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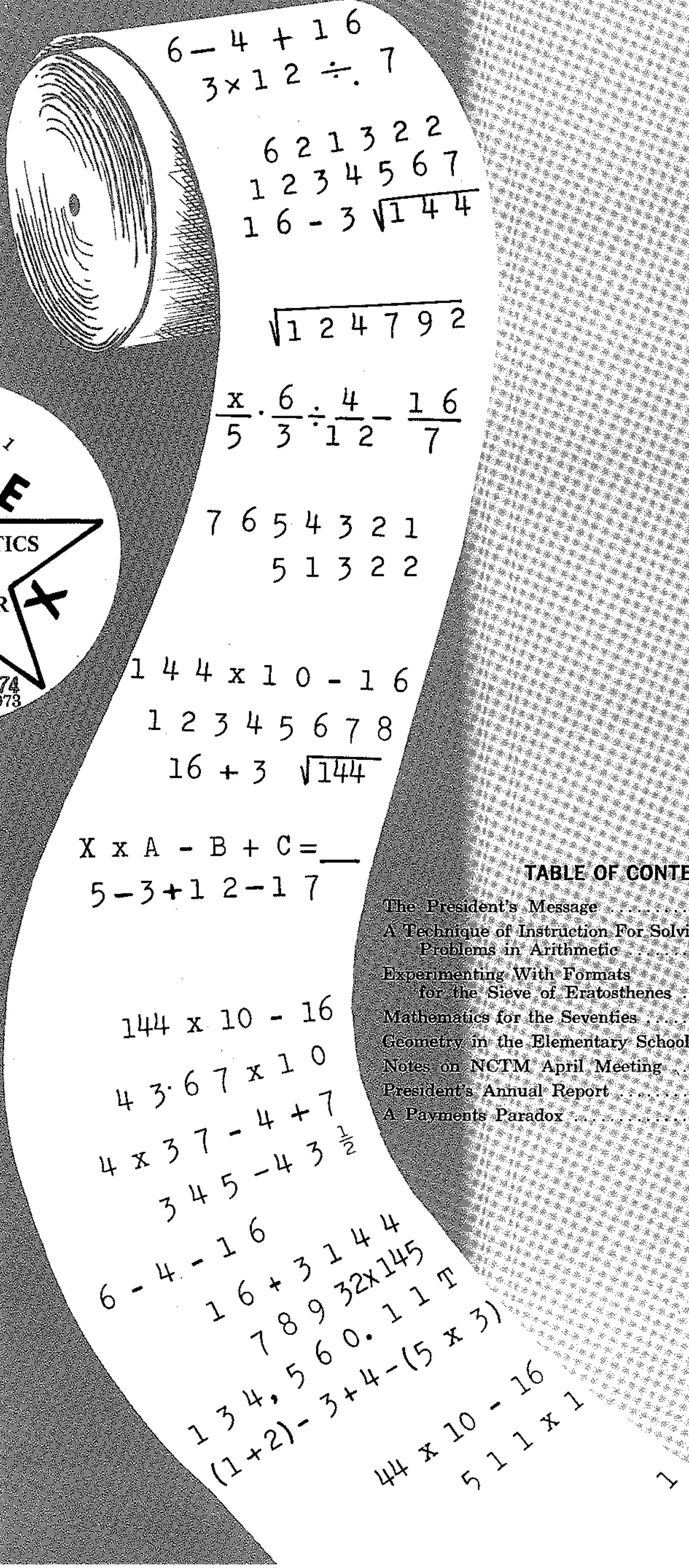

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TEXAS MATHEMATICS TEACHER is the official journal of the Texas Council of Teachers of Mathematics. The views expressed are the contributor's own and are not necessarily those of the publisher or the editor. All manuscripts and correspondence about this publication should be addressed to Mr. J. William Brown, *Texas Mathematics Teacher*, 100 So. Glasgow Drive, Dallas, Texas 75214.

TEXAS MATHEMATICS TEACHER is published quarterly by the Texas Council of Teachers of Mathematics. Payment of membership fee of $3.00 entitles members to all regular Council Publications.

President's Message

Where to begin is the question! There are so many, many people we want to thank for the very excellent convention we have just had. Certainly we do not want to leave anyone out, but even at the risk of that, we do want to say, on behalf of Texas Council of Teachers of Mathematics, thank you and we are indeed grateful to

Dr. Bill Guy, *general chairman*
Mrs. Betty Chipman, *program chairman*
Dr. Irene St. Clair, *registration chairman*
Mr. Eddie Likins, *exhibits chairman*
Mrs. Carlie Estefan, *financial chairman*
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and countless others who did so much to make this truly the best convention we've ever had. Hopefully, you gained some very practical ideas you could put to work "on Monday." Share your ideas and enthusiasm in your building! As we begin to plan for next year's conference, we urge you to send us your suggestions. There was truly something for everyone this year. Next year we want this too.

We are pleased to welcome two new officers to the Texas Council Executive Committee. Bill Ashworth, department chairman, Dobie High School in Pasadena, is the new president-elect. John Savell, department chairman, Lanier High School in Austin, is the new vice president representing senior high school. He takes over from Thomas Hall whose term expired.

A constitution committee was established at the annual meeting. If you have comments or suggestions regarding any part of the constitution, please write Bill Ashworth, Chairman, Constitution Committee Dobie High School, Pasadena Independent School District, P. O. Box 1799, Pasadena, Texas.

