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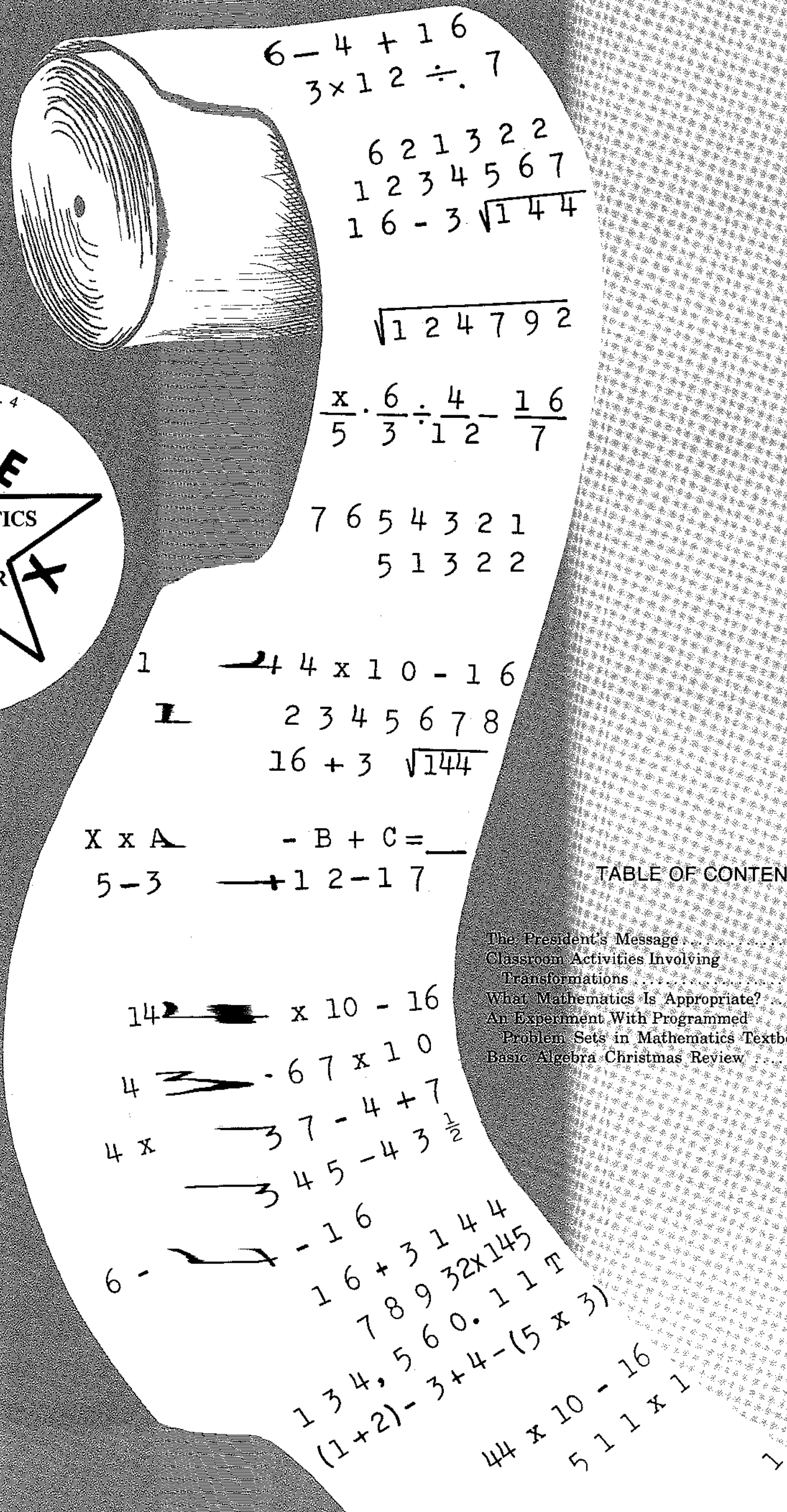


TABLE OF CONTENTS

	Page
The President's Message	3
Classroom Activities Involving Transformations	4
What Mathematics Is Appropriate?	8
An Experiment With Programmed Problem Sets in Mathematics Textbooks	11
Basic Algebra Christmas Review	14

Classroom Activities Involving Transformations

DR. FORREST L. COLTHARP
Kansas State College of Pittsburg

"Tell it like it is!" is the demand being made by the youth of today. In all areas of society, in politics, in religion, in personal relations, and even in the classroom the younger generation is expecting us to "Tell it like it is." We often do just the opposite however, in elementary school mathematics. How many of us have been guilty of telling first or second grade students — "You never subtract a larger number from a smaller number?" Even if we do not tell such an obvious falsehood, we sometimes use incorrect procedures and "sugar-coated" terminology.

One of the most difficult tasks is the "unlearning" of incorrect information. Those of us who have updated their traditional backgrounds in mathematics have experienced this difficulty. We have much more trouble with the modern concepts than the children since they're encountering it for the first time. We must struggle to replace misconceptions and memorized procedure with understanding and justifiable mathematics.

Even though we may recognize these difficulties as students, when on the other side of the desk, we are guilty of causing them. Is it really easier to refer to the principle that justifies the fact that 6 x 9 gives the same result as 9 x 6, the order principle? I have found that primary age students have less difficulty with the words, commutative and associative, than they have with my name. Repetitive use of correct terminology by the teacher will do much to associate the term with the procedure, property or topic under discussion.

The concern of this writer is with the usage of correct terminology to describe certain procedures used in elementary school geometry. We often want to compare the sizes of line segments and the shapes and size of other geometric figures to determine if they are congruent. We do so by moving, sliding, turning, flipping and other means to compare one geometric figure with another. These movements can be mathematically justified by transformations.

Intuitively, a transformation is the association of a geometric set of points with a new set determined by "moving" the given set. Historically, transformations are significant because Euclid spoke of superimposing one triangle upon another to determine if they were congruent. At the present time, concepts of transformations are being treated rigorously and formally in advance courses of mathematics. The concepts and language of transformation geometry are intuitively simple and have an increasingly important part to play in mathematics ed[...] tion. Due to European success in teaching, trans[...] mations are also becoming popular in new mater[...] in the United States.

The most common transformations are thos[...] which the whole plane is somehow moved wit[...] stretching or twisting so that shapes and size[...] geometric figures are not altered. It can be pro[...] in a rigorous presentation that the only basic tr[...] formations that preserve size and shape are t[...] called *motions* or *isometries*. The basic motions [...] translation, rotation and reflection. There is al[...] trivial motion called the *identity* transforma[...] which leaves a figure unchanged. This motion o[...] is the result of other motions.

