at all levels could take a lesson from industry, particularly the automotive industry. As the necessity for a smaller engine to produce more power arose, this industry produced a more efficient engine with a much greater compression ratio which produced more power per cubic inch of engine. Not only this, but in order to produce more engines and motor vehicles, the automotive industry has taken advantage of technology in order to become a more efficient and effective operation.

In order to teach more effectively, teachers of mathematics must make better use of *their* time. This can be accomplished by using techniques and materials which will enable them to be more effective and efficient in the classroom. To do this, they must also use creative and innovative teaching strategies, taking advantage of recent advances in technology.

Are Teachers Taking Advantage of New Techniques?

Is it not true that the teachers of today are meking the best possible use of materials and techniques available to them? Goodlad, citing a research study, conducted in sixty-seven urban school systems across the United States, says NO! Using a list of several "reasonable expectations," Goodlad and other experts observed the classrooms of scores of teachers and reported that in the vast majority of cases the teachers not only were not using innovative techniques in their teaching but seemed to be unaware that such techniques were available.[2]

Other educators have been concerned with the problems of the schools occasioned by the necessity to compress more teaching into a relatively small amount of time. Broudy cites the need for subject matter which is more easily generalized. He believes that subject matter should be taught in such a way that it will facilitate the transfer of training.[3]

Corrigan contends that teaching should be such that children will learn to be comfortable in a society of change.[4] Davies of the U. S. Office of Education believes that the view of our society is changing from a provincial one to a multi-cultural, dynamic perspective.[5] Yet, Stranavage indicates that our lockstep, self-contained classroom idea of education is intended for preserving a steady-state society.[6]

One Solution to the Problem

There is no one simple solution to all these problems, but several innovative teaching strategies, known to most teachers by name only, offer real benefits to teachers who are willing to try them. One such strategy is team teaching. Team teaching is an instructional strategy involving two or more teachers in a regular, cooperative effort to plan, present and evaluate an educational program for a group of students.

Since the interest here is in more effective, efficient instruction, perhaps it would be helpful to list some of the needs of schools which teachers most often cite as necessary for improving instruction:

(1) Increased planning time;
(2) Utilization of better teaching strategies;
(3) Increased subject matter competence;
(4) Increased individulization of instruction;
(5) Improved human relations within the classroom;
(6) Better integration of subject matter areas;
(7) Extra time needed for discovery approach.

How does team teaching increase planning time? This is one of the most familiar aspects of team teaching. It is will known that some concepts can be presented to very large groups equally as well as they can be presented to small groups. Since one teacher can present a concept to a very large group, with the help of technical aids, other teachers have free time to plan for small group sessions. Teaching to large groups is usually done in rotation so that each teacher will have an equal amount of planning time.

In addition to teaching in rotation, a para-professional team member can release the teachers from many of the "busy-work" chores of teaching, (more about differentiated staffing in a later issue). The para-professional, being a member of the team, would attend team planning sessions and most of the class sessions in which the team is teaching and would therefore, have a better perspective of the whole process of instruction than would the para-professional working "free-lance" for several regular classroom teachers.

How does team teaching improve teaching strategies? The teacher who leaves the comforts and security of his own "private" classroom for the varied and challenging experiences of working closely with other teachers, runs the risk of revealing his inadequacies in the knowledge and use of teaching strategies. However, he also exposes himself to the best teaching strategies which his fellow team members have to offer. His competence is certain to improve as a result of this exposure.

In addition to sharing ideas, if one teacher is able to inspire an exciting and competitive atmosphere within the classroom, consider the possibilities of utilizing the interactions of three or four teachers in the same classroom to establish an effective learning atmosphere.

Again, the para-professional as a team member becomes important. Many effective teaching strategies depend upon the production of goods and materials to be use in the classroom. The para-professional team member releases the teachers from this task.

How can team teaching increase subject matter competence? This too is one of the more familiar characteristic advantages of team teaching. Members of the team who are particularly competent in a specific topic or area of the subject matter can assert leadership on these topics. Other team members will benefit from the knowledge of the more competent member. The students will benefit because the total subject matter understanding of the team members will be increased.

Since college curriculums vary, the team members will all bring different points of view on the subject matter topics to the team. This will result in a more generalized presentation of topics and will benefit the students by providing them with understandings which can be more easily transferred.

In a classroom, a good teacher can usually present a topic from two or three different points of view and thereby, increase the probability that each student will understand at least one of the approaches. In a team teaching situation, if there are three teachers involved, then the probability of understanding is increased by a factor of three.

How does team teaching improve individualization of instruction? If there are two or three teachers in a team, then intra-classroom grouping is not only possible but is quite easy. Grouping can be done to change the teacher-pupil ratio, to accommodate children of different performance levels, or to treat problems which may have been revealed by the administration of a diagnostic test.