You will be interested in the history of the Texas Council of Teachers of Mathematics that will be completed soon. We are grateful to Keene Van Order for his fine work in this endeavor.

Workshops will again be held across the state. Be on the look out for publicity and sign up early. This is one of the most beneficial services the Texas Council offers. Teachers are glad to take their time to engage in an exciting half-day with math.

Are you getting the *Texas Mathematics Teacher*, or are you borrowing it from a friend? Every issue brings more and more practical ideas for all levels. We are indeed grateful to those who have taken their time to contribute many, many excellent articles. Join TCTM now and get *your* copy of the journal.

52nd ANNUAL MEETING
of
National Council of Teachers
of Mathematics
April 17-20, 1974
at
Chalfonte-Haddon Hall
The Convention Center on the Boardwalk
between
Steel Pier and Central Pier
Atlantic City, New Jersey
Theme for the Week
THE METRIC SYSTEM

A Technique of Instruction for Solving Word Problems in Arithmetic

by **MAX JERMAN**
Seattle Pacific College

The notion that simply giving students lots of problems to solve is the best way to teach problem solving in arithmetic, is not supported by research literature. This is not to say that students should not be given many problems to solve, but rather to say that a program of systematic instruction should precede the presentation of sets of problem-solving exercises to students. Although there is a lack of consistent findings in reports of problem-solving research, I shall review some of the attributes of successful programs of instruction in problem-solving which researchers have repeatedly identified as significant, in terms of learning to solve problems in general as well as word-problem exercises in arithmetic. Then I shall report on the results of tests of some programs which have attempted to incorporate these attributes.

Crutchfield (1966) listed several characteristics which he considered to be important components for any program of instrutcion in problem-solving. Included among the components mentioned were such things as: (1) the value of defining the problem properly; (2) techniques for identifying the problem properly; (3) generating and testing many ideas without fear of making mistakes; (4) looking everywhere for clues and sources of ideas; (5) provide immediate feedback to a student's responses to confirm or guide his thinking; (6) freedom from direct group or teacher pressure to keep up with others; and (7) checkpoints for individual diagnosis of sequential progress in learning to solve problems.

Crutchfield was concerned with generalized instruction in problem-solving rather than instruction in any particular academic content area. Nevertheless, the principles have been found effective in instruction in problem-solving in arithmetic as well. One exception to the generally favorable findings is a study by Treffinger and Ripple (1966) which did not provide evidence for transfer from the *Productive Thinking Program*, which was coauthored by Covington, Crutchfield, and Davis (1966), to problem-solving exercises in arithmetic for students in grades 4 through 7. However, this may have been due to the low level of reliability and high level of difficulty of the problem-solving tests used in the study. A general problem with some of the studies of methods of teaching problem-solving skills is that they have used only one-step problem exercises in their criterion tests. It may be that the effect of the special training students received was masked by the lack of complexity in the types of problems students were asked to solve.

This may have been the case in a study by Wilson (1964). He sought to compare the effectiveness of two approaches, an action-sequence approach and a wanted-given approach. Although he reported that the wanted-given approach was the most effective for fifth-grade students, his test instruments contained only one-step problems. Wilson's technique was to lead the student to; (1) recognize the wanted-given structure of the problem; (2) express the structure by using a single direct equation; and (3) solve the equation. Students in Wilson's study were taught four basic types of equations which were to be applied to problems requiring the four basic operations. He used a part-whole terminology in his instructional program.

A modified version of Wilson's wanted-given program was tested experimentally by Jerman (1971). Instead of teaching students, fifth-graders that there are four types of problems, they were taught that there were basically two types, sum-type problems and product-type problems. If a problem is a sum-type problem, then it is of the form $a + b = c$. This is the first step. The wanted-given aspect enters as one attempts to determine whether it is a sum or an addend that is wanted. If a sum is wanted, then the operation required is that of simply adding the given addends. If one of the addends is wanted, then the required

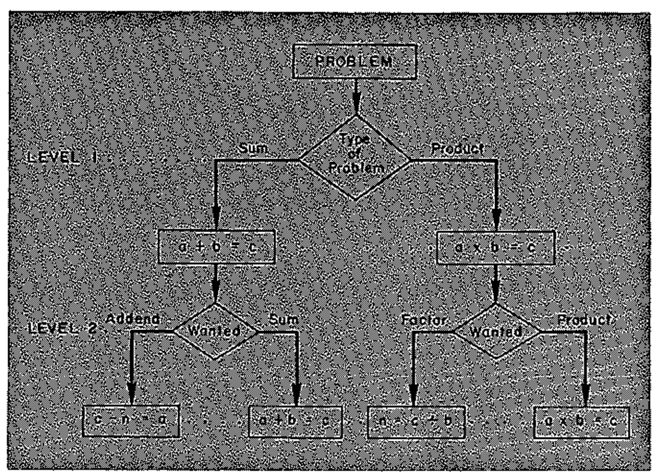

Figure 1. Decision structure of the Modified Wanted-Given Program for simple one-step verbal problem solving indicating decision levels.

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operation is subtracting the given addend from the sum to find the missing addend. In a similar manner, students were taught to identify and solve product problems.