A *translation* is a motion which involves a m[...] ment of each point the same distance in the s[...] direction by a "sliding" of the plane (see Figu[...] (a)). A *rotation* moves each point of the figure aro[...] some fixed point, called the *center of rotation*, [...] turning the plane (see Figure 1 (b)). The mo[...] *reflection* involves the movements of each p[...] across some line, called the *axis of symmetry*, [...] the plane is reflected in the line as a mirror [...] Figure 1 (c)).

Figure 1 →

There are many examples in elementary sc[...] geometry where the language of transformation[...] clarify the experience and mathematically justify[...] procedure. For example, if every point of a lin[...] translated a fixed distance (see Figure 2 (a)), [...] line and its image are parallel. For similar res[...] if an angle $\angle$ ABC is translated along the ray [...] so that B′ is the image of B, then line AB is par[...] to A′B′. This fact we normally accredit to congr[...] corresponding angles (see Figure 2 (b)). If an isosc[...] triangle with two congruent sides is reflected al[...] an axis of symmetry as prictured in Figure 2 [...] we not only confirm the sides $\overline{AC}$ and $\overline{BC}$ are [...] gruent but also establish that $\angle$ A is congruen[...] $\angle$ B. We also observe that the axis bisects the [...] and the angle opposite the base. Since the linear [...] $\angle$ 1 and $\angle$ 2 are congruent, we also conclude [...] the axis is perpendicular to the base.

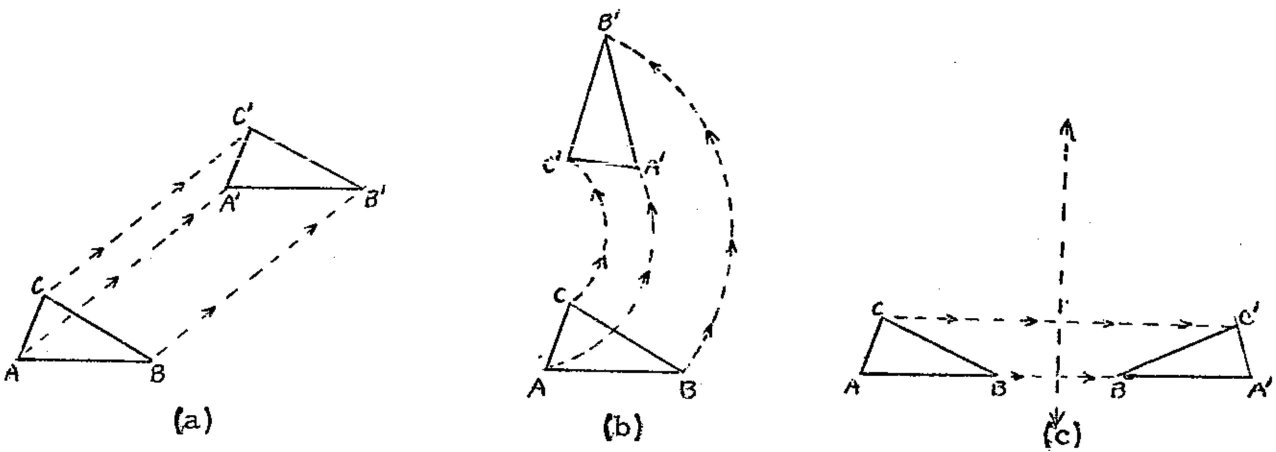


Figure 1

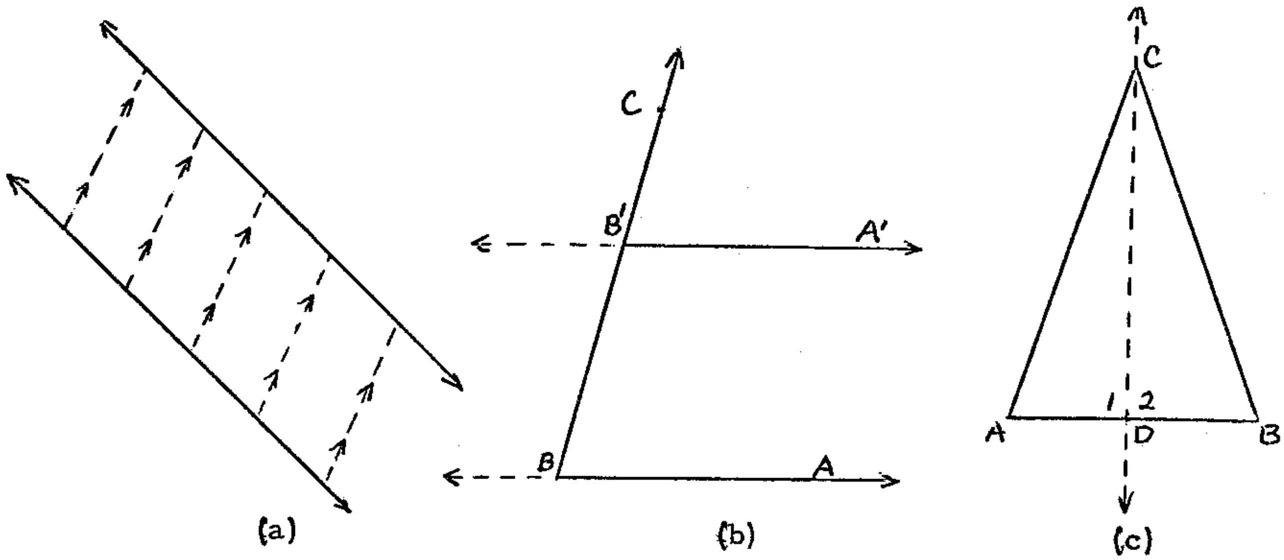


Figure 2

Figure 2

If we examine various quadrilaterals we find that basic isometries also help describe some of their properties. If a square is reflected in the diagonal $\overline{NP}$ (see Figure 3)a)), we observe that opposite angles are congruent and opposite sides congruent. If reflected about another axis of symmetry $\overline{RS}$, we observe that consecutive angles are congruent, and if we rotate side $\overline{MP}$ one-fourth of a revolution clockwise about point M, we find $\overline{MP}$ is congruent to $\overline{MN}$. If a parallelogram is rotated one-half of a revolution about the point X where the diagonals interesect, we observe that triangles formed by a diagonal are congruent (see Figure 3 (b)). From this observation we conclude that opposite angles and opposite sides are congruent. If we translate $\angle$A along side $\overline{AB}$ such that A′ (or B) is the image of A, we observe that consecutive angles of a parallelogram are supplementary or form linear pairs.