The individualization of instruction also implies the use of special materials and, again, the para-professional team member can release the regular teachers from the task of constructing them.

How does teaching improve human relations in the classroom? It may not! Much depends upon the team members and their ability to work together. However, teams can be constructed in special ways to meet the needs of special student populations. Because of recent civil rights decisions, many teachers are now teaching in multi-cultural settings. By teaching and planning with someone of another race or culture, a teacher can gain a deeper understanding of why members of that race react as they do. In the classroom, children are more willing to work cooperatively with those of another race or culture if they observe a multi-cultural teaching team working effectively together. If language is a barrier in the classroom, a bilingual team member can help solve that problem.

Research has shown that in the childhood years, children need close contact with adults of both sexes for optimal psychological development. Since many children come from homes where only one parent is present, a sexually mixed team is beneficial.

As every teacher knows, there are some instances in the clsass room in which a one-to-one relationship between teacher and pupil is necessary because of human relations problems. Team teaching provides this opportunity without the necessity of a teacher deserting the rest of his class.

How does team teaching facilitate the integration of subject matter areas? This question has become one of great importance for mathematics educators. Many educators believe that both mathematics and science can be learned more effectively if they are taught in an integrated fashion. This conclusion has lead to the development of integrated curriculums for both grade school and high school levels. However, finding a teacher who is well enough qualified in both mathematics and science to teach an integrated curriculum is difficult indeed.

Team teaching helps to solve this problem. A team consisting of a math specialist, a science specialist and a papa-professional can effectively present an integrated curriculum. There is, however, a danger present in this type of organization. The math specialist, feeling more comfortable in his own subject matter area, tends to teach all of the mathematics, and the science specialist tends to teach all of the science for the same reason. This tendency results in the segregation of the integrated curriculum and the benefits of the integrated curriculum are lost. Careful, cooperative planning is necessary to prevent this type of segregation from occurring.

How does team teaching facilitate the use of the discovery approach? This question is closely related to the previous one. Research has shown that the use of discovery-inquiry teaching in mathematics results in better retention on the part of the students.[7] The use of an integrated science-math curriculum greatly facilitates the use of the discovery method in teaching mathematics.

Other characteristics of team teaching also facilitate the use of the discovery method in teaching mathematics. The sharing of ideas by teachers helps them to design effective problematic situations with which to confront students in the classroom. Intra-classroom grouping, increased individual attention, and the presentation of many different points of view are all characteristics of team teaching which facilitate the use of the discovery approach. The discovery approach to learning plus a mobile, versatile, dynamic classroom atmosphere created by an effective teaching team all combine to help the student become secure in a rapidly changing environment.

In conclusion, the most important ingredient in effective team teaching comprehensive, cooperative team planning and respectful trust and faith by every team member for every other team member.

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(John L. Creswell, Associate Program Director, National Science Foundation; Dr. David Fitzgerald, Mathematics Education Department, Temple University, Philadelphia.)

MATHEMATICAL FOUNDATIONS

By Julia Comber

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None will dispute the importance of a good foundation to a child's education. This is laid in the first two years of school life, so the importance of early education cannot be overstressed. It also cannot be refuted that:

1) all children are different and
2) young children learn by doing.

Having agreed on these two important propositions, it is becoming increasingly accepted, particularly in primary schools in the United Kingdom, that children should work in small groups or individually at their own pace in order to progress naturally and with certainty.

One may well ask why change 'old math'? It may have served us well in the past, but education is itself a living and growing process and does not remain static. Social conditions have altered, the age of automation is upon us, and computers and calculating machines have come to stay, coping with the every-day computation of business. Thus there is less need for people who can perform computations swiftly, and more need for those able to assess situations and formulate and solve problems. These citizens of the future are in our schools at present, and to them we owe a duty in fitting them for such conditions.

Much learning in the past has been done by rote and without the understanding needed to progress to the next stage with confidence and indeed with enjoyment. In order to perform even simple operations such as $7+5 = 12$, the children should first have understood such concepts as: matching, invariance of number, ordering, inclusion, union and place value.

Let us look more closely into these basic concepts. For example no one in the world has ever seen a 'seven', which is a figment of man's imagination. We come to accept the idea of seven (or, of course, any other number) by experiences which involve *handling* this quantity of many different concrete objects such as beads, shells, sweets, apples etc. and also by *seeing* such groups in many different contexts. The first year at school should provide these experiences in many situations until the child is ready to match (or put into a one to one correspondence), two sets of objects. When he is completely satisfied that set A contains the same number of objects as set B, no matter how they are placed or grouped, then he has acquired the concept of the invariance (or conservation) of number. Once this concept has really been acquired, and there are means of checking on this (see reference 1), then experiences in the other basic concepts should follow. The old way of drilling children with number facts, regardless of their stage of understanding, has rarely proved satisfactory. Concepts cannot be taught; but, the child who is given the right experiences by an understanding teacher will not only acquire the concepts but also apply them to the skills with confidence.