The effectiveness of this approach seems to accrue from: (1) keeping the number of rules to be remembered as few as possible, namely two as suggested by the reasearch of Thiel (1939) and Mitchell (1932); (2) providing an interesting instructional dialogue, via programmed text using cartoon characters with whom students can readily identify as suggested by Crutchfield and others; (3) providing immediate feedback for each step of each problem as suggested by Crutchfield and the research on programmed instruction and computer-assisted instruction, and (4) demonstration of some alternate ways to solve various problems without laboring the point. As shown in Figure 2, there is no need for the student to learn new rules to handle multiple-step problems.

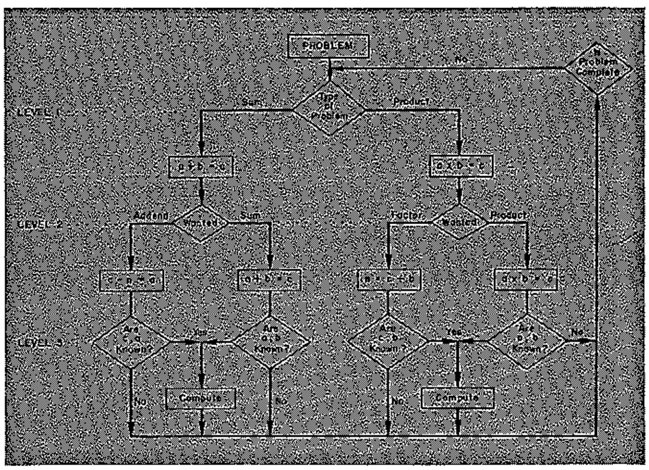

Figure 2. Decision structure of the Modfied Wanted-Given Program for multi-step verbal problem solving.

The same rules can be applied in each case, thereby reducing the complexity of the problem-solving task.

In the study reported by Jerman (1971), which compared student achievement on multi-step arithmetic word problems by students who were given the *Productive Thinking Program*, the Modified Wanted-Given Program and a control group, the results indicated that both groups which received instruction in problem solving scored significantly higher ($p < .001$) than the control group in terms of their ability to use the correct procedure. As suspected, the effects of instruction in the *Productive Thinking Program* became evident and significant on tests which included problems requiring several steps for solution. Students in the modified Wanted-Given Program scored significantly higher ($p < .005$) than those in the *Productive Thinking Program* in terms of using the correct procedure to solve the problems. This seemed to indicate that while instruction in general problem solving produced significant differences as compared to a control group, that specific instruction in problem solving in a mathematical context produced further significant gains.

Covington and Crutchfield (1965) and Olton et al. (1967) reported the observation that students achieved up to 50 percent higher scores on post-test measures of creative thinking and problem-solving in classes where the teacher discussed each lesson. In a follow-up study just completed, Zulick (1973) studied the effects of teacher discussion vs. non-discussion with two classes using the revised edition of the Modified Wanted-Given Program (Jerman, 1973). The revised version of the Modified Wanted-Given Program consisted of 10 programmed lessons, as compared to 16 in the original version, together with two problem cards for each lesson which contained from 8 to 10 exercises. The students were directed to work through the programmed booklets, checking all their work with the answers provided and complete the 4-item post-test contained in the back of each lesson booklet. If the student met criteria on the post-test, 3 out of 4 items correct, he was directed to the "shorties" problem card which contained a variety of short exercises, 10 in number, which were intended to provide practice for the concepts taught in the lesson. If a student met criteria on the "shorties," 8 out of 10 correct, he was directed to the "challengers" problem card which contained a variety of problems, 8 in number, of varying numbers of steps. Any student failing to meet criteria in the instructional booklet was directed to a remedial help section of the booklet for an alternate approach to the topic. Any student failing to meet criteria on the problem cards was directed to restudy the instructional booklet and to try again. Three classes of remedial seventh-grade students were used in her experiment. Two of the classes were selected at random to serve as treatment groups. In one class she discussed each lesson at the end of the period and worked through one or two of the problems with the class. The other class received no discussion but simply worked through the programmed materials on their own. Zulick found no significant differences, due to discussion, in mean scores for the two treatment groups. However, both groups scored significantly higher than the control group on both the post-test and follow-up test of 20 word problems each.

This second study did lend support to the effectiveness of the approach in terms of correct responses on criterion measures rather than simply the number of times the correct procedure was used as was the case in the Jerman (1971) study. The results of the follow-up study also cast some doubt on the effectiveness of class discussion in a problem solving context in mathematics of the type used in this study. Further study needs to be undertaken to identify the factors that inhibit or enhance the effectiveness of discussion of programmed instructional materials in problem solving in arithmetic.

The revised edition of the Modified Wanted-Given Program contained what this writer believes to be some of the most effective components of instruction in problem solving in arithmetic. These are:

1. Require the student to remember no more than two rules such as sum-type or product-type identification.
2. Provide extensive instruction on important terms such as sum, product, addend, and factor and the means of solving simple equations for the unknown, wanted, term.
3. Provide for immediate feedback for each step of the solution to each problem.
4. Provide motivation by the use of cartoon characters or some other means to enhance the probability that the student will identify with some aspect of the instructional program.
5. Provide sequential progress check tests so that the student has an opportunity to evaluate his own growth in problem solving ability.
6. Provide an obviously graded set of word problem exercises, requiring more than one step for solution, at regular intervals for practice.
7. Provide a wide variety of exercises that are occasionally humorous or silly to keep the instructional approach from becoming too heavy.