Figure 3 *(next page)*

You may have observed that in some instances when we moved a figure it was indistinguishable from itself in its original position. In such a case, the figure is *invarient under the transformation;* that is, each point of the figure is moved into a point of the figure. For example, the isosceles triangle (Figure 2 (c)) was invarient under a reflection in the axis of symmetry $\overleftrightarrow{CD}$. The square (figure 3 (a)) was invarient under a reflection in either axis of symmetry $\overleftrightarrow{PN}$ or $\overleftrightarrow{RS}$, while the parallelograms (Figure 3 (b)) is invarient under a rotation about the point X.

Isometries are more easily identified by considering several types of transformations that are not motions. Figure 4 illustrates three such transformations that do not preserve size and shape.

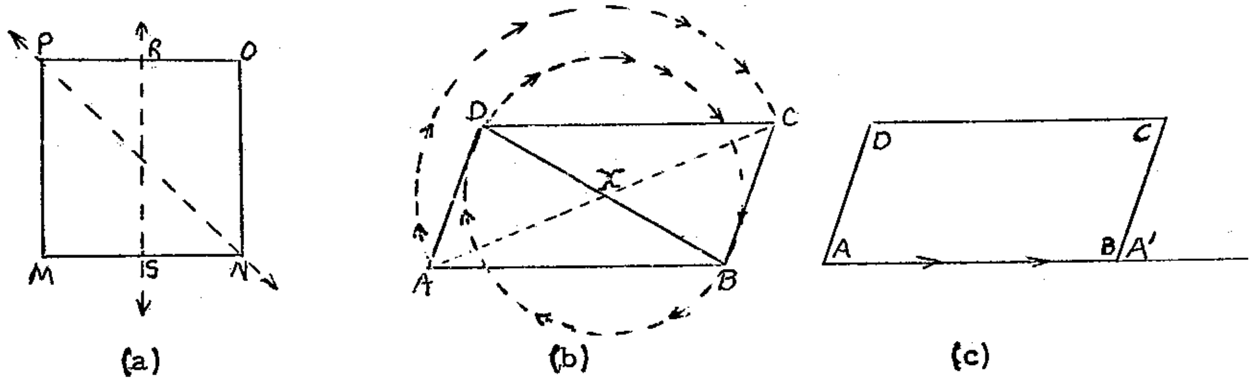
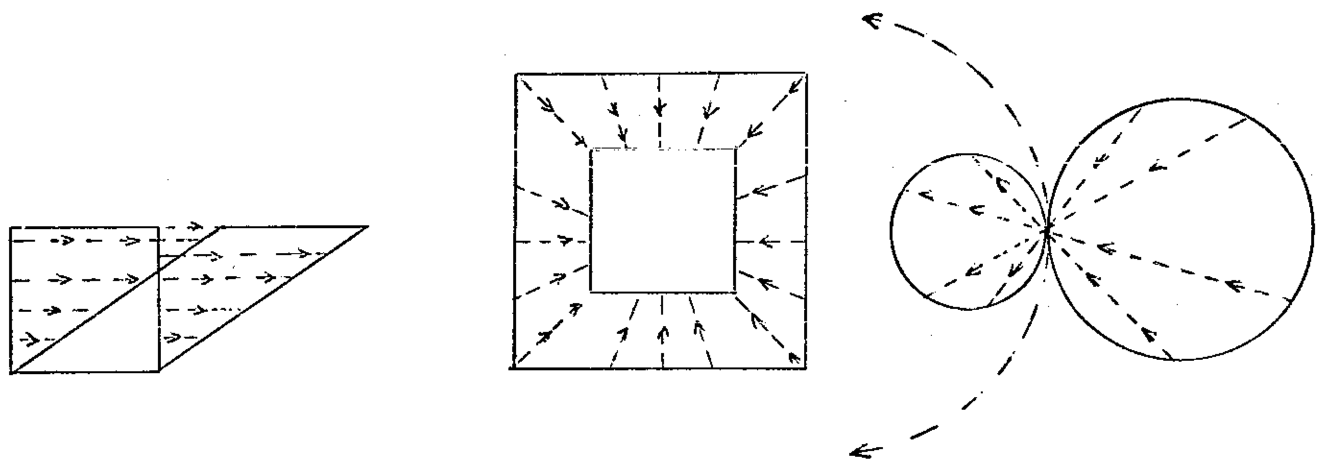


Figure 3



Shear Dilatation Involution

Figure 4

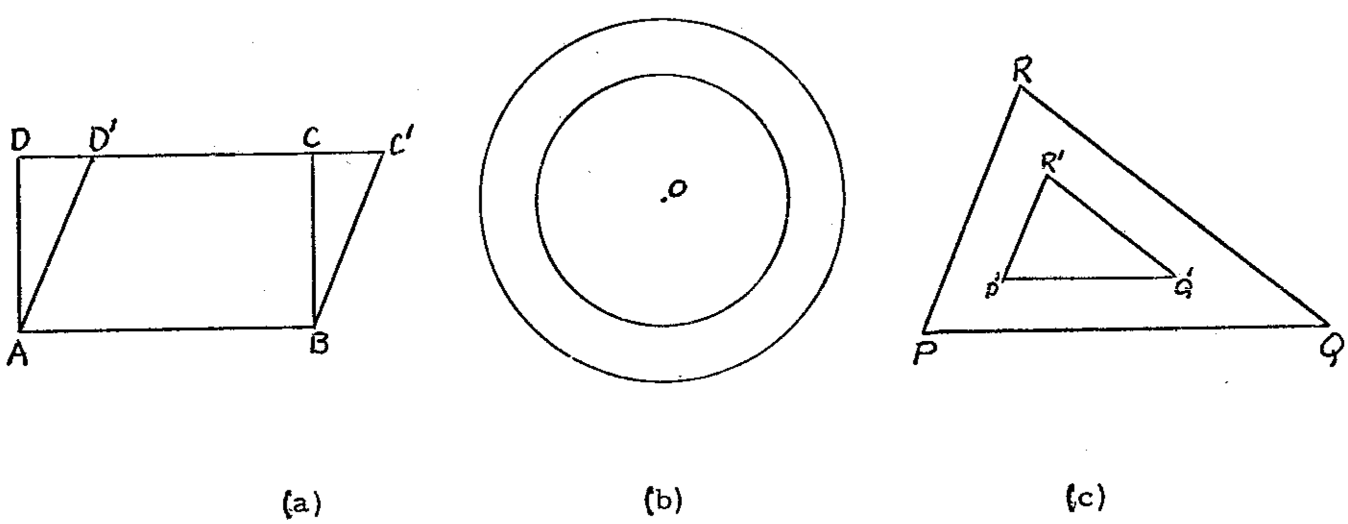


Figure 5

If we consider a rectangle ABCD and a parallelogram ABC'D' (see Figure 5 (a)) with common base and altitude, we have an example of the transformation shear which preserves size (area), but not shape. Two concentric circles (Figure 5 (b)) or two similar triangles (Figure 5 (c)) are examples of the transformation dilatation which preserve shape but not size. These are classroom examples of transformations which are not isometries — do not preserve size and/or shape.