In spite of the computers previously mentioned, children will still need to be able to do many of the traditional sums. Skills and concepts sould go hand-in-hand; skills without concepts lead to confusion and despair, whereas concepts without skills give justification to critics who complain that 'they can't do their sums now'.

One way in which children are different is that they acquire ideas at different rates and in different ways, and there is also not a definite linear progression of concepts. Some children, because of environment and experiences, often prove to be further ahead with spatial concepts than with those leading to number; and, when we consider mathematical foundations, we must of necessity remember that the geometric ideas are just as important as the arithmetic ones. In the past, far too many text books and teachers have regarded mathematics as purely 'number' for young children; but an awareness of shapes in the world around them comes at a very early age and can be developed along lines leading to geometric and spatial ideas far earlier than has hitherto been thought possible.

There is, in fact, a 'tree' of concepts with two main branches, leading to numerical and spatial ideas respectively. On the numerical side, for example, matching leads on to invariance of number and sorting, in that order. Sorting, in turn, leads, by parallel branches, to the idea of union of sets and to inclusion. A separate sub-branch leads from comparison of two objects towards ordering. Both sequences, matching, sorting, etc., and comparisons, ordering, etc., are needed for an understanding of numbers and operations on them, such as addition.

Returning to our problem $7+5 = 12$, having discussed matching and invariance above, we now consider the concept of ordering. Before dealing with numbers like 5 and 7 it is necessary to know them as members of a sequence, 1, 2, 3, 4, 5, 6, 7 . . . But even before considering the ordering of numbers, experiences should include ordering many objects according to size, weight, thickness and other criteria which children will quickly decide. After many such experiences the ordering of numbers can be discussed.

Again, sorting is a necessary prerequisite for our problem of $7+5 = 12$. Broadly speaking, the whole of mathematics is sets and relations; and, the idea of sorting things into sets is fundamental to young children, who begin this activity from a very early pre-school age anyway. And so sorting objects into sets in many different ways comes very naturally, leading on to the abstraction of, for example; the 'fiveness' of five. This, in turn, leads on to sorting in terms of numbers, using concrete objects, so that it is seen, in many different contexts, that 7 objects and another 5 objects together form a set of 12 objects. Thus the child is led on to the rather difficult concept of 'inclusion', of realizing that the 5 objects are included in the set of 12, and on to the idea that 5 is less that 7 in the ordering of 5, 6, 7, etc. As well as sorting, leading to the idea of inclusion, children will sort and count the objects together, thus leading to the union of two sets, and after many experiences will extract such sums as $7+5 = 12$.

However, even before setting down the equation, great care should be taken to ensure that the = sign is thoroughly understood as symbolizing a relationship that is reflexive, symmetric and transitive. Naturally very young children would not be expected to understand such terms, but careful avoidance on the part of the teacher of the premature or wrong use of symbols will prevent such misunderstandings as 'my doll = 4 ozs.' The wise teacher will avoid these special mathematical symbols until such time as the child is dealing with actual numbers, then $7+5 = 12$ becomes a true statement. The teaching of place value has been lightly skimmed over in the past, and 'carrying sums' became meaning less to many a six- and seven-year old. But, again, with many experiences with concrete objects such as bundling sticks into tens, and good use of the number line *before* writing down equations, confusion many be avoided always, providing the teacher has the skill and patience to wait and to know when the child has reached the stage of understanding.

To those concerned with mathematical education, these stages of understanding, lengthy as they may seem in print, are necessary and desirable, in order to lay firm foundations and to impart to the child the delights of the pattern, order and relationships of the mathematics which lies in the world around him. However, to some opposed to 'discovery' methods (a greatly over-worked term), the idea of providing the child with many varied experiences may seem to be a waste of time, when all that appears to be needed is to put out problem of $7+5 = 12$ on the board and let the child copy it and similar ones many times over. It is all a matter of attitude on the part not only of the teacher but of the child also. Drill for skill has produced too large a proportion of students, particularly girls, who 'hate maths' and give up the subject at the earliest opportunity.

The spatial concepts, leading eventually to the invariance of length, mass, weight, etc., should be introduced along the same lines as the numerical ones. All these experiences will be linked with the other areas which form part of the child's education, with art, craft, language, etc. (see reference 2). For young children do not think, as adults do, in terms of periods on a timetable; mathematics should not, indeed *cannot* be contained in separate compartments. In fact, mathematics is with everything (see reference 3).

Having established right attitudes, and that the acquisition of certain known concepts is the accepted way for sound preparation for mathematical education, certain problems must be faced and overcome. These must include recording, assessing, use of apparatus and the in-service training of teachers who may not be well versed in the newer ideas.