As can be seen, the list of components above represents a synthesis of the ideas and suggestions put forward by many people including Crutchfield and his colleagues, Wilson, and the author's own experience. Taken together, they seem to provide a sound basis for the development of an effective instructional program in problem solving in the elementary school.

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Experimenting with Formats for the Sieve of Eratosthenes

by **ROGER OSBORN**
The University of Texas at Austin

Most teachers of mathematics — particularly in the seventh and eighth grades — recognize the Sieve of Eratosthemes as a procedure (algorithm) for producing the primes in a list of all natural numbers from 2 to n, n $>$ 2. It is not the purpose of this discussion to give the details of the steps in the sieve and the proofs of their validity, but, rather, to suggest that some formats for following the rules are more efficient than others, and that at least one format leads to auxiliary results of sufficient interest to merit some study of (or at least inquiry into) these side effects.

The common format found in textbooks for arranging the matrix of numbers in preparation for applying the rules of the sieve is to have the first row contain the numbers (this means, of course, their representations, but the number/numeral argument is counterproductive here) 2 through 10, and all subsequent rows contain ten members:

	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20.

This traditional arrangement into tens is doubtless dictated by our "hang-up" about arranging everything with the same base that our numeral system uses.

Let me suggest an experiment, first for you the reader, and then later, possibly, for some of your pupils. A single piece of paper (or stencil) is adequate for this experiment. On the sheet, arrange the numbers 2 to 120 in rows of ten (except for the first row) as indicated above, in rows of six:

```
    2   3   4   5   6
7   8   9  10  11  12   , in rows of seven, and
13 14  15  . . .          in rows of two.
```

[Obviously, other choices are available: rows of five, or of twelve, or of twenty-one, etc., but the ones suggested in the preceding sentence should suffice. Further, the arrangement in rows of two will fill a narrow vertical strip of the page, and you may not get all of the numbers up to 120 on the page, but you should have enough for the experiment.]

Now begin applying the rules for the sieve. With your pencil, draw a ring around the 2 to show that it is to be kept, and scratch out every second number thereafter (every subsequent multiple of 2). Do this in every one of the four formats suggested. If you conveniently can do so, find the pattern in each format of where these multiples of 2 are located so that the scratching out may be done efficiently by one or several long pencil strokes.

Put a ring around the 3, the least number remaining on the list which has not yet been encircled or scratched out. Start counting from the 3 and scratch off from the original list every third number thereafter (these are the subsequent multiples of 3 and, as such, are not primes). Again, if for each format you can find the pattern of where these multiples of 3 are located, then you will be able to use one or more long pencil strokes to scratch them out.

At this stage of the procedure, regardless of format, approximately two-thirds of the members of the original list have been scratched out. At this stage, which of the formats seems to have been most convenient to use?

Keep the 5 which remains near the head of the list. Show this by putting a ring around it. Scratch out every fifth number (of the original list) thereafter — the subsequent multiples of 5. What is the pattern for their locations? Try to describe it to yourself for each of the experimental formats. Try to scratch out these multiples of 5 with long pencil strokes.

Keep the 7 (near the top of the list) and scratch out the subsequent multiples of 7. What is the pattern of the locations of these multiples of 7 in each of the formats? Can you again use long pencil strokes to scratch out several per stroke?

Keep the 11, and scratch out the subsequent multiples of 11. Where are they? [I realize that, by this stage of the procedure, the sieve has already cast aside all composite numbers from the original list and has kept only those of the original numbers which are prime, but please bear with the instructions and questions anyway.] Can these multiples of 11 be scratched out with long pencil strokes? What adjective or noun would you use to describe these strokes?

Continue at least through keeping 13 and striking out all subsequent multiples of 13, keeping 17 and scratching out subsequent multiples of 17, and doing likewise for 19.

Did one of the original formats display more recognizable patterns in the scratching out process than did the others? *My* answer is an unqualified "yes." The arrangement in rows of six should have had the greatest number of discernable patterns, and it will be this format about which the remaining comments and questions will be concerned. The "long pencil strokes" described above should be all diagonals of varying inclination, some declining to the left, some to the right. The multiples of 5 are on diagonals declining to the left, one column over and one row down (except for having to start a new diagonal in the right-most column every once in a while . . . how often?). The multiples of 13 are on diagonals declining to the right, one column over, two rows down. How would you describe the diagonals which strike out the multiples of 17? Of 19?

In which of the columns do the 2 and the 3 occur? In which columns do *all* of the rest of the primes occur? In which column do multiples of 6 occur? Would your answers to the second and third of these preceding questions lead you to suspect that *all* primes greater than 3 are either one greater or one less than some multiple of 6?

Do you see any other patterns developing in the table with rows of six?

After answering these last several questions for the table of rows of six, try to answer them for each of the other formats.

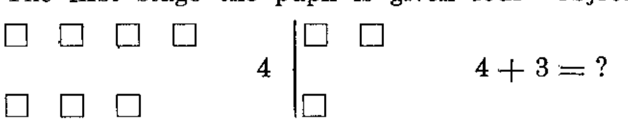
Even if, after comparing formats, you still prefer the tens' format for the Sieve of Eratosthenes, I hope you see both the validity of experimentation with the formmat *and* the usefulness of the search for patterns. Only through experimentation can improvements in notation and procedure be found; only through recognition of patterns can progress be made in our understanding of mathematics.