Activities at the elementary level often involve two sets of geometric shapes where the child is asked to "match" the figures which are the same size and shape. This experience involves performing two or more of the basic isometries or motions. If we perform two motions in succession we obtain the *composition* which is a motion also. The order that motions are performed is important and not generally reversible, as the following exercise illustrates. Given the triangle ABC, we translate the figure the distance and direction indicated by the arrow from P to Q, and then rotate the figure one-fourth of a complete revolution about the point A (see Figure 6 (a)). The same two motions but in reverse order are illustrated in Figure 6 (b)).

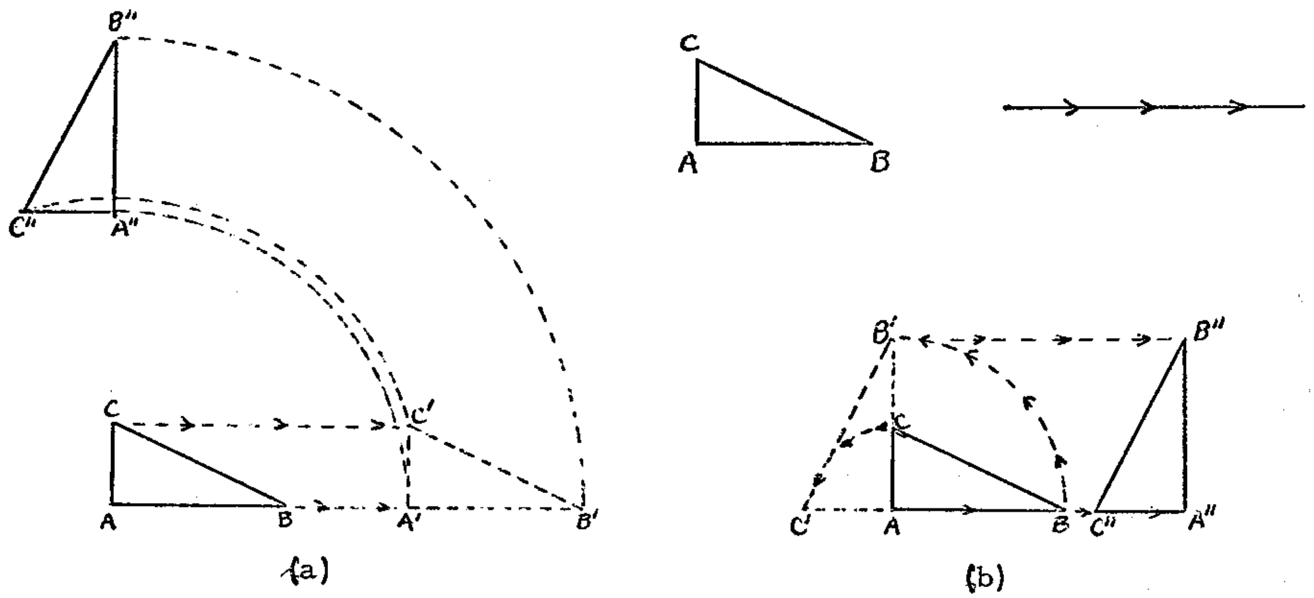


Figure 6)

In Figure 6, the plane was redrawn for each part of the exercise. We note that the resulting positions of the triangles were different in each case, but in both cases the triangles were unchanged in size and shape. The following second grade activity involves the "matching" of a set of geometric figures with the correspondingly congruent figure in a second set. Observe that figure A can be "matched" to the corresponding figure in the second set by a rotation and translation. If we reflect figure B in its side and then translate, we can determine the corresponding figure B'. Try your hand at determining the composites of isometries necessary to match the remaining figures.

In conclusion, let me again appeal to the classroom teacher to become familiar with the terminology of transformations, and to use these terms to justify the activities involving the movement of geometric sets of points. Remember to "Tell it like it is!"

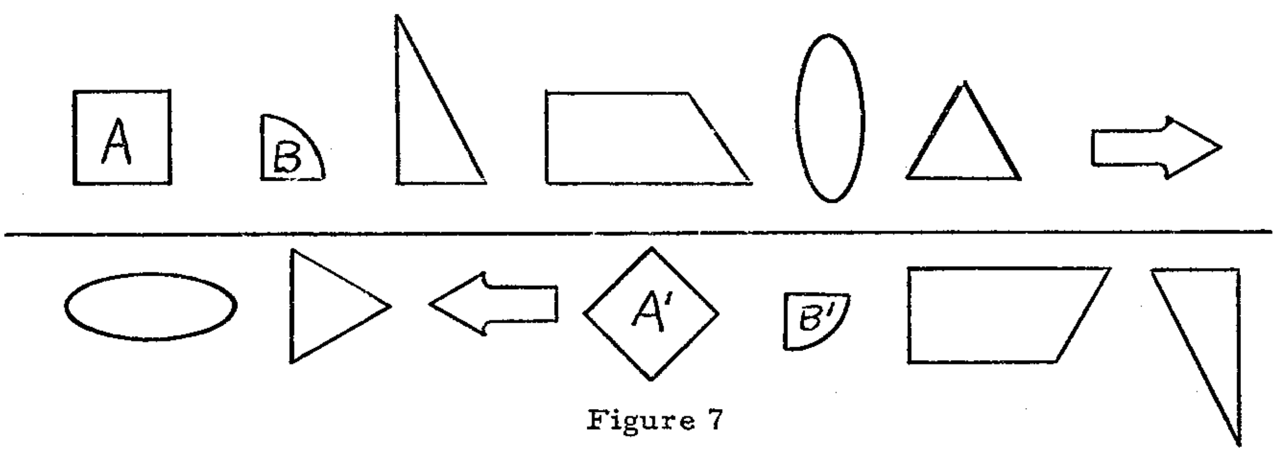


Figure 7

WHAT MATHEMATICS IS APPROPRIATE?

Dr. Charles D. Gallant
St. Francis Xavier University
Antigonish, Nova Scotia, Canada

This talk was originally entitled "The Nature of Mathematics." The working subtitle, has been "What Mathematics is Appropriate?" I intend to relate the two.

I believe there are two basic points of agreement as to what mathematics is appropriate for secondary students, and students at the grade 10-12 level, in particular.

First, mathematics educators share the belief that each student must have the opportunity to achieve mathematical literacy. That is, he must have sufficient mathematical concepts, principles and skills to function in modern society. This concept of math literacy is an ever changing one. If educators are convinced that mathematical literacy includes some knowledge of probability and statistics, or computer science, or business mathematics, et cetera, then these topics are appropriate. In addition, since the curriculum at the grades 10-12 level features an interplay of science or social studies, et cetera, with mathematics, this factor again determines a changing level of mathematical literacy. However, this point is not my main concern.