Keeping simple but telling records of each child's progress is most necessary or much time and effort will be wasted. Doubtless many teachers have found their own methods of recording; and, certainly, local educational authorities present us with record forms to be completed. However, newer methods have outgrown some of the old records forms which in fact gave very little useful information. Much thought should be given to passing on relevant information, which will enable the next teacher to pick up the child's stage of understanding in the minimum time. Forms to assist teachers of the younger children (5 to 8 years) have been devised (see reference 4). Guide lines for checking the acquisition of concepts may be found in a particularly useful series of books which were produced through collaboration between the Nuffield Project and Piaget's Institute at Geneva (see reference 5).

The use of specific apparatus with young children should be, naturally, a matter for individual teachers to decide; but, care should be taken to avoid the over-sophisticated structured apparatus. The too early use of coloured rods, or blocks of varying shapes can, in the hands of the inexperienced, lead to the mathematics being restricted to the material instead of the material being an accesory of the mathematics. For the young child, little if any special apparatus is needed, for mathematics can be found in their toys, in junk, around their classroom and indeed with their class-mates (see reference 6).

The in-service training of teachers is being dealt with very thoroughly in the United Kingdom through courses at Teachers' Centres, television programmes, lectures, demonstrations and visits between schools. Many excellent publications are now on the market, and those listed in the references are just some which have been found to be good material for laying mathematical foundations.

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IDEAS
For Elementary Mathematics Teachers

By SISTER MAY PETRONIA
Mount Mary College
Milwaukee, Wisconsin

When we go back to school, teachers wonder whether students remember anything they were taught last year. Yet we all realize that we forget a great deal of what we have learned if we do not in some way make continuous use of it. Mathematics is no exception! What do we do with students at different grade levels who no longer can give automatic response to the basic facts in addition, subtraction, multiplication, and division, presuming they once could? Some contend that in a modern program of mathematics one no longer needs to provide students with drill. In my opinion, this not true. It is a mistake. Surely, drill should not come before understanding. If at all possible, drill should be made interesting, exciting, and challenging so that it might be enjoyable and even fun. If one does not like the word drill, perhaps he could substitute "sustained attack." Regardless of what word one uses, I think that students of mathematics at all levels need some type of practice as a means of fixing ideas. So let us take new courage and work with a will to have those we teach again attain the goal of being able to give automatic response to the basic facts in the different operations. It is a must!

In this article, I certainly do not claim to have ideas which are entirely new, but I do claim go have helpful ideas which I would like to share with you. I also think these ideas will appeal to you as teachers because they can be readily carried out in the classroom without a great deal of preparation and materials. They are simple ideas! My aim is to suggest some ideas for each of the four operations.

ADDITION

A teacher might use "skip " addition in various ways. For example, he might tell the students to take the number 9 and keep adding 8 until they reach 129. How the students record this would depend upon the skill they have acquired for addition and the grade level.

Some students may make their record like this:

```
 9
 8
17
 8
25
 8
33
49   and so on.
```

Others might simply write: 9, 17, 25, 33, 41, 49, 57, 65, 73, 81, 89, 97, 105, 113, 121, 129, or they might record the sums vertically. "Skip" addition often produces beautiful patterns! Have students be on the alert for them. In the above example, notice the digits in one's place in each of the numerals — 9, 7, 5, 3, 1, 9, 7, and so on.

To have students require more speed with no loss in accuracy, and hopefully to have them practice on their own outside of school time, a teacher might try this approach. Many teachers have found it successful. Ask the class to take a number, say 18, and keep adding 7. Time them. Let them continue adding for one full minute or whatever time limit you deem appropriate. Then have the entire class stand and recite the answers together. As soon as a student has a wrong answer or no longer has an answer, he should be seated. The student who remains standing longest is the winner.

Teachers might extend this idea by using "skip" addition with fractions — say, start with ½ and keep adding ½. Thus: ½, 1, 1 ½, 2, and so on. Or try one a bit more challenging — start with ½ and keep adding 1/3. Why not have students do the same with decimals? The result might just be amazing!

Just one more idea for having students get practice in addition! It is not only interesting but fun. Have the students write the name for some number. Then have them reverse the digits in the numeral that names the number to obtain the name for a second number. Add the two numbers named. Then reverse the digits in the numeral that names the number which is the sum of the first two numbers. Again add the numbers named. Continue this procedure as often as necessary; a number is eventually reached whose name reads the same from both ends. For example, let's try 1971.

```
 1971
 1791
 3762
 2673
 6435
 5346
11781
18711
30492
29403
59895
```

If you really want fun, try 98. I would caution any teacher who uses this idea in his classroom to have tried the examples himself previously. Some numbers require many, many, many additions. If the additions do not become too lengthy, teachers might use this idea for races. One way to carry this out would be to select two teams of students. Then let one member of each team work at the board, giving them a certain number; the rest of the class may work the same example at their seats. Each team member endeavors to complete the addition correctly and FIRST. The one who does just that earns a point for his team. Why don't you try it with your class? Students do enjoy it!