Mathematics for the Seventies

by **LOLA J. MAY, Ph.D.**
Mathematics Consultant
Winnetka Public Schools, Illinois

The teaching of mathematics at all levels should be exciting for the emphasis is now on teaching strategies. No longer is there a need to worry about number versus numeral, how to spell commutative, or the right and left distributive property. Now the need is to learn more about creating activities that help build models for learners. Morris Klein is right when he states the issue is not traditional mathematics versus modern mathematics but how to teach mathematics. Many of the so called trouble spots in mathematics are created because teachers and textbooks miss some of the essential teaching steps in learning a concept or a skill. These steps can be taught by skillful teachers.

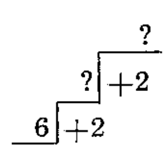
A trouble spot in the primary levels is where pupils continue to count all the objects presented in two sets. Then the jump is made from this to trying to do addition with only abstract symbols. The first stage the pupil is given four objects



How many? (7) How many? (7) How many? (7)
First stage Second stage Third stage

and three objects and asked, "How many?". The pupil counts 1, 2, 3, 4, 5, 6, 7. The second stage should be where the number of the first set is given so the pupil does not count the objects and then counts only the objects in the second set. The counting goes 4, 5, 6, 7. Then the third stage of seeing the number of both sets and just giving the sum. Going from concrete for both sets to abstract for both sets is a big jump and the second stage is needed for many pupils.

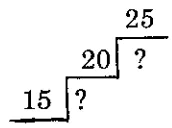
Another trouble spot is sequential counting in the primary grades. Again the pupils need a model to help them understand what they should do to find the missing numbers in the sequence. Stairs can be shown and in the first stage the pupils write the number that goes with each step. In the example the pupil sees that you go from one step to the other by adding 2. They can complete the sequence 6, 8, 10, ? ? by continuing to add 2. Many examples like this with adding 5, 10, 3, and 4 should be used. The next stage is to show the stairs with the numbers written on top of each



6, 8, 10, ? , ?

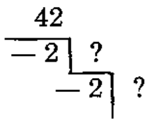
step and the pupil has to tell how high is each step. Once the pupil has decided the rule is add 5, then the sequence 15, 20, 25, ? ? can be completed.

After many examples like this are presented the pupils are ready to look at sequences where the

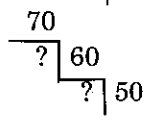


15, 20, 25, ?, ?

rule is add some number and decide on their own what they need to add. The same stages are repeated for subtracting a given number in a sequence. Once this has been done the pupils have a model to help them get started with easy



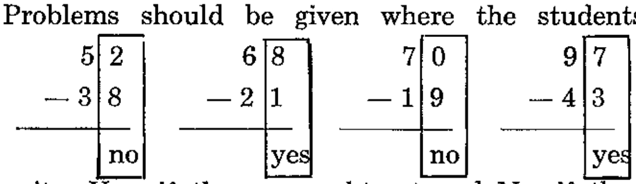
42, 40, 38, ?, ?



70, 60, 50, ?, ?

sequences and will be better prepared to tackle sequences that are more difficult.

Before students get involved in learning to rename in subtraction they need to be trained to look at the problem and make some decisions. Problems should be given where the students



write, Yes, if they can subtract and No, if they cannot subtract. They should not solve the problems. After doing about four pages like the examples shown the students are ready to learn how to subtract the problems where they have been writing no.

Teaching pupils to be able to solve missing addend problems has been a real problem for many years. Missing addends can only be meaningful after the pupil knows the addition and subtraction facts and also knows how to analyze a sentence. In the example below the teacher needs to ask some questions. Is 13 a sum or an addend? Is 9

$$\square + 9 = 13$$

A A S

a sum or an addend? Is the missing number a sum or an addend? Will you add or subtract to find the missing number? Do it. Is 12 a sum or an addend? Is 7 a sum or an addend? Is the missing number

$$12 - \square = 7$$

S A A

a sum or an addend? Will you add or subtract to find the missing number? Do it.

As the pupils learn to first look at the sentences and decide what they are looking for and then make the decisions, they can find the missing number. This type of training is very valuable in mathematics for the student who first looks and analyzes a problem before he starts to solve the problem.

The examples given are only a few of the trouble spots but by looking at them one can be aware of some of the teaching that is needed to help students become successful in learning life survival skills in mathematics. If one honestly believes students do not fail but only teachers fail to find the trouble spots then the challenge is a real one. Creative teachers learn from the teachers next door. All of us who teach must share our experiences and learn from each other. This can be an exciting experience for the great teachers always want to continue to grow.

Geometry in the Elementary School

by LELON R. CAPPS

University of Kansas, Lawrence, Kansas

In modern programs for elementary school mathematics instruction one significant change has been the inclusion of more geometry. Unfortunately, the inclusion of a topic in the curriculum does not guarantee that it will be taught, and more importantly, learned. There are many factors that influence what is happening to geometry in our modern programs and it is the purpose of this article to provide some information on the "state of the subject.

How Did It Start?

Historically geometry gained its place in the curriculum over a long period of time. First courses in geometry appeared in high schools as early as 1821 but a course in geometry was not commonplace in high schools until it appeared as an entrance requirement to college. Yale required a course in geometry for entrance in 1865 while Princeton, Michigan and Cornell introduced the requirement in 1868 with Harvard following in 1870.