The second principle of agreement among mathematics educators is that students must be given the option to continue or to terminate their study of mathematics. This is a far more complex principle. The mathematics which is appropriate, must:

1) enable a student to continue his mathematical education in accordance with his abilities and objectives, and
2) allow a student to terminate his mathematics education with an adequate appreciation of the nature of mathematics.

This appears to be a tall order. To resolve this difficulty, let us identify some characteristics of mathematics. I believe these characteristics clearly dictate what mathematics is appropriate.

There are many definitions of mathematics. None of the definitions are completely satisfactory. Rather than identify mathematics, they characterize mathematics, i.e., they reveal interesting facets or viewpoints of the subject. For example, mathematics is a language; mathematics is the discovery and study of analogies; mathematics is the study of relationships; mathematics is the study of structures; mathematics is problem solving; et cetera.

The following definition although logically unsatisfactory, best suits my purposes:

Mathematics is whatever mathematicians are doing. I call particular attention to the word "doing". What exactly are mathematicians doing? They are abstracting; they are searching for patterns; they are conjecturing; they are generalizing; they are analyzing; they are synthesizing; et cetera. In addition to these processes, mathematicians are formalizing their findings – this is the process of axiomization, or the creation of deductive structures. These are processes which, I believe, best characterize what mathematicians are doing, and hence, best characterize mathematics. ***These processes, then, expose the true nature of mathematics.***

To teach mathematics means more than teaching mathematical content. Mathematics *is a verb*. To teach mathematics means to teach someone to *do mathematics*, i.e., *to learn the processes involved*. It is not significant, for our purposes, that the active mathematician employs these processes at a very sophisticated level. What is significant, is that the student is doing the same thing, i.e., employing the same processes as the active mathematician does when the latter is dealing with sheaves, or functors, or banach spaces, et cetera. Only the content or product differs; the processes do not.

Let us turn attention, now, to several of the processes.

A. ABSTRACTION: A process of **set** formation. Abstraction occurs, e.g., in concept formation, when one recognizes a commonality among the elements of a referent set of a concept.

EXAMPLES:

a)

Nimbie	Non-Nimbie
1230	1660
1410	150
1005	222
2202	

What is a nimbie?

b) Homogeneous Polynomial

$x^2 + y^2$	$y + x^2$
$y - x$	$x + y - 1$
y	$y + 2$
$x^5 + x^4y + z^5$	$y + x^4z$

What is a homogeneous polynomial?

What mathematics, then, is appropriate? Generally speaking, whatever topics which will enable a student to become aware of the processes and *proficient in their use*. Perhaps one should qualify this further. The mathematical topics should be enjoyable, and useful as product knowledge.

Should we teach calculus in high school? Should we teach matrix theory? Should we teach number theory, affine geometry, algebraic geometry, analysis, topology, et cetera? All of these topics and more, have been proposed. My first reaction is NO!

Mathematics educators have argued that in order to develop skill in mathematical processes, one must, first, have adequate knowledge of concepts, principles and skills. Students at the grade 10-12 level already have considerable mathematical knowledge. Why introduce more concepts — more principles — and more skills? Why not enrich the content they already have been exposed to? Why not pose problems which will require that they organize their previous knowledge? Why not have students explore all the areas mentioned above, insofar as they enrich what they have already learned, instead of excluding any of them? There are so many sources for enrichment: *The Mathematics Teacher,* Mathematics Magazine; several Yearbooks of the Council are devoted exclusively to such material; the many curriculum projects in the United States and Canada.

The use of "enrichment material" serves a double advantage. First, the students ***can*** be introduced to many branches of mathematics as extensions of their school mathematics. Secondly, the students can more easily address themselves to the processes, since the requisite mathematical knowledge is not new to them.

Students at the grade 10-12 level, and at university, are not competent with respect to the processes. Their mathematics education is at the level of recognition, recall, and algorithmic thinking. Their thinking is reproductive, rather than productive. As mathematics teachers, we speak of active inquiry, guided discovery, adventurous thinking and exploration. These activities feature processes which are often stifled by our attempts to cover content.

If students participate, actively, in the recreation of mathematics, they may learn the processes. In turn, they may possibly create mathematics, or at least be able to pursue mathematics, more successfully, if they so desire.

As teachers of mathematics, we have examined, or perhaps designed, objectives for mathematics education. Many of these objectives are process objectives. Perhaps the most frequently occuring objective is the problem solving ability or what is considered the same, the ability to reason. Problem solving involves the very processes we have discussed. Problems impel students to think, and learning stems from thinking and not from recalling; in other words, reasoning is not algorithmic. Problems enable a student to view content in ways which are most conducive to new insights into relationships.

In conclusion, it is my opinion that a given unit in mathematics is appropriate if it is useful, enjoyable and gives rise to a body of problems which can achieve process objectives.

B. GENERALIZATION: There are two different views of the process of generalization.

1) *Generalization:* A process of generating general statements, i.e., for which there are many instances. The process usually involves abstraction from the set of instances.

EXAMPLES:

a) $\sqrt{2}$ is irrational
$\sqrt{3}$ is irrational
$\rightarrow \sqrt{p}$ is irrational, p a prime
$\sqrt{5}$ is irrational
$\sqrt{7}$ is irrational

b) $\frac{2}{3} - \frac{1}{2} = \frac{1}{6}$

$\frac{3}{4} - \frac{2}{3} = \frac{1}{12}$

$\rightarrow \frac{n}{n+1} - \frac{n-1}{n} = \frac{1}{n(n+1)}$

$\frac{4}{5} - \frac{3}{4} = \frac{1}{20}$

$\frac{5}{6} - \frac{4}{5} = \frac{1}{30}$

2) *Generalization:* A process of set extension so that the referent set of the old concept or general statement is isomorphic to a subset of the referent set of the new concept or general statement.

EXAMPLES:

a) Obtaining the complex numbers from the reals.
b) Extending the affine plane to the projective plane.
c) Obtaining the rationals from the integers.
d) Extending "$\sqrt{p}$ is irrational" to "$\sqrt{n}$ is irrational," where n is not a perfect square.

C. ANALYSIS: A process of breaking down or reducing a structure to its component parts. Examination of these components parts is adequate in itself to understand some facet of the structure.

EXAMPLES:

a) If $n \varepsilon J$ then $n^2 - 1$ generates only one prime number.

b) What is the effect of adjoining a zero at the right end of a binary number?