SUBTRACTION

"Skip" subtraction provides the same practice and patterns as "SKIP" addition. It can be handled in the same manner.

For example, start with 81 and keep subtracting 7 until you reach 11. It will look like this: 81, 74, 67, 60, 53, 46, 39, 32, 25, 18, 11.

Or start with 9 and keep subtracting ¾ until you reach 1½, 9, 8¼, 7½, 6¾, 6, 5¼, 4½, 3¾, 3, 2¼, 1½.

How about decimal fractions?

Another idea which students find exciting and which can be used in various ways in a classroom is this. Write the names of four numbers in a row, leaving space between (see example below). Take the difference between the first and the second numbers and write its name in the first column; take the difference between the second and the third numbers and write its name in the second column; take the difference between the third and the fourth numbers and write its name in the third column; take the difference between the fourth and the first numbers and write its name in the fourth column. Continue finding the difference in this manner. Eventually a point will be reached where the four numbers named will be equal. For example, take

64	129	95	37
65	34	58	27
31	24	31	38
7	7	7	7

REMARKABLE, is it not?

Why not extend this idea and use numbers other than counting numbers! For example,

3 5/8	1/2	7 3/4	9 1/4
3 1/8	7 1/4	1 1/2	5 5/8
4 1/8	5 3/4	4 1/8	2 1/2
1 5/8	1 5/8	1 5/8	1 5/8

You may want to see if your students can discover some simpler way for cases like the one above. Can you?

MULTIPLICATION

Tables of various kinds provide ample and interesting practice, sometimes in more than operation. Let me exhibit a few, for which I will invent names so as to have a means of referring to them.

There are regular tables like the ones which follow:

x	2	5	3	4
4				
1				
3				
5				

x	9	5	8	6	7
4					
8					
6					
9					
7					

The lower the grade level, the simpler and shorter one would make the table, of course. Incidentally, if you are a teacher of the intermediate grades or if you teach at the junior high level, did you ever have your students construct a table for the one hundred basic facts of multiplication? If you did not, it might be an excellent idea to do so. Then let the students see how many observations they can make by looking at the table carefully. I am certain you will be amazed at the many discoveries they can find.

Other than regular tables, a teacher might use what I call "puzzle" tables. These always hold more fascination for students. Examples of such are given below.

x			4
5			
3		9	
	8		16

x		9		
				20
	18	27		15
7			56	
8	48	72		

A teacher might let the students construct puzzle tables and then let some of their classmates complete them.

A third type of table is the sum-product table situated in somewhat of a puzzle setting as can be seen below.

a	7		8	9	8
Sum			15	15	
Product		81			40
b	6	9			

Naturally tables like these could be used for numbers other than counting numbers. Here is an example.

a	1/2		2/3	5/6	3/4
Sum		4/5		2 1/2	
Product			1/3		7/8
b	1/3	1/2			

A way of giving students practice in both multiplication and subtraction is what I like to call naming numbers in the "determinant" way. This could be carried out in one of two ways. A teacher could write a determinant and explain how one finds the number represented by the determinant. Then he might ask students to find the simplest name for the numbers named by different determinants. For example,

$$\begin{vmatrix} 7 & 4 \\ 2 & 5 \end{vmatrix} = (7 \times 5) - (2 \times 4) = 35 - 8 = 27$$

or

$$\begin{vmatrix} 2\ 1/2 & 4 \\ 3 & 7 \end{vmatrix} = (2\ 1/2 \times 7) - (3 \times 4) = 17\ 1/2 - 12 = 5\ 1/2.$$

At other times a teacher could ask students to construct determinants which name certain numbers. For example, suppose the teacher asks for determinants which name the number 7. How many such can be found? Here are a few.

$$\begin{vmatrix} 8 & 7 \\ 7 & 7 \end{vmatrix} \text{ or } \begin{vmatrix} 10 & 11 \\ 3 & 4 \end{vmatrix} \text{ or } \begin{vmatrix} 7\ 1/2 & 2\ 1/4 \\ 8 & 3\ 1/3 \end{vmatrix}$$

DIVISION

Division is undoubtedly the process in which students need most practice and yet it is the one operation for which it is more difficult to find ways of supplying that practice. There is one way, indeed not really new, but one which does provide considerable practice in the basic facts of division. One might think of it as dividing one number by another after the numbers are expressed in terms of their factors. Here are several examples.

$$\frac{24 \times 72 \times 81 \times 21 \times 32 \times 49}{6 \times 84 \times 18 \times 8 \times 42 \times 63} = ?$$