As might be expected, the first high school courses used college level texts. Thus, the treatment of geometry was a formal one. The intuitive approach had its inception in about 1859 when Thomas Hill suggested that intuitive geometry in the elementary grades provide the basis for arithmetic and that all elementary children should be exposed to geometry prior to the studies of formal geometry at the high school level.

Thus, the rationale for including geometry in elementary school programs is by no means recent. Bernard Marks' statement of 1871 is still relevant:

> How it ever came to pass that Arithmetic should be taught to the extent attained in the grammar schools of the civilized world, while Geometry is wholly excluded from them is a problem for which the author of this little book has often sought a solution, but with only this result, vis., that Arithmetic, being considered an elementary branch, is included in all systems of elementary instruction; but Geometry, being regarded as a higher branch, is reserved for systems of advanced education, and is, on that account reached by but very few of the many who need it (Marks, 1871).

The Committee of 10 followed in 1892, with the recommendation that intuitive geometry be introduced at the elementary level. It was the report of that committee that began the trend of more geometry in elementary school programs.

How Much Geometry Is There In our Elementary Program?

A survey of five leading textbooks recently was done by Peterson. In each series, the percent of pages devoted to geometry was determined for grades three, four, five, and six. The range of percent for each of the grade levels follows:

3rd — 10.0% — 19.5%
4th — 10.0% — 22.2%
5th — 13.3% — 24.2%
6th — 15.3% — 27.7%

As stated previously, the fact that, on the average, 15% of a program contains geometry is not a guarantee that it is being taught to students or that students are learning it when it is taught. Thus, we need to examine some issues related to having the content presented to and learned by the students. Several questions merit discussion.

Should the Geometry be Massed in One or Two Comprehensive Units or Should it be Distributed in Lessons of 2 or 3 Pages Interspaced Throughout a Program?

Common sense would seem to dictate that geometry content be developed in an integrated fashion. That is, the topic or topics should be provided enough space and time to allow for ample discussion and exposure with some opportunity to digest and interrelate several concepts that are interdependent. Teaching a lesson here and a lesson there would not seem to allow for the relating of the concepts taught.

Are Students Capable of Learning the Concepts Being Presented?

Numerous studies have been conducted to indicate that the basic concepts of geometry now included in programs can be learned by students. However, people frequently will point to an isolated statement to prove that it can't be done. For example, Piaget claims that the abstract operations which enable a child to understand coordinate geometry do not develop until ages eleven to thirteen. Yet, we know that third and fourth grade students use concepts of coordinate geometry most readily in constructing and interpreting maps and graphs. One almost is led to believe that in searching for reasons not to do something all that is needed is the statement of one authority that will exhibit a discrepancy between that statement and practice.

What About Teaching Geomery to the "Culturally Deprived"?

Johnson taught a three week unit in topology and geometry to both Anglo American and disadvantaged Mexican-American second grade students. She found no relationship between ethnic membership and achievement in geometry. Bring also compared achievement in geometry between Caucasians and other ethnic groups. He found no significant difference in gain scores between groups.

One could conclude that geometry may be one of the best topics to teach to ethnic groups and "culturally deprived." In the writer's opinion, we should operate on the assumption that all students can learn and refrain from using ethnic group membership and cultural deprivation as reasons for *not* achieving our goals in teaching mathematics.

Do Teachers Have the Necessary Knowledge of Geometry to Teach it Effectively?

Bailey found that 70% of a group of prospective elementary teachers obtained a score of less than 70% on geometry concepts taught in elementary mathematics texts. Obviously, there are reasons for their low scores. Many elementary teachers have not taken formal coursework in geometry since they were in high school. This, coupled with the fact that geometry did not receive emphasis in the elementary mathematics curriculum they were teaching would most likely result in low scores in geometry content tests.

However, the results also may reflect, to a great extent, an attitude toward geometry that is widespread among elementary level teachers. Because of an inadequate background, many teachers fear the area of geometry. Too often, this fear manifests itself in teachers omitting the geometry lessons in their programs. The net result of this behavior will have disastrous consequences if this omission of geometry continues. It can be explained in the following way.

At the junior and senior high school levels, the geometry and trigonometry is being integrated into the curriculum as part of the Algebra I and Algebra II courses. The end result of this integration processes will, in many cases, lead to the discontinuance of separate geometry and trigonometry courses at these levels.

Thus, students who enter Algebra I and II will need some background experience in geometry that will include concepts beyond measurement. Current elementary programs have included many of these concepts at the intuitive level. Thus, we can see why there is greater emphasis on geometry in the elementary curriculum.

More importantly, we see the cruciality of providing all elementary level students with introductory experiences to the geometry content that is included.

In drawing this discussion to a close, it might be well to provide a general listing of the geometry content in elementary programs at grade 4, 5, 6. After examining this table, we need to give serious thought to answering the questions that follow it.