Decimal	Binary		Binary	Decimal
(1)	1	—	10	(2)
(2)	10	—	100	(4)
(5)	101	—	1010	(10)

c) Show that $p^2 = 2q^2$ is impossible in integers.
Use the number of divisors of a square.
i.e.; if $n = p_1^{\alpha_1} \ldots p_k^{\alpha_k}$, then $ND(n) = (\alpha_1 + 1)(\alpha_2 + 1) \ldots (\alpha_k + 1)$.

Hint: $ND(p^2)$ is odd; $ND(2q^2)$ is even.

d) Consider 5 points in the plane, no three of which are collinear. In how many ways can we join these points so each point is connected to exactly 3 others?

e) The average of 8 rational numbers is 12. After one of these numbers has been erased, the average of the remaining seven numbers is 9. What number has been erased?

D. SYNTHESIS: A process of creation of new structures. The analysis of several structures leads to a new organization of the parts. Usually additional and fairly remote principles are required for solution.

EXAMPLES:

a) Find an integer N s.t. 1, 3, 8, N, have the property that the product of any pairs is a square less one.

b) If $(-2)^2 = 4$; $(-2)^3 = -8$;
what sign does:
$(-2)^{2.5}$ have? $(-2)^{\sqrt{2}}$ have?

c) For any prime p, prove that the sum of the integers 1, 2, 3, . . ., p-1 is a multiple of p. Can you generalize this result?

d) If $\sqrt{2}$ is an irrational number, prove that, for any rational x, $x + \sqrt{2}$ must also be irrational.

e) In how many ways can the fraction ½ be written as the sum of two positive fractions with numerators equal to one and denominators a natural number?

f) If $n = x^2 + y^2$ where $n, x, y \ \varepsilon\ J^+ = [0,1,2, \ldots]$ prove that $2n = a^2 + b^2$ where $a, b \ \varepsilon\ J^+$.

g) The diagonal of a given square is 12 inches. What is the area of the square?

WORKED EXAMPLES:

f) Let $J^+ = [0, 1, 2, 3, \ldots]$. If $n = x^2 + y^2$ where $n, x, y \ \varepsilon\ J^+$, prove that $2n = a^2 + b^2$ where $a, b \ \varepsilon\ J^+$.

EXAMPLES:

$8 = 2^2 + 2^2$ $\quad 13 = 2^2 + 3^2$
$16 = 4^2 + 0^2$ $\quad 26 = 1^2 + 5^2$
$52 = 4^2 + 6^2$ $\quad 20 = 2^2 + 4^2$
$104 = 10^2 + 2^2$ $\quad 40 = 2^2 + 6^2$
$80 = 4^2 + 8^2$
$160 = 4^2 + 12^2$

PROOF:

If $n = x^2 + y^2$ then
$2n = 2x^2 + 2y^2$

* significant step

$\overset{*}{=} (x+y)^2 + (x-y)^2$
$= a^2 + b^2$

c) $p = 5$
$1 + 2 + 3 + 4 = 10 = 2.\underline{5}$

$p = 7$
$1 + 2 + 3 + 4 + 5 + 6 = 21 = 3.\underline{7}$

$p = 11$
$1 + 2 + 3 + 4 + 5 + \ldots + 10 = 55 = 5.\underline{11}$

In general: $\sum_{i=1}^{p-1} i = \frac{p(p-1)}{2}$

* Synthesis

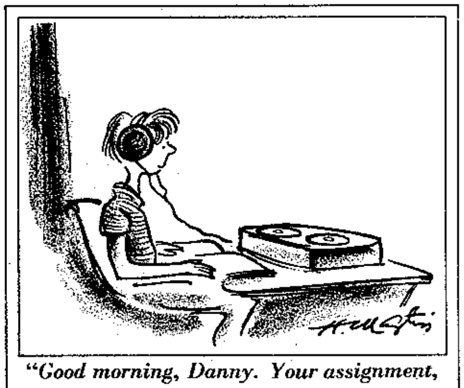
Since p is prime $p - 1$ is even,
therefore $\frac{p-1}{2}$ is an integer k
therefore, $\sum_{i=1}^{p-1} i = kp$, i.e., a multiple of p.

Generalization:

Result holds for any odd integer p, rather than just primes.

REFERENCES

1. "Objectives for Mathematics Learning — Some Ideas for the Teacher," **Ontario Institute for Studies in Education,** Bulletin No. 3, 1968 S. Avital and S. Shettleworth.
2. The Mathematics Student Journal, **N. C. T. M.**; several issues.
3. **Readings in Secondary School Mathematics,** D. Aichele and R. Reys, Prindle, Weber and Schmidt, Inc.



"Good morning, Danny. Your assignment, should you decide to accept it, will be problems 7 through 25 on page 28 . . ."

An Experiment With Programmed Problems Sets in Mathematics Textbooks

FLOYD VEST AND MERIDON V. GARNER
NORTH TEXAS STATE UNIVERSITY, DENTON, TEXAS

While in the process of preparing a mathematics textbook for prospective elementary teachers, the authors explored the possibility of applying programming techniques to the design of problem sets accompanying a textbook. The problem sets were arranged in a workbook format with automatics association of short answers with questions being provided by blanks immediately next to the problem statement. Space for answers was provided in the form of blanks in sentences in such a way that the completed sentence indicated the significance of the answer in relation to the problem statement. Time saving and informative aids in the form of preprinted diagrams and not usually found in textbooks were provided. Information about appropriate steps for solving problems and their order was supplied by blanks, diagrams, hints, and boxes immediately adjacent to problems. Such aids provided for smaller steps within the problem solving sequence and a larger number of items of explicit information supplied in immediate association with each problem. In repetitious series of problems, a gradual change from dependent to independent problem solving was provided by the gradual omission of such prompts.

This combination of expository sections in a textbook and highly programmed problem sets falls within the domain of May's suggestion for materials which seem to incorporate "... maintain the present exposition problem pattern (5, p. 21)" and Lasswell's suggestion for small programmed units to accompany texts (4.) The programmed problem sets also meet the condidtions recommended by Gagné for the writing and assigning of homework exercises. Gagné argued that "... homework assignments need to be of a nature that *all* students can readily master. Their purpose is not to distinguish between bright and dull students; rather, it is to ensure that all students have attained a specific set of prerequisite knowledges (3, p. 294)."

It was conjectured that many of the assumed advantages of programmed materials could be found in these highly programmed problem sets. In particular, students would have a higher level of success, less anxiety associated with extended periods of uncertainty in attempts at independent mathematical inquiry, more self-confidence, and a better general attitude toward mathematics. It also was conjectured that students using the programmed problem sets would achieve as well in the area of recall of facts and principles as students who are using conventional problem sets.

The often considered essential programming technique of providing immediate reinforcement in the form of answers to problems was omitted. It was felt that as a result of careful programming, high levels of explicit information, and the often self-checking nature of mathematics, sufficient reinforcement for the student would be derived from these factors and the prompt return of graded problem sets.