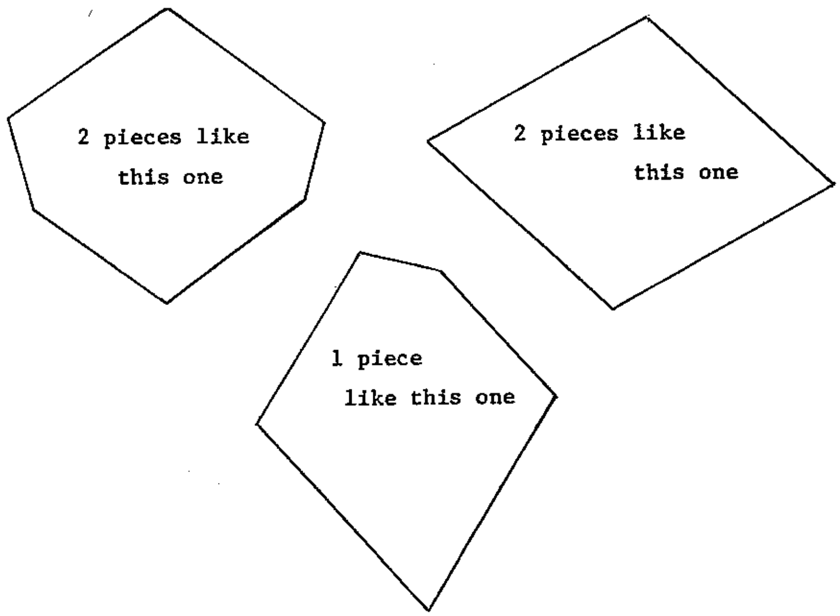
The expression at the left names the number 24.

$$\frac{27 \times 56 \times 28 \times 30 \times 36 \times 45}{20 \times 54 \times 21 \times 63 \times 60 \times 24} = ?$$

The expression at the left names the number 1.

One could use this as a means of competition in the classroom. Let the students choose two teams. Then have one member of each team go to the board, dictate a division to them, and see which student comes up first with the simplest name for the number named by the division.

In closing, could you like to have a puzzle to challenge your students? If you would, then let them try this "Star Puzzle." I first obtained mine from Fun Incorporated, Chicago, Illinois. It contains five pieces of wood (heavy paper will do as well or better) in shapes such as you see below. The pieces are to be put together to form a five-pointed star. It is advertised as a star puzzle for folks who don't give up, and the advertisement is true!



The Secondary School Mathematics Curriculum Improvement Study

DESIGN AND ASSESSMENT

Teachers College, Columbia University
Howard F. Fehr, *Director*

The Classical Viewpoint

Traditional mathematics grew out of the need for understanding the physical environment and out of this practical knowledge there was created an idealization of physical space (Euclidean geometry) and a system for counting and measuring (whole and rational numbers). As commerce, navigation, and exploration grew, the need for further mathematics led to higher specializations called algebra and analysis. Almost 300 years ago all this knowledge was organized into four branches of mathematics — arithmetic, algebra, geometry, and analysis — each considered a closed and separate field of investigation. Under each of these categories a proliferation of subjects came into being. For example, under arithmetic there arose courses in numerical calculation, combinatorial analysis, Diophantine analysis, probability, theory of numbers, statistics, etc. and similarly for the other three divisions. This organization became the manner of presenting mathematics as a subject for school instruction. It has held sway for over 200 years.

The Contemporary Viewpoint.

Mathematics, as a branch of knowledge, no longer holds to this classical point of view. How the contemporary point of view emerged is a long story of the gradual developments and insights brought to light over a hundred or more years. From the developments in understanding numbers organizing algebraic systems, and creating new geometric spaces along with the emergence of Cantor's set theory, Hilbert's formalism, and the concept of structure, a new contemporary viewpoint of mathematics has come into being. As early as the 1930's the Bourbaki recognized that certain fundamental concepts underpinned all the branches of mathematics, and that structural concepts gave possibilities for organizing all the traditional branches into a general inter-related set of structures. As a consequence they rebuilt all the existing mathematics using the basic notions of sets, relations and mappings, into two great structures: — the algebraic such as groups, rings, fields, vector spaces; and the topological such as metric and linear space, compact space, normed and vector space. This structuring uncovered a richness of inter-relations between the subjects, concepts, and theories previously hidden by the traditional separation, and which permitted unity to all mathematical study.

It is this contemporary viewpoint of mathematics that shows the need, and shows how, to restructure our school mathematics program. It would not have been possible to make an intelligent change in curriculum construction without this twentieth century development in pure mathematics. Contemporary mathematics may be described as the study of ordered pairs.

(Set, Structure).