*CONTENT ANALYSIS, GRADES 4, 5, AND 6

	Average No. of Lessons	Percent
General Metric Geometry	.23	.96
Length	3.10	12.93
Perimeter	.83	3.46
Circumference	.47	1.96
Area	3.37	14.05
Volume	1.27	5.30
Liquid Measure	.43	1.79
Angle Measure	.90	3.75
General Non-Metric Geometry	1.20	5.00
Curves (lines, line segments, rays)	2.27	9.47
Angles	1.13	4.71
Circles and Ellipses	1.57	6.55
Congruence and Similarity	1.30	5.42
Intersections of Geometric Figures	.20	.83
Polygons Except Triangles	1.63	6.80
Triangles and Pythagorean Relationship	.97	4.04
Space Figures	2.08	8.67
Transformations	.73	3.04
Symmetry	.30	1.25
Miscellaneous	-	-
Total	23.98	

*Above table based on analysis of five major textbooks used in U.S.

Are elementary teachers adequately teaching the geometry content in the curriculum?

Can we afford to omit the lessons on Geometry in the elementary school mathematics curriculum?

Will our elementary school students be adequately prepared for the junior and senior high school mathematics courses?

Who will be responsible?

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Notes Re: NCTM Meeting in April

1. Annual meeting registration fees are $9 for members and $12 for nonmembers, with a $2 discount for those registering in advance.
2. Full-time students who have not taught professionally, the non-teaching spouse or child of a registrant, NCTM board members, NCTM past presidents, speakers, and commercial exhibitors will be issued badges in lieu of registration.
3. An elementary school teacher preregistration, at $10 per badge, will be issued by mail to schools. The number of elementary teachers from the same school attending at the same time is limited to the number of badges purchased by mail through the school principal.
4. An elementary school district may negotiate a per head registration fee for their elementary teachers with headquarters of NCTM, predicated by their agreement to the following—
   - A. To release all elementary teachers within the negotiating school district.
   - B. That payment of the fee will be made by the negotiating school district.
5. Advance registrations postmarked after thirty (30) days before the convention will be returned. No refunds will be made on cancellations received after the convention begins. Make room reservations by mail through—Chalfonte-Haddon Hall, North Carolina Avenue, Atlantic City, N.J. 08404.

President's Annual Report

November 2, 1973

The Texas Council of Teachers of Mathematics began its new year in February, 1973, with the annual meeting at CASMT. The 1972-73 TCTM year was late in starting due to the postponement of CASMT until this February date.

In April, TCTM co-hosted the annual meeting of the National Council of Teachers of Mathematics. Many members of TCTM contributed countless hours to the very fine convention held in Houston. The executive board of TCTM met during the convention.

This year has been most profitable even though it was short. We look forward to a continued eventful year of mathematics for students through TCTM efforts. Activities for the year included

- Co-sponsoring workshops
  —with Coastal Bend Council of Teachers of Mathematics at Gregory-Portland High School, Portland, Texas
  —with Pasadena Math Council at Dobie High School, Pasadena, Texas
- Presenting a mathematics program at the State TSTA Convention for the Mathematics Section
- Providing speakers for other workshops and mathematics meetings across the state
- Assisting Rio Grande Council in its election of officers and programs for 1973-74
- Co-sponsoring the annual Conference for the Advancement of Mathematics Teachers held in Austin

It was a good 1972-73 year. It will be an even better 1973-74 for boys and girls in Texas as they study mathematics. TCTM will do all it can to make this happen. Remember, TCTM is You—all of us, together!

A PAYMENTS PARADOX

DAVID R. DUNCAN and BONNIE H. LITWILLER

Associate Professors of Mathematics
University of Northern Iowa
Cedar Falls, Iowa

Mathematics teachers and students are often interested in problems which appear to be straightforward but have surprising solutions. An interesting example is illustrated by this problem. Which of the following jobs would provide you with the better salary:

Job A—$8000 the first year and a raise of $800 for each succeeding year.

Job B—$4000 the first six months and a raise of $200 for each six months period

It is evident that in the first year Job A provides the better salary. Routine arithmetic calculations yield these results:

Job A will pay $8000 for the first year while Job B will pay $4000 the first six months and $4200 the second six months for a total salary of $8200 for the first year.

A plausible conjecture, however, is that beginning in the second year, the salary for Job A will surpass that of Job B. After all, one would think that $800 increase once a year is twice as good as two increases of $200 for a year! Routine calculations, however, yield a surprising result. Salaries earned during the second year are:

Job A—$8800

Job B—$4400 + $4600 (Recall a salary increase of $200 for every six months period) or $9000

Similarily the salaries during the third year are:

Job A—$9600

Job B—$4800 + $5000 or $9800

Note that for each of the first three years Job B paid $200 more per year than Job A; thus, a $200 raise each six month period produces a better total increase than an annual raise of $800. You will find that this is also true for all succeeding years.

Now consider a second example:

Job A—$8000 the first year and a raise of $800 for each succeeding year. (Note that this is the same data as in the first example.)

Job B—$2000 the first three months (a quarter of one year) and $50 raise for each succeeding quarter.