The Problem

The problem then became the comparison of the effects of traditional problem sets to that of highly programmed problem sets in terms of mathematical achievement and attitude toward mathematics.

Procedure

In this pilot study, a control group of 21 students was taught using a textbook with traditional problem sets. An experimental group of 23 students was taught using the same text material with the exception that the problem sets were presented in a highly programmed workbook format. The two groups used the problem sets as homework assignments. The individual problems in the two types of problem sets were identical with the exception of level of programming and format. In both classes the homework was collected, graded, and handed back to each student at the next class meeting. Both classes were taught by the same teacher from a daily lecture plan that was prepared in advance. The two classes were treated the same with respect to grading, attendance policies and testing.

The subjects were female sophomore and junior level elementary education majors enrolled in a CUPM Level I, "Structure of the Number System" mathematics course (1). The experimental and control groups consisted of intact classes drawn together through the usual registration procedures.

The evaluative instruments were Mathematics Inventory (a multiple choice teacher — made test of recall of facts and principles with a split-half corrected reliability coefficient of .83) and the Dutton Scale for Attitude Toward Arithmetic (a Thurstone scale for measuring attitude toward arithmetic with a reliability coefficient based on test-and-retest of .94) (2.) The Mathematics Inventory test was a 100 item test with four alternatives per multiple choice question. The test was modeled after several standardized tests which have been developed for testing elementary teachers in the area of modern school mathematics. It was designed to systematically cover the content of the course with 16 of the questions testing knowledge of sets and number, 14 questions on the system of whole number, 17 related to elementary number theory, 27 on systems of numeration and 26 questions representing knowledge of the system of rational numbers. At both the beginning and end of the semester, all subjects were given Mathematics Inventory and the Dutton Scale for Attitude Toward Arithmetic.

In order to develop further information about the differences between the treatments, an attempt was made to estimate the level of success for the two types of homework problem sets. The aim in the development of the highly programmed material was a minimum of 90 per cent success for each student on each problem set. This aim was accomplished with the exception of a few problem sets for which the departure was minimal. The average percentage score for the traditional problem sets for students over the whole semester's work was 84 per cent with a range from 92 per cent to 43 per cent.

Results

Since the experimental and control groups consisted of intact classes resulting from registration procedures, the treatments were compared on the basis of residualized gains resulting from pre- and post-administration of the two tests. The two classes were not significantly different at the .05 level as determined by scores on the Otis Quick-Scoring Mental Ability Test. The results of the statistical analysis of the data on achievement is presented in Table 1.

TABLE 1
Means of Residualized Gains on Mathematics Inventory and Dutton Scale for Attitude Toward Arithmetic

	Exp. Group N = 23		Control Group N = 21		t	p
	Mean	SD	Mean	SD		
Mathematics Inventory	-3.99	8.01	4.37	5.28	-3.95	p< .05
Dutton	- .24	1.01	.27	.64	-1.94	p< .1 p< .05

As is indicated by the table, the mean residualized gain on Mathematics Inventory for the experimental group was significantly less than that of the control group at better than the .05 level and the mean residualized gain on the Dutton Scale for Attitude Toward Arithmetic for the experimental group was also significantly less than that of the control group at better than the .1 level. The control group seems to have achieved better than the experimental group in both mathematical knowledge and atttitude toward arithmetic.

It is interesting to note that both groups had a positive change in attitude toward arithmetic which was significant at better than the .05 level. The means of the pre- and post-administration of the Dutton Scale for Attitude Toward Arithmetic are reported in Table 2. This data indicates indirectly the high level of interest and general characteristics of the experimental conditions.

(See Table 2, next page)

Summary and Conclusions

Although the degree of programming of problem sets and level of success on these problem sets was greater under the experimental treatment, it seems that the students under this treatment did not achieve as well in terms of mathematical knowledge and attitude toward arithmetic as those under the more traditional control treatment. Thus the combination of a higher level of independent behavior and a lower level of success associated with the conventional problem sets was reasonable for the students involved. This pilot study suggests an apparently fruitful area of investigation. Further experimental study of programming and behavior could be more precisely classified and compared.

BIBLIOGRAPHY

1. Committee on the Undergraduate Programs in Mathematics, *Course Guides for the Training of Teachers of Elementary School Mathematics,* (Fourth Draft), Berkeley, California, Committee on the Undergraduate Programs in Mathematics, 1964.

TABLE 2
Means for Pre- and Post-Administration of the Dutton Scale for Attitude Toward Arithmetic

	N	Pre-Dutton Mean	Post-Dutton Mean	SD of Difference	$\underline{t}$	p
Experimental Group	23	5.06	6.17	1.16	-4.50	$p<.05$
Control Group	21	5.27	6.80	1.21	-5.62	$p<.05$

2. Dutton, Wilbur H., "Attitude Changes of Prospective Elementary School Teachers Toward Arithmetic," *The Arithmetic Teacher*, IX (December, 1962), 418-424. (The scale used in this study is in Table 1 on page 419.)

3. Gagne´, Robert M., *The Conditions of Learning,* New York, Holt, Rinehart and Winston, Inc., 1965.

4. Lasswell, Edmund C., "An Open Letter to Authors and Publishers," *The Mathematics Teacher*, LIX (December, 1966), 739-740.

5. May, Kenneth O., *Programmed Learning and Mathematical Education, A CEM Study,* Buffalo, Mathematical Association of America, 1965.

Basic Algebra Christmas Review

JANICE L. GAYLORD
Chairman, Mathematics Department
Bellaire High School
Bellaire, Texas

The motivation for the following type of review is that it produces immediate results. As the student answers true or false questions he simultaneously draws the indicated line segment and a picture unfolds before him. Upon completion (and sometimes a bit before) he can tell how well he has performed. He can also refer back to an incorrect answer by noting a misplaced segment in the final picture.