For example, elementary algebra eventually becomes a study of the Set of real numbers and its structure of order, density, binary operations and functions. Geometry is a study of spaces as

(Set Structure)

where the set is a collection of elements called points, with certain defined subsets, and the structure is that of coincidence, betweenness, parallelism, perpendicularity and transformations.

The Goals of School Mathematics Instruction.

The mathematics we teach to our student *today* should be relevant to their needs in the society in which they will live *tomorrow*. Thus the mathematics we teach should reflect the manner in which the subject is conceived contemporarily. To this end we must first of all be concerned with the formation of the intellect—the capacity to do cognitive thinking. What we teach must develop the human mind in its capacity to *understand and interpret* numerical, spatial, and logical situations in the physical universe and life within it, and to approach problems from a scientific, questioning, and analytic attitude. All our students must come to know of mathematics what it is as conceived by mathematicians today, what material it deals with, what types of thinking (not only axiomatic) it uses, what it accomplishes, and how it is invading almost all other domains of human activity. This is the primary purpose of the SSMCIS program.

Of course our instruction must have an "informational and skill" dimension also, inasmuch as it is changed with transmitting from one generation to the next, that inherited knowledge and skill to use it, which is considered basic and useful in the years ahead. This information should be acquired during the process of developing mathematical thinking. This target of instruction permits us to drop much of the traditional content so long considered useful and to choose more general and more unifying concepts as basic.

Thirdly it is the usefulness of our subject that has maintained it as a main discipline of educational endeavor. Our instruction serves to develop the capacity of the human mind for observation, selection, generalization, abstraction, and construction of models and procedures for use in solving problems in the other disciplines. Unless the study of mathematics can operate to clarify and to solve human problems it indeed has only narrow value.

To recapitulate, the goals of SSMCIS are to develop within the minds of students the contemporary viewpoint with regard to the nature and structure of mathematics, given an abundant store of useful knowledge and skills, and enable students to solve problems through mathematical models.

The Design of Experimentation.

To initiate the development of a curriculum to meet the foregoing goals, a group of distinguished mathematicians and mathematical educators from Europe and America were convened to flowchart a program-sequence and scope for children aged 11½ to 17½ years. The base of the program was to be the ideas of sets, relations, mappings and operations. On these fundamental concepts were built the structures, group, ring, field, and vector space. The realizations of these structures — the number systems and the several geometries — were at the core of the spiral approach and all the activities that take place in the number systems and geometries form the important concepts and uses of mathematics. This unified organization not only permits the calculus to be studied in the 11th and 12 th school years but it also permits the study of genuine modern applications to probability, statistics, computer oriented problems, linear programming, numerical analysis and the usual application of calculus to mechanics and kinematics.

Each year of study was preceded by a summer in-service institute where 20 teachers were trained to understand the mathematics and how to teach to secure the desired goals. The first year of each course required two teachers in each experimental class — one observing student and teacher reaction and then the other. At intervals these teachers reported back to the experimental consultants the results of the teaching. During the next summer the course was revised and rewritten to meet all criticisms or stumbling blocks. After a second revision, the course was put in final form and released to the public. Thus, essentially each course has undergone three years of tryout before achieving its final form. The outcomes were tested by both behavioral and affective domain objectives defined by the general goals of the program.

The Algebra Study.

The algebra study begins with finite arithemetics as numbers on a clock, and develops the fundamental notions of operation, commutativity, associativity, identity element, and inverse element.

Besides the traditional binary operations of addition, multiplication, power, and their inverses, other operations are examined, for example those of max, min, Pythagorean, lcm, gcf, mid-point, tri-point, composition, and so on. At all times sets of numbers, points, or elements that constitute a group are singled out. In turn the number systems — naturals, integers, rationals, reals and complex, are introduced semi-formally and associated with the structute groups, rings, and fields. Matrices are introduced in Grade 9 with illustrations of their many uses, for instance as transformation operators on points in two and three space.

After mappings of various sets (numbers on a line, elements in a plane, etc.) are studied, functions are developed on each of the several number systems. Operations on functions, and graphs of functions are examined. Special functions are singled out for extensive treatment, for example, the polynomials (quadratics in particular), rational functions, trigonometic functions (2 kinds), logarithmic and exponential functions, the identity and inverse functions. All this algebra comes into extensive use in the development of the calculus.

The *geometry program* is a prominent and important one in SSMCIS, in which however, there is none of the usual treatment of synthetic Euclidean geometry. There is a review of the common line figures in a plane, measurement of line segments and angles, coordinatization of a line and lattice points in a plane. By the use of drawings, paper folding, mirrors, and physical devices, transformations of the plane are introduced as bijections. Reflections in a line and a point, translations, rotations (half turns), and glide reflections, with their preserving property lead to the group of isometries. In turn these transformations are used to develop many of the properties of plane Euclidean geometry. Dilations are also introduced and related to similarity. Subsequently the transformations are studies in a coordinatized plane and related to 2 x 2 matrix operators.