The salaries earned for each of the three years have been computed:

During the first year:

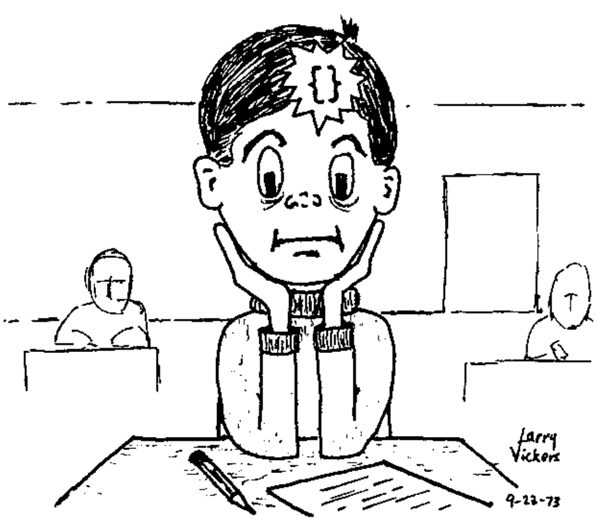
Job A—$8000

Job B—$2000 + $2050 + $2100 + $2150 or $8300

During the second year:

Job A—$8800

Job B—$2200 + $2250 + $2300 + $2350 or $9100



Borrowed from
The Oklahoma Council of Teachers of Mathematics Newsletter
(Fall 1973)

During the third year:

Job A—$9600

Job B—$2400 + $2450 + $2500 + $2550 or $9900

Two observations should be made at this point.

1. Job B pays $300 more than Job A each year. Again this is surprising since a common error is to think that four quarterly raises of $50 is the same as a yearly raise of $200 which is certainly less than $800.

2. If the yearly wage for Job A is thought of as being paid in four equal quarterly payments, then the *first* quarterly payments for the first three years are: $2000 for the first year, $2200 for the second year, and $2400 for the third year. Note that these are also the same wages as earned on Job B for the first quarter of the respective years. Job B then surpasses Job A for the remaining paying periods of each year. The same is true in example one also.

Let us now generalize the problem and consider Job B first. The year will be divided into "n" paying periods of equal length; these periods will be called "n-periods". For instance in example one, "2-periods" were used while in example two "4-periods" were used.

Job B—A worker earns X/n for the first n-period and receives a raise of r for each succeeding n-period.

Job A—A worker earns X for the first year and receives a raise of $n^2 \cdot r$ for each succeeding year.

During the first year Job A earns X. Job B: earns:

$$X/n + (X/n + r) + (X/n + 2r) + \cdots$$
$$+ (X/n + (n\text{-}1)r) =$$
$$n(X/n) + r + 2r + \cdots + (n\text{-}1)r =$$
$$X + r(1 + 2 + \cdots + (n\text{-}1)) =$$
$$X + \frac{r(n\text{-}1) \cdot n}{2}$$

$$\left(\text{Recall } 1 + 2 + 3 \cdots + k = \frac{k(k+1) \cdot)}{2}\right.$$

During the second year Job A earns $X + n^2r$. Job B earns:

$$[X/n + nr] + [X/n + (n + 1)r]$$
$$+ X/n + (n + 2)r] + \cdots + [X/n + (2n\text{-}1)r] =$$
$$n(X/n + nr) + (r + 2r + 3r + \cdots$$
$$+ (n\text{-}1)r) = X + n^2r + \frac{r(n\text{-}1)n}{2}$$

For these two years and each succeeding year Job B pays $\frac{r(n-1)n}{2}$ more than Job A. The previous two examples were special cases of the general development. In the first example, $x = 8000$, $n = 2$, $r = 200$; in the second example $X = 8000$, $n = 4$, $r = 50$. For the first "n-period" of each year Jobs A and B pay the same and the advantage of Job B occurs during the remaining pay periods of the year. For the first n-period of the second year, Job A receives $\frac{n^2 \cdot r}{n}$ or $n \cdot r$ of his yearly raise. His salary for the first n-period would be $X/n + nr$; this is $1/n$ th of the base salary plus $1/n$ th of the yearly raise. Compare this with the first n-period of Job B for the second year.

In a cursory examination of this situation one might incorrectly conclude that a raise of r received by a worker n times a year would have approximately the same result as a raise of $n \cdot r$ for a whole year. The calculations reveal surprisingly that even a raise of $n^2 \cdot r$ each year is not sufficient to balance n raises of r during a year with the given starting salaries.

References: The first example is adapted from a problem in *Elementary Analysis: A Modern Approach* (Prentice Hall, 1960) by H. C. Trimble and Fred W. Lott.

PROFESSIONAL MEMBERSHIP APPLICATION

Date: ________ School: ________ School Address: ________

Position: ☐ teacher, ☐ department head, ☐ supervisor, ☐ student,* ☐ other (specify) ________

Level: ☐ elementary, ☐ junior high school, ☐ high school, ☐ junior college, ☐ college, ☐ other (specify) ________

Other Information: ________

		Amount Paid
Texas Council of Teachers of Mathematics	☐ New membership ☐ Renewal membership	3.00
Local ORGANIZATION: ________	☐ New membership ☐ Renewal membership	
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	$ 9.00, dues and one journal ☐ *Arithmetic Teacher* or ☐ *Mathematics Teacher*	
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	5.00 additional for subscription to *Journal for Research in Mathematics Education* (NCTM members only)	
	.50 additional for individual subscription to *Mathematics Student Journal* (NCTM members only)	
	The membership dues payment includes $4.00 for a subscription to either the *Mathematics Teacher* or the *Arithmetic Teacher* and 25¢ for a subscription to the *Newsletter*. Life membership and institutional subscription information available on request from the Washington office.	

* I certify that I have never taught professionally ________ *(Student signature)* *Enclose One Check for Total Amount Due* →

Fill out, and mail to Dr. Floyd Vest, Mathematics Department, North Texas State University, Denton, Texas 76203.

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