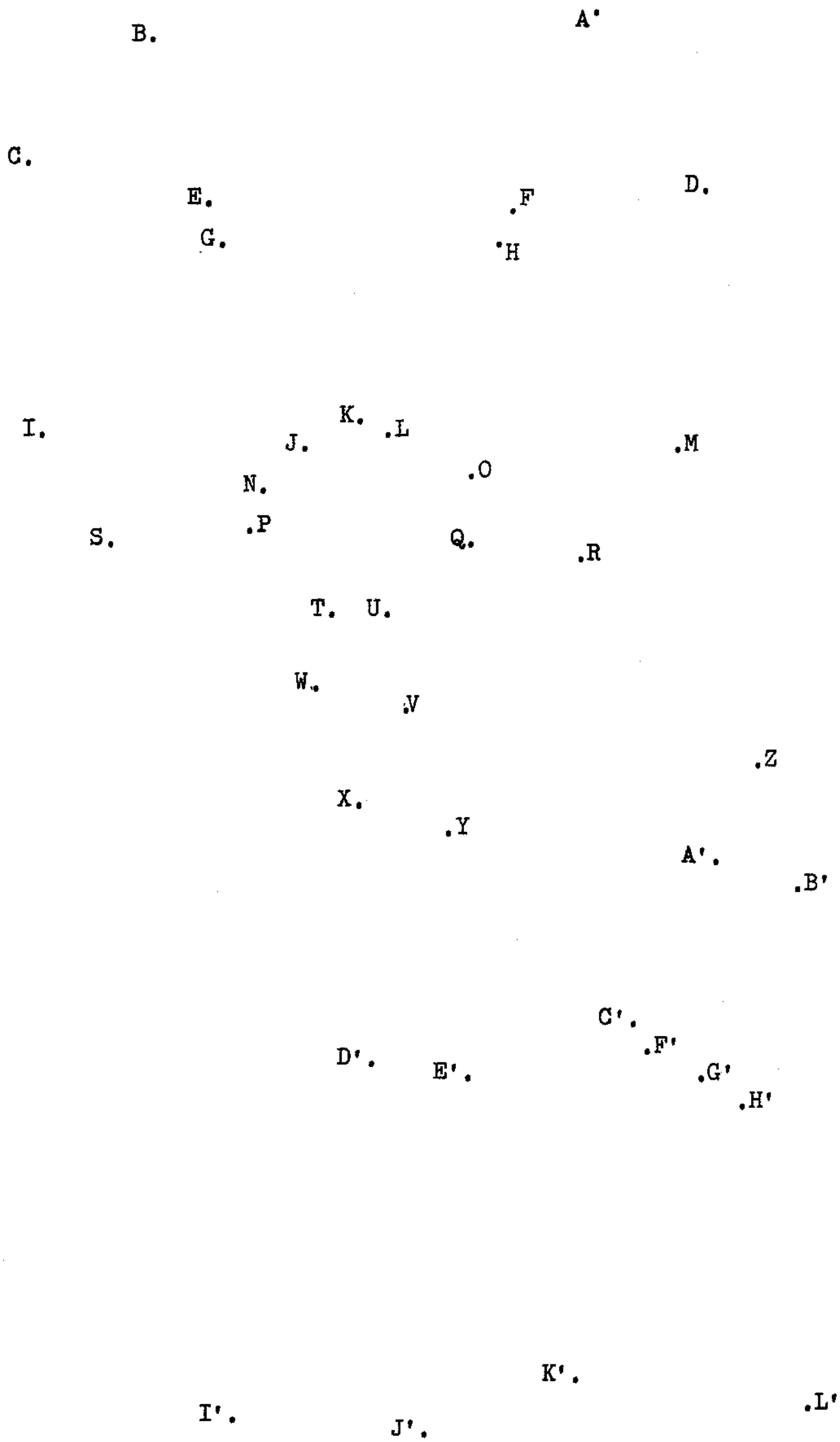
One of the conveniences of this type of questioning is that the same picture and columns of true-false responses may be used as a review for any subject at any level. The test is converted by merely substituting appropriate true-false statements on the new material for the corresponding previously true or false statements. However, we have learned by experience that it is best to have a variety of pictures available as students seem to resent a repeat picture. In this regard, children's "Connect the Dots" books are an inexpensive resource of pictures.

For five years now, I (and many fellow teachers) have used a review of this type in my classes on the last day of school prior to dismissal for Christmas holidays. It has been found that the students from sophomores to seniors will work on this picture review when their minds may otherwise wander with the pre-holiday spirit. The following picture is appropriate for the Christmas season but you'll have to review to see the result!

BASIC ALGEBRA CHRISTMAS REVIEW

		If TRUE connect	If FALSE connect
1.	3x is a monomial	IS	IP
2.	$8 - (-8) = 0$	E'D'	E'J'
3.	$\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$	DM	DH
4.	If $x = 3$, then $\frac{2}{3}x = \frac{2}{9}$	G'F'	G'K'
5.	$(3x)(3x) = 9x^2$	QX	QU
6.	$(x^2)(x^3) = x^5$	BE	BC
7.	$4x(a + b) = 4ax + b$	A'B'	A'Z
8.	$-7 < -6 - (-1)$	JN	JK
9.	$(5a)(-5b) = -25ab$	WI'	WD'
10.	In the expression x^3, x is called the base.	AK	AD
11.	$x^2/x^3 = x$	H'B'	H'L'
12.	$(8m + 12n) - (-m - 3n) = 9m + 15n$	MO	MQ
13.	$(6)^2 = 12$	CI	CG
14.	$(.3)(.3) = (.9)$	A'G'	A'Y
15.	$(0)(5) = 0$	PX	PT
16.	$\frac{3}{4} \div 2 = \frac{3}{2}$	D'Y	D'J'
17.	$5 + 4 - (3 - 1) = 7$	DF	DA
18.	$(4y)^5 = 4y^5$	F'G'	F'K'
19.	xy is a binomial	QO	QR
20.	$5 + (3)(2) = 16$	JK	JG
21.	$(x - y)^2 = x^2 - y^2$	B'H'	B'L'
22.	$0.1 = \frac{10}{100}$	IN	IJ
23.	If $x = -6$ and $y = 3$, then $(\frac{x}{y})^4 = 16$.	C'E'	C'K'
24.	$18 - 35 = 17$	AH	AF
25.	$x + x = x^2$	C'J'	C'F'
26.	$(y^2)^3 = y^6$	MR	ML
27.	$(a - b)(a - b) = a^2 + 2ab + b^2$	CI	CE
28.	$\frac{gt}{gt} = 0$	VU	VY
29.	If $y = 3$, then $4y^2 = 144$.	PT	PS
30.	$x^2y^6/xy^8 = x/y^2$	D'I'	D'Y
31.	$3 - (-3) = 6$	HL	HF
32.	$4x + 5 < 1$ is an equation.	G'B'	G'H'
33.	In the expression $\frac{1}{3}x + y$, $\frac{1}{3} + y$ is called the coefficient of x.	LU	LO
34.	If $x - 4a = a$, then $x = 5a$.	BK	BC
35.	$\frac{x + y}{3} = \frac{x}{3} + \frac{y}{3}$	B'Z	B'H'

BASIC ALGEBRA CHRISTMAS REVIEW



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Fill out, and mail to Dr. Floyd Vest, Mathematics Department, North Texas State University, Denton, Texas 76203.

N O W ! !

TEXAS MATHEMATICS TEACHER
J. William Brown, Editor
Texas Council of
Teachers Of Mathematics
Woodrow Wilson High School
100 S. Glasgow Drive
DALLAS, TEXAS 75214

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