A synthetic axiomatic affine plane geometry, with three axioms and several definitions is developed in grade 8, from which 15 to 20 theorems are proved. Finite models (4 points, 9 points, committee structure) are given and the theorems interpreted in these models. Subsequently this geometry is extended to the coordinatized affine plane, and informally to 3-space affine geometry. There are units of study on measures of areas and volumes of common geometric regions, and an informal study of 3-dimensional Euclidean space. Our students study space by transformations, by coordinates, by synthetic methods—and by vectors. The latter enters through.

6. *The linear algebra program* which is new to secondary school mathematics. This study begins with the algebra of matrices, the solution of systems of linear equations by the Gauss-Jordan methods which is then adapted to a tableau algorism. Systems are expressed in matrix notation. Vectors are initially introduced as ordered pairs, ordered triples, or generally ordered n-tuples of real numbers. Addition and scalar multiplication of vectors leads to the geometry of affine lines and vector lines in a plane, and then of lines and planes in 3-space (or n-space).

This study culminates in the definition of an abstract vector space structure which is exhibited by many models of earlier study. The study of subspaces completes this first approach. A graphical interpretation is given to the solution of linear programming problems.

The subsequent study returns to linear equations represented in parametric form, the elimination of parameters to obtain the standard form and vice-versa. Then linear sums or linear combinations, the generation of spaces, (spanning) linear dependence and independence, basis and dimension are examined. In all this study, an associated geometric affine vector space serves for illustration. With the introduction of the inner product, orthogonality is defined, the norm and distance functions developed and the Euclidean vector space of 2 and 3 dimensions is achieved. A final chapter in the 11th school year develops linear mappings (as bijections), the concepts of kernel, range, and co-domain of linear mappings. The study concludes with linear programming using the simplex method.

7. *Probability and Statistics* appear as chapter of instruction in every grade, seven through twelve. The study begins with an a-posteriori approach by recording the occurrence of out-comes in experiments, their relative frequency and the assignment of a probability measure. With dice, spinners, coins, etc. an a-priori probability assignment is made on the expected uniformity of outcomes. The collection of numerical data and its graphical representation by histograms and frequency polygons leads to measures of central tendency — mode, median, arithmetic mean, and to variance — range and standard deviation. Here notation and its use is introduced. In the ninth year, probability is introduced in a formal manner with the axioms of a probability field — the outcome set, the power set, the probability space and the probability measure. Events are related to subsets of the outcome set (the power set and their intersections. Combinatorics (Permutation and combinations) are treated sufficiently to provide material for probability problems.

In the tenth year the study is extended to conditional probability, from which arise the concepts of dependent and independent events, Bayes' theorem, random variables and mathematical expectation. In courses V and VI, the study of probability includes many genuine applications of the subject, — simulations, Bernoulli experiments, the binomial distribution, and Markov chains. With the calculus, the theory is expanded to include infinite probability space. A booklet on Statistical Distributions with hypothesis testing provides additional study on the use of the Poisson, exponential, and other distributions.

8. Analysis (the calculus) begins in Course V with a contemporary approach to continuity via the metric topology of a line and a plane. By the use of topological concepts of neighborhoods, and the

bijection of one into the other, continuity at a point and over an interval is developed intuitively and then formalized. This theory is then used to define limits (at a point) without recourse to sequences. The derivative of functions is approached by linear approximations to the graphs of the functions, followed by the usual development of techniques, theory, and applications of differentiation. The integral calculus is introduced through summations of rectangular areas that bound a function over piecewise step intervals. The analysis program thus has a contemporary approach and covers all the material demanded by the CEEB Advanced Placement A-B Examination, (although it is not an aim of our program).

9. For a brief period in grade 8, *logic* is studied as a separate entity. Here the study treats Quantifiers V and E, connectives 'and' and 'or', the negation "not", implications, bi-implications, their symbols and their relations through the media of truth tables. The unit closes with an explanation of inference schemes (and a hint of the algebra of logic). This study is not used for the purpose of formalizing all subsequent study, but rather as a means for verifying theorems which are proved upon given conditions.

10. The computer program starts with flowcharting in grade 7. At the start of grade 10, BASIC is taught, programs for solving numerical problems are written, and finally they are given to the computer through relayed consoles. Thereafter every mathematical topic subject to numerical computation procedures includes problems for programming and computer solution.

The modification of this six year program, by including more applications, by offering more opportunity to solve problems, by concretization of the more abstract concepts and structures, and by eliminating those parts that can be studied subsequent to leaving high school can form the core program — the common mathematical knowledge for all future citizens. The great mass of people do not need a different mathematics, for as Lichnerowicz so brilliantly stated: There is only one *mathematic* — the same for all people.